ROUTE LOCATION AND DESIGN

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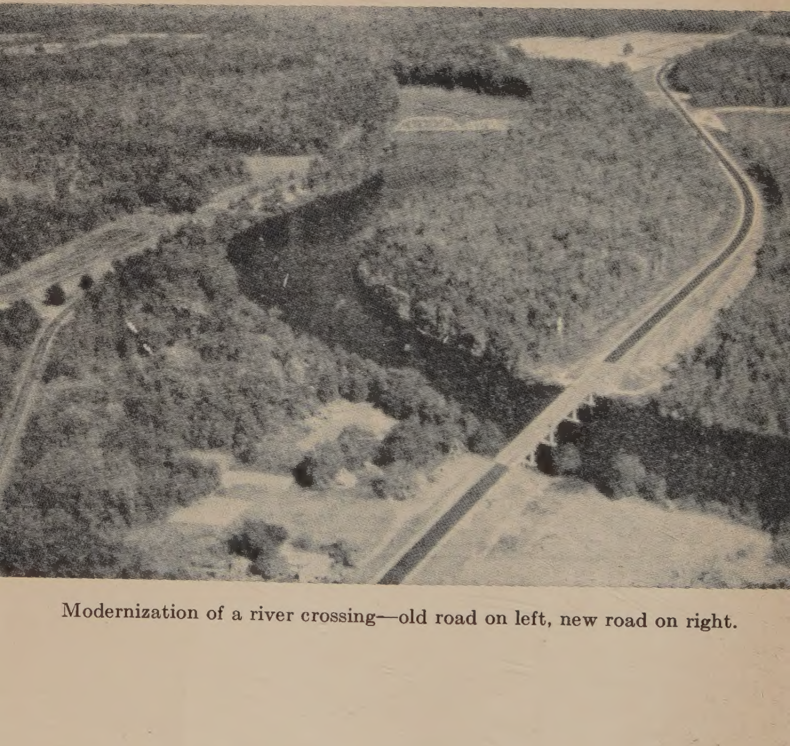

Modernization of a river crossing—old road on left, new road on right.

ROUTE LOCATION AND DESIGN

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Fifth Edition

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ROUTE LOCATION AND DESIGN

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PREFACE

The fifth edition, under the appropriate new title *Route Location and Design*, embodies constructive changes in the previous fourth edition consisting of up-to-date additions, corrections, and replacements. It contains approximately 65 more pages than the previous edition. It includes 47 tables (all carefully checked for accuracy), 16 chapters, and 150 illustrations, among which are 4 full-page charts. In addition, 12 types of interchanges are shown, and there are 16 pictures showing different views of the interstate system as well as several noteworthy achievements in the field of highways and bridges.

This new work is, indeed, an improvement in every way on the preceding edition, which itself has been called a classic.

Chapter 1, outlining the various transportation systems, has been rewritten to take into account changes that have taken place during the past decade, with reference to the interstate system and the AASHO Design Standards for Highways other than Freeways.

Chapter 3 contains a new and lucid treatment of the layout of circular curves—including outer and inner concentric curves, tangent offsets—and the location of a point on the curve by deflection angle and distance from the PI.

Chapter 4 contains a clearer presentation of compound and broken-back curves. It also incorporates additional examples.

In Chapter 6 (Transition Spirals) is included a new section dealing with nonparallel outer and inner spirals where the radial offsets are variable.

Chapter 7 (Spiraled Compound Curves) includes a new section dealing with nonparallel outer and inner connecting spirals where the radial distances are variable.

New sections have been added to Chapter 8 (Elements of Highway Safety and Design) entitled (*a*) Scenic Values in Planning and Design, and (*b*) Urban Design for Building and Traffic Efficiency.

Chapter 9 (Intersections and Interchanges) is new throughout.

Chapter 11 (Electronic Equipment and Applications) is a radical and up-to-date revision of the former work.

Chapter 12 (Drainage Surveys and Facilities) is, for the most part, a newly written chapter.

Three new tables (8, 9, and 10) have been added; all were carefully checked.

The author wishes to thank L. A. Colbert, electronic design engineer of the North Carolina State Highway Commission, Thomas P. Noe, vice-president of Carolina Steel, Inc., and Dr. Fred Brooks, chairman of the Department of Computer Science, University of North Carolina, Chapel Hill, for timely corrections and revisions of Chapter 11.

Acknowledgment is also due the American Association of State Highway Officials (AASHO), and the Location Divisions of the California, North Carolina, and Virginia Highway Departments.

THOMAS F. HICKERSON

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CHAPTER 1

TRANSPORTATION SYSTEMS

1. Introduction. Transport has always been one of the main factors in the spread of civilization. In this age of space exploration and automation the provision of an efficient and properly maintained transportation system is nonetheless vital at all stages of national and international development.

The so-called transportation systems considered here are primarily *railroads*, *highways*, *air transport*, *pipelines*, and *waterways*. In a broad sense the classification also includes electric transmission lines for power, telephone, or telegraph; cableways and belt conveyers; aqueducts and flumes. The location and construction of these systems are governed by *office plans* based on *field surveys*, which may consist of actual ground surveys, aerial surveys, or a combination of these methods.

The universally accepted advantages of aerial photography and mapping as applied to all phases of route location and design coupled with the introduction and adaptation of electronic computers have revolutionized engineering procedures. However, the old-time ground-survey methods have not been entirely supplanted. They will be needed frequently as a check on aerial surveys, particularly for those areas where visibility of the terrain is obscured by trees or houses. Also, ground-survey operations are still required in the layout of curves and the setting of construction stakes.

Along with considerations of economy, feasibility, physical features of the terrain, purpose of the project, right of way, alternate locations available, these systems are alike in the requirement that the route be as *direct* as possible between controlling points with *gradients* and *curvature* held to a reasonable minimum. It is desirable of course that the *location* of the facility be governed by sound engineering principles in order to justify possible future extension or improvement.

2. Controlling Factors. Route location, particularly for railroads and highways, is influenced by a number of factors, prominent among which are the traffic to be accommodated and the physical features of the terrain, generally classified as *flat*, *rolling* (light to heavy), and *mountainous* (light to heavy). From the standpoint of the drainage pattern of the country, routes may consist of a *valley location*, a *ridge location*, or a *cross-country location*.

In flat country the line might be straight between termini; but frequently a change of direction (and curvature) is introduced to reach strategic points, to relieve the monotony of driving, or to avoid certain areas necessitating costly or undesirable property severances, such as the location of a right of way through or too close to cemeteries, churches, schools, etc.

In rolling and mountainous country a typical railroad route will be a valley location with easy gradients and considerable curvature following the general direction of a stream receiving the drainage of the territory. Flood damage from the main stream and its tributaries is likely even if the line is elevated above the high-water level, as it should be.

A ridge location possesses many desirable features: easy drainage, minimum cuts and fills, and gradients that may be within reasonable limits. But such ideal conditions seldom prevail for extended distances, since the general direction of a ridge rarely coincides with that of the route.

A cross-country location necessitating cuts and fills and bridges is the rule rather than the exception on highways, now that safety and convenience of operation call for long tangents and reduced gradients joined by slight curvature. Many existing highways of this undulating type provide inadequate vertical visibility for safe driving. Attention is called to the importance of long vertical as well as horizontal sight distance.

On routes crossing mountainous areas, the gaps or saddles in the ridges are definite points of control from which the line may proceed downward on either side of the watershed according to the maximum allowable gradient. Such a line has the characteristics of sidehill location with uniformly falling grades, curvature conforming somewhat to the shape of the ground, and light to heavy balanced cuts and fills. Loops, although objectionable, are sometimes necessary in the "development" process of introducing distance in order to reach a summit without exceeding a maximum allowable grade. A distance of 1 mile on a continuous 1.00 percent grade,* for example, will rise 52.8 ft; a distance of 1 mile on a 2 percent grade will rise 105.6 ft; and so on.

The surveying operations (Chap. 2) applying in general to all these systems, but more specifically to railroads and highways, are classified as (1) the *reconnaissance*, (2) the *preliminary survey*, and (3) the *location survey*. Also, there is a *construction survey* as a guide to the contractor preceding actual construction of the facility. This survey includes the resetting, if need be, of the location center-line stakes with sufficient offset stakes, and special surveys for the layout of culverts or bridges, grade elimination structures, turnouts, possible changes in design during construction, finish grade stakes, etc.

3. Railroads. The engineering procedure establishing the line and grade of railroads is essentially the same as that for highways. But relatively greater refinements of gradients and curvature are necessary for successful railroad operation.

Aside from the extra tractive force required to accelerate a train, the drawbar pull of a locomotive in hauling a train of cars must overcome three types of resistances: (1) *train resistance*, (2) *grade resistance*, and (3) *curve resistance*.

Train resistance refers to the variety of opposing forces that must be overcome by the tractive power of the locomotive on a straight, level track. Among the causes of these resistances are (1) internal friction offered by the parts of the locomotive itself; (2) journal friction, rolling resistance, and the resistance resulting from track deflection under load—all increasing with the axle load; (3) flange resistance and air resistance that increase with speed. Numerous tests have shown that the train resistance to constant speed on a straight, level track is about 6 lb per ton of train.

Grade resistance is one of the most important factors to be considered in railroad location. Resolving the weight W (where W = weight of car) into components parallel and perpendicular to the track, inclined at an angle α degrees to the horizontal, as shown in Fig. 1, we have

---

* Percent grade (or gradient) indicates *rate of grade*, or the vertical rise or fall in a distance of 100 ft (1 station).

$$F = W \sin \alpha = W \tan \alpha \text{ (closely).} \qquad (a)$$

If $W = 1$ ton $= 2{,}000$ lb and $\tan \alpha = 0.01$ (1 percent grade), then $F = 20$ lb. Hence the grade resistance is at least 20 lb per ton per percent grade. Assuming the train resistance to be 6 lb per ton at constant speed, it follows that the total tonnage which can be hauled up a 0.3 percent incline is about half that which could be hauled on a straight, level track by the same locomotive.

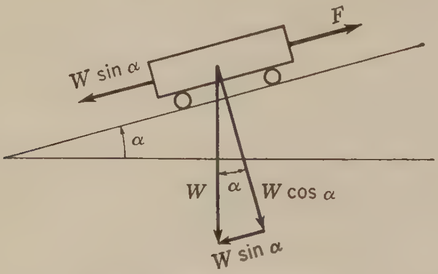


FIG. 1. Grade resistance.

The "ruling grade" or "limiting grade" is defined as the maximum gradient over which a tonnage train can be hauled with one locomotive; hence it will limit train length. However, the ruling grade is not the maximum grade if we take into account "momentum" grades and "pusher grades."

When a ruling grade is reduced to allow for curvature resistance, it is said to be *compensated for curvature.* The American Railway Engineering Association (AREA) recommends a compensation of around 0.04 percent per degree of curve (see page 57 for definition of degree of curve). Thus if a 3° curve is superimposed on a 1.2 percent grade, the compensated grade is limited to $1.20 - 3 \times 0.04 = 1.08$ percent.

Steeper grades (up to a reasonable maximum) are permissible on highways, but the speed of loaded trucks is greatly reduced thereby, particularly on long ascents. Hence, from the standpoint of interference with the operation of passenger cars and the efficiency of truck transportation per se, ruling grades are a factor in highway location, as in railroad location, but to a lesser degree.

On railway main lines, grades not exceeding 1.00 percent predominate; grades in excess of 2.2 percent are rare. Over fills the grade might be level, but in long cuts the grade should be not less than 0.30 percent to provide proper drainage.

Curve resistance tends to increase with the degree of curve. It is assumed to be around 0.8 per ton per degree of curve, or the equivalent of a 0.04 percent grade for each degree of curve. Since the car wheels are rigidly fastened to the axles, there must be some slippage while rounding curves, because of the difference in length of the outer and inner rail. This slippage (longitudinal and lateral) results in frictional resistance. Also, there is more or less flange pressure, the forward flange against the outer rail and the rear flange against the inner rail.

Relocations reducing both the degree of curvature and the total central angle on existing lines allow greater speeds and thereby secure greater operating economy. Among the trunk-line improvements in the last decade is the grade and alignment revision of two portions of the Missouri Pacific Railroad, which reduced grades from a maximum of 2.45 to 1.25 percent compensated, saved 3 miles of distance, and reduced the total curvature (sum of the central angles) by almost 900°.

The problems of curvature reduction have assumed greater importance with the advent of high-speed (greater than 70 mph) operation. The electric and diesel-electric locomotives, now widely used, are well adapted to fast schedules covering long distances for both passenger and freight traffic. They possess advantages of smooth starting, fast acceleration, ability to exert full power at slow or fast speeds, and serviceability over longer periods than in the case of steam locomotives.

Material such as crushed stone placed on the subgrade serves essentially the same purpose, whether it is for a highway or a railroad. When used in the latter case, it is called ballast, and it gives stability to the track, causes smoother riding, facilitates drainage, lessens dust, and reduces weeding and other maintenance problems.

The railroad right of way is the strip of land on which the roadbed and other facilities are constructed. Common widths in open country vary from 50 to 100 ft, and may extend even to 400 ft. The wider and deeper the cut or fill or the flatter the slopes, the more land will be required. The need for double tracking and other expansion should not be overlooked. Within station limits and yards, sufficient land is needed for tracks, structures, and other facilities.

No new major trunk lines are likely in view of the rapidly declining passenger traffic as a consequence of competing air and automobile travel; but there will continue to be an occasional construction of spur lines and industrial tracks to open and serve new sources of raw material and industry. Aside from curve and grade reductions on existing lines, the big problem for the American railway engineers is the maintenance of facilities against high speeds and heavy equipment.

4. Highways. Highway location and design involve considerations of curvature, gradients, drainage, soil conditions, safety, sight distance, width, and roadside safeguards and surfaces to withstand the impact of traffic. But an important highway is not properly located and designed unless the following information is at hand: a traffic census, origin-destination and delay studies, the expected traffic load within the lifetime of the highway pavement, the relation of the highway itself to future development of the system, design standards to satisfy the needs of expected traffic over a period of years, and an adequate right of way.

Wherever feasible, moderate curvature and gradients are desirable. By many motorists curvature is preferred to long, straight stretches, the argument being that it promotes alertness and relieves the monotony of driving—which in itself might cause sleepiness leading to accidents. But in some sections of the country the topography is such that a sweeping curved alignment will come about naturally, while in other sections, avoiding long tangents may result in layouts which will look *forced* when viewed from high locations.

An unexpected change from one standard of alignment to a lower standard, as for a traffic circle or a curve at the end of a long tangent, is a source of danger and may be responsible for many accidents. The same is true of hidden reversed curvature or a combination of horizontal curve with a summit vertical curve.

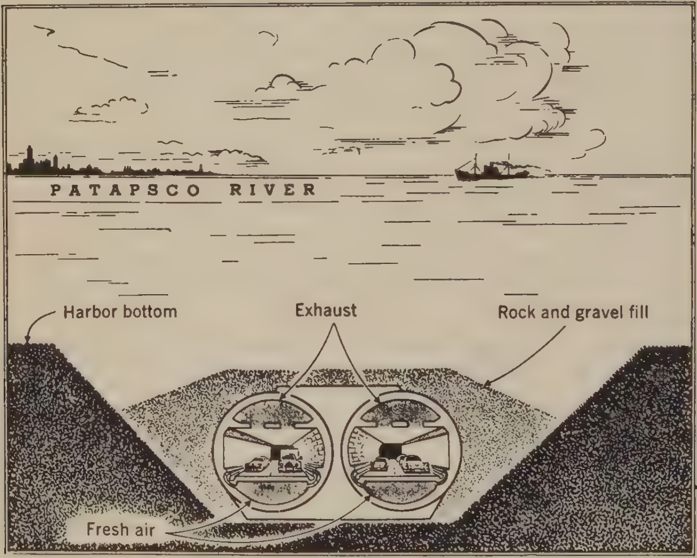
The aesthetic appearance of a highway depends upon its design as well as upon the treatment of the roadside areas. Straight lines and graceful curves are pleasing to the eye, while the reverse is true of kinks in the line or ill-fitting curves. A sufficient right of way should be acquired for future development of the highway and as a step toward protection against undesirable roadside establishments which frequently destroy full traffic capacity and create excessive hazards. If a four-lane highway is contemplated in the future, adequate right of way should be acquired for the purpose and the two-lane highway located in a position such that it will serve later as one side of a four-lane highway. While acquisitions are being made, it will be possible to retain in the right of way the narrow strips, gores, and other parcels of land cut off from the original tract. This land may be utilized for highway purposes in the future or for roadside planting and beautification. Too much standardization is often disadvantageous; thus in rolling topography, a divided highway is superior if the designer thinks and works in terms of a one-way road rather than of one center line for a fixed cross section—as is particularly helpful in intersection designs of expressways through cities. The right of way is not constant but varies considerably so that wide medians are available in rural areas where land costs are low. In some cases stands of timberland and water courses might be included in the median area.

In regions subject to heavy snowfall, the highway should be located with reference to preventing or reducing the drifting of snow. Ground subject to landslides should be avoided if possible.

Excessive highway grades are undesirable for three reasons: (1) speed reduction of loaded vehicles, (2) danger of descent, and (3) possible erosion of the road surface, shoulders, or ditches. It is known that loaded trucks are appreciably slowed down by gradients exceeding 3 percent.

Along with the needed extra width for narrow roads and streets around curves may be mentioned *acceleration* and *deceleration* lanes at the approach and runoff from intersection structures and at other places where extra lanes of constant or variable widths are desirable for cars entering and leaving the traffic stream. Also, an extra lane is advisable for two-lane highways on steep ascending grades for the exclusive use of slow-moving truck traffic.

Tunnels are quite expensive in first cost, but they provide shorter distance, better alignment, and better grades. The time and distance savings for large volumes of traffic can amount to tremendous sums over a period of years and may justify the construction of a tunnel. As an outstanding example, the city of Boston, after building its airport, found it useful to construct a vehicular tunnel beneath inner Boston Harbor, bringing the airport within 2 miles by road from the business district and supplanting the original access to the airport through congested city streets and over a slow ferry or long circuitous routes around an arm of the harbor.



Baltimore Harbor tunnel abolishes traffic bottleneck.

With modern grading methods and the use of scraping and hauling units including bulldozers, road graders, and power shovels coordinated with hauling, dumping, and spreading equipment, it is now possible to accomplish projects requiring heavy earth and rock excavations in a short time and at less cost per cubic yard than formerly. Although modern earth-moving equipment and methods permit depths of cut and heights of fills far greater than would have been considered possible some years ago, consideration must be given to the fact that very wide areas of right of way are required for deep excavations. If the location is through an urban district where property developments are involved and where values are correspondingly high, it may be better to excavate an opening, build the tunnel structure, and then fill back material so that existing improvements are only temporarily disrupted. An example of this nature was encountered on the Funston Avenue tunnel approach to the Golden Gate bridge in San Francisco.

Over a vast mileage of the two-lane secondary highways in this country there is still urgent need for greater widths, reduced gradients, easier curvatures, and stronger pavements to withstand safely increased speeds and the loads of continuously increasing traffic volumes. When traffic rises above the capacity of a two-lane pavement, opinion predominantly favors the construction of a four-lane divided pavement. Although three-lane highways may be found, usually in the vicinity of cities, they are considered dangerous because the middle lane is used by opposing traffic.

There is an increasing demand from both the town authorities and the motorists on through highways for *bypass* routes to avoid delays and hazards incident to street congestion. In cases where the bypass is not feasible, city expressways consisting of an expensive enlargement of the capacity of such city arteries have been constructed with accompanying ramps and intersection structures.

As the speed and volume traffic have grown, both congestion and accident hazards have increased to such an alarming extent as to force the highway engineer to the consideration and development of an entirely new kind of public highway facility—the *controlled-access highway*. Designed for the specific purpose of preventing early obsolescence due to roadside encroachments while serving large volumes of motor vehicles in a manner providing the greatest comfort, convenience, and safety, its salient features include divided roadways, with median strips wide enough to avoid the glare of oncoming lights and the danger of collision, the elimination of crossing and marginal conflicts with other streams of motor vehicles, and geometrical relationships of alignment and grade which provide adequate sight distances and high operating speeds.

If a major highway is urgently needed, is economically justified, and is not attainable from the usual sources except on a piecemeal basis over a period of many years, there is sound argument for its construction as a *toll highway*. Notable examples are the Pennsylvania Turnpike, the Merritt Parkway, the Maine Turnpike, the New Jersey Turnpike, the New York State Thruway, the Ohio Turnpike, and a number of separate *toll bridges* at different parts of the nation. Bonds at low interest rates, when backed by the toll revenue from such superhighways and by the credit of the state, are attractive to bankers, insurance companies, and other lending agencies. Most of the turnpikes will probably be fitted into the national interstate system of superhighways. That system will stretch a 41,000-mile network of toll-free, modern highways to provide uninterrupted driving from border to border and coast to coast.

An *expressway* is a divided arterial highway for through traffic with full or partial control of access and generally with grade separations at intersections. A significant distinction between a major street and an expressway is the distance between crossings and access connections. Whereas a major street may include numerous private driveways, an expressway has relatively few, if any. Expressways cover the wide range in facilities between the two extremes of major streets and freeways.

A *freeway* is an expressway with full control of access. The term "freeway" has no connection with whether the highway is a toll or free facility. Rather, the term refers to freedom of movement without traffic signals. All crossings are separated by bridges, pedestrians are excluded, and abutting property is denied access. Carefully designed ramps take care of all access and exit. The limitation or control of access to public highways is probably the major feature distinguishing modern highways from those built in earlier periods. This concept has been slow to evolve, but some of the American states have had such favorable acceptance of the principle that they exercise access control whenever land is purchased for highway relocations.

A *parkway* is a type of arterial highway provided for non-commercial traffic, with full or partial control of access and usually located within a park or a ribbon of parklike development. Usually there is special attention in design to features such as flat and rounded slopes, preservation of native ground cover, landscaping, scenic overlooks, etc.

Garden State Parkway in New Jersey. Trees in narrow medians are omitted because they might add a fatality to cars out of control.

A free-flowing highway, as a *freeway*, will attract traffic, even though the distance is longer, for time will be saved; or some will prefer the ease of driving on free-flowing facilities when the time might be longer.

The pattern of the system or network of arterial highways in a city evolves from the over-all transportation needs. Each route should be located to serve the particular corridor of traffic and at the same time fit the terrain, use of land, and other factors. Routes separately determined combine to form the pattern for the area.

Whenever conventional roads and streets cannot safely and efficiently serve traffic demands between centers of heavy traffic generation, freeways should be located and built to meet these demands. Under favorable conditions, the conventional city street can carry approximately 600 vehicles per lane per hour. One lane of a freeway can carry at least 1,500 vehicles per lane per hour and do it more safely.

Freeways are specified for the Interstate System which has been defined between major areas of traffic generation. The task of locating this system in, through, around, and between traffic generators involves many complex problems. Perhaps never before have highway engineers been in position where correct decisions can accomplish so much good but where wrong decisions can be so costly. General locations can be determined by the techniques of origin-destination studies. Specific locations are influenced by topography, right of way, and construction costs.

The Federal-Aid Highway Act of 1956 authorizing the location, design, and construction of the vast system of interstate and feeder highways calls for the highest standards. A pamphlet published by the AASHO (revised May 15, 1965) gives these standards—which are to be regarded as minimum standards—outlined as follows.

(1) The Interstate System

(*a*) Each section of interstate highway shall be designed to serve safely and efficiently the volumes of passenger vehicles, buses, and trucks, including tractor-trailer and semitrailer combinations and corresponding military equipment, estimated to be that which will exist in the design year which hereafter is to be 20 years beyond that in which the plans, specifications and estimates for actual construction of the section are approved, including attracted, generated, and development traffic on the basis that the entire system is completed.

The peak-hour traffic used as a basis for design shall be as high as the 30th highest hourly volume of the design year, hereafter referred to as the design hourly volume, "DHV." Unless otherwise specified, DHV is the total, two-direction volume of mixed traffic.

(*b*) On all sections of the Interstate System, access shall be controlled by acquiring access rights outright prior to construction or by the construction of frontage roads, or both. *Control of access* is required for all sections of the Interstate System. Under all of the following conditions, intersections at grade may be permitted in sparsely settled rural areas which are a sufficient distance from municipalities or other traffic-generating areas to be outside their influence, and where no appreciable hazard is created thereby.

1. The Interstate highway is a two-lane highway having a DHV of less than 500.
2. Each intersection at grade is with a public road or private driveway with little potential for traffic increase and on which the current ADT does not exceed 50 vehicles.
3. Such intersections do not exceed two per side of the Interstate highway per mile.
4. Sufficient additional corner right of way at each intersection at grade is acquired to insure that access connections on the cross-road are sufficiently removed to minimize interference with the Interstate highway.
5. The right to eliminate, terminate, or reroute each such public road or private driveway is vested in the appropriate public authority at the time of initial construction.

Where a grade separation is called for under these standards and extraordinary conditions exist under which a grade separation would not be in the public interest, an intersection at grade may be permitted through agreement between the State highway department and the Secretary of Commerce.

(*c*) There will be no *railroad grade crossings* for all through traffic lines.

(*d*) All *at-grade intersections* of public highways and private driveways shall be eliminated, or the connecting road terminated, rerouted, or intercepted by frontage roads, except as otherwise provided under Control of Access.

(*e*) The *design speed* shall be at least 70, 60, and 50 mph for flat, rolling, and mountainous topography, respectively, and depending upon the nature of terrain and development; and the design speed in *urban areas* should be at least 50 mph.

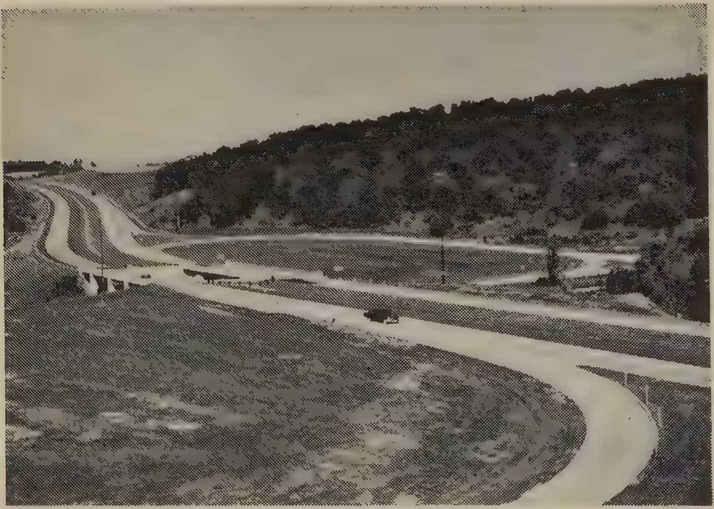
(*f*) *Curvature superelevation*, *sight distance*, and allied features, such as *transition curves*, should be correlated with the design speed in accordance with high standards; where it is not feasible to provide enough *safe passing opportunities*, a *divided highway* should be used instead.

(*g*) For design speeds of 70, 60, and 50 mph, *gradients* generally shall not be steeper than three, four, and five percent, respectively.

Gradients two percent steeper may be provided in rugged terrain.

(*h*) *Traffic lanes* shall not be less than 12 feet wide.

Where the DHV exceeds 700 or exceeds a lower two-lane design capacity applicable for the conditions on a particular section, the highway shall be a divided highway. For lower volumes, the highway shall be a two-lane highway so designed and located on the right of way that an additional two-lane pavement can be added in the future to form a divided highway.



View of grade separation on U.S. 240, Hyattsville, Md. Interstate Highway System.

Efficiency and capacity of two-lane highways may be increased by providing added climbing lanes on upgrades where critical lengths of grade are exceeded or by providing more frequent and longer sections safe for passing.

(*i*) *Medians* in rural areas in flat and rolling topography shall be at least 36 ft wide; *medians* in *urban* and *mountainous areas* shall be at least 16 ft wide; narrower medians may be provided in urban areas of high right-of-way cost, on long and costly bridges, and in rugged mountainous terrain, but no median shall be less than four feet wide.

Curbs or other devices may be used where necessary to prevent traffic from crossing the median.

Where continuous barrier curbs are used on narrow medians, such curbs shall be offset at least one foot from the edge of the through-traffic lane. Where vertical elements more than 12 inches high, other than abutments, piers, or walls, are located in a median, there shall be a lateral clearance of at least three and one-half feet from the edge of through traffic lane to the face of such element.

(*j*) Shoulders usable by all classes of vehicles in all weather shall be provided on the right of traffic. The usable width of shoulder shall be not less than 10 feet. In mountainous terrain involving high cost for additional width, the usable width of shoulder may be less but at least six feet. Usable width of shoulder is measured from edge of through-traffic lane to intersection of shoulder and fill or ditch slope except where such slope is steeper than 4:1 where it is measured to beginning of rounding.

(*k*) *Side slopes* should be 4:1 or flatter where feasible and not steeper than 2:1 except in rock excavation or other special conditions.

(*l*) Fixed minimum widths of *right-of-way* are not given because wide widths are desirable, conditions may make narrower widths necessary, and right-of-way need not be of constant width.

In rural areas right-of-way widths should be not less than the following, plus additional widths needed for heavy cuts and fills:

Type of highway	Minimum width (ft)	
	Without frontage roads	With frontage roads
2-lane	150	250
4-lane divided	150	250
6-lane divided	175	275
8-lane divided	200	300

In urban areas right-of-way width shall be not less than that required for the necessary cross section elements, including median, pavements, shoulders, outer separations, ramps, frontage roads, slopes, walls, border areas, and other requisite appurtenances.

(*m*) All *culverts* are to be of sufficient length to accommodate the pavements, median, and shoulders.

(*n*) *Bridges* and *overpasses*, preferably of deck construction, should be located to fit the over-all alignment and profile of the highway.

On all rural sections the clear height of structures shall be not les than 16 feet over the entire roadway width, including the usabl width of shoulders. At urban places the 16-foot clearance shall appl to a single routing. On other urban routes the clear height of structures shall be not less than 14 feet. Allowance should be made for an contemplated resurfacing.

The width of all bridges including grade separation structure between inside faces of curbs, when used, or between rails when curb are not used, shall equal the full roadway width on the approache including the usable width of shoulders, when the length along th parapet or rail on the bridge structure is less than 250 feet. Curbs o full width bridges shall not exceed 18 inches in width. Longer bridge may have the same width provided the DHV exceeds 1,125 passenge vehicles per lane in urban areas and 750 passenger vehicles per lane i rural areas and no span is greater than 200 feet.

On bridges other than those specified in the preceding paragrap offsets to face of parapet or rail shall be at least $3\frac{1}{2}$ feet measured fro the edge of through traffic lane. Barrier curbs on such bridges an curbs on approach highways, if used, shall be offset at least 2 fee These dimensions apply on right and left.

The lateral clearance from the edge of through-traffic lanes to th face of walls or abutments and piers at underpasses shall be the usab shoulder width but not less than eight feet on the right and four an one-half feet on the left.

A safety walk shall be provided in tunnels and on long-span structures on which the full approach roadway width, including shoulder is not continued.

(2) AASHO GEOMETRIC DESIGN STANDARDS FOR HIGHWAYS OTHER THAN FREEWAYS

[*Revised, May 15, 1965*]

Introduction. Geometric design of highways deals with the dime sions of the visible features of the highway such as the alignmen sight distances, widths, slopes, grades, etc., as distinguished fro structural design which deals with the thickness, composition materials, load carrying capacity, etc.

Frequently, available finances do not permit the construction of th ideal in highway design. Where economy is necessary it is suggest that it be practiced on some feature other than the principal geometr features. The roadway section can be improved and widened reasonable cost. The surfacing can be widened and strengthened any future date that finances will permit. But the geometric features of alignment, grade, and sight distance, when once molded into the landscape and tied down by right-of-way and surfacing, are most difficult and expensive to correct. It appears, therefore, that in discussion and decision on these principal geometric features a generous factor of safety should be added, and unquestioned adequacy rather than strict economy should be the criterion.

These standards are intended to be applied as general design controls regardless of the system of which the highway is a part. The use of more liberal values than the indicated minimum values herein is recommended where conditions are favorable and costs are not excessive.

Rural Highways; Design Hourly Volume. Highways shall be designed for specific traffic volumes as determined by accepted procedures. The design hourly volume, DHV, will be the basis for design except that the current annual average daily traffic, ADT, may be the basis for design for low volume roads. Normally this will be the 30th highest hourly volume for the design year. Usually the design year is about 20 years from date of construction, but may be within a range of 15 to 25 years. A *design speed* of 30, 40, 50, 60, or 70 mph, as appropriate for conditions, shall be selected. *Maximum grades* (percent) are given in the table below.

Type of topography	Design speed (mph)				
	30	40	50	60	70
Flat.	6	5	4	3	3
Rolling.	7	6	5	4	4
Mountainous.	9	8	7	6	6

Alignment between control points should be of as high standard as is commensurate with the topography, the existing traffic, and the probable future traffic in order that future improvements may be made with a minimum of investment loss due to obsolescence. Sudden changes between curves of widely different radii or between long tangents and sharp curves should be avoided by the use of curves of gradually increasing or decreasing radii without at the same time introducing an appearance of forced alignment. In relatively level topography on multilane highways the use of long curves of very small degree are preferred to long tangents connected by relatively shar curves, except that on 2-lane roads tangents are less a deterrent t passing than flat radius curves even where adequate passing sigh distance is provided on the latter. Where crest vertical curves an horizontal curves occur at the same location, there should be above minimum sight distance design, and the relation between horizonta and vertical sight distance should be checked to assure that the horizontal curve is visible as drivers approach.

TABLE A. MAXIMUM DEGREE OF CURVE AND MINIMUM RADIUS FOR LIMITING VALUES OF e AND f*

Design speed (mph)	Max. e	Total $(e+f)$	Minimum radius (ft)	Max. degree of curve (rounded) (°
30	.06	.22	273	21.0
40	.06	.21	508	11.5
50	.06	.20	833	7.0
60	.06	.19	1,263	4.5
70	.06	.18	1,815	3.0
30	.12	.28	214	26.5
40	.12	.27	395	14.5
50	.12	.26	641	9.0
60	.12	.25	960	6.0
70	.12	.24	1,361	4.0

* e = Rate of roadway superelevation foot per foot
f = Side friction factor for design

Depending on the maximum *superelevation* value, the maximum *curvature* for different design speeds is shown in Table A.

For small central or deflection angles the curve should be long enough to avoid the appearance of a kink. For a central angle of 5 degrees the curve should be at least 500 feet long and for each decrease of one degree in the central angle the length should be increased at least 100 feet.

Pavement crown shall be a minimum which will provide proper drainage. *Cross slope* normally should be as shown in Table C.

Minimum *stopping sight distances* for all highways and *passing sight distance* for 2-lane highways shall be as shown in Table B. Criteria for measuring sight distance, both vertical and horizontal, shall be:

For *stopping sight distance*, height of eye 3.75 feet and height of object 0.5 foot. For passing sight distance height of eye 3.75 feet and height of object 4.5 feet.

A *climbing lane*, or added lane for upgrade slow-moving vehicles, should be provided where critical length of grade is exceeded, the percentage of trucks is high, and the traffic volume is high in relation to capacity.

TABLE B. MINIMUM SIGHT DISTANCES IN FEET

Design speed (mph)	30	40	50	60	70
Stopping sight distance					
Stopping distance (ft)	200	275	350	475	600
Min. *K* value† for:					
Crest vertical curve	28	55	85	160	255
Sag vertical curve	35	55	75	105	145
Passing sight distance					
Passing distance (ft), 2-lane	800	1,300	1,700	2,000	2,300
Min. *K* value for:					
Crest vertical curve	190	510	870	1,210	1,610

† *K* value is a coefficient by which the algebraic difference in grade may be multiplied to determine the length in feet of the vertical curve which will provide minimum sight distance.

TABLE C. NORMAL PAVEMENT CROSS SLOPES

Surface type	Range in rate of cross slope	
	Inch per foot	Foot per foot
High	$\frac{1}{8}$–$\frac{1}{4}$	.01–.02
Intermediate	$\frac{3}{16}$–$\frac{3}{8}$	.015–.03
Low	$\frac{1}{4}$–$\frac{1}{2}$	.02–.04

The *number of lanes* shall be determined on the basis of highway *capacity* and *design volume.*

For low volume roads, 2-lane roads are appropriate regardless of other conditions.

Where the design volume exceeds the capacity of an existing 2-lane road or a proposed design for a 2-lane road, the following measures shall be considered:

1. Improve alignment and profile to decrease gradient and increase the percentage of total length of highway with sufficient sight distance to afford opportunities for passing.
2. Improve capacity by the addition of climbing lanes or construction of 4-lane sections.
3. Improve capacity through intersections by added lanes.

TABLE D. MINIMUM WIDTHS OF SURFACING FOR TWO-LANE RURAL HIGHWAYS

Design speed (mph)	Minimum widths of surfacing in feet for design volumes‡ of:				
	Current ADT 50–250	Current ADT 250–400	Current ADT 400–750		
			DHV 100–200	DHV 200–400	DHV 400 & over
30	20	20	20	22	24
40	20	20	22	22	24
50	20	20	22	24	24
60	20	22	22	24	24
70	20	22	24	24	24

‡ For *multilane highways, lane widths* shall be 12 feet.

4. Modify design to provide a 4-lane highway initially or as planned stage construction.

For roads with current ADT under 50 vehicles per day, the *minimum width of roadway* between intersections of side slopes shall be 26 feet. For all other roads the minimum width of roadway shall be the sum of the minimum width of surfacing and usable shoulder widths as described above.

The usable *width of shoulder* is that which can accommodate a vehicle partially or wholly when it is stopped. Wherever feasible it is to be surfaced or stabilized. Where the traveled way and shoulders are surfaced alike some delineation between the two is desirable.

Where guardrail, wall, or other vertical element is installed the lateral distance to the face of such vertical element shall be at least equal to the minimum usable shoulder width.

Where normal usable shoulder width is 6 feet or more a reduction of 2 feet may be made in cut sections in difficult terrain.

TABLE E. MINIMUM WIDTHS OF USABLE SHOULDERS FOR RURAL HIGHWAYS

Design volume¶		Minimum usable shoulder width (ft)
Current ADT	DHV	
50–250		4
250–400		4
400–750	100–200	6
	200–400	8
	Over 400	10

¶ For low volume roads, use of a roadway width that is the sum of values in Tables 6 and 7 is acceptable regardless of the actual width of shoulder.

Bridges to Remain in Place. Where an existing highway is to be reconstructed, an existing bridge which fits the proposed alignment and profile may remain in place when its structural capacity in terms of design loading and clear roadway width are at least equal to the values shown as desirable minimums for the applicable traffic volume as given in the following table. Existing bridges with lower values but prefera-

Traffic		Design loading structural capacity		Roadway clear width (ft)	
Current ADT	DHV	Desirable minimum	Minimum	Desirable minimum	Minimum
0–50		H-12	H-8	16	14
50–250		H-15	H-10	18	16
250–400		H-15	H-10	20	16
400–750	100–200	H-15	H-12	22	18
	200–400	H-15	H-15	24	20
	Over 400	H-20	H-15	24	22

bly not less than those shown as minimum may remain in place if considered adequate by the agency responsible for the operation, maintenance, and replacement of the structures.

Multilane Highways. Where a 2-lane two-way highway is not sufficient to carry the estimated DHV, a multilane highway should be provided. Rural multilane highways should be divided. A highway is not normally considered to be a divided highway unless two or more lanes are provided in each direction of travel and the median is 4 feet or more in width and constructed or marked in a manner to discourage its use by moving vehicles except in emergencies.

Multilane undivided highways may be permissible (a) as segments of an otherwise 2- or 3-lane highway where there would be substantial lengths with sight distances insufficient for safe passing, (b) where lanes are added in both directions on long steep grades, and (c) where the roadside is heavily developed and local service is one of the primary objectives. In each case a divided highway is preferable. If desired, a paved flush but contrasting median may be provided.

Medians. There are many advantages in wide medians and widths of 60 or more feet should be provided where feasible. The minimum width of median to effectively separate traffic moving in opposite directions is about 4 feet. For protection of vehicles at grade crossings the width of median should be at least 30 to 40 feet. Where left-turn lanes are to be provided in the median area a width of 22 feet or more is preferable and at least 14 feet should be provided.

The median adjacent to the left edge of each pavement should be designed to enable vehicles to safely encroach thereon in emergencies.

Where a divided highway is being considered, the advantages of independent roadway design should be explored to secure a pleasing and economical design that fits the topography.

Right-of-way Width. Right-of-way width should be not less than that required for all elements of the cross section and appropriate border areas. Additional width may be necessary for construction.

Side Slopes. Slopes should be as flat as feasible. Flat side slopes increase safety by providing maneuver area in emergencies, are more stable than steep slopes and simplify the establishment and maintenance of plant growth. The intersection of slope planes should be well rounded.

Where right-of-way is available it frequently is less costly to flatten embankment slopes to 4:1 or flatter for appreciable depths, instead of constructing guardrail. Maintenance cost is also reduced.

Curbs. Except where necessary to control traffic or drainage, the use of curbs on roadways should be avoided for safety in enabling vehicles to veer from the pavement in emergencies, for economy and for simplicity. Where barrier curbs are continuous along a highway they should be offset at least 1 foot and preferably at least 2 feet from edge of traffic lane and so carried across all structures. Barrier curbs introduced on bridges or intermittently elsewhere should be offset at least 2 feet and preferably at least 3 feet.

Interstate I-89, near Waterbury, Vt.

Normally median curbs are not used with medians of about 16 feet or more in width except where a median turning lane is provided. Medians of lesser width may be curbed or treated in some other way to provide a more positive separation.

Vertical clearance at underpasses should be at least 14 feet over the entire roadway width, to which an allowance of 4 to 6 inches should be added for resurfacing.

The minimum *horizontal clearance* both right and left on structures, from the edge of traffic lane to the face of parapet or railing shall be not less than 3.5 feet and preferably 4.5 feet. For low volume roads the clearance may be one foot less.

Except on low volume roads, the normal roadway section, including usable shoulders, should be carried over all structures 50 feet or less in length and preferably those up to 150 feet long.

Where the ratio of DHV to capacity is 0.75 or greater the width of bridges should equal the roadway width on the approaches, including the usable width of shoulders, even though the length exceeds 150 feet.

The foregoing clearances need not apply to long-span structures on which, due to their high unit cost the selection of the cross section dimensions should be subject to individual economic studies.

At underpasses the lateral distance from the edge of through traffic lanes to the face of walls, piers or abutments should be at least the usable shoulder width but not less than 6 feet on the right and 4.5 feet on the left.

Access from roadsides to the roadways of arterial highways should be controlled by regulations.

URBAN HIGHWAYS

The previous design controls applicable to rural highways are applicable generally to urban highways. The following additional controls cover items of difference.

1. Pavement should have where feasible sufficient width to accommodate the traffic volume without exceeding capacity during the design hour. Design capacity should be calculated on the basis of roadway and traffic conditions, taking into consideration such variables as frequency of intersections, volume of cross traffic at intersections, parking practices, turning movements, and volume of truck and bus traffic.

2. The width and number of lanes on major streets is frequently governed by practical limitations, such as existing building lines and other limitations on available width of right-of-way. Lanes should be 12 feet wide where feasible, but not less than 11 feet.

Where the directional distribution during peak hours exceeds 60 percent on undivided two-way streets, reverse flow lanes should be considered.

Advantage may be taken of a street with an odd number of lanes, as a 5- or 7-lane street, to convert the center lane to a storage lane for left turning vehicles by pavement marking at appropriate intersections. This arrangement is especially fitting on streets under traffic signal control with a separate signal phase for left turns. On sections having sufficient width, a full median and median lanes would be in order.

The operation of streets for one-way traffic is an accepted standard practice. Geometric standards for two-way streets are applicable also to one-way streets.

3. Where there are continuous barrier curbs but no shoulders or parking lanes the barrier curbs should be offset at least 1 foot and preferably at least 2 feet from edge of traffic lane.

Barrier curbs are fitting on narrow medians if midblock turns are to be prohibited.

4. Diagonal parking adjacent to through traffic lanes is hazardous and shall not be permitted. Where parallel parking is contemplated, parking lanes should be wherever feasible at least 10 feet in width, and preferably 12 feet.

5. Air Transport. As compared with other systems, airways can furnish unusually rapid service for passengers, mail, and small shipments of lightweight, valuable commodities where speed of delivery is a controlling factor. The scheduled airlines now constitute the primary passenger carrier in the nation, exceeding both the railroads and intercity buses in terms of passenger miles but not number of passengers. A conservative prediction is that between now and 1975, the air carrier revenue passenger miles will triple, the number of passengers will triple, the volume of domestic civil air cargo will increase fourfold, and business flying will double.

The location and design of the airport itself require expert planning to provide the most satisfactory accommodations for arriving and departing passengers and the efficient handling of cargo. Also, there should be sufficient parking space and good surface transportation service between the airport and the real centers of departure or destination.

Sites for runways are favorable if they can be graded and drained at a reasonable cost and if the flight requirements of the planes to use the terminal are satisfied. The location of runways is affected by the topography of the site, obstructions adjacent to airport, approach zones, free circulation of traffic, and future expansion. The test of a good master plan is "its ability to accommodate itself to changes and advances in technological developments." Aside from operational accidents and incidents on the runway, the plan will continue to be affected by snow, birds, slush, turbulence and vortices, failure of approach aids, airport power failure, poor taxiing techniques, mishaps at gate positions, in-flight emergencies such as stuck undercarriage or one engine out, a sick passenger, or a private pilot who has lost his way.

In the political-economic environments in which planes are operated, the private pilot has and will probably retain access to airports used by the airlines. His equipment may not be built, installed, or maintained to the same standards as that used in airlines or large corporation aircraft, and his proficiency may be less than that of the professional pilot. Other items of interest to the airport planner are wind conditions, prevalence of fog, and the smoke from industrial centers near the site.

The safety factors which affect aircraft operation should never be compromised. Aside from the installation of nationwide radar traffic control equipment, the foremost of these is the freedom of the site from obstructions that would constitute a hazard to aircraft flying in the vicinity and flying in the approach and transition zones. Future growth or possible future erection of obstructions in the area should be guarded against at the time the property is acquired. This may be accomplished by purchasing sufficient land under the approach and transition zones or by purchasing avigation easements.

Excerpts from the ASCE 1957 "Jet-age Airport Conference"

Jet transports will not make obsolete all the airports in a single day, but they are going to require some changes at many airports, such as size and design of terminal buildings, the runway and taxi layout, ramp facilities and ramp management, and airport detection radar. When frequent schedules unload and load a greater volume of passengers, a different size terminal is needed and one that is better designed for the efficient flow of passengers to and from the airplanes. On the ramp the jet brings new problems, a chief one being fueling equipment and methods. A jet can demand 20,000 gallons of fuel at a major terminal of one of its long-range flights.

Airport surface movements must be speeded up. This is one way of increasing the capacity of an airport. It is done by making it possible for the big, new planes to make turns off the runway at 60 or 70 miles an hour instead of slowing down to 10 or 20 miles per hour.

Traffic on an airport surface can become badly snarled when the controller cannot see the limits of the airport and when the pilot himself can get lost in the darkness or thick weather. In fact, an airport can be closed to all traffic if a single plane is moving about over its surface and the controller and pilot do not know where it is. The answer is Airport Surface Detection Equipment. This equipment is a short-range radar by which the controller can move planes on the field by "seeing" them on his radar.

Most metropolitan airports require more land than was anticipated in the past; acquisition of adjoining property in most cases can be done only at prohibitive expense. Location planning of airport installations which allow a choice should try to anticipate and forecast the increasing land-space demands which will accompany this changing and growing industry. Primarily it must be accepted that in an industry changing as rapidly as air transportation, the best forecast will be imperfect. Wherever possible the designer must avoid creating airport facilities which cannot be expanded and modified without loss when changing kinds of aircraft and increased quantities of people must be accommodated.

It appears advisable that the designer should try to build today what the aviation industry will need in 1975; what airport accommodations these travelers will need; how many automobile parking spaces, how many square feet of lobby area, of restaurant and concession area, of airline operations space, how many airplane loading positions will be required.

One of the major advantages of air-travel is speed. The air-passenger desires to stay at an airport as short a time as possible. The airport is just a way station as far as he is concerned, a necessary part of traveling a long distance. As airline service improves because of more frequent departures and more precise scheduling, the present passenger-occupancy at the terminals will diminish. If there is less time spent waiting, there will be less lobby space required per enplaned passenger.

The airport problem is not solely limited to providing adequate facilities to handle the airplanes and passengers on the ground. It goes further than providing sufficient landing and terminal facilities, including long enough runways and efficient taxiway systems with high speed turn-offs and warm-up areas. The problem of increasing airport capacity through stepping up the airport acceptance rate and the establishment of lowest possible minima, require consideration of other problems relating to getting the airplane into and out of the airport. *Airport approach* problems must, therefore, be given consideration along with other airport matters. The presently expanded air traffic and the increased spread of urbanization with its resulting encroachment on existing airports by residential, commercial and industrial development have highlighted the necessity of present and continuous consideration of airport approach protection problems. The radio/television industry must have tall antenna towers in order to provide the degree of service expected; however, these tall skeletal structures and their supporting guy wires are hazardous to aviation because they are hard to mark and hard for pilots to see. A committee is now developing standards to be used as a guide by both the radio/television and aviation interests in their consideration of mutual problems relating to the height and location of proposed new antenna structures and new airports. The criteria being developed by this committee have the following objectives: (*a*) to provide protection for low altitude intercity routes, (*b*) to provide increased protection for airways and much-used fly ways, and (*c*) to provide additional protection for areas in the vicinity of airports.

Airport owners seek to protect the approaches to their airports by two general methods. The most effective and positive method of protection of airport approaches is the acquisition of interests in land, with such interest varying from a fee simple absolute to avigation or aerial easements. The second general method establishes a limitation on the height of structures which may be put on land by legislation enacted under the police power. This method of approach protection, commonly referred to as airport zoning, is justified primarily as a regulation in the interest of public health and safety.

Galley service will be relatively unchanged from present practice except that the galleys will have separate entrances on the opposite side of the airplane from the passenger loading doors which will allow simultaneous passenger and galley loading.

Plans are being finalized by some airlines to incorporate cargo containers into the cargo hold which will be removable such that the container will be preloaded in the baggage room, hauled out to the airplane in a baggage train, and hoisted into the cargo compartment. This system will eliminate the baggage handling which contributes so much to the baggage damage experienced today. It will also materially assist in speeding up the baggage delivery. The containers would hold up to 40 pieces of average luggage depending upon the airplane. When the first jet services are introduced the passenger loading will be handled by conventional mobile passenger loading stands at both front and rear passenger doors.

In a broad sense, a terminal is the functional link between ground and air. It must accept all sorts and types of undisciplined streams of passengers, visitors, baggage, freight, express, vehicles and aircraft which, before the terminal can be called efficient, must be directed into specific channels tailored to satisfy the particular requirements involved. At Los Angeles Airport, the passenger arrives directly at the airline's ticketing building; his ticket and baggage are processed; and he reaches that airline's plane loading ramps by direct and separate channel access from that building to the ramps. This permits individual, independent operation for any given airline, and direct, unencumbered departure and take-off for the passenger. The economic advantage is obviously great; a means of orderly expansion of the components are provided, and components are constructed only as required which avoids the possibility of "over-building." A forecast was made, too, that the size and number of communities within the greater Los Angeles metropolitan area would create an impetus for use of helicopters to service the airport; and a method of transporting people by helicopter from smaller, outlying airports to the various airline satellites within Los Angeles International was reviewed and incorporated into the plan.

During the Conference, manufacturers of large turbojet powered aircraft presented considerable information on both the benefits and problems associated with the introduction of these craft into commercial airline service. Among the problems is the determination of adequate runway lengths. Generalized curves of Takeoff and Landing Runway Length Requirements are available from the aircraft manufacturers. But this information alone can be misleading because other factors peculiar to each airport should be considered; namely, runway width and orientation, runway and taxiway load bearing capacity, taxiway width and form, ramp maneuvering space, and nuisance abatement.

The runway length is affected by the following: (1) Civil Air Regulations, (2) airport elevation, (3) airport obstructions, (4) runway gradient, (5) weight of aircraft, (6) airplane performance characteristics, (7) reserve fuel policy, (8) cruise control procedures, (9) altitudes to be flown, (10) surface temperature, (11) surface winds, etc.

Only the takeoffs for long non-stop trips will require the longest runways. The primary runways of a few cities are now (year 1957) as follows: Boston, 10,000 ft; Los Angeles, 8,560 ft; New York International, 9,420 ft; Denver, 10,000 ft. The secondary runway lengths are normally 85 to 90 percent of that on the primary runway.

Ten years ago, little or no thought or attention was given to outside noise characteristics in aircraft design. Today, in behalf of its jet-age neighbors, the entire industry—from manufacturing president to crew flight—is determined to do everything possible to solve this problem; and the effort will not stop with only the installation of suppressors on the engines.

It is generally agreed that the arrival of the jet transport will mark the beginning of another surge in the vigorous growth of air transportation. It is this growth for which most airports must prepare, rather than the jet itself, since perhaps not more than 50 communities will be involved directly with the first phases of pure jet operation. However, all will be faced with airport expansion problems of some kind, such as the difficulty of overcoming topography or approach conditions, obstructions, gradient or other undesirable features, where terminal area is restrained on all sides by highways, hangars, runways, industrial plants, and the like; also the need to accommodate a new industry which desires an airport site in order to simplify air shipment of products or to facilitate the air travel of company executives or technicians by airline or corporate aircraft; or the need to increase the airport capacity rate by development of a parallel runway in order to avoid saturation or obsolescence.

A solution to the expansion problem has often led to a major revision of the airport configuration as an alternate to the construction of a replacement or to the continuation of the airport under severe handicaps—either operational or financial. Fortunately, many such adjustments in the airport configuration have been possible by taking advantage of the "single runway" concept and phasing out one or more of the existing multiple runways. Wherever its application is feasible, considerable property and pavement may be released for other uses. Situations where limitations of terrain and real estate did not preclude the re-design of an airport configuration with sufficient capacity to accommodate the traffic forecasts may be cited those at Portland, Me., Hartford, Conn., Concord, N.H., Albany, N.Y., and Port Columbus, Ohio.

An efficient instrument runway pattern is the "open-parallel" design where the parallels are separated by the terminal building and the ramp area. Arrivals and departures are automatically segregated and any cross traffic is confined to the ramp area. With this arrangement, takeoffs and approaches are conducted independently and virtually simultaneously on their respective runways. The average IFR (instrument flying rules) capacity for this configuration is estimated at 65 operations per hour which is just short of the theoretical rate.

6. Pipelines. Now grouped among the transportation systems of the nation, pipelines provide direct, low-cost, dependable movement of petroleum products, and also of water and sewage over shorter distances. In the oil and utility products industry, the advantages of pipelines over other means of transportation is obvious when one realizes that with a pipeline there are no empty vehicles to return and no cars to build or service. Delivery is continuous and the amounts to be delivered can be predicted accurately. The system is in the ground and free from most of the maintenance problems of surface transportation; however, there are certain hazards that will be mentioned later. Theft is almost impossible and the labor required for operation is comparatively small. The industry is characterized by high capital costs and low operating expenses.

The pipelines were among the first of the industries to adopt radio and microwave facilities for their dispatching, message transmission, and remote control of station operations. By the use of remote controls, electric pumps, and compressors, a line becomes almost automatic in operation; thus an oil line in Ohio is now remotely controlled by a dispatcher in New York City.

Pipe lengths have increased from the early 10-ft joints of cast pipe to 40-ft joints of 30-in. and larger steel pipe of today. Some of the smaller-sized pipes come in joints up to 60 ft in length. Less resistance to flow results as the diameter becomes larger and the curvature—horizontally and vertically—becomes easier.

The modern ditching or trenching machine can excavate a trench 4 ft wide and from 5 to 7 ft deep at a rate of a mile and a half a day, depending upon the type of soil and the amount of rock encountered in the ditch line. Customary pipeline specifications call for a minimum cover of 30 in. over the top of pipe, so that a 30-in. line will require a ditch 5 ft deep minimum. For every mile of such line approximately 3,900 cu yd of excavation is piled up on the bank and then moved back into the ditch after placing the pipe.

The year 1950 marked the completion of a 30-in. natural gas main from Texas to New York City—a distance of 1,840 miles—and a wood-stave pipeline in Oregon 12 ft inside diameter and 1,700 ft in length, mostly on curves, to convey water under 45- and 50-ft heads. By the end of 1959, the total pipelines transporting petroleum in the United States exceeded 200,000 miles.

Each river crossing regardless of the size of pipe constitutes a separate engineering problem. Difficulties also arise if a highway crosses the pipeline right of way and reduces the cover so as to necessitate either expensive encasement or the cutting and relocating portions of the pipe.

In view of the hazardous nature of high-pressure gas pipelines, the following requirements, based on a national safety record extending over the past 38 years, have been imposed:

(1) Valves should be used at each end of the pipelines on long bridges; (2) initial testing near highway traffic must be carefully controlled; (3) pipeline connections to highway bridges should be light flexible hangers; (4) they should not be located upstream from bridges; (5) they should be laid at safe depths and the locations marked; (6) encasements may be necessary for the pipeline under highway embankments; (7) speculative hazards should be considered, such as (*a*) increase of pressure without notice, (*b*) depreciation of pipe by corrosion and erosion, (*c*) rupture of pipeline by flood, lightning, earthquake, landslides or land subsidence, (*d*) rupture of pipeline by allision of uncontrolled vehicles, (*e*) demolition of pipeline by arson or sabotage.

Sewer mains serving cities present relatively more difficulties than water mains of the same or larger size for the reason that they are usually gravity operated and must be laid on uniform grade between manholes and with changes in direction occurring only at the manholes.

7. Waterways. Location surveys for canals, aqueducts, and flumes are similar to those for other routes. Relatively slight grades are common for canals. This is particularly true on locations where soil erosion might take place, unless the bottom and sides are lined with concrete or other material. In general, there is much more flexibility in alignment than in grade. Difficult problems arise when a highway or railroad must cross over or under a canal. The curvature of ship canals must not be too sharp for vessels to negotiate the turns. Sometimes it is feasible for extra width to compensate for excessive curvature.

Waterways handle bulk freight at low cost but lack speed of delivery. In some instances the economy of artificial waterways as compared with railway or truck transportation is subject to debate.

An important purely artificial waterway in this country is the New York State Barge Canal, the construction of which was an engineering feat involving numerous locks, bridges, deep excavations, and so on, combined with the relocation of other systems.

The present Welland Canal is the latest in a series of four which have been built in the Niagara River Peninsula to overcome the obstacle to navigation presented by the Niagara River, its Rapids, and Niagara Falls. This ship canal connects Lake Erie to Lake Ontario, a distance of about 28 miles and a difference in elevation of 326.5 ft. Seven concrete locks are used each with an average lift of 46.5 ft. The locks are 80 ft wide, 859 ft long between gates, and with a depth of water of 30 ft over the sills. The usable length for navigation is 715 ft, and the widest ship which has transited the canal is 75 ft. Twenty bridges, fourteen highway and six railway, span this canal. Eleven of these are the vertical-lift type, giving a clearance of 120 ft from the surface of the water. This canal is 27 ft deep, 200 ft wide at the bottom, and 310 ft at the water line, with the banks on 2:1 slope. To prevent damage to the banks by wave action, they have been protected by a concrete wash wall 12 in. thick extending from 5 ft below to 5 ft above water line, where excavation was done in the dry, and by a layer of broken-stone protection at all other sections. The banks have proved unstable at various locations and minor slides have occurred. These have been remedied by flattening the slope by dredging, relieving the top of the bank, or placing drains in the banks to reduce the moisture content. This canal is part of the St. Lawrence Seaway extending into the Great Lakes, a great stairway for ships creating ocean ports 2,000 miles inland and 602 ft above the sea.

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CHAPTER 2

ROUTE SURVEYS BY GROUND AND AERIAL METHODS

8. Scope. From the general to the specific, a typical procedure to determine the accepted route for a railroad, highway, pipeline, or any other transportation system is as follows: (1) an area reconnaissance is made by thoroughly examining a wide area from one terminal point to the other for the most feasible route alternatives; (2) a reconnaissance survey and comparison of the route alternatives in sufficient detail for selection of the best route; (3) a preliminary survey of the best route in sufficient detail for the preparation of plans and estimates of construction quantities; (4) completion of plans for the final location; (5) the setting of stakes for construction of the facility including drainage structures, right-of-way lines, etc. From the beginning of the first of these stages to the completion of work in the fourth stage, there is progressive decrease in scope and increase in accuracy within feasible practical limits.

The objectives are the same whether the project is a new location or a relocation of short sections of existing routes. The methods to be followed will vary somewhat with the nature of the project and the standards to be achieved.

The foregoing surveying operations are usually classified as the reconnaissance, the preliminary, and the location survey.

Reconnaissance Surveys

9. Ground and Air Reconnaissance. Aided by available maps and stereoscopic aerial photographic coverage of the area, the instruments used in ground reconnaissance need rarely be more than a prismatic compass, an Abney hand level, and an aneroid barometer.

The Abney level, when set at 0.0, serves the purpose of a *Locke level;* in addition, it is valuable for running trial lines on a sidehill terrain. With the vertical arc set at any desired percent grade (or corresponding vertical angle), a forward point on the ground (independent of distance) is established when a rodman moves until the rod reading is at the "height of eye." In mountainous country it is better to start at the top (at a pass or gap) and proceed downward.

In difficult situations covering small areas where no suitable maps are available, it is sometimes necessary to make complete surveys, construct appropriate maps, and project (on paper) routes for comparative study and selection prior to the actual layout on the ground.

Maps of the area supplemented by aerial photographs are exceedingly helpful, the latter giving a greater wealth of detail than could be expected from ground surveys alone. Most of the United States has been photographed from the air at least once. Up-to-date information on the existence and nature of available aerial maps may be obtained from the Map Information Office of the U.S. Geological Survey, Washington, D.C.

Already many state highway departments, including the department in California, use aerial photographs in the planning and reconnaissance stages of practically every project. These maps do not show contours, but they are invaluable for many planning purposes. In flat terrain, they are sufficient for projecting the final location. When quantitative information which cannot be obtained from the photographs (or mosaics) is required, the first source is existing topographic maps, such as the U.S. Geological Survey quadrangle maps. It should be stressed, however, that the location, as determined by aerial photographs supplemented by small-scale topographic mapping, where required, is approximate only and subject to later refinement during the design stage.

10. Aerial Mapping Processes; Definitions of Terms. Photogrammetry is the science or art of obtaining measurements by means of photography, the definitions here being applied only to aerial photography, usually with the camera axis vertical and the flight altitude constant as indicated by barometer readings. A continuous series of overlapping photographs in a given direction is called a *flight strip*. The photographs may be *contact prints*, which, because of the usual 60 percent overlap, form stereoscopic pairs that may be examined under a stereoscope, by viewing simultaneously one of a pair of overlapping photographs with the right eye and the same object on the other photograph with the left eye, thus causing the terrain to appear in three dimensions.

A photographic *mosaic* is an assembly of aerial photographs to form a continuous photographic representation of the area. *Controlled mosaics* are those brought to a uniform scale through identification of points on the ground at known distances apart. If the photographs are matched without reference to ground control, the map is *uncontrolled.*

Strip mosaics find wide application in highway, railroad, pipeline, transmission-line, and aqueduct studies and in certain types of property surveys. Uncontrolled mosaics are used extensively for reconnaissance surveys. Controlled mosaics find their widest application in accurate property surveys, city and regional planning activities, and natural-resources studies.

Stereoscopic vision is facilitated by the use of the instrument called a *stereoscope.* Thus if two photographs of the same area and object, taken from different exposure stations, are properly oriented and viewed simultaneously, the observer has the impression of seeing a relief model of the terrain covered by the photographs. Use of the stereoscope greatly facilitates correct interpretation of aerial photographs, because the three-dimensional views invariably give a clearer conception of the relief and elevation of the natural terrain, and clarify the details of man-made structures. Minor but important differences in elevation, such as embankments, cuts, ditches, and stream banks, are readily detected, and approximate elevation differences estimated, by means of simple stereoscopic observation.

Interpreting aerial photographs is a skill acquired through experience and practice. The stereoscope should always be used when overlapping pairs are available. Many cultural features are identified by the regularity of their outline or alignment, such as roads, railroads, canals, power lines, and fences. Objects such as radio towers, water tanks, bridges, and tall buildings are clearly revealed by their shadows in the photograph, even though the image of the feature itself does not serve to identify it. A bridge may appear on the photograph as a narrow strip, whereas the shadow may indicate whether it is of arch, truss, cantilever, or suspension construction. Natural features are irregular, in contrast to the geometric pattern of man-made features. Streams are easily identified by their winding courses, variable widths, and the pattern of vegetation along their banks. The shore line of water features can be studied in great detail on aerial photographs.

Evergreen forests appear as dark, dense masses whether the photography is taken in the summer or winter. Deciduous forests appear dark and dense on summer photography; but, on photographs taken in winter, when the leaves are off, the ground is clearly visible through the trees. For this reason, most mapping photography of wooded areas is obtained in the winter or early spring.

The interpretation of photographs for special purposes, such as the geology or drainage of the area, requires detailed study of tones and textures and their relationship to the vegetative cover or land forms.

The term *photogrammetry* implies the use of photography for map making in surveying. A *planimetric* map shows the plan or horizontal position of ground features, while a *topographic* map shows in addition relief or elevations indicated by contour lines.

The preparation of topographic maps by photogrammetric methods is one of the most important applications of photogrammetry. This work must be performed by specially trained key personnel using specialized photographic equipment; and in conjunction with this mapping process, are field surveys to establish vertical and horizontal controls for a limited number of points on the ground that have been identified on the photographs. To satisfy U.S. National Map Standards for accuracy, the planimetric features shown on the map shall be within 0.02 in. at the map scale and 90 percent of all contours shall be accurate to plus or minus one-half contour interval and no contour line shall be more than one contour interval from its true position.

Long-distance measurements will be greatly simplified by the use of radarlike devices such as the tellurometer or geodimeter which measure the time required for a radio microwave or a beam of light to travel between portable units at either end of the line to be measured. Extensive literature in the field of photogrammetry is available, notably the publications of the American Society of Photogrammetry.

The steps of preliminary aerial topographic mapping are (1) photographic mission; (2) ground-control surveying; (3) stereoplotting; and (4) drafting. The methods are substantially the same for planimetric maps except that a large proportion of stereoplotting time is saved. Plotting must still be done stereoscopically so that all horizontal points will bear the correct relationship to each other.

Photogrammetry has its *limitations*. Smoke or mist may interfere with photography, and conditions are unfavorable where the features are hidden by snow or a dense growth of evergreen trees. Ground surveys are necessary for locating contours in flat terrain (if they are essential) and for locating features not visible in the photographs, such as private cemeteries, property corners, wells, pipelines, etc. Measurement of critical elevations at existing structures is one function that rarely can be performed by photogrammetry. The amount of ground surveys necessary for such fill-in data is slight, however, and can be made a specific part of the instructions to field parties.

The map scales used for reconnaissance range from 1 in. = 200 ft to 1 in. = 2,000 ft with contour intervals of 5, 10, and 20 ft. For use in the design of interchanges or other features, the scale may be as large as 1 in. = 20 ft with 1- or 2-ft contour interval. A scale of 1 in. = 200 ft with 5-ft contours has been widely used for preliminary line projections.

11. Advantages of Aerial Maps Summarized. Besides less cost and more detail, aerial mapping has many unique advantages not obtained in ground surveys, among which are the following: (1) with a wide area, often extending to a mile, a better opportunity is presented for comparison and selection of the best route; (2) by avoiding trespass on private property for running preliminary trial lines, less friction and a better bargaining result when acquiring right of way for the final location of the route; (3) the map provides an inventory of surface features showing blighted areas, drainage patterns, sources of construction materials, traffic density, and the location of existing trees or other features for landscaping; (4) combinations of photogrammetric and automatic digital-computer operations yield earthwork cut-and-fill determinations for highways and railways independently of cross-section data obtained from ground surveys.

Many of the highway departments have their own photogrammetry divisions, manned by experts who can advise on the correct mapping applications for each survey situation, while other organizations rely upon the commercial air-mapping companies, several of which are also competent in the practice of highway and route engineering.

Preliminary Surveys

12. Foreword. The preliminary survey, if run on the ground independently of aerial methods, is an instrumental survey (with transit, level, and other equipment) of the line tentatively established by the reconnaissance. This line is an accurate traverse which serves as a reference for obtaining topographical information and other data pertaining to the strip of terrain within which the route will probably be located. From the cross-section notes of the preliminary survey, a contour map may be prepared and on it projected the "paper location," which becomes, with probable minor adjustments during the staking-out process, the final location.

In contrast to the foregoing procedure, modern freeways with their wide rights of way, complex interchanges, and high standards of alignment and grade require an entirely different location technique than the conventional highways of ten or fifteen years ago. Fortunately, developments in the fields of photogrammetry and control surveys have kept pace with these constantly increasing requirements and have provided products which the highway engineer can use to solve his location problems.

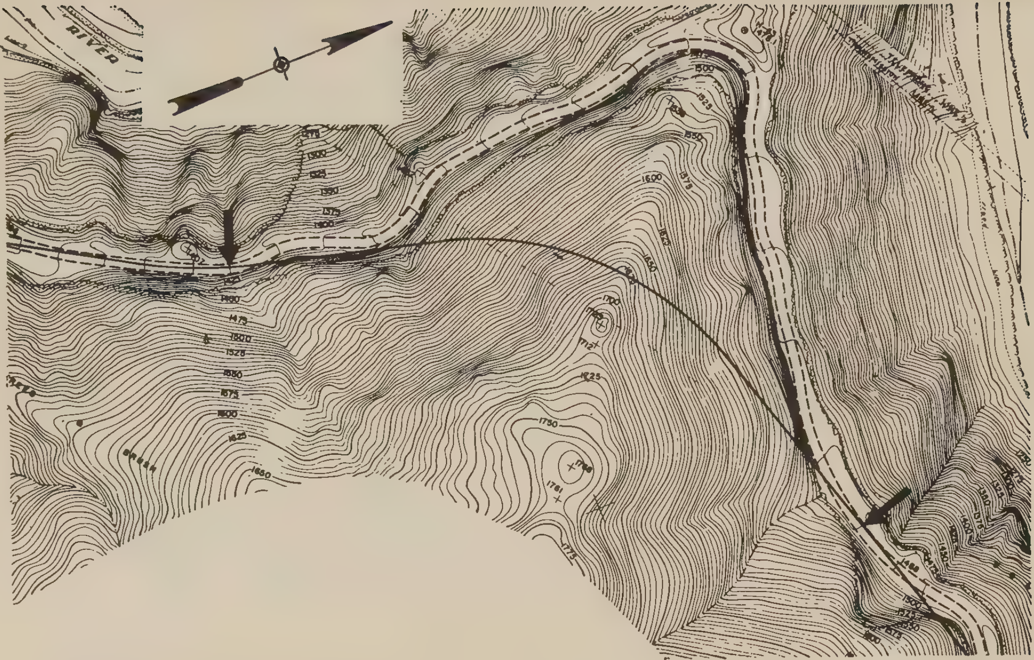
Location practice in some sections of the country has followed the trend started by engineers for several of the large turnpikes a number of years ago in obtaining a strip of topographic mapping a mile in width at a scale of 1 in. = 200 ft with 5-ft contours. The maps were used to study various alternative lines and select the best location. The final line was not calculated in the office but was staked in the field by scaling ties from planimetric features to the graphic projection on the map. The locator made such adjustments as were found necessary in the field and proceeded with profiles and cross sections in the conventional field-survey manner.

Further refinements of this method are being used in several of the states. They generally consist of a follow-up photographic flight at low altitude after the proposed center line has been staked in the field. From this low-altitude photography a planimetric map at 1 in. = 50 ft is prepared and photogrammetric cross sections for use in detailed design work are developed in the stereoplotter.

After the approximate location of the route has been determined (using aerial photography), the next step in California practice is to obtain large-scale, small-contour-interval photogrammetric maps for determination of the final line and detailed design of the facility. Such mapping is generally at a scale of 1 in. = 50 ft with 2-ft contours or with photogrammetric spot elevations at 100-ft intervals in level terrain where contours would not provide sufficient accuracy. A scale of 1 in. = 100 ft with 5-ft contours is occasionally used for design work in mountainous terrain. The mapping is generally from 1,000 to 1,400 ft in width with additional width at interchange sites, thus allowing the designer considerable latitude in determining the final line. He can make this determination with the large-scale detailed mapping at hand and does not have to rely on small-scale mapping for his information. Furthermore, he is not influenced by a line previously staked on the ground to which he feels he must adhere if possible.

Preliminary studies of horizontal alignment are often done on the basis of graphical analysis, that is, by the use of splines, curve templates, ship curves, and straightedges. These tools are also useful in establishing smooth grade lines or profiles for pavement edges, interchanges, and many other features.

13. Procedure: Preliminary Transit Line. The preliminary traverse, independent of aerial surveys, is a series of straight lines with distances, or stationing, continuous from the beginning to the end of the survey and angles carefully read and checked by double centering. Thus Sta. 12 + 47.2 means 1,247.2 from the beginning and not from the preceding turning point; Sta. 13 would be set 52.8 ft beyond the angle point; etc.


RIVER
1500
1525
1550
1575
1600
1625
1650
1700
1712
1725
1750
1761
1775
1475
1450
1400
1375
1350
1325

Center line of projected new highway replacing old road (double dashed line) in rugged California terrain.

FORM OF NOTES

PRELIMINARY SURVEY OF ROAD

Station	Angle, *L*	Angle, *R*	C.C.	M.C.	Distance
9					
+75.3		15°-18′	*S*21°-31′*E*	*S*21°-30′*E*	505.1′
8					
7					
6					
5					
4					
+70.2	18°-19′		*S*36°-49′*E*	*S*37°-00′*E*	370.2′
3					
2					
1					
0+00				*S*18°-30′*E*	
	18°-19′	15°-18′			

Fig. 2

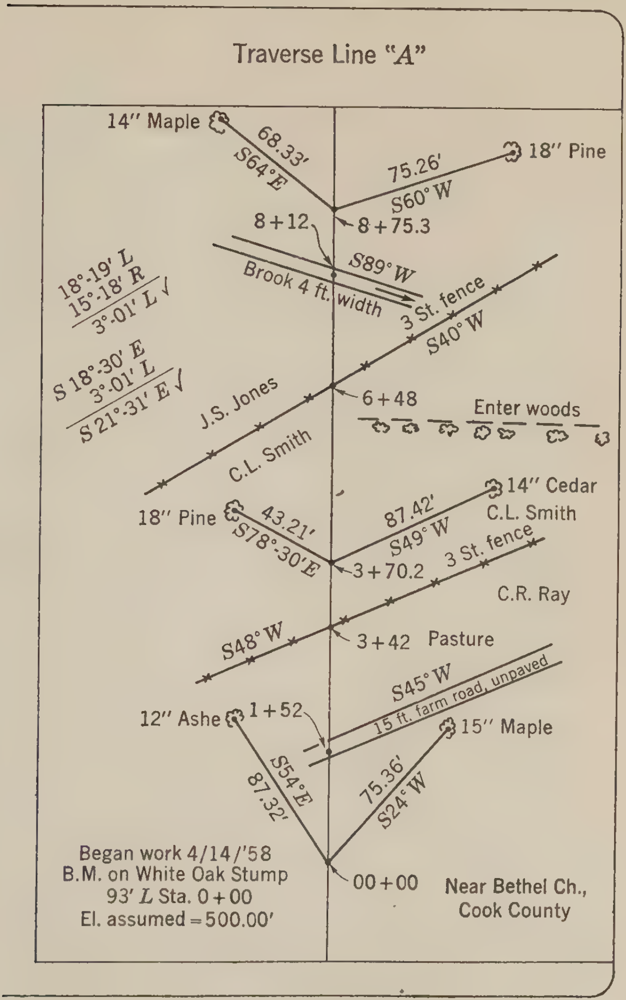

Traverse Line "A"
14" Maple
68.33'
S64°E
75.26'
S60° W
18" Pine
8+12
8+75.3
S89° W
Brook 4 ft. width
18°-19' L
15°-18' R
3°-01' L √
S 18°-30' E
3°-01' L
S 21°-31' E √
3 St. fence
S40° W
J.S. Jones
6+48
Enter woods
C.L. Smith
14" Cedar
C.L. Smith
18" Pine
43.21'
S78°-30'E
87.42'
S49° W
3+70.2
3 St. fence
C.R. Ray
S48° W
3+42
Pasture
S45° W
15 ft. farm road, unpaved
12" Ashe
1+52
15" Maple
S54°E
87.32'
75.36'
S24° W
Began work 4/14/'58
B.M. on White Oak Stump
93' L Sta. 0+00
El. assumed = 500.00'
00+00
Near Bethel Ch.,
Cook County

FIG. 2 (*Continued*)

Distances are usually measured with a 100-ft tape. Stakes are set at intervals of 100 ft, that is, at each station. Hubs driven flush with the ground and sloping guard stakes mark the angle points. On the basis of the first compass bearing, the subsequent bearings are calculated to agree with the deflection angles. These "calculated" bearings should agree reasonably well with the compass bearings, as indicated in Fig. 2.

In prolonging straight lines, double centering is important.

All established railroad or highway alignment should be referred to the true meridian, determined by the sun or star observations.

All notes should be kept in a notebook and should never be erased. When mistakes are made, mark through them and write the corrections close by. When alternative lines are run, label them "Line A," "Line B," etc., to avoid confusion.

14. Referencing Transit Points. Two ways of referencing points in order that they may be reestablished, if lost, are shown below.

Method A. Measure carefully the distance from the given point *P* to three different objects, say, a nail at the root of a tree, as shown in Fig. 3*a*. It is important to choose convenient objects

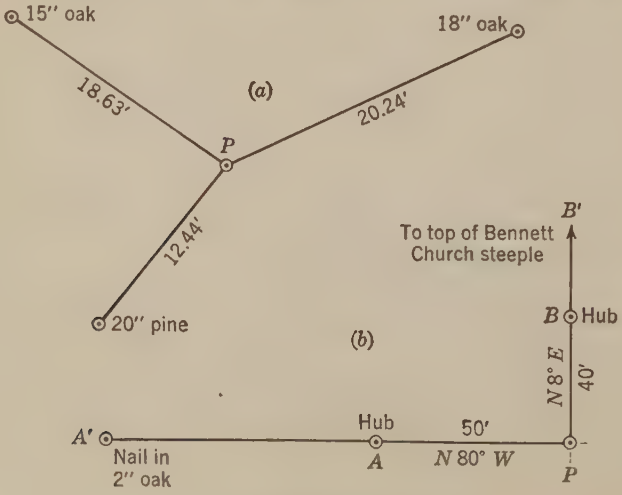


FIG. 3. Referencing point *P*.

less than 100 ft away which can be found later and which subtend angles at point P not too small for a precise intersection.

This method is simple and rapid, but it *should not be used in the case of final location when the points will fall within the limits of construction.*

Method B. Two "tacked" hubs (A and B in Fig. 3*b*) are set at convenient distances from point P, each exactly in line with distant points A' and B', which may be nails in trees or, preferably, distant objects, such as church steeples. These hubs should be placed where they will not be disturbed during construction, and the angle APB should approximate a right angle for the best intersection.

Point P can be relocated by setting the transit at points A and B, in turn, and finding the intersection of lines AA' and BB'.

15. Levels for Profile and Bench Marks. For the purpose of drawing a profile of the transit line, the level party takes readings at each station, at all breaks in the ground, and at critical points, such as road and stream intersections.

Bench marks should be established at intervals of about 1,000 ft and at, or near, all drainage or underpass structures. All elevations should, if feasible, be based on U.S. Geological or Coast and Geodetic Survey data. Check levels should be run over the entire line, and the usual allowable error in profile levels between consecutive bench marks is the square root of the distance (in miles) times 0.05 ft.

All rod readings at ground points are taken to the nearest 0.1 ft, while those on turning points and bench marks are taken to the nearest 0.01 ft. Table A shows the usual form of recording profile leveling notes.

16. Organization and Duties of Surveying Parties. The three surveying parties for either a preliminary or a location survey consist usually of (1) the *transit party* of four to seven men—chief of party, transitman, and two or more helpers, who are chainmen, stakemen, and axmen; (2) the *level party* of two or three men—levelman, rodman, and perhaps note keeper; and (3) the *topography* or *cross-section party* of three or four men—levelman, rodman, chainman, and recorder.

The *chief of party* must be experienced in general surveying

TABLE A. FORM OF LEVEL NOTES

Sta.	B.S.	H.I.	Rod	F.S.	Elev.	Remarks
B.M.	(+) 4.32	504.32		(−)	500.00	B.M. on W. Oak Stump 75′ L. Sta. 0 + 00.
0 + 00			1.3		503.0	
0 + 50			2.5		501.8	
1			6.7		497.6	
2			10.1		494.2	
2 + 10			12.01		492.31	
⊙	1.64	493.97		11.99	492.33	
3			2.0		492.0	
3 + 50			6.1		487.9	
4			11.4		482.6	
⊙	0.14	483.52		10.59	483.38	
5			0.5		483.0	Spring Branch
6 + 00 ⊙				2.31	481.21	
Totals..	+6.10			−24.89 + 6.10		500.00 481.21
Total difference			=	18.79	=	18.79

work and should have a good eye for choosing the most favorable ground. He should possess sound judgment, executive ability, and diplomacy. In general, his duties are to handle the reconnaissance, do the planning for the party, designate the duties of each man, and accept general responsibility for the whole procedure. As a rule, he works ahead of the party, determines the location of the center line, the angle points, character of the curves, and makes such notations as are necessary to furnish complete information for future use. He or a designated member of his party interviews property owners as to their land lines.

Having on hand the proper equipment and materials is an item that requires the constant attention of a chief of party. He should anticipate and estimate the needs of his party for the work to be done. In a typical state or county organization, he makes daily or weekly reports directly to the chief locating engineer under whose general direction the survey is made.

When working on private property, the chief of party should instruct his men to be particularly careful to avoid any property damage, such as the unnecessary cutting of trees, breaking fences, tramping down crops, flowers, and shrubs, failure to close gates, blocking driveways, or other careless actions that will annoy owners or cause damage.

The *transitman* reads and records all angles, bearings, and distances, and sees that the axmen and chainmen are in line. He should be able to make sketches and take field notes neatly, speedily, and in clear and concise form. It is his duty to see that the instrument is constantly in good adjustment. The transitman frequently acts as chief of party.

Unless a special topography party is organized for the purpose, the transitman should record neatly and clearly on the right-hand page any topography notes that may have influence in determining the final location, such as the position and bearing of streams, drainage structures, property lines, intersecting roads or railroads, and pipelines.

The *head chainman* carries a range rod and the front end of the tape, which should be held firmly with one hand while the rod is moved into line with the other. He should always put himself nearly in line before receiving a signal from the transitman; and he must be able to manipulate the tape and rod accurately and rapidly using plumb lines when advisable. He should exercise good judgment in selecting transit points and taking pluses. It is his duty to direct the axmen in clearing the line ahead and to see that the stakes are marked correctly. A proper understanding should be had with the transitman as to signals. The head chainman should be the most energetic man available for the job, as he regulates the speed of the party during much of the time.

The *rear chainman* holds his end of the tape over the point last set after seeing that the tape is taut and free of kinks. Sometimes it may be his duty to record in a notebook the pluses to fences, streams, property lines, and other features to be reported later. He takes care of the tape and is held responsible for its condition.

The *back rodman*, when signaled by the transitman, holds a range pole or plumb line over the backsight point for orientation. He then comes forward and brings whatever excess equipment may be on hand after having driven a stake at the backsight station.

The *stakeman* carries the stakes, which should be plainly labeled with the correct station number as directed by the head chainman. At each turning point, a tacked hub should be driven nearly flush with the ground; within about a foot of it there should be a sloping guard stake on which is written the correct station number.

The *axmen* are supposed to clear the line of brush and trees so that the transit and level parties can proceed without too much delay; but no trees should be destroyed that are actually not in the line of vision.

The *level party* includes three men: levelman, rodman, and notekeeper. If only two men do the work, the levelman records his data. Aside from running levels over the line, he establishes bench marks at about every 1,000 ft.

A typical set of level notes is shown in Table A (page 47). It should be seen that the difference in the sum of the backsights (plus) and front sights (minus) equals the difference in elevation of the turning points. The right-hand page (not shown) is for miscellaneous notes and descriptions of bench marks.

Some engineers prefer level notes running *up* the page, in order that any sketches or cross-section notes on the right-hand page may appear to be in a more natural sequence.

The *rodman* carries the rod and holds it vertical on the ground at all stations and at breaks along the profile. He should not overlook readings for the elevation of nearby streams, visible high-water marks, intersecting roads, etc. The rodman carries a small notebook in which he enters the readings on all turning points.

The *topographer* (see page 52) should be one of the most capable and experienced men in the party organization. He will have one or two assistants. Frequently the transitman or the levelman will do the work of the topographer.

The so-called *cross-section party* includes at least a *levelman* and a *rodman.* The instruments used are ordinarily a *hand level, rod,* and *tape.* If the ground is not very abrupt, an engineer's level is sometimes substituted for the hand level.

Table B. Simple Form of Cross-section Notes

Sta.	Elev.	Left	℄	Right
30	115.6	$\frac{+6.5}{28}$ $\frac{+4.6}{18}$ $\frac{+0.9}{10}$	$\frac{0}{0}$	$\frac{-0.5}{8}$ $\frac{-0.6}{16}$ $\frac{-1.2}{28}$
29	118.4	$\frac{+6.1}{26}$ $\frac{+4.1}{16}$ $\frac{+1.0}{12}$	$\frac{0}{0}$	$\frac{-0.4}{10}$ $\frac{-0.6}{18}$ $\frac{-1.0}{30}$
28	120.1	$\frac{+4.8}{30}$ $\frac{+2.4}{16}$ $\frac{+0.5}{10}$	$\frac{0}{0}$	$\frac{0.0}{12}$ $\frac{-1.0}{20}$ $\frac{-0.8}{35}$
27	120.8	$\frac{+2.8}{25}$ $\frac{+1.2}{12}$ $\frac{0.0}{9}$	$\frac{0}{0}$	$\frac{-2.9}{10}$ $\frac{-5.0}{15}$ $\frac{-5.0}{32}$
26	121.2	$\frac{+1.7}{32}$ $\frac{+1.2}{23}$ $\frac{+1.1}{15}$ $\frac{0.0}{8}$	$\frac{0}{0}$	$\frac{-3.1}{12}$ $\frac{-6.1}{16}$ $\frac{-5.7}{20}$ $\frac{-4.4}{30}$

A simple form of *cross-section notes* is shown in Table B. The results are given in the form of a fraction in which the numerator is the difference in elevation of point on the ground relative to the center-line station, and the denominator is the distance from the center line. Care should be taken to use the proper sign—*plus* to indicate points above and *minus* to indicate points below the line. Thus, observing that the notes run up the page, we see that at Sta. 28, a point 30 ft to the left of the center line is 4.8 ft *above* Sta. 28, and a point 35 ft to the right is 0.8 ft *below* this station. The intermediate points represent breaks in the ground.

Another form of cross-section notes applying to Sta. 27 (in Table B) is given in Table C. By this method, all rod readings on the cross profile are recorded, from which actual (not relative) elevations are obtained.

TABLE C. ANOTHER FORM OF CROSS-SECTION NOTES

Sta.	Elev.	H.I.	Left			℄	Right		
27	120.8	126.12	123.6 $\frac{2.5}{25}$	122.0 $\frac{4.1}{12}$	120.8 $\frac{5.3}{9}$	120.8 $\frac{5.3}{0}$	117.9 $\frac{8.2}{10}$	115.8 $\frac{10.3}{15}$	115.8 $\frac{10.3}{32}$

17. Contours. Contour sketches or maps may be determined by several methods: (1) *by means of level* (or *hand level*), *rod, and tape*, (2) *by plotting from slope or cross-section notes*, (3) *by means of transit* (or *plane table*) *and stadia*, (4) *by aerial methods.*

With the actual elevation of each station known (determined previously by the level party), the position of contours at right angles to the surveyed center line (or in any known direction) is fixed by ground measurements; between these points, the contours are sketched in by eye.

To illustrate, assume the "height of eye" (HI) of the level telescope to be 5.2 ft (as would be obtained by placing the rod in a vertical position alongside the level); then if the level is set over a station whose elevation is, say, 592.6 ft, his HI will be at elevation 597.8 ft. Hence to locate the 590-ft contour, the rodman should continue moving until the rod reads 7.8 ft; to locate the 595-ft contour, he moves until the rod reads 2.8 ft. It will therefore be seen that the rod reading for fixing a point on the contour is found by subtracting its elevation from that of the HI, and the rod is to be moved until this reading is obtained. The horizontal distance to this point is then measured and recorded as part of a set of notes running up the page in the form of fractions of which the numerator denotes the proper contour and the denominator is the distance from the center-line station.

The levelman may continue this operation in the same direction, usually perpendicular to the center line, by moving to an established point on the contour and proceeding with a new HI to locate the next contour.

A *hand level* is often used instead of the engineer's level with greater speed and with an accuracy within 0.1 ft. A 5-ft staff is used to support a Locke level. This procedure is like that in the above example except that the height of eye is fixed at, say, 5 ft instead of 5.2 ft, or whatever the height is after the setup.

The contour interval is usually 5 ft, but it may be *more* for steep slopes or *less* for rather flat terrain. In the latter case, with comparatively long sights, the engineer's level would be preferable.

The chief characteristics of contours, illustrated in Fig. 4, may be summarized as follows:*

1. All points on any one contour have the same elevation, as at *A*.
2. Every contour closes on itself, either within or beyond the limits of the map. If the contour does not close within the limits of the map, it will run to the edge of the map, as at *B*.
3. A contour that closes within the limits of the map indicates either a summit or a depression. In depressions there will usually be found a pond or a lake; but where there is no water the contours are usually marked in some way to indicate a depression, as at *C*.
4. Contours can never cross each other except where there is an overhanging cliff, and here there must be two intersections, as at *D*. Such cases seldom occur.
5. On a uniform slope, contours are spaced equally, as at *E*. The steeper the slope, the smaller is the spacing; the flatter the slope, the larger is the spacing.
6. On a plane surface they are straight and parallel to each other, as at *F*.
7. In crossing a valley, the contours run up the valley on one side and, turning at the stream, run back on the other side, as at *G*. Since the contours are always at right angles to the lines of steepest slopes, they are at right angles to the thread of the stream at the point of crossing.

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* T. Abrams, "Essentials of Aerial Surveying and Photo Interpretation," McGraw-Hill Book Company, Inc., New York, 1944.

8. Contours cross the ridge lines (watersheds) at right angles, as at *H*.

9. In general, the curve of the contour in a valley is convex toward the stream.

10. All contours are multiples of their intervals, and every fifth one is usually made heavier.

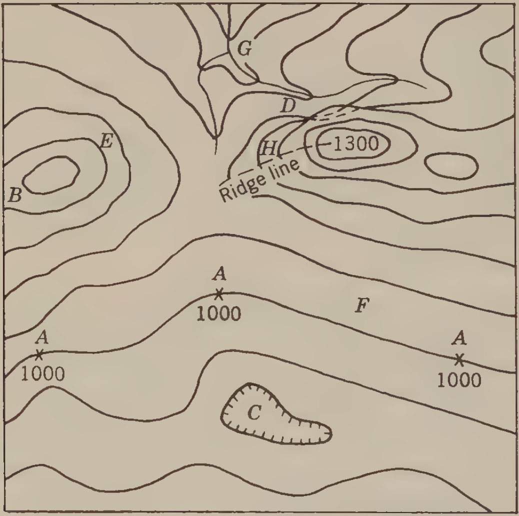

FIG. 4. Contours.

18. Topography. The word "topography" is ordinarily understood to include not only the physical features of the terrain but also the various objects and obstructions which must be considered in connection with the project, such as other intersecting routes, property lines, pole lines, buildings, trees, bridges, and culverts.

The simplest method of locating a tree, corner of a building, or any point near the reference line is by *pluses* and *perpendicular offsets*. When this method is not feasible, a point may be located by angles and distances, or by angles from each of two stations.

The topography notes and the accompanying sketches are sometimes recorded on the right-hand page of field books. A better method, when there are many data, is to plot the topography in the field on 19- by 25-in. sheets of cross-section paper mounted on a topographer's sketch board.

LOCATION SURVEYS

19. Scope. The location survey consists in laying out on the ground the final center line which has been projected on paper or otherwise predetermined. The tangents, curves, and drainage structures are established by means of a continuous transit survey taking into account whatever adjustments in the line and grade appear to be advisable. All topography together with property corners included in the right of way is measured from the surveyed center line as the staking of the line progresses. In taking the ground-surface cross sections of rolling country, it is good practice to put them directly on tracing sheets which provide scales of 1 in. = 10 ft horizontally and vertically. For flatter country, a scale of 1 in. = 5 ft is preferred.

A coordinate system is advantageous as a control in stakeout operations. Such a system often starts with the preliminary ground survey. Where the through highway is not on a state coordinate system, a local system can be established by arbitrarily assigning a grid and coordinates to one selected control point and computing the coordinates of all other points along the project.

Using modern methods, excavation quantities are calculated by an electronic computer which combines terrain notes picked off the contour maps with roadbed notes tabulated by the designer.

20. Field Notes. The field notes must be clear and concise and yet comprehensive enough for making accurate and complete plans of the project.

It is important that the levelman check every item of his notes and that check levels are run over the entire line. Slight deviations, within permissible limits, may be adjusted over the line.

The transit notes are kept in special field notebooks. The left page, seen in Fig. 5, is used for recording stations, angles, curve deflections, curve data, and calculated bearings; the right page (not shown) is used for topography, sketches, details of references for hubs, location of all property lines crossing or touching the right of way, names of property owners, and anything which will affect the cost of construction.

An accuracy in the measurement of distances of 1 in 5,000 is desirable, but rarely realized by ordinary chaining. Double-reversal methods should be used at angle points and at intermediate transit points on long tangents. The calculated bearings should be based on sun or star observations, and "mean sea level" as determined by the U.S. Coast and Geodetic Survey should be used as a datum for leveling if possible. In this manner, the highway or railroad location survey notes, if properly tied in with prominent marks of the terrain, the state coordinate system, and the national network of geodetic control, will supply data for permanent maps of the country.

TRANSIT NOTES

Point	Station	Deflections	Magnetic bearings Curve data	Calculated bearings
P.O.T.	100	0° 00.0′	*N* 42° 00′ *E*	*N* 42° 04′ *E*
○ S.T.	93+14.33	Etc.	Spiraled curve data: $\Delta = 49° 36'$, $l_s = 500$, $\theta_s = 12°$, $D_c = 4.8° = 4° 48'$, $T_s = 805.22$, $E_s = 130.9$, $\Delta - 2\theta_s = 25° 36'$, $L_c = 533.33$	
⊙ C.S.	88+14.33			
○ S.C.	82+81.0	3° 59.9′		
	82	2° 48.5′		
	81	1° 37.7′		
	80	0° 46.0′		
	79	0° 13.6′		
	78	0° 00.4′		
○ T.S.	77+81.00	0° 00.0′		
○ P.I.	88+86.22	49° 36.0′ *R*	*N* 42° 00′ *E*	*N* 42° 04′ *E*
P.O.T.	75+00.0	0° 00.0′	*N* 7° 30′ *W*	*N* 7° 32′ *W*
○ P.T.	65+72.49	6° 37.5′	Circular curve data: $\Delta = 13° 15.0'$ $D = 1° 30'$, $R = 3819.72$, $T = 443.64$, $E = 25.68$, $L = 883.33$	
	65	6° 05.0′		
	64	5° 20.0′		
	63	4° 35.0′		
	62	3° 50.0′		
	61	3° 05.0′		
	60	2° 20.0′		
	59	1° 35.0′		
	58	0° 50.0′		
	57	0° 05.0′		
⊙ P.C.	56+89.16	0° 00.0′		
○ P.I.	61+32.80	13° 15.0′ *L*	*N* 7° 30′ *W*	*N* 7° 32′ *W*
P.O.T.	50+00.0	0° 00.0′	*N* 5° 30′ *E*	*N* 5° 43′ *E*

Fig. 5

The extent to which this ideal is reached depends largely on the availability of triangulation stations and bench marks and on their use.

Transit points (on hubs) should be referenced so that they may be relocated readily. In addition to the precise referencing of intersection points, the beginning and ending of all curves, and points on long tangents, the place of beginning and ending of a survey should be referenced generally as to location, by giving the approximate distances to nearby towns or other well-known local points.

If the center-line points come on an existing road, a parallel offset line should be established for guidance during construction.

On the front page of a notebook, opposite the flyleaf, should appear the name and number of the survey. Certain information of a very general nature will be given here, such as a description of the survey, the names of the members of the party, the notebooks used, the dates, and the initials of the field computation checkers.

The indexing of field books should be kept up as each survey is completed, and all notes should be intelligently and clearly entered, so that the same may be readily worked up by others who are not familiar with the survey.

Figure 6 shows a typical notebook complying with the standards of the New Jersey Highway Department.

21. Equalities. Equalities or equations in station numbers should be avoided as much as possible. When they are absolutely necessary, they should be carefully explained on the flyleaf, so that a total stranger can understand them. Thus, Sta. 374 + 12.5 of this line = Sta. 342 + 16.4 of the line ahead; if lines are designated by letters, Sta. 374 + 12.5 of line A = Sta. 342 + 16.4 of line B, ahead.

It is even more important to avoid equalities in level datum. For this reason it is desirable, whenever practicable, to base all levels on sea-level datum, as obtained from government bench marks. When this is impracticable, a datum may be assumed, but it must be adhered to throughout the survey.

In tying one line into another, the new line should be run for at least two courses over the old one, and the equality of stationing noted at each hub in the field book and otherwise explained, so that anyone can understand or plot the work. These equalities should also be marked on stakes securely driven near each hub.

22. Soil and Rock Sounding. In the case of new construction, notes should be made of the type of materials at various points along the survey, as indicated by soundings or borings, for the purpose of estimating a preliminary classification of the relative quantities of earth, loose rock, hardpan, or rock to be encountered. In many of the organized highway departments, this work is done in cooperation with geologists and the soil examination laboratory.

Available materials for construction, stone quarries, and gravel pits, with distances and character of road to be traveled over, should be noted. Often a pit or quarry unknown to county authorities or division engineers is located by use of aerial photographs of the region.

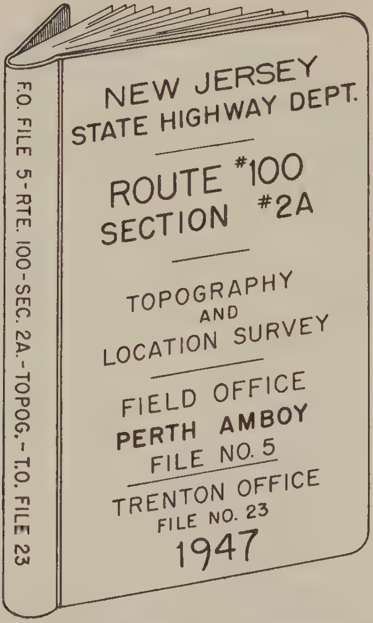
F.O. FILE 5-RTE. 100-SEC. 2A.-TOPOG.-T.O. FILE 23
NEW JERSEY
STATE HIGHWAY DEPT.
ROUTE #100
SECTION #2A
TOPOGRAPHY
AND
LOCATION SURVEY
FIELD OFFICE
PERTH AMBOY
FILE NO. 5
TRENTON OFFICE
FILE NO. 23
1947

FIG. 6. Sample of information to be shown on field books.

23. Topography and Right-of-way Data. The following items are considered in preparing a right-of-way plan: location of boundary lines, full names of property owners, classification of lands through which the survey passes, area of land cut off, location of water affecting pasture rights, stations and pluses of property lines, area (in acres) of the right of way to be purchased, legal description of the same, ties to at least one monument, and net length of center line across the property.

Where trees, buildings, telephone poles, wells, and other features of similar nature are within the right-of-way lines and their removal or changing affects property settlements, these items should be carefully shown on the plan to their true location and dimensions.

It is advisable to check the name of the property owner in the county records and to investigate the titles and also to fill out the tax-valuation section of the appraisal report from the county records. This service is usually rendered by the legal or right-of-way division of the organization.

24. Cross Sections. Cross sections should be taken at each 100-ft station, at all breaks in the center-line profile, and oftener if necessary to determine more accurately the volumes of excavation and embankment. Cross sections should extend for a distance on either side of the center line to allow the road template to be superimposed upon them, whether the grade of the road at that point involves cut, fill, widening, superelevating, or shifting of the center line.

Cross sections should also be taken at each culvert, and the distance out should be extended in case a relocation or cleaning out of the runoff channel is contemplated.

There is a tendency to take too few cross sections, particularly in rough country where rapid changes occur from cut to fill. The Rhodes Stadia Arc has been used to advantage in cross sectioning.

When there is a considerable time interval between the surveys for design and for actual construction, it is advisable to cross-section anew while setting slope stakes. Later, upon the completion of the grading operations, a final section taken directly over the location of the cross section prior to construction will give the yardage which should be calculated before the final payments are made (if the work is done by contract).

A complete record should be kept of any changes of grade and alignment and of any authorized additions or deductions of yardage during the progress of the work.

Borrow pits, channel changes, etc., must be carefully cross-sectioned before the contractor begins work on them. Upon completion of the work, the pits should be cross-sectioned again for final determination of yardage.

Note: With photogrammetry and electronic equipment most, if not all, of these procedures for cross sections can be discarded.

25. Grade Lines. The grade lines should be established with a view to securing a profile with long, flat tangents joined by long, vertical curves providing good visibility and smooth riding. To fit the ground and expedite drainage, the changes in grade from plus to minus would occur in cuts, while the changes from minus to plus would occur in fills. Trial grade lines may be drawn in pencil until the final line is established. If a thread is held taut between desirable control points, one may study the effect of shifting grades without drawing pencil lines.

If the established gradient is such that the balancing of cuts and fills can be accomplished without exceeding hauls of around 5,000 ft, and if the requirements of drainage, visibility, and general appearances are satisfactory, then it is fairly certain that the balancing will be economical. Ordinarily, the volumes of cuts should exceed those of fills, from 10 to 20 percent, because of shrinkage and other losses. For excavations involving spongy soils, heavy sod, or vegetation, a greater allowance should be made.

Information as to the unit cost of handling excavated material is necessary in order to determine whether balancing, wasting, or borrowing is most economical. Wasted materials can often be used to widen fills.

Level grades, or a change from a steeper to a flatter grade, should be avoided in cuts to ensure necessary drainage. Vertical curves of considerable length should be introduced where the total change of grade exceeds $\frac{1}{2}$ to 1 percent. Short choppy grades should in all cases be avoided.

The grade line is often fixed by requirements of greater importance than that of balancing cuts and fills. Thus:

1. In rockwork (if classified) it is generally cheaper to have the embankment quantities in excess, where earth, at a cheaper rate, may be borrowed.
2. It may be cheaper to borrow rather than excavate the equivalent roadway material that is relatively more valuable.
3. In towns the highway grade may be adjusted for street intersections or lowered to receive drainage from the abutting property; in flat, swampy country it may be raised so that the fills are in excess, in order to give better foundation and drainage.
4. It may be necessary to continue a maximum rate of grade in order to reach a summit.
5. The grades of approaches to bridges, underpasses, or intersecting roads must conform to these structures.
6. The grade along sidehill construction may need adjustment to avoid areas where landslides are likely.

26. Construction Stakes. When actual construction of the facility is about to begin, it is the engineer's duty to see that a sufficient number of substantial construction stakes are set correctly for the guidance of the builder or contractor. Further details are presented in Chap. 12.

27. Special Surveys. Ground surveys for minor improvements, such as the partial reconstruction or resurfacing of a highway, may come under this classification. Special surveys also include land surveys for (1) right-of-way descriptions; (2) grade-crossing eliminations (allowing a vertical clearance under structures of at least 14 ft above the traffic lanes of highways and 22 ft above the top rail of railroads); (3) site surveys to obtain information needed for the design of large culverts, bridges, interchanges, and complex intersections at grade; and (4) subsoil studies.

28. Explanation. Much of the work in connection with highway planning is usually done on paper. This calls for maps of the area upon which the proposed route is tentatively projected before being finally established on the ground.

Then on the basis of field surveys, data are available for preparing a detailed set of plans (or maps) including title sheet, typical pavement cross sections, plans and profiles, cross sections of excavation and embankment, structural plans, summary sheets, and such departmental standards as are necessary for the project.

All plans in accordance with the standards of the U.S. Bureau of Public Roads are on sheets measuring 22 by 36 in. with a binding margin 2 in. wide on the left-hand edge.

29. Scales. The plan is usually drawn to a scale of 1 in. = 100 ft, while the profile of the center line has a scale of 1 in. = 100 ft horizontally and 1 in. = 10 ft vertically. The yardage cross sections are drawn to a scale of 1 in. = 5 ft (or 1 in. = 10 ft) both horizontally and vertically. Variations from these scales occur in special cases, depending on the topography, type of work, and the data required.

30. Typical Sections. A typical cross section of the pavement should be placed on the first sheet directly following the title page. This section, on a suitable scale, should show the various slopes for cut and fill, the crown of the finished pavement, the width of the shoulders, the thickness of the various courses of pavement or resurfacing, the position of the profile grade line, the center line of pavement, and the limits of the right of way.

Widening and superelevation tables must be made for each curve and should be shown on special sheets immediately before the yardage cross-section sheets.

Figure 7 shows a typical cross section with conventional terms applied to two-lane highways.

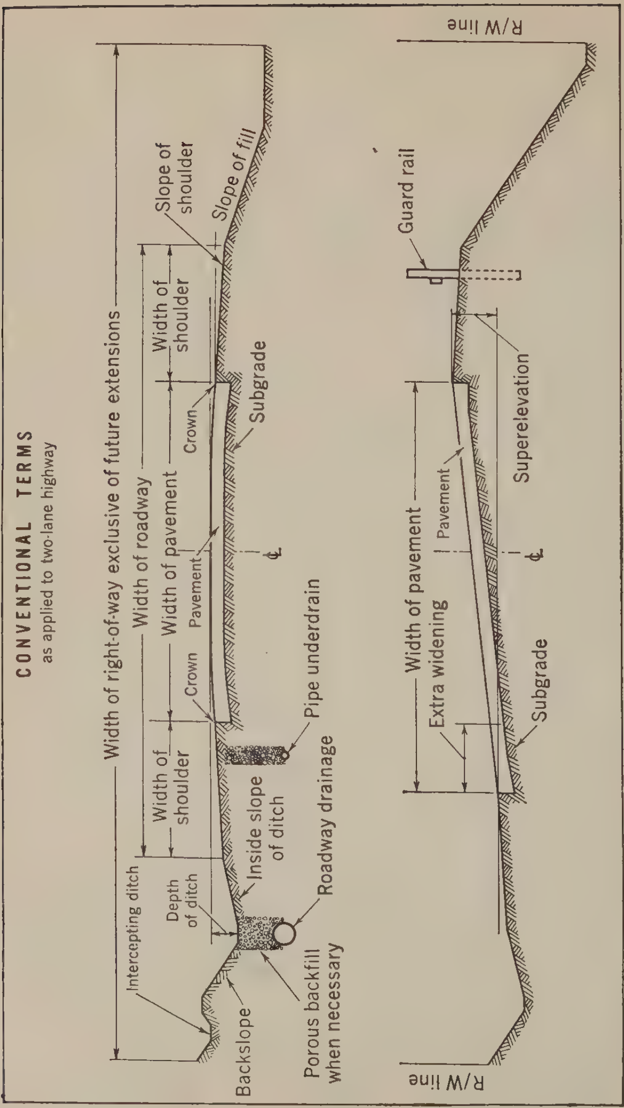
CONVENTIONAL TERMS
as applied to two-lane highway
Width of right-of-way exclusive of future extensions
Width of roadway
Width of pavement
Width of shoulder
Width of shoulder
Slope of shoulder
Slope of fill
Crown
Crown
Pavement
Subgrade
℄
Pipe underdrain
Intercepting ditch
Depth of ditch
Inside slope of ditch
Roadway drainage
Backslope
Porous backfill when necessary
R/W line
R/W line
Guard rail
Superelevation
Width of pavement
Pavement
℄
Extra widening
Subgrade

Fig. 7

31. Data on Plan. The plan of the road (see Fig. 8) shows the center line of construction; bearing of tangents; angles of intersection; station of beginning and ending of curves; full data for each curve including intersection angle; equalities or equations of stationing (if any); streams and railroads on or within 500 ft of the center line; property lines; names of property owners; intersecting roads; location, size, and purpose of all pipelines; location, size, and direction of flow of present and proposed culverts and bridges; structures affected by the proposed construction; the location of all unusual and special features, such as guard rails, retaining walls, ditch protection, manholes, catch basins, inlets, and connections.

The plan sheets should show the balance points of yardage, together with a subsummary of the same between balance points. These plans should also include excavation, embankment, waste, and borrow.

For minor changes and for locations requiring new or additional rights of way, the margins of the existing road should be shown by broken lines. Where major relocations of roadway are made, the old road should be shown for at least 500 ft on either side of the center line of the proposed new road. Plans should be platted with the stationing from left to right. A north point is placed upon each sheet.

Each set of plans shows the alignment and profile of the existing road for such distance beyond each end of the project as shall be necessary to avoid any future changes in alignment or grade in the project.

32. Data on Profile. All elevations should be based on sea-level datum carried from the nearest U.S. Geological Survey bench mark. The profile should show datum lines; the profile of the surface of the proposed center line; tops of rails of all grade crossings; elevations of all inlets and outlets of existing culverts; flow lines of all tile drains; profile of intersecting streams; low- and high-water marks, with data of same; elevations of existing bridge footings, seats, and floors; pipelines and conduits; driveways; profile of intersecting roads for at least 500 ft on each side of the center line; ground elevations at all buildings that may affect the grade of the improvements; clearance of all overhead structures and cables above grade; the proposed grade line, together with the percentage of all grades and the location and elevation of all intersections thereof; the length of vertical curves; the elevation of footings for new structures; the elevation and complete description of all bench marks.

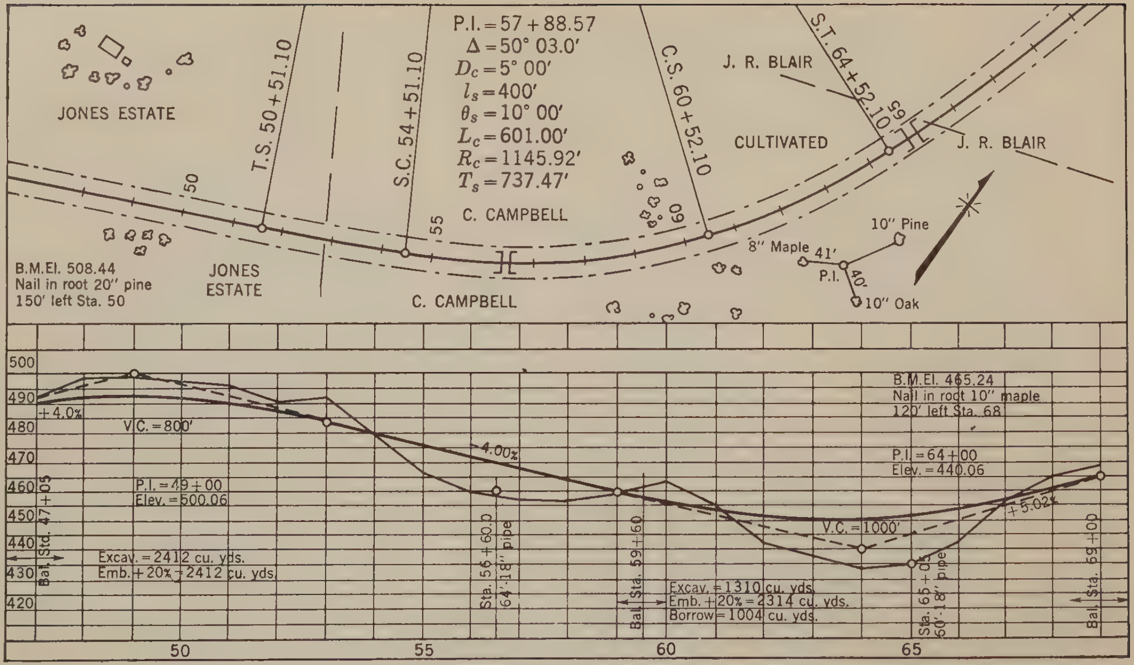
JONES ESTATE
T.S. 50+51.10
S.C. 54+51.10
P.I. = 57+88.57
Δ = 50° 03.0′
D_c = 5° 00′
l_s = 400′
θ_s = 10° 00′
L_c = 601.00′
R_c = 1145.92′
T_s = 737.47′
C.S. 60+52.10
J. R. BLAIR
S.T. 64+52.10
65
CULTIVATED
J. R. BLAIR
50
55
60
C. CAMPBELL
10″ Pine
8″ Maple 41′
40′
P.I.
10″ Oak
B.M.El. 508.44
Nail in root 20″ pine
150′ left Sta. 50
JONES ESTATE
C. CAMPBELL
500
490
480
470
460
450
440
430
420
+4.0%
V.C. = 800′
P.I. = 49+00
Elev. = 500.06
Bal. Sta. 47+05
Excav. = 2412 cu. yds.
Emb. + 20% = 2412 cu. yds.
−4.00%
Sta. 56+60.0
64′-18″ pipe
Bal. Sta. 59+60
Excav. = 1310 cu. yds.
Emb. + 20% = 2314 cu. yds.
Borrow = 1004 cu. yds.
B.M.El. 465.24
Nail in root 10″ maple
120′ left Sta. 68
P.I. = 64+00
Elev. = 440.06
V.C. = 1000′
+5.02%
Sta. 65+05
60′-18″ pipe
Bal. Sta. 69+00
50
55
60
65

Fig. 8. Plan and profile for a highway.

State Route 17, Connecticut, showing 20-ft median and 8-ft gravel shoulders. The new planting which will prevent headlight glare.

References

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2. Whitmore, George D., and Morris M. Thompson: "Abbett's American Civil Engineering Practice," vol. 1, John Wiley & Sons, Inc., New York, 1956.
3. Manual of Photogrammetry, American Society of Photogrammetry, Washington, D.C., 1952.
4. California Practice, California Department of Public Works, Sacramento, Calif., 1965.
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CHAPTER 3

CIRCULAR CURVES

Part I: General Topics

33. Classification. The surveyed center line of roads consists of a series of *straight lines* and *curves*. Technically, the straight lines are called *tangents*, and a curve uniting two intersecting tangents is known by its *radius* or by the angle subtended at the center of an *arc* (or *chord*) *of* 100 *ft*.

Circular curves are classified as *simple*, *compound*, and *reverse*.

34. Functions or Parts of Curves. A *simple curve* is a circular arc joining two tangents. Figure 9 shows such a curve with all its related parts, or functions, which will now be explained.

1. If the tangents be produced, they will meet at a *point of intersection*, called PI (or vertex, V).

2. Proceeding from left to right around the curve, point A, the *beginning* of the curve is called the PC (*point of curvature*), while point B, the *end* of the curve, is called the PT (*point of tangency*).

3. The external angle of deflection between the tangents is called the *intersection angle*, or the *angle*. This angle (Δ) is equal to the central angle subtended by the arc AB.

4. In Fig. 9, the distance VA ($= VB$) from the PI to the PC (or to the PT) is called the *tangent distance* T. Then

$$T = R \tan \tfrac{1}{2}\Delta. \tag{1}$$

5. The straight-line distance AB from the beginning to the end of the curve is called the *long chord*, or LC. From Fig. 9,

$$\sin \tfrac{1}{2}\Delta = \frac{AD}{OA} = \frac{\frac{1}{2}(\text{LC})}{R}$$

from which

$$\text{LC} = 2R \sin \tfrac{1}{2}\Delta. \tag{2}$$

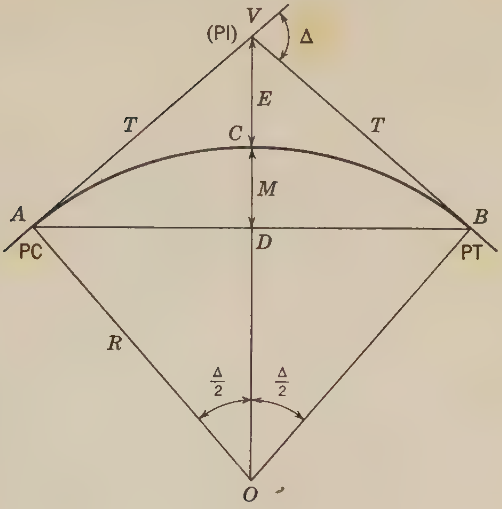

FIG. 9. Parts of a circular curve.

6. The *external distance* E is the distance from the PI to the middle of the curve. From Fig. 9,

$$\cos \tfrac{1}{2}\Delta = \frac{OA}{OV} = \frac{R}{R + E},$$

from which $$E = R(\sec \tfrac{1}{2}\Delta - 1) = R \operatorname{exsec} \tfrac{1}{2}\Delta. \qquad (3)$$

Combining Eqs. (4) and (6), it follows that

$$E = T \tan \tfrac{1}{4}\Delta. \qquad (4)$$

7. The *middle ordinate* M is the length of the ordinate from the middle of the long chord to the middle of the curve. From Fig. 9,

$$\cos \tfrac{1}{2}\Delta = \frac{OD}{OA} = \frac{R - M}{R},$$

from which $$M = R(1 - \cos \tfrac{1}{2}\Delta) = R \operatorname{vers} \tfrac{1}{2}\Delta. \qquad (5)$$

8. The curvature of a circular arc is fixed by its *radius* or by the *degree of curve.*

9. *Degree of curve D* is defined as (*a*) the *central angle which subtends a* 100-*ft arc* (Fig. 10) or (*b*) the *central angle which subtends a* 100-*ft chord* (Fig. 11).

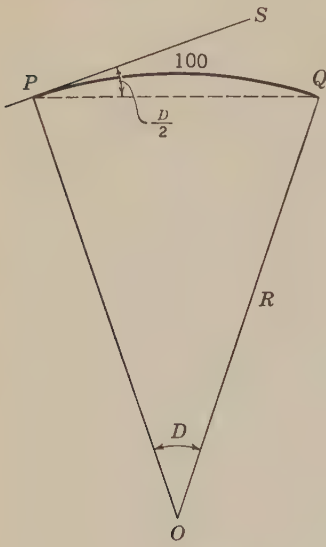

FIG. 10. Arc definition.

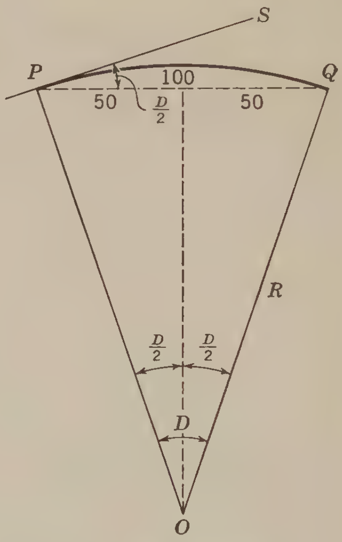

FIG. 11. Chord definition.

a. From Fig. 10 (*arc* definition)

$$R = \frac{100}{D}\ (D \text{ in radians}) = \frac{5{,}729.578}{D}\ (D \text{ in degrees})$$

or from the proportion (Fig. 10),

$$100:2\pi R = D^\circ:360^\circ,$$

$$R = \frac{5{,}729.578}{D}\ (\textit{arc}\ \text{definition}), \tag{6}$$

showing that R varies inversely as D.

Letting $D = 1^\circ$, it is seen that the radius of a 1° curve as defined by the *arc* definition is 5,729.578 ft.

b. In Fig. 11, we have $\sin \frac{1}{2}D = 50/R$, from which

$$R = \frac{50}{\sin \frac{1}{2}D} \text{ (\textit{chord} definition).} \tag{7}$$

If $D = 1°$, Eq. (7) gives $R = 5{,}729.648$ ft; that is, the radius of a 1° curve as defined by the *chord* definition is 5,729.648 ft.

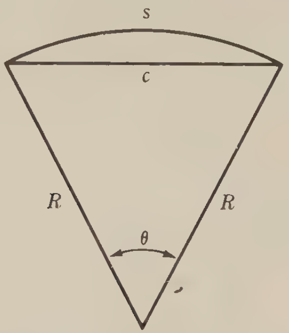

FIG. 12

From either Eq. (6) or (7), we have, approximately,

$$R = \frac{5{,}730}{D} \qquad \text{or} \qquad D = \frac{5{,}730}{R}. \tag{8}$$

Table 5 gives the radii R for various values of D according to either the *arc* or the *chord* definition. It is seen that the difference is small for curves of small degree.

10. *Chord lengths* c are always less than the corresponding arc lengths s (Fig. 12). Using the appropriate value of R, we have, by either the *arc* or *chord* definition,

$$\textit{True length} \text{ of chord} = 2R \sin \tfrac{1}{2}\theta. \tag{9}$$

From Figs. (10) and (12), we have the proportion

$$\frac{\theta}{D} = \frac{s}{100} \qquad \text{or} \qquad \theta = \frac{s}{100} D;$$

hence

$$c \text{ (\textit{arc} definition)} = 2R \sin \left(\frac{1}{2} \frac{s}{100} D\right). \tag{10}$$

By means of Eq. (10), Table 7 gives chord lengths for various arcs.

The *nominal chord length* (*chord* definition) of a chord (or subchord) is 100 θ/D. This is an arbitrary definition without theoretical basis, since chords are not exactly proportional to the corresponding central angles, but it has long been used in railroad practice. For example, a *nominal* subchord of 60 ft will subtend an angle of $\frac{60}{100}D$; its true length will be slightly more that 60 ft, and may be computed by Eq. (9). With a 10° curve and a nominal chord length of 50 ft, the true length is 50.05 ft (see Table 10).

11. *Arc lengths* s in terms of chord lengths c (*arc* definition) may be found from Eq. (10). Thus $c = 2R \sin [\frac{1}{2}(s/100)D]$; $\sin [\frac{1}{2}(s/100)D] = c/2R$; $\frac{1}{2}s/100\ D = \text{arc} \sin (c/2R)$, where $R = 5{,}729.578/D$. Hence

$$s = \frac{200}{D} \text{arc} \sin \frac{c}{2R} = \frac{200}{D} \text{arc} \sin \left(\frac{cD}{11{,}459.156}\right). \qquad (11)$$

Table 9 based on Eq. (11) gives arcs for various chord lengths; thus if $D = 50°$ and $c = 60$ ft, $s = 60.71$ ft.

12. *Arc lengths* s in terms of *chord lengths* c (*chord* definition) may be found as follows. From Figs. 12 and 11, we have

$$c = 2R \sin \tfrac{1}{2}\theta; \qquad (a)$$

$$100 = 2R \sin \tfrac{1}{2}D. \qquad (b)$$

Dividing Eq. (a) by Eq. (b) and reducing, we get

$$c = 100 \frac{\sin \frac{1}{2}\theta}{\sin \frac{1}{2}D}. \qquad (c)$$

Then $\sin \frac{1}{2}\theta = \dfrac{c \sin \frac{1}{2}D}{100}$; $\quad \theta = 2\left[\text{arc} \sin \left(\dfrac{c}{100} \sin \dfrac{1}{2} D\right)\right]$. $\qquad (d)$

Since $s = R\theta \times 0.0174532925$, where θ is measured in radians, Eq. (d) becomes

$$s = 2R(0.0174532925) \text{ arc} \sin \left(\frac{c}{100} \sin \frac{1}{2} D\right). \qquad (e)$$

Then substituting $R = 50/\sin \frac{1}{2}D$ in Eq. (*e*), we get

$$s = \frac{1.74532925}{\sin \frac{1}{2}D} \text{ arc sin } \left(\frac{c}{100} \sin \frac{1}{2} D\right). \tag{12}$$

Table 11 is based on Eq. (12); if $D = 30°$ and $c = 80$ ft, $s = 80.58$ ft.

13. The *length of curve L* (*arc* definition) is the distance around the curve from A (the PC) to B (the PT) as shown in Fig. 9. Since any two arcs of the same circle are proportional to the opposite central angles, we have $L/100 = \Delta/D$, from which

$$L = 100 \frac{\Delta}{D}. \tag{13}$$

Equation (13) is used also for the *chord* definition, but it implies the *total length* as if measured along the 100-ft chords of an inscribed polygon. If the curve begins or ends with a subchord, true subchord lengths (not nominal) should be used.

Example. Given $\Delta = 27°24'$, $D = 3°42'$, to find the *length of curve*.

Solution. By Eq. (13), $L = 100 \dfrac{27.4}{3.7} = 740.540$ ft.

The curve has seven 100-ft chords and a nominal chord of 40.54 ft. The true chord should be 40.55 ft.

By Table 13, the actual arc is $R\Delta$ (Δ in radians) $= 50/\sin 1°51'$ $(0.4782202) = 740.669$ ft. The excess is therefore $740.669 - 740.540 = 0.129$ ft.

14. *Arc excess* representing the difference between an arc and the subtended chord is formulated in Appendix A. It follows that

$$\textit{Arc excess (arc definition)} = \left(\frac{s}{100}\right)^3 \left[0.127 \left(\frac{D}{10}\right)^2\right] \text{(approx.)}. \tag{14}$$

If $s = 50$ ft and $D = 30°$, Eq. (14) gives

$$\textit{Arc excess} = 0.125[0.127\ (9)] = 0.143 \text{ ft.}$$

Hence the chord length for a 50-ft arc

$$(D = 30°) = 50.00 - 0.14 = 49.86 \text{ ft},$$

which checks with Table 7.

It will be seen that Eq. (14) is true to the nearest hundredth of a foot for 50-ft arcs up to $D = 64°$ and for 100-ft arcs up to $D = 30°$, as shown in Table 7.

For the *chord* definition,

$$\textit{Arc excess} = 0.127 \left(\frac{D}{10}\right)^2 \text{ (approx.)}. \qquad (15)$$

Equation (15) is true to the nearest hundredth of a foot for 100-ft arcs up to $D = 15°$, as shown in Table 11.

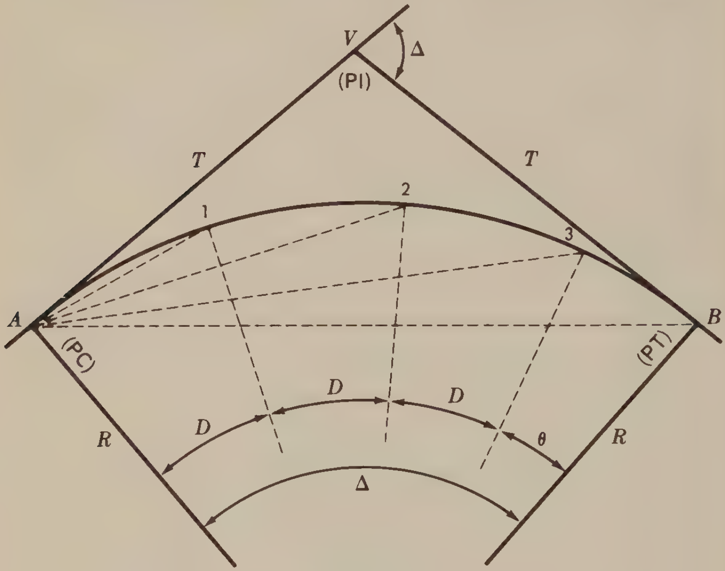

FIG. 13

35. Elements of Curve Layout. The angle between a secant and a tangent, or between two secants intersecting an arc, is measured by one-half the intercepted arc. Thus, in Fig. 13, the angle $VA1$ is one-half the arc $AO1$; the angle $VA2$ is one-half the arc $AO2$; etc.

Beginning at point A (the PC), if the first chord $A1$ is a full-length chord, we may deflect $VA1$ $(= \frac{1}{2}D)$, and point 1 is on the curve, the arc $A1$ being 100 ft (*arc* definition) or the chord $A1$ being 100 ft (*chord* definition). For Sta. 2, deflect an *additional* angle $A12$ ($= \frac{1}{2}D$), and with one end of the tape at point 1, swing the other end until the 100-ft *arc* 12 or *chord* 12 (point 2) is on the line $A2$. Point 2 is then on the curve, etc.

If the final chord $3B$ (see Fig. 13) is a subchord, the *additional deflection* ($= \frac{1}{2}\theta$) is less than $\frac{1}{2}D$. The last deflection (BAV) is $\frac{1}{2}\Delta$.

It is important in all cases, and particularly when a curve begins or ends with a subchord and the deflections are odd quantities, that the last additional deflection should be carefully computed and added to the previous deflection to see that the total equals $\frac{1}{2}\Delta$. If point B has been previously established, by an independent measurement along the forward tangent VB, the line AB should cross point B; if not, due to possible cumulative errors in the curve layout, point B should govern the alignment as the PT of the curve.

Example 1. Given $\Delta = 8°$ and $D = 2°12'$. Assuming Sta. 42 to be at the PC of the curve, it is required to find the length of curve L and the curve data, on the basis of the *arc* definition. Note: *unless stated otherwise, all examples will be presented as per arc definition.*

Solution. $L = 100\,\Delta/D = 100\left(\dfrac{8}{2.2}\right) = 363.636$ ft.

By Table 2, the tangents AV and VB for $\Delta = 8°$ and $D = 2°12'$ equal T for 1° curve$/D = \dfrac{400.651}{2.2} = 182.114$ ft. As a check,

$$T = R \tan \tfrac{1}{2}\Delta = \frac{5729.578}{2.2}\,(0.0699268) = 182.114 \text{ ft.}$$

The stationing of point V (the PI) may now be determined. Thus

$$\begin{array}{rl} \text{PC} = & 42 + 00 \\ T = & 1 + 82.114 \\ \hline \text{PI} = & 43 + 82.114 \end{array}$$

The curve begins with three 100-ft arcs (each subtended by a chord of 99.99 ft) and ends with an arc of 63.64 ft (subtended by a chord of 63.64 ft, as indicated in Table 7).

Beginning at Sta. 42 + 00, the deflection is $\frac{1}{2}D = 1°06'$ to Sta. 43. For each extra 100 ft the additional deflection is 1°06′. The final additional deflection for the subchord of 63.636 ft is $\frac{63.636}{100}(1°06') = 0.63636(1.1°) = 0.7000° = 0°42'$. As a check, $3 \times (1°06') + 0°42' = 4°(= \frac{1}{2}\Delta)$. The deflections are

PC Sta. $42 + 00 \qquad = 0°00'$
$43 \qquad 0° + 1°06' = 1°06'$
$44 \qquad 1°06' + 1°06' = 2°12'$
$45 \qquad 2°12' + 1°06' = 3°18'$
PT $45 + 63.636 \quad 3°18' + 0°42' = 4°00'$.

Note: The last deflection should be $\frac{1}{2}\Delta = \frac{1}{2}(8°) = 4°00'$.

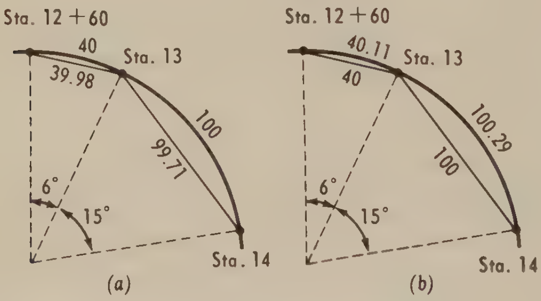

FIG. 14. Chords, subchords, etc., for both definitions of D (= 15°). (*a*) Arc definition. $R = 381.97$. (*b*) Chord definition. $R = 383.06$.

Example 2. Given $D = 15°$, PC at Sta. $12 + 60$. Required: deflection angle and subchord from PC to Sta. 13 and Sta. 14, on the basis of both definitions of the degree of curve.

Solution (see Fig. 14).

a. Arc Definition. The deflection to Sta. 13 for a 40-ft arc is

$$\frac{40}{100} \times \frac{D}{2} = 3°00'.$$

By Table 7 or Eq. (9), the chord for a 40-ft arc is 39.98 ft; the chord for a 100-ft arc is 99.71 ft.

b. Chord Definition. The deflection to Sta. 13 for 0.4 of a station (or nominal chord of 40 ft) is $\frac{40}{100} \times D/2 = 3°00'$. The nominal chord to Sta. 13 is $\frac{6}{15} \times 100 = 40$ ft, and the corresponding arc is $\frac{40}{100} \times 100.286$ (from Table 11) $= 40.114$ ft.

36. The Unit Curve. The *lengths* of two curves subtending the *same central angle* Δ are in direct proportion to the corresponding radii. Hence,

$$\frac{L}{L'} = \frac{R}{R'}.$$

Since $T = R \tan \frac{1}{2}\Delta$ and $T' = R' \tan \frac{1}{2}\Delta$, we have

$$\frac{T}{T'} = \frac{R}{R'} = \frac{L}{L'}.$$

Similarly, it may be shown that *all* parts of one curve are proportional to the corresponding parts of any other curve having the same central angle.

The functions of curves are therefore multiples of any one part which may be called the *unit curve.* Thus we may have (*a*) the 1° *curve*, which is widely used in the United States, (*b*) the *curve of unit length*, and (*c*) the *curve of unit radius*, which is standard practice in England and many other foreign countries.

a. The 1° *Curve.* If $D = 1°$, then $R = 5{,}729.58$ (*arc* definition) or $R = 5{,}729.65$ (*chord* definition). Rounding off to $R = 5{,}729.6$, it is seen that tables based on the 1° curve *apply to either definition.*

Then, $T \text{ (for 1° curve)} = 5{,}729.6 \tan \frac{1}{2}\Delta$,
$E \text{ (for 1° curve)} = 5{,}729.6 \operatorname{exsec} \frac{1}{2}\Delta$, etc.

Table 2 gives tangents and externals to a 1° curve when the intersection angle Δ is known.

The appropriate values of T and E for any case are the tabular quantities *divided* by the actual degree of curve D.

Example 1. Given $\Delta = 47°20'$, $D = 12°$. Required: T and E.

Solution. By Table 2,

$$T = \frac{T \text{ (for 1° curve)}}{12} = \frac{2{,}511.13}{12} = 209.26 \text{ ft},$$

and

$$E = \frac{E \text{ (for 1° curve)}}{12} = \frac{526.13}{12} = 43.84 \text{ ft}.$$

The foregoing values of T and E are *exact* for the *arc* definition; for the *chord* definition, *corrections* are to be *added* to both T and E. Thus, by Table 3, a correction of 0.38 ft is to be added to T, giving $T = 209.64$ ft.

b. Unit Arc of 100 *Ft.* If the unit curve is defined as one whose arc length is 100 ft, then

$$L \text{ (unit curve)} = 100 = 100\frac{\Delta}{D},$$

from which $D = \Delta$, and the radius of unit curve (arc definition) = 5,729.58/Δ.

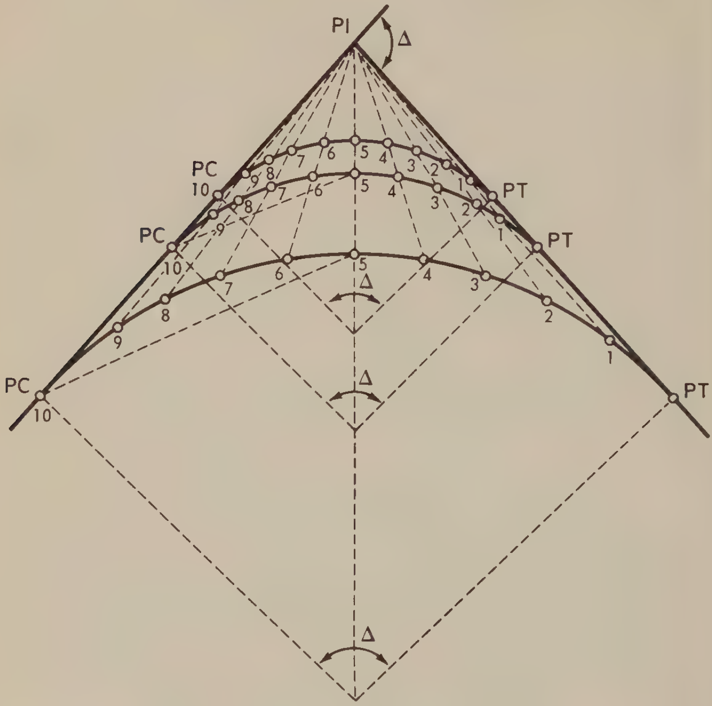

FIG. 15. Showing that deflection angles are independent of curve length. See explanation of Table 43.

Table 1 (first part) gives values of R, T, E, LC, and M for the unit curve of length 100 ft. For curves longer than 100 ft, these tabular quantities must be *multiplied* by the length of curve in stations. The degree of curve is obtained by *dividing* the values of Δ by the length of curve in stations.

By the use of Table 1, curves whose lengths are a whole number of feet may be found conveniently, so that when divided into 10 equal parts (see Fig. 21), fractional arc lengths are avoided. This table is also particularly advantageous in laying curves by the method of offsets (see Fig. 24) and in connection with metric curves.

For any external angle Δ, the length of curve L is usually fixed by desired values of some one part of the curve, such as E, T, D, R, LC, or M.

Example 2. Given $\Delta = 40°50'$ and $E = 47$ ft (approx.). Required: L and the other parts of the curve.

Solution. From an examination of the ground, the external distance E should be about 47 ft. In Table 1, $E = 9.405$ ft when $L = 100$ ft. Then,

$$L = \frac{47}{E\ (\text{for unit curve})} = \frac{47}{9.405} = 4.997 \text{ stations} = 499.7 \text{ ft.}$$

Rounding off, $L = 500$ ft. Accordingly,

$$D = \frac{\Delta}{5} = \frac{40.833}{5} = 8.167° = 8°10',$$

and, by Table 1, we get

$$\begin{aligned} R &= 140.32 \times 5 = 701.6 \text{ ft}, \\ T &= 52.230 \times 5 = 261.15 \text{ ft}, \\ E &= 9.405 \times 5 = 47.025 \text{ ft}, \\ \text{LC} &= 97.897 \times 5 = 489.48 \text{ ft}, \\ \text{M. ord.} &= 8.815 \times 5 = 44.075 \text{ ft.} \end{aligned}$$

c. Unit Radius. Letting $\Delta°$ = the central angle subtending an arc of radius R, we have the proportion

$$\frac{\text{Arc}}{2\pi R} = \frac{\Delta°}{360°}$$

from which

$$\text{Arc length} = \frac{\pi}{180} \times R\Delta° = \frac{R\Delta°}{57.29578} \tag{16}$$

If $R = 1$, Table 13 gives arc lengths for various values of the central angle Δ.

Example 3. Given $\Delta = 36°40'30''$ and $R = 500$ ft. What is the arc length?

Solution. By Eq. (16),

$$\text{Arc length} = \frac{500 \times 36.675}{57.29578} = 320.05 \text{ ft.}$$

As a check, from Table 13,

Angle	*Arc* ($R = 1$)
36°	0.6283185
40′	0.0116355
30″	0.0001454
	0.6400994

Then Arc length = 500 × 0.6401 = 320.05 ft.

If $R = 10{,}000$, Table 1 (second part) gives tangents and externals for various values of the intersection angle Δ. For any other radius, the quantities are in proportion. Thus, if $R = 1{,}000$ ft, the corresponding values of T and E are one-tenth of the tabular quantities.

Example 4. Given $\Delta = 40°20'$ and $R = 4{,}000$ ft. Required: T and E.

Solution. By Table 1, T (for radius of 10,000) = 3,672.68. Then,

$$T \text{ (for } R = 4{,}000) = \frac{4{,}000}{10{,}000} \times 3{,}672.68 = 1{,}469.07 \text{ ft.}$$

Also E (for radius of 10,000) = 653.1. Then,

$$E \text{ (for } R = 4{,}000) = \frac{4{,}000}{10{,}000} \times 653.1 = 261.24 \text{ ft.}$$

Using the appropriate radius (Table 5), T and E may conform to either definition of D.

37. Even-radius Curves. The selection and layout of curves on the basis of the *radius* instead of the *degree of curve* are preferred by a few highway departments in the United States, as is the practice in England.

The deflection angle for one station (arc definition) is

$$\tfrac{1}{2}D = \frac{1}{2}\,\frac{5{,}729.578}{R} \text{ (degrees)} = \frac{171{,}887.3}{R} \text{ (minutes).}$$

Hence

$$\text{Deflection per ft of arc} = \frac{1{,}718.873}{R} \text{ (minutes).} \tag{17}$$

Example 1. Given $R = 500$ ft. Required: deflection angle for one station (100-ft arc).

Solution. Using Eq. (17), or by Table 14, the deflection per ft of arc = 3.438 min. Then the deflection per station = 343.8′ = 5°43.8′.

Example 2. Given $\Delta = 40°50'$ and $E = 47$ ft (approx.). Required: integral value of R and the corresponding T and E.

Solution. By Table 1 (second part), $E = 670.3$ for $\Delta = 40°50'$ and $R = 10{,}000$. Then

$$R = \frac{47}{670.3} \times 10{,}000 = 701.2 \text{ ft.}$$

Rounding off, $R = 700$. Accordingly, by Table 1,

$$T = \frac{700}{10{,}000} \times 3{,}722.28 = 260.56 \text{ ft}$$

$$E = \frac{700}{10{,}000} \times 670.30 = 46.92 \text{ ft.}$$

38. Curve Selection and Field Procedure. To satisfy the requirements of safety with greater speeds on highways and railways, the curvature must be reduced to the minimum allowed by the topography. Too often the degree of curve is rounded off to the nearest whole degree when fractional values would not only improve the alignment but result in an appreciable saving in distance.

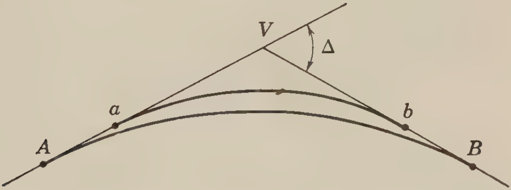

FIG. 16. Curve is shortened and flattened by following path *AB* instead of path *AabB*.

In Fig. 16, let the degree of curve D, the radius R (feet), and the tangent T (feet) refer to the flatter curve AB, while d, r, and t are corresponding parts of the sharper curve ab, the external deflection angle being Δ (degrees).

It follows that

$$T = VA = VB = R \tan \tfrac{1}{2}\Delta; \qquad t = Va = Vb = r \tan \tfrac{1}{2}\Delta.$$

Also, $Aa = bB = T - t$.

Assuming unlimited tangents and the same Δ, the distance saved by the use of the flatter curve is

$$(Aa + \text{curve } ab + bB) - \text{curve } AB$$
$$= \left[2(T - t) + 100\left(\frac{\Delta}{d}\right)\right] - 100\left(\frac{\Delta}{D}\right). \qquad (a)$$

Noting that

$$R = \frac{5{,}729.58}{D} \qquad r = \frac{5{,}729.58}{d},$$

substituting in (*a*), and reducing yields

$$\text{Distance saved} = \left(\frac{1}{D} - \frac{1}{d}\right) \times [11{,}459.16(\tan \tfrac{1}{2}\Delta) - 100\Delta], \qquad (18)$$

where D, d, and Δ are in degrees and the distance saved is in feet.

Thus if $\Delta = 40°20'$, $D = 0°24'$ $(= 0.4°)$, and $d = 1°$, the distance saved by using the flatter curve is

$$\left(\frac{1}{0.4} - \frac{1}{1}\right) \times [11{,}459.16(0.3672680) - 4{,}033.333] = 262.87 \text{ ft.}$$

Further, If $\Delta = 22°$, a $0°20'$ curve is 54.87 ft shorter than a 1° curve. Also, if $\Delta = 90°$, a $1°20'$ curve is 614.8 ft shorter than a 2° curve.

Chord corrections are generally unnecessary on curves flatter than about 4°. An inspection of Table 7 shows that the following chords may be taken equal to the corresponding arcs without appreciable error:

100-ft chords up to 4° curves,
50-ft chords up to 10° curves,
25-ft chords up to 25° curves,
10-ft chords up to 100° curves.

The current railroad handbooks on curves define the degree of curve as the central angle subtended by a 100-ft chord (Fig. 11). In highway engineering practice, it is believed this chord definition has been universally discarded in favor of the arc definition (Fig. 10), since it is simpler and leads to no confusion when dealing with short-radius curves. For flat curves (say those under 3°) practically the same results are obtained by either definition.

All the curve tables given here apply to either definition except Tables 3, 4, 10, and 17, which refer specifically to chord definition.

39. Exercises. When any two parts of a circular curve are given, the other functions of the curve may be found by formulas or tables, as will be seen from the cases that follow.

a. To Find L When Δ and D Are Known. If $\Delta = 20°24' = 1{,}224'$ and if $D = 1°40' = 100'$, then

$$L = 100\left(\frac{\Delta}{D}\right) = 100\left(\frac{1{,}224}{100}\right) = 1{,}224 \text{ ft.}$$

b. To Find R When D Is Known. Suppose $D = 4°30'$. Then by Table 5, R (arc definition) = 1,273.24 ft, and R (chord definition) = 1,273.57 ft.

c. To Find D When R Is Known. Suppose $R = 550$ ft. Then by Table 6, D (arc definition) = 10.41741° = 10°25′.

d. To Find T and E When Δ and R Are Known. If $\Delta = 40°12'$ and $R = 1{,}000$ ft, then, by Table 1, we get

$$T = \frac{3{,}659.48}{10} = 365.95 \text{ ft;}$$

as a check,

$$T = R \tan \tfrac{1}{2}\Delta = 1{,}000 \times \tan 20°06' = 365.95 \text{ ft.}$$

$$E = \frac{648.56}{10} = 64.86 \text{ ft.;}$$

as a check,

$$E = R \operatorname{exsec} \tfrac{1}{2}\Delta = 1{,}000 \times \operatorname{exsec} 20°06' = 64.86 \text{ ft.}$$

e. To Find D (or R) When Δ and E Are Known. Referring to Fig. 9, let the two tangents AV and VB intersect at an angle $\Delta = 42°30'$. Suppose, after careful consideration of the terrain, point C opposite V (the PI) is selected as a desirable position for the center of curve to run and that VC, the external distance E, measures about 42 ft. By Table 2, E (for 1° curve) = 417.99 ft; then $D = 417.99/42 = 9.95°$. Rounding off, $D = 10°$; then

$$E \text{ (revised)} = \frac{417.99}{10} = 41.80 \text{ ft}$$

$$R \text{ (approx.)} = \frac{5{,}730}{10} = 573 \text{ ft.}$$

More exactly (from Table 5),

$$R \text{ (arc definition)} = 572.96 \text{ ft;}$$
$$R \text{ (chord definition)} = 573.69 \text{ ft.}$$

f. To Find D (or R) When Δ and T Are Known. Given $\Delta = 42°30'$ and the tangent lengths are not to exceed 203 ft. From Table 2, T (for 1° curve) = 2,228.11 ft; then

$$D = \frac{2{,}228.11}{203} = 10.98°.$$

Rounding off, $D = 11°$; then

$$T = \frac{2{,}228.11}{11} = 202.56 \text{ ft,}$$

which is permissible for either definition of D, since the *correction* (Table 3) is 0.29 ft, giving T (chord definition) = 202.56 + 0.31 = 202.87 ft.

g. To Find (for Arc Definition) D, T, L, LC, and M when Δ and E Are Known. Given $\Delta = 25°06'$ and E = about 46 ft. By Table 2, E (for 1° curve) = 140.249 ft; then

$$D = \frac{140.249}{46} = 3.05°.$$

Rounding off, $D = 3°$; then

$$E \text{ (exact)} = \frac{140.249}{3} = 46.75 \text{ ft,}$$

which is satisfactory. Next,

$$T = \frac{T \text{ (for 1° curve)}}{3} = \frac{1{,}275.46}{3} = 425.15 \text{ ft}$$
$$L = 100\left(\frac{\Delta}{D}\right) = 836.67 \text{ ft.}$$

Multiplying the values of LC and M (from Table 1) by the length of curve, in stations, we get

$$\text{LC} = 99.202 \times 8.3667 = 830.00 \text{ ft;}$$
$$M = 5.454 \times 8.3667 = 45.63 \text{ ft.}$$

h. To Find L, T, and E When Δ and D Are Known. Given $\Delta = 24°32'$ and $D = 0°15'$ (= 0.25°). By Table 2,

$$T = \frac{T \text{ (for 1° curve)}}{0.25} = \frac{1{,}245.76}{0.25} = 4{,}983.04 \text{ ft}$$

and

$$E = \frac{E \text{ (for 1° curve)}}{0.25} = \frac{133.866}{0.25} = 535.46 \text{ ft}$$

$$L = 100\left(\frac{\Delta}{D}\right) = 9{,}813.33 \text{ ft.}$$

40. Field Procedure in Locating PC and PT. Local conditions may cause a special procedure, but ordinarily the tangents, as AV and BV in Fig. 17, are run to an intersection at V, the PI, and the external angle Δ is measured and checked. This implies that the forward tangent VB is fixed by strategic points ahead; otherwise subsequent field adjustments of the line may be necessary, which, although advisable, are time-consuming and expensive.

In determining the degree of curve D, careful consideration must be given to speed and safety of the expected traffic as well as the topographical features of the terrain. Frequently D is selected indirectly from desirable or controlling values of E and T to fit the ground best.

With a transit set at the PI the PC and PT should be established independently so as to be at equal tangent distances T from the PI. It is important that these points at the beginning and the end of the curve should be exactly in line with distant points (POT) on the tangents VA and VB produced. Ordinarily the PT would be located first, while the line of sight is fixed in that direction.

For special cases, such as an inaccessible PI, the foregoing procedure should be modified, as will be shown later.

41. Locating Curves by the Deflection Method

1. *When Transit Is Set at the PC.* It is assumed that the external deflection angle Δ is known, that the stationing has been continued to the PI, and that all parts of the curve have been calculated by means of either a fixed E or T, or an assumed D or R. Then the proper station numbers of the PC and PT can be determined, as will be shown later.

Procedure (Fig. 17). Assume that the transit is set at the PC, with vernier reading 0° on initial tangent AV, and that points 1, 2, 3, 4 are full stations to be established on the curve.

Turn the telescope until the vernier is clamped to read angle VA_1 (one-half angle d). Then when the rod is held at the proper chord length and the tape is swung about A as center until the rod is on the line of sight, point 1 on the curve is located.

Next, turn the telescope until the vernier is clamped to read angle VA_2, which equals the preceding deflection plus $\frac{1}{2}D$. Then when the rod is held at the proper chord length for a full station and the tape is swung about point 1 as center until the rod is on the line of sight, point 2 on the curve is located.

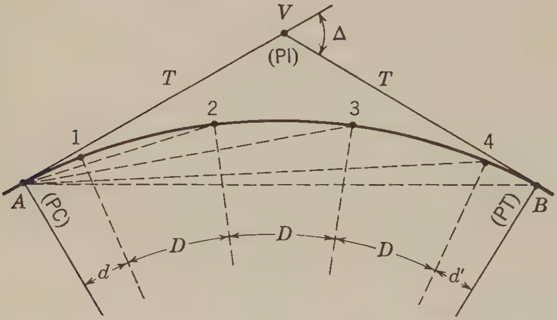

FIG. 17. Locating points 1, 2, 3, 4 by deflection angles.

The remaining points along the curve are located in a similar manner by successive measurements from the preceding point, the corresponding deflections increasing by increments. It is important to establish the stations carefully so as to avoid cumulative errors. The final deflection angle from A to B (PT) should equal $\frac{1}{2}\Delta$, and the last chord length should reach the PT.

2. *When Transit Is Set at the PT.* If the entire curve is visible from the PT, one setup of the transit may be avoided if the curve is run in entirely by deflections from the PT instead of the PC, since the instrument would be in position for proceeding forward on the next tangent.

When a transit is set at the PT, with the telescope in its normal position, the instrument is properly oriented either by sighting on the PI, vernier reading $\frac{1}{2}\Delta$, or by sighting on the PC, vernier reading 0°00′.

Assuming that the instrument is oriented as stated above, the curve notes are the same whether deflections are from the PC or PT. (See example which follows.)

For long curves, it is better to run in the second half by starting the measurements and deflections from the PT; for in this way, any small errors of surveying can be adjusted at the middle of the curve where a slight deviation in the alignment is of less consequence than at or near points of tangency to the curve.

Example. Given $\Delta = 12°20'$ and the PI at Sta. 47 + 50.64. Required: complete curve data for layout by deflections.

Solution. Let it be assumed that T is not to exceed 375 ft. By Table 2, T (for 1° curve) = 619.059 ft. Hence,

$$D = \frac{T \text{ (for } 1° \text{ curve)}}{375} = \frac{619.059}{375} = 1.65°.$$

Rounding off, $D = 1°40'$.

More exactly,
$$T = \frac{619.059}{\frac{5}{3}} = 371.44 \text{ ft},$$

which, being less than 375 ft, is permissible.

Also,
$$L = 100\,\frac{\Delta}{D} = 100\,\frac{12°20'}{1°40'} = 100\,\frac{740'}{100'} = 740.0 \text{ ft}.$$

PI = Sta. 47 + 50.64
T = 3 + 71.44
PC = Sta. 43 + 79.2
L = 7 + 40.0
PT = Sta. 51 + 19.2

Deflection per 100 ft = $\frac{1}{2}D = 0°50' = 50'$.
Deflection per ft = 0.50′.
Transit set at PC (or PT), vernier reading 0° (or $\frac{1}{2}\Delta$) *on tangent.*

Station	*Deflection*
43 + 79.2 PC	0°00′
44	0°10.4′
45	1°00.4′
46	1°50.4′
47	2°40.4′
48	3°30.4′
49	4°20.4′
50	5°10.4′
51	6°00.4′
51 + 19.2 PT	6°10.0′ (check) = $\frac{1}{2}\Delta$

Note: Owing to the small degree of curve, all *corrections* for T or subchord are insignificant; hence the curve data apply accurately to either definition of D.

3. *When Transit Is Set at Intermediate Point on Curve.* It is assumed that the first part of a curve has been located by deflections from the PC, but the next part of the curve is not visible from the PC and must be located by deflections from some intermediate point which is usually a full station.

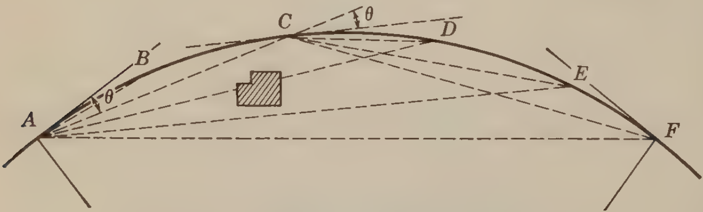

FIG. 18. Deflection angles from point C.

Transit set at point C visible from the PC. With the transit set at any intermediate point, say point C in Fig. 18, there are two methods of procedure depending upon how the vernier is set for a backsight on preceding curve point A, the PC.

Method a. Backsight on A with the telescope inverted and vernier reading the total deflection from A to C (angle θ), the angle being reset on opposite side of 0°. Plunge the telescope, unclamp the upper plate, and turn until the angle reads 0°; then the line of sight is on the forward tangent to curve at C as indicated by the dashed line in Fig. 18. The forward points D, E, etc., are located by setting the proper deflection angles from the auxiliary tangent at C.

Method b. Set the vernier to read 0° and backsight on A with the telescope inverted. Reverse the telescope, unclamp the upper plate, and set the vernier to read deflections to forward points as if they were located from the PC.

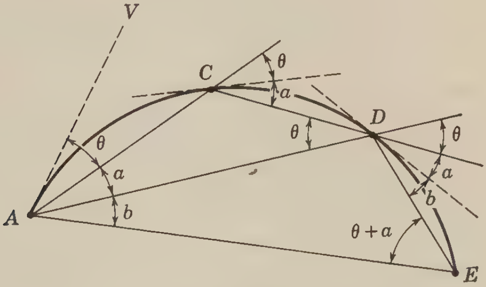

FIG. 19. Deflection angles from point C.

This method possesses the advantage of using the original notes on the basis of complete layout from the PC and is, therefore, recommended.

A study of Fig. 19 explains the geometrical principles involved. The angle between chord AC (extended) and chord CD equals $(\theta + a)$, or angle VAD, the total deflection angle from A to D.

Transit set at D not visible from the PC. Let the transit be on any other curve point, say D, from which the PC is not visible. As in previous situations, the instrument may be oriented in two ways: (a) the vernier set to read 0° on local tangent; (b) the vernier set so it will read the proper deflections to any forward point, using notes for deflections from the PC. The detailed procedure is as follows:

Method a. With transit at D (Fig. 19), backsight on any previous point, say C, with vernier (on opposite side of 0°) reading the *difference* between the deflection from A to D and that from A to C. Plunge the telescope, unclamp the upper plate, and turn until the vernier reads $a°$; then the line of sight is on tangent to curve at D. This is true because the difference between deflection angles to points C and D is $(\theta + a) - \theta = a$.

Method b. With the transit set at D (Fig. 19), invert the telescope and backsight on point C with vernier reading $\theta°$, which is the deflection angle from A to C. Plunge the telescope, unclamp the upper plate, and set the vernier to read the deflection to the next station, as if it were located from A.

The transit is now *oriented to the curve at D.* The forward deflection angle to point E (Fig. 19) is $(\theta + a + b)$, same as angle VAE.

Therefore, in order to use the deflections as per a complete layout from the PC, when the transit is on an intermediate curve point, *backsight to any station with vernier set to read the deflection from the PC to that station.*

42. Curve Locations from the PI. Referring to Fig. 20 the transit is set at V, the point of intersection of the tangents, and the telescope is directed to any point P on the circular curve AB.

Let α = angle PVB, or PVA when point P is on other side of M,
θ = angle POB, etc.

$$\tan \alpha = \frac{PN}{VN} = \frac{DB}{VB - NB} = \frac{R \text{ vers } \theta}{T - R \sin \theta}. \qquad (a)$$

Substituting $T = R \tan \frac{1}{2}\Delta$ and dividing the numerator and denominator by R, we have

$$\tan \alpha = \frac{\text{vers } \theta}{\tan \frac{1}{2}\Delta - \sin \theta}. \qquad (19)$$

The solution of the above formula gives deflections from the tangent VB, or VA, to any point on the curve. *These deflections are independent of the radius or length of curve.*

Imagine the entire curve divided into 10 equal parts (see Fig. 21), starting at either the PT or the PC.

Let α_1 and θ_1 represent the corresponding angles to point 1; α_2 and θ_2 the angles to point 2; etc.

It is obvious from Fig. 21 that $\theta_1 = \frac{1}{10}\Delta$, $\theta_2 = \frac{2}{10}\Delta$, and so on. By means of Eq. (19), the deflections α_1, α_2, etc., to each of the 10 points of equal division along the circular arc for various values of Δ have been computed and compiled in Table 46. Interpolations over ranges of 1° in Δ give results as closely as 1′ (or less) in the deflection angle.

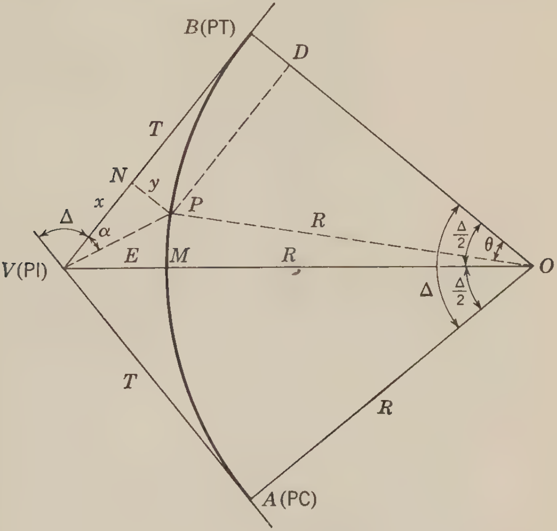

FIG. 20. Locating point P from the PI.

The middle point of the curve (point 5) is located independently by the deflection angle α_5, or its equivalent $(90 - \frac{1}{2}\Delta)$, and the external distance E. All other points on the curve are located by deflections taken directly from Table 46, and chord measurements *along the curve* (not from the PI).

Assuming the PI to be accessible, this method has the advantages that the curve may be staked out in one operation while the transit is set and oriented at the PI for the purpose of finding the angle Δ; thus much time is saved.

The points on the curve are at equal distances apart, but they are not *full stations.* For this reason, the usefulness of the method is restricted to *special situations.*

The distance j (Fig. 21) to the outer (or inner) PI is

$$j = \tfrac{1}{2}w \sec \tfrac{1}{2}\Delta,$$

in which $\frac{1}{2}w$ is the radial distance from the center-line curve to the outer (or inner) parallel curve.

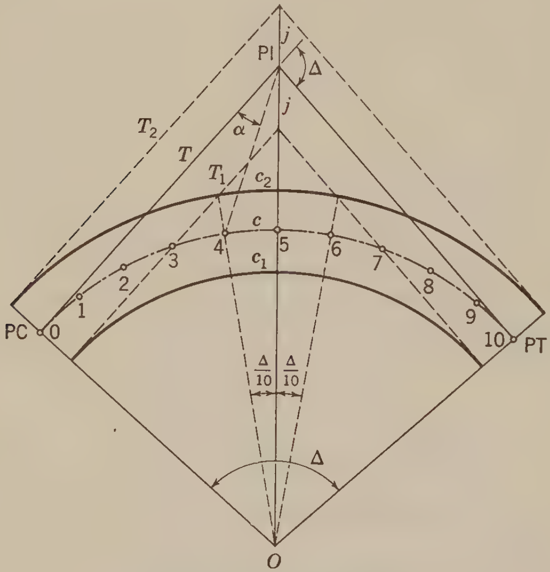

Fig. 21. The 10-point curve.

With the transit set at the outer (or inner) PI, the deflections to corresponding points are the *same* as those for the center-line curve (see Table 46).

The arcs c_1 and c_2 are proportional to the corresponding radii; that is,

$$c_1 = c\,\frac{R_1}{R}, \qquad c_2 = c\,\frac{R_2}{R},$$

in which R_1 and R_2 are the radii for the inner and outer parallel (concentric) curves. See Art. 47.

Example. Given $\Delta = 40°00'$, PI at Sta. 62 + 11.8. It is required to write down curve data on the basis of location from the PI.

Solution. From Table 1, for $L = 100$ ft, $E = 9.193$ ft, $T = 52.135$ ft. Suppose $E = 46.5$ ft fits the ground.

Hence
$$\text{Ratio} = \frac{46.5}{9.2} = 5.05.$$

Let $L = 500$ ft; then $E = 9.193 \times 5 = 45.96$ ft.

Also, $T = 52.135 \times 5 = 260.7$ ft, and $D = \frac{40}{5} = 8°$.

$$\begin{aligned} \text{PI} &= 62 + 11.8 \\ T &= \underline{\ \ 2 + 60.7} \\ \text{PC} &= 59 + 51.1 \\ L &= \underline{\ \ 5\qquad\quad} \\ \text{PT} &= 64 + 51.1 \end{aligned}$$

Referring to Table 46 with $\Delta = 40°00'$ as argument, the deflections are as given in Table A and the equal chords are each 50 ft.

Transit set at the PI, vernier reading 0°00′ *on initial tangent.*

TABLE A

Point	Station	Deflection
PC	59 + 51.1	0°00′
1	60 + 01.1	0°29′
2	60 + 51.1	2°29′
3	61 + 01.1	7°58′
4	61 + 51.1	23°41′
5	62 + 01.1 ($E = 46.0$)	70°00′
6	62 + 51.1	116°19′
7	63 + 01.1	132°02′
8	63 + 51.1	137°31′
9	64 + 01.1	139°31′
10 (PT)	64 + 51.1	140°00′

As a check in taking deflections from the table, it should be noted hat the first + ninth = second + eighth = third + seventh = fourth + sixth = $180 - \Delta = 140°$.

43. Passing a Curve through a Fixed Point. Given the angle Δ (Fig. 20) and two tangents of undetermined length, it is required to find the radius of a curve that will pass through a fixed point, say P (Fig. 20). This point is located by the angle α and the distance VP from the PI, or it may be located by the coordinates x and y.

In the triangle VOP,

$$PVO = \tfrac{1}{2}(180 - \Delta) - \alpha = 90 - \left(\alpha + \frac{\Delta}{2}\right),$$

$$POV = \frac{\Delta}{2} - \theta;$$

hence

$$VPO = 90 + (\alpha + \theta).$$

By the law of sines,

$$\frac{\sin VPO}{\sin PVO} = \frac{OV}{OP},$$

that is,

$$\frac{\cos(\alpha + \theta)}{\cos(\alpha + \Delta/2)} = \frac{R \sec(\Delta/2)}{R} = \sec\frac{\Delta}{2},$$

from which

$$\cos(\alpha + \theta) = \sec\frac{\Delta}{2}\cos\left(\alpha + \frac{\Delta}{2}\right) = \frac{\cos(\alpha + \Delta/2)}{\cos(\Delta/2)}, \tag{20}$$

from which θ may be determined; then

$$R = \frac{y}{\text{vers}\ \theta}. \tag{21}$$

Example. Given $\Delta = 48°$, $\alpha = 10°12'$, $y = 20$ ft. Required: the radius of curve R, and also the tangent T.

Solution. By Eq. (20),

$$\cos(\alpha + \theta) = \frac{0.8270806}{0.9135455} = 0.9053524,$$

giving $(\alpha + \theta) = 25°07'45''$. Deducting α, we get $\theta = 14°55'45''$. Then

$$R = \frac{20}{0.0337550} = 592.505 \text{ ft},$$

from which $$D = \frac{5{,}729.58}{592.505} = 9.67°.$$

$$T = \frac{T \text{ (for 1° curve)}}{9.67} = \frac{2{,}550.97}{9.67} = 263.80 \text{ ft.}$$

44. Locating Quarter-points on a Circular Curve. With a transit set at the PI, the quarter-points on a circular curve (Fig. 22) are located quickly and accurately by means of Table 47.

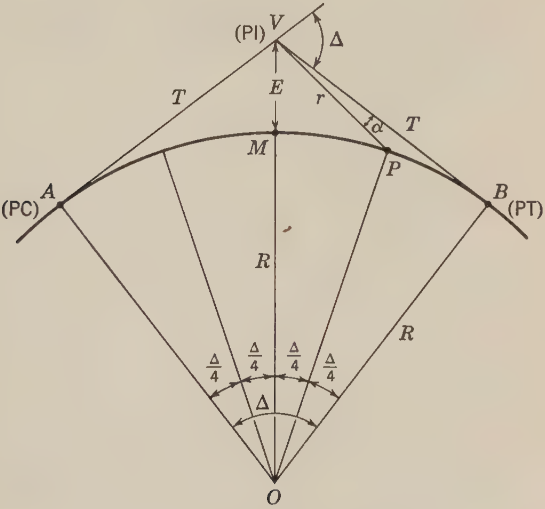

FIG. 22. Locating P, a quarter-point.

This table gives the direction α and the distance r from the PI to the first and third quarter-points. For a given intersection angle Δ, the angles α remain the *same* on curves of any length, but the values of r, which are based on a 1° curve, are to be *divided* by the actual degree of curve D.

The mid-point of the curve is located by α_5 $[= \frac{1}{2}(180 - \Delta)]$ and the external distance E.

This method is useful in fitting a curve to the ground and locating a curve for grading work or for spur tracks.

Example. With a transit set at the PI, it is required to locate the quarter-points of the curve when $\Delta = 44°$ and $D = 12°$.

Solution. Referring to Table 47, $\alpha = 4°55.5'$ and r (for 1° curve) = 1,226.2. Then the actual distance r from the PI to the first (or third) quarter-points is 1,226.2 ÷ 12 = 102.18 ft.

By turning off the angle α from each tangent in turn and measuring the distance r, these points are established.

Since (from Table 2), $E = 450.0/12 = 37.5$ ft, the mid-point of the curve is established by turning off 68°00′ from the tangent and measuring 37.5 ft.

The PC and PT of the curve would be set in the usual way.

Comparing Fig. 22 with Fig. 20, it is seen that $\theta = \Delta/4$.

In the triangle PVO, Fig. 20,

$$\frac{\sin PVO}{\sin POV} = \frac{OP}{VP} = \frac{R}{r}; \tag{a}$$

$$\frac{\cos\,(\alpha + \Delta/2)}{\sin\,(\Delta/2 - \theta)} = \frac{R}{y/\sin\,\alpha} = \frac{R\sin\,\alpha}{R\text{ vers }\theta}. \tag{b}$$

Rearranging Eq. (*b*), noting that $\theta = \Delta/4$, and vers θ

$$= 1 - \cos\,\Delta/4,$$

we get

$$\sin\,\alpha = \frac{1 - \cos\,\Delta/4}{\sin\,\Delta/4}\left[\cos\left(\alpha + \frac{\Delta}{2}\right)\right] \tag{c}$$

$$= \frac{1 - \cos\,\Delta/4}{\sin\,\Delta/4}\left(\cos\,\alpha\,\cos\frac{\Delta}{2} - \sin\,\alpha\,\sin\frac{\Delta}{2}\right). \tag{d}$$

Dividing Eq. (*d*) by cos α, we get

$$\tan\,\alpha = \frac{1 - \cos\,\Delta/4}{\sin\,\Delta/4}\left(\cos\frac{\Delta}{2} - \tan\,\alpha\,\sin\frac{\Delta}{2}\right). \tag{e}$$

Factoring Eq. (*e*), we get

$$\tan\,\alpha\left[1 + \frac{1 - \cos\,\Delta/4}{\sin\,\Delta/4}\left(\sin\frac{\Delta}{2}\right)\right] = \frac{1 - \cos\,\Delta/4}{\sin\,\Delta/4}\left(\cos\frac{\Delta}{2}\right). \tag{f}$$

Noting that $\sin \Delta/2 = 2 \sin \Delta/4 \cos \Delta/4$, and solving Eq. ($f$), we get

$$\tan \alpha = \frac{(1 - \cos \Delta/4)(\cos \Delta/2)}{\sin \Delta/4[1 + (1 - \cos \Delta/4)(2 \cos \Delta/4)]}. \tag{22}$$

Applying Eq. (22), Table 47 has been compiled for values of Δ from 10° to 120°. The angle α is independent of the curve length.

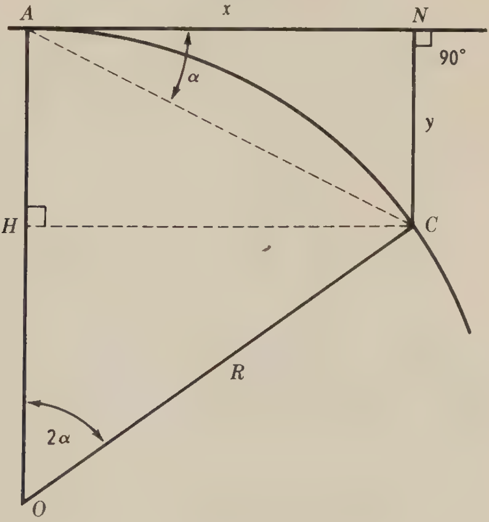

FIG. 23.

However, this is not the case for the values of r in this table, which are for a 1° curve.

From Fig. 22, we have

$$r = \frac{y}{\sin \alpha} = \frac{R \text{ vers } \Delta/4}{\sin \alpha}. \tag{23}$$

For a 1° curve, $r = (5{,}729.578/D)$ (vers $\Delta/4$)/$\sin \alpha$.

45. Locating Curves by the Offset Method

a. In Fig. 23, it is required to find the perpendicular $NP(= y)$, when $AN(= x)$ and the radius are known.

From the right triangle ODP, $OD = \sqrt{R^2 - x^2}$; y $(= AD) = OA - OD$. Hence $y = R - \sqrt{R^2 - x^2}$, or

$$y = R\left[1 - \sqrt{1 - \left(\frac{x}{R}\right)^2}\right]. \tag{24}$$

Example 1. Given $R = 1{,}000$ ft and $x = 100$ ft, to find y.

Solution. $x = \dfrac{100}{1{,}000} = 0.1$. Then by Eq. (24),

$$y = R(1 - \sqrt{1 - 0.01}) = R(1 - \sqrt{0.99}) \\ = R(1 - 0.99499) = 5.01 \text{ ft.}$$

Example 2. Given a 3° curve. It is required to find the perpendicular offset y when $x = 300$ ft.

Solution. $R = \dfrac{5{,}729.578}{3} = 1{,}943.193$ ft; $x/R = 0.15438507$; $(x/R)^2 = 0.0238345$. Then by Eq. (24),

$$y = R(1 - \sqrt{1 - 0.023845}) = R(1 - \sqrt{0.9761655}) \\ = R(1 - 0.98801) = 23.299 \text{ ft.}$$

As a check, $\sin \theta = x/R = 0.15438507$, giving $\theta = 8°52'52''$, from which $y = R \text{ vers } \theta = R(0.0119892) = 23.297$ ft.

Other similar procedures will follow for locating circular curves by offset measurements: (*b*) by tangent offset and distances; (*c*) by offset from prolonged chords; (*d*) by middle ordinates.

The tape alone may be used, but greater precision is to be expected with the transit and tape, particularly in carrying out the first of these methods. In the great majority of cases, the deflection method is preferable, but there are possible situations—a dense forest, for example—where the curve might be set out by offsets requiring less clearing for the lines of vision.

b. Offsets from the Tangent. In Fig. 24, it is required to locate any point P (P_1, P_2, P_3, etc.) along the curve by means of the tangent distance AN $(= x)$ along the tangent and the perpendicular offset $(= y)$, where $y_1 = N_1P_1$, $y_2 = N_2P_2$, $y_3 = N_3P_3$, etc.

Let θ_1, θ_2, θ_3, etc. = central angles of the arcs AP_1, P_1P_2, P_2P_3, etc. If the points P_1, P_2, P_3, etc., are full stations, then the θ's $= D$, the degree of curve, except when A (the PC) is not a full station, where $\theta_1 = (c/100) \times D$.

A study of Fig. 24 indicates three procedures, any one of which might be preferable, depending on circumstances. These are:

1. $AN_1 = R \sin \theta_1$, $N_1P_1 = R \operatorname{vers} \theta_1$;
$AN_2 = R \sin (\theta_1 + \theta_2)$, $N_2P_2 = R \operatorname{vers} (\theta_1 + \theta_2)$;
$AN_3 = R \sin (\theta_1 + \theta_2 + \theta_3)$, $N_3P_3 = R \operatorname{vers} (\theta_1 + \theta_2 + \theta_3)$.
2. $AN_1 = AP_1 \cos \frac{1}{2}\theta_1$, $N_1P_1 = AP_1 \sin \frac{1}{2}\theta_1$;
$AN_2 = AN_1 + P_1P_2 \cos (\theta_1 + \frac{1}{2}\theta_2)$;
$N_2P_2 = N_1P_1 + P_1P_2 \sin (\theta_1 + \frac{1}{2}\theta_2)$;
$AN_3 = AN_2 + P_2P_3 \cos (\theta_1 + \theta_2 + \frac{1}{2}\theta_3)$,
$N_3P_3 = N_2P_2 + P_2P_3 \sin (\theta_1 + \theta_2 + \frac{1}{2}\theta_3)$.
3. AN = one-half the long chord for twice the arc AP, that is, LC (in Table 1) for twice arc AP multiplied by length of arc AP, in stations.

NP = middle ordinate for twice the arc AP; that is, middle ordinate (in Table 1) for twice arc AP multiplied by twice length of arc AP, in stations.

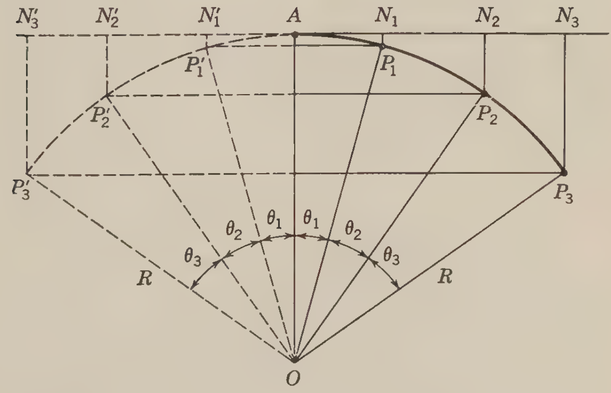

FIG. 24. Locating P_1, P_2, P_3, etc., by tangent offsets.

Example. Given $\Delta = 30°30'$, $D = 5°$, $L = 610$ ft, PC at Sta. 47 + 60. Required: full stations along the curve (arc definition).

Note: Only the first three stations will be considered here; those beyond Sta. 50 might be referred to the forward tangent (at the PT), thus avoiding long offsets and probably giving more precise alignment at the end of the curve.

Solution. $R = 1{,}145.92$ ft, $\theta_1 = 2°$, $\theta_2 = \theta_3 = D = 5°$. Chord (100-ft arc) = 99.97.

1. $AN_1 = R \sin 2° = 39.99$ ft, $N_1P_1 = R \text{ vers } 2° = 0.70$ ft;
   $AN_2 = R \sin 7° = 139.65$ ft, $N_2P_2 = R \text{ vers } 7° = 8.54$ ft;
   $AN_3 = R \sin 12° = 238.25$ ft, $N_3P_3 = R \text{ vers } 12° = 25.04$ ft.
2. $AN_1 = 40 \cos 1° = 39.99$ ft, $N_1P_1 = 40 \sin 1° = 0.70$ ft;
   $AN_2 = 39.99 + 99.97 \cos 4.5° = 39.99 + 99.66 = 139.65$ ft;
   $N_2P_2 = 0.70 + 99.97 \sin 4.5° = 0.70 + 7.84 = 8.54$ ft;
   $AN_3 = 139.65 + 99.97 \cos 9.5° = 139.65 + 98.60 = 238.25$ ft;
   $N_3P_3 = 8.54 + 99.97 \sin 9.5° = 8.54 + 16.50 = 25.04$ ft.
3. By Table 1, $AN_1 = 99.980 \times 0.4 = 39.99$ ft;
   $AN_2 = 99.751 \times 1.4 = 139.65$ ft;
   $AN_3 = 99.271 \times 2.4 = 238.25$ ft;
   $N_1P_1 = 0.873 \times 2 \times 0.4 = 0.70$ ft;
   $N_2P_2 = 3.050 \times 2 \times 1.4 = 8.54$ ft;
   $N_3P_3 = 5.217 \times 2 \times 2.4 = 25.04$ ft.

It is seen that the three different procedures give identical results.

c. Offsets from Prolonged Chords. In Fig. 25, assume point P to be established by the offset y from the initial tangent. Let c = chord AP = chord PQ; θ = corresponding central angle. Then if the chord AP is prolonged by eye until $PQ' = AP = c$, point Q on the curve can be correctly established, since the distances PQ (= c) and

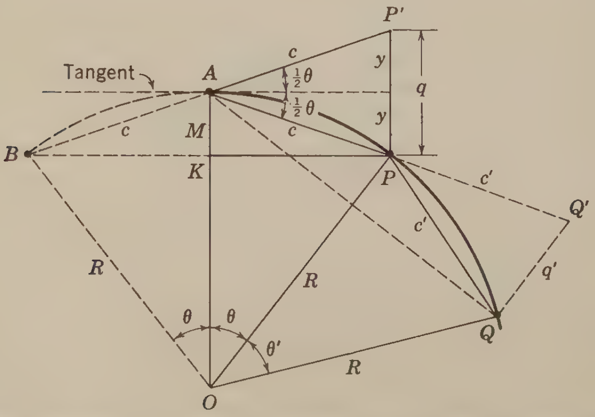

Fig. 25. Locating P (or Q) by "deflection distance" from prolonged chord.

$Q'Q$ ($= 2y$) are known. By prolonging PQ, the next point on the curve may be set in a similar manner.

The chord offset $Q'Q$ (or $P'P$) is called "deflection distance," or q.

If the chords are equal, it follows that $Q'Q = P'P = 2y = 2M =$ twice the middle ordinate for twice the arc PQ. Hence $q = 2M =$ (*middle ordinate from Table 1 for twice the subtended angle*) × (*twice the arc length in stations*). Also, we have the formula

$$q = 2R \text{ vers } \theta. \tag{25}$$

If the chords are *unequal*, the same general rule applies. Thus, if θ' is the angle subtended by chord PQ ($= c'$), then $PQ = c'$, and q', the "deflection distance," is determined by $2\theta'$ and twice the arc PQ.

Another formula for deflection distance may be derived from the similarity of triangles APP' and OAB, where

$$\text{Chord } AB = \text{chord } AP = c.$$

Thus,

$$P'P : AB = AP : OA \quad \text{or} \quad q : c = c : R,$$

giving

$$q = \frac{c^2}{R}. \tag{26}$$

Noting further that $q = P'P = 2y = 2R \text{ vers } \theta$, it follows that

q (for 1° curve, 100-ft arc) $= 2(5{,}730 \text{ vers } 1°) = 1.745$ ft,
q (for 2° curve) $= 3.49$ ft,
q (for 3° curve) $= 5.24$ ft,

and so on closely in proportion up to $D = 10°$.

Thus, if $D = 2.5°$, $q = 2.5 \times 1.745 = 4.362$ ft $=$ deflection distance for 100-ft chord. Also, since q varies as the *square* of the chord [see Eq. (26)], the deflection distance for a 40-ft chord of 2.5° curve is $4.362 \times (40/100)^2 = 0.70$ ft.

d. Middle Ordinates. For any chord AB (Fig. 26) of length c subtending a central angle Δ, the middle ordinate is

$$M = R \text{ vers } \tfrac{1}{2}\Delta = \tfrac{1}{2}c \tan \tfrac{1}{4}\Delta. \tag{27}$$

Points along the curve may be located by offsets from the long chord AB. Ordinarily the curve would be divided into an even number of parts, say 4, 6, 8, or 10.

1. Curve divided into four equal parts.

Let P_1, P_1 be the quarter-points (not shown in Fig. 26), and let N_1, N_1 be the corresponding projections on chord AB. Also, imagine chord P_1P_1 to intersect DC at D_1.

Since chord $AB = 2R \sin \frac{1}{2}\Delta$, chord P_1P_1 ($= N_1N_1$) $= 2R \sin \frac{1}{4}\Delta$,

then $$AN_1 = BN_1 = R \sin \tfrac{1}{2}\Delta - R \sin \tfrac{1}{4}\Delta. \qquad (a)$$

Also, since $DC = R \text{ vers } \frac{1}{2}\Delta$, it follows that $D_1C_1 = R \text{ vers } \frac{1}{4}\Delta$;

hence $$N_1P_1 = M - R \text{ vers } \tfrac{1}{4}\Delta. \qquad (b)$$

2. Curve divided into eight equal parts.

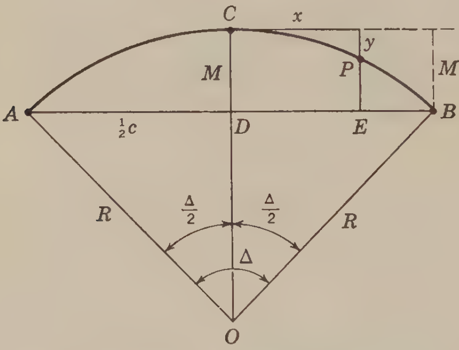

FIG. 26. Locating P by offset from long chord.

Let P_1, P_2, and P_3 be the points of equal division along the curve (not shown); then since chord $AB = 2R \sin \frac{1}{2}\Delta$ and $DC = R \text{ vers } \frac{1}{2}\Delta$, it follows that

$$AN_1 = BN_1 = R \sin \tfrac{1}{2}\Delta - R \sin \tfrac{3}{8}\Delta; \qquad N_1P_1 = M - R \text{ vers } \tfrac{3}{8}\Delta;$$
$$AN_2 = BN_2 = R \sin \tfrac{1}{2}\Delta - R \sin \tfrac{1}{4}\Delta; \qquad N_2P_2 = M - R \text{ vers } \tfrac{1}{4}\Delta;$$
$$AN_3 = BN_3 = R \sin \tfrac{1}{2}\Delta - R \sin \tfrac{1}{8}\Delta; \qquad N_3P_3 = M - R \text{ vers } \tfrac{1}{8}\Delta.$$

46. Distance from a Tangent to a Circular Curve.* Given a circular curve ACC' with center at O (Fig. 27) and the tangent AN ($= t$). It is required to find the distance NC($= r$), or NC'($= r'$), when the angle ANC($= \theta$) is known and greater or less than 90°.

Complete Fig. 27 with lines AB and HOF perpendicular to NCC'; draw auxiliary line ON; let $s/2 = CF = FC'$.

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* See author's discussion of paper by John J. Durkin in *Civil Eng.*, p. 69, February, 1965.

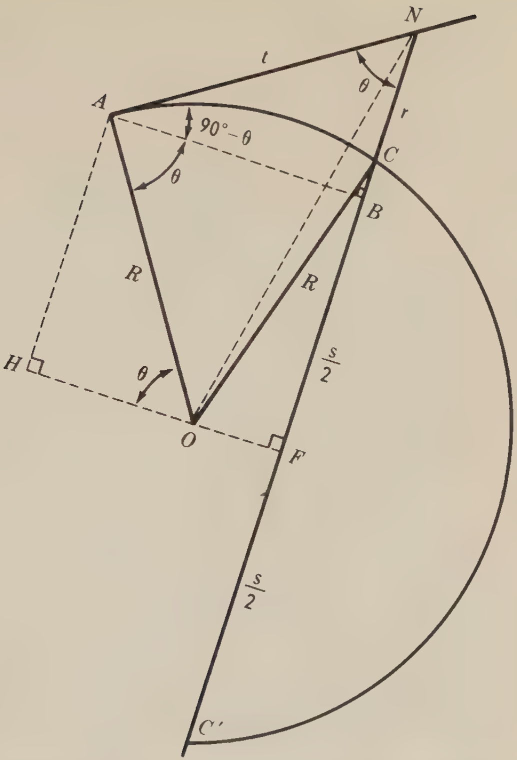

N
t
θ
r
A
90° − θ
C
θ
B
R
R
H
θ
$\frac{s}{2}$
O
F
$\frac{s}{2}$
C′

FIG. 27. Distance NC (or NC') from tangent AN to circular curve.

From the three right triangles ONA, ONF, and OCF, we get

$$\overline{ON}^2 = t^2 + R^2; \qquad (a)$$

$$\overline{ON}^2 = \left(r + \frac{s}{2}\right)^2 + \overline{OF}^2; \qquad (b)$$

$$\overline{OF}^2 = R^2 - \left(\frac{s}{2}\right)^2. \qquad (c)$$

Combining (a) and (b), we get

$$t^2 + R^2 = \left(r + \frac{s}{2}\right)^2 + \overline{OF}^2. \qquad (d)$$

Substitute (*c*) in (*d*), giving

$$t^2 + R^2 = \left(r + \frac{s}{2}\right)^2 + R^2 - \left(\frac{s}{2}\right)^2, \text{ from which}$$

$$r^2 + (r)(s) - t^2 = 0. \tag{e}$$

Treating Eq. (*e*) as a quadratic and reducing, we get

$$r = -\frac{s}{2} + \sqrt{\left(\frac{s}{2}\right)^2 + t^2}. \tag{f}$$

From (*c*), $\left(\frac{s}{2}\right)^2 = R^2 - \overline{OF}^2$, where $OF = HF - HO = t \sin \theta - R \cos \theta$. Then

$$\left(\frac{s}{2}\right)^2 = R^2 - (t \sin \theta - R \cos \theta)^2, \tag{g}$$

or

$$\frac{s}{2} = \pm \sqrt{R^2 - (t \sin \theta - R \cos \theta)^2}. \tag{h}$$

Substituting (*h*) in (*f*) and letting $J^2 = R^2 - (t \sin \theta - R \cos \theta)^2$, we have

$$r = NC = \pm J + \sqrt{J^2 + t^2},$$

or

$$r = J\left[-1 + \sqrt{1 + \left(\frac{t}{J}\right)^2}\right], \tag{28}$$

or

$$r' = NC' = J\left[1 + \sqrt{1 + \left(\frac{t}{J}\right)^2}\right]. \tag{29}$$

If $\theta = 90°$, then, since $\sin 90° = 1$ and $\cos 90° = 0$, Eq. (*h*) becomes $s/2 = \sqrt{R^2 - t^2}$ and Eq. (*f*) becomes

$$r = NC = -\sqrt{R^2 - t^2} + R = R[1 - \sqrt{1 - (t/R)^2}],$$

which agrees with Eq. (24).

Example. Given $\theta = 60°$, $t = 200$ ft, and $R = 1{,}200$ ft, to find r and r'.

Solution. $J^2 = (1{,}200)^2 - (173.49524 - 600)^2 = (100)^2(144 - 18.2153904) = (100)^2(125.78461)$. Then $J = 100(11.215392)$, $(t/J)^2 = 0.03180027$, and $\sqrt{1 + (t/J)^2} = \sqrt{1.03180027} = 1.015775$. By Eq. (28), $r = NC = 17.692$ ft; by Eq. (29), $r' = NC' = 2{,}260.8$ ft.

47. Outer and Inner Concentric Curves. Frequently, it is necessary to locate outer and inner concentric (or parallel) circular arcs, such as property lines, curb lines, and offset curves for reference during construction of the facility. The full lengths of these arcs are also desirable.

Referring to Fig. 28, let

Δ = total central angle;

L, R, D = length, radius, and degree of curve of center-line curve;

L_o, R_o, D_o = corresponding parts of outside curve;

L_i, R_i, D_i = corresponding parts of inside curve;

c = arc (or chord) subtended by central angle θ;

c_o, c_i = outside and inside arcs subtended by θ;

w = radial distance from center to outer or inner curve.

Since the length of a circular arc equals its radius multiplied by the central angle (in radians), we have

$$L = R \times \text{angle } \Delta;\ L_o = R_o \times \text{angle } \Delta;\ L_i = R_i \times \text{angle } \Delta. \quad (30)$$

Substituting $R_o = R + w$ and $R_i = R - w$, we get

$$L_o = L + w \times \text{angle } \Delta = L + a, \quad (31)$$

$$L_i = L - w \times \text{angle } \Delta = L - a, \quad (32)$$

in which $a = w \times \text{angle } \Delta = 0.017453w\Delta$ (angle Δ in degrees).

Note: $L_o + L_i = 2L$; $L_o - L_i = 2a$.

Example 1. Given $w = 50$ ft, $\Delta = 40°12'30''$, to find L_o and L_i.

Solution

Angle	*Radians* (*Table* 13)
40°	0.6981317
12′	0.0034907
30″	0.0001454
	0.7017678

$$a = 50 \times 0.7017678 = 35.09 \text{ ft.}$$

Then $L_o = L + 35.09$ ft; $L_i = L - 35.09$ ft.

Note: Using degrees instead of radians, we get

$$a = 50 \times 0.017453 \times 40.20833 = 35.09 \text{ ft.}$$

Example 2. Given $\Delta = 40°12'30''$, $D = 6°$, R (Table 5) $= 954.93$. Required: length of outside curve and the value of c_o corresponding to 50-ft arcs on the center-line curve.

Solution. Assume $R_o = 1{,}000$ ft; then $w = 1{,}000 - 954.93 = 45.07$ ft. Hence, by Table 13, $L_o = 1{,}000 \times 0.7017678 = 701.77$ ft and $c_o = 1{,}000 \times 0.0523599 = 52.36$ ft, or $c_o = 50 + 45.07 \times 0.0523599 = 52.36$ ft, where 0.0523599 (radian) is the central angle of the 50-ft arc.

Location of Outer and Inner Curves. Two methods are in use for staking the outer and inner curves: (*a*) by *offsets*, that is, by setting points at the desired radial distance from the center-line curve and at the proper chord length from adjacent points on the concentric curve; (*b*) by *deflection angles* from stations on the outside or inside curves. Example 3 which follows will show two ways of applying the method of offsets: (1) when the curve is divided into equal parts; (2) when points on the concentric curves are located opposite full stations on the center-line curve.

Example 3. Given $\Delta = 30°12'$, $D = 5°$, $L = 604$ ft, PC at Sta. 82 + 40, and $w = 20$ ft. Required: the location of the concentric curves.

1. *Assume the curve divided into* 10 *equal parts.* Since $\theta = \Delta/10 = 3.02°$, the chord correction $= 20 \times (0.017453) \times 3.02 = 1.0542$ ft. Then $c_o = 60.4 + 1.054 = 61.454$; $c_i = 60.4 - 1.054 = 59.346$ ft.

Also, $L_o = 614.54$ ft; $L_i = 593.46$ ft.

2. *Assume the concentric curves referred to full stations.* Chord correction $= w \times 0.017453 \times \theta$, where $\theta = c/L \times \Delta = c \times D/100$ (Fig. 28). Then chord correction $= 20 \times 0.0174533 \times (c/100 \times D)$.

Station	c (ft)	$c \times D/100$	Chord correction	Concentric curves
82 + 40	0	0	0	
83	60	3.0	1.047	$c1_o$ = 61.047; $c1_i$ = 58.953
84	100	5.0	1.745	c_o = 101.745; c_i = 98.255
85	100	5.0	1.745	c_o = 101.745; c_i = 98.255
86	100	5.0	1.745	c_o = 101.745; c_i = 98.255
87	100	5.0	1.745	c_o = 101.745; c_i = 98.255
88	100	5.0	1.745	c_o = 101.745; c_i = 98.255
88 + 44	44	2.2	0.768	$c2_o$ = 44.768; $c2_i$ = 43.232
PT			10.540	614.540 593.460 ft

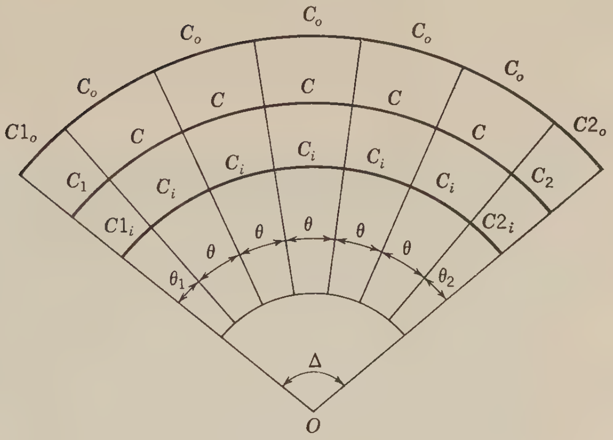

C_o
C_o
C_o
C_o
C_o
$C1_o$
$C2_o$
C
C
C
C
C
C_1
C_2
C_i
C_i
C_i
C_i
C_i
$C1_i$
$C2_i$
θ
θ
θ
θ
θ
θ_1
θ_2
Δ
O

FIG. 28. Outer and inner concentric curves.

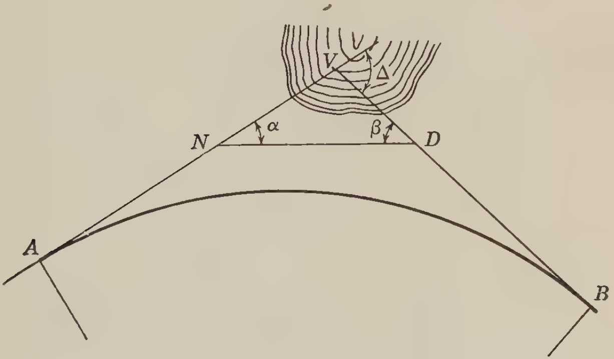

V
Δ
N
α
β
D
A
B

FIG. 29. Procedure where PI is inaccessible.

48. To Locate a Curve When PI Is Inaccessible. In Fig. 29, it is required to determine Δ and other data necessary for locating a curve connecting the given tangents AV and VB, point V being inaccessible.

Select two points N and D on tangents AV and VB. Measure angle VND, distance ND, and angle VDN. Then deflection angle $\Delta = VND + VDN = \alpha + \beta$.

Solve triangle VND for both NV and VD by the law of sines.

Hence $AN = T - NV$ and $BD = T - DV$, where T depends upon the degree of curve adopted; if it is desirable that the curve should begin at, or near, some point A, then $T = AN + NV$, from which D is determined.

49. Metric Curves. Under the metric system a chain of 100 meters is too long, and a chain of 10 meters is too short, but the 20-meter chain is considered satisfactory. If a station is assumed to be 10 meters in length, then a full chain is two stations, and ordinarily only every other station need be staked.

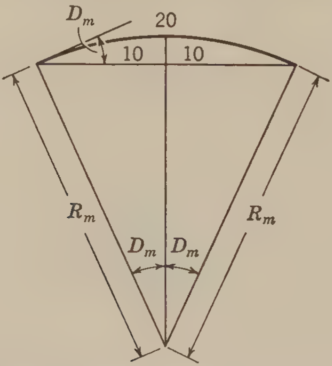


FIG. 30. Metric degree of curve.

Two definitions for the "degree of curve" are in use: (1) angle at the center subtended by an arc (or chord) of 20 meters; (2) angle at the center subtended by an arc (or chord) of 10 meters. The latter definition is preferred because of its simplicity and adaptability to the tables given herein.

In some foreign countries the metric curve is designated by its radius.

Accordingly, in Fig. 30, if D_m = degree of curve and

$$R_m = \text{radius (meters)},$$

we have the proportion

$$D_m{}^\circ.360^\circ = 10:2\pi R_m,$$

giving
$$D_m = \frac{572.958}{R_m} \tag{33}$$

or
$$R_m = \frac{572.958}{D_m} \text{ (meters).} \tag{34}$$

If $D_m = 1°$, $R_m = 572.958$ meters.

Therefore the functions (in meters) of the unit 1° *metric curve* are one-tenth of the corresponding functions (in feet) of the unit 1° *foot curve.*

Hence Table 2 may be used to find the tangents and externals to the 1° metric curve, or Table 1 for the functions of the unit curve of length 10 meters.

Letting L_m = length of curve (in meters), we have the proportion

$$L_m : 10 = \Delta : D_m$$

giving
$$L_m = 10 \frac{D_m}{\Delta}. \tag{35}$$

Example. Given $\Delta = 25°00'$ and external distance E from PI to point where middle of curve should run is 7.0 meters. It is required to find complete curve data.

Solution. Entering Table 2 with $\Delta = 25°00'$ as argument, we have E (in meters) for 1° curve $= \frac{1}{10}$ (139.111) = 13.91.

Hence $D_m = 13.911/7.0 = 1.99°$. Let $D_m = 2°$; then $E = 13.911/2 = 6.9511$ meters.

$$\text{Also,} \quad T = \frac{\frac{1}{10}(T \text{ for } 1° \text{ curve})}{2} = \frac{127.022}{2} = 63.511 \text{ meters,}$$

$$\text{and} \quad L_m = 10\frac{\Delta}{D_m} = \frac{10 \times 25.0}{2} = 125.0 \text{ meters.}$$

The values of LC and M may be found by multiplying the tabular values by $L_m/100$; thus, by Table 1,

$$\text{LC} = 99.209 \times 1.25 = 124.011 \text{ meters,}$$

and
$$M = 5.432 \times 1.25 = 6.79 \text{ meters.}$$

The curve will be located by the usual method of deflections, using 20-meter chords, and setting stakes at every other station.

Deflection for 20 meters = 2°00′ = 120′; for 1 meter = 6′.

Thus, if the PC is at Sta. 41 + 6.24 (410 + 6.24 meters) and the transit is set at the PC, vernier reading 0°00′ on tangent, the deflections are as follows:

Station (1 *Sta.* = 10 *meters*)	*Deflections*
41 + 6.24 (PC)	0°00′
42	0°22.6′
44	2°22.6′
46	4°22.6′
48	6°22.6′
50	8°22.6′
52	10°22.6′
54	12°22.6′
54 + 1.24 (PT)	12°30.0′ check

PC = Sta. 41 + 6.24
L_m = 12 + 5.0
PT = Sta. 54 + 1.24

A comparison between *metric* curves and *foot* curves may be made as follows: For the same radius, *the ratio of metric curves to foot curves* is $\frac{10 \text{ meters}}{100 \text{ ft}} = \frac{32.808 \text{ ft}}{100 \text{ ft}} = 0.32808$, or about as 1 to 3; that is D_m is about $\frac{1}{3}D$.

In order that the chord may not differ appreciably from the corresponding arc, the following chords may be used:

20-meter chords up to 2° curves of radius = 286.5 meters (= 940 ft).
15-meter chords up to 3° curves of radius = 191.0 meters (= 627 ft).
12-meter chords up to 4° curves of radius = 143.2 meters (= 470 ft).
10-meter chords up to 5° curves of radius = 114.6 meters (= 376 ft).
8-meter chords up to 7.5° curves of radius = 76.4 meters (= 251 ft).
6-meter chords up to 10.5° curves of radius = 54.6 meters (= 179 ft).
5-meter chords up to 12° curves of radius = 47.7 meters (= 156 ft).
4-meter chords up to 15° curves of radius = 38.2 meters (= 125 ft).
2-meter chords for curves sharper than 15°.

It should be noted that all the circular- and spiral-curve tables in the foot system are applicable to the metric-system curves, where the length of unit curve is 10 *meters instead of* 100 *ft.*

View of U.S. 441 showing variable median and existing trees and stream saved in the median; also shown is a fringe of the Gatlinburg, Tenn., Foothills Parkway Spur.

PART II: SPECIAL TOPICS

50. Scope. The problems considered here are typical of those resulting from shifts of tangents causing minor relocations. The intersection of a straight line with a circle is also considered, the main purpose being the tying of property lines to curves for definite data in right-of-way descriptions.

Many other cases testing the ingenuity of the engineer might arise in connection with curve adjustments to fit the situation. The solution usually involves applications of geometry and trigonometry, but complications may often be avoided by simple graphical analysis. Preliminary graphical studies in an office are preferable to the more costly trial-and-error field surveys requiring the time of a full party.

51. Relocating Forward Tangent; Determining Radius

Case 1. Given a simple curve joining two tangents. If the forward tangent is shifted parallel to itself outward a distance p (Fig. 31) without changing the radius, it is required to locate the new PC.

Solution. Let AB be the old curve and $A'B'$ the new curve. Also, let $p = BD$ = perpendicular distance between tangents;

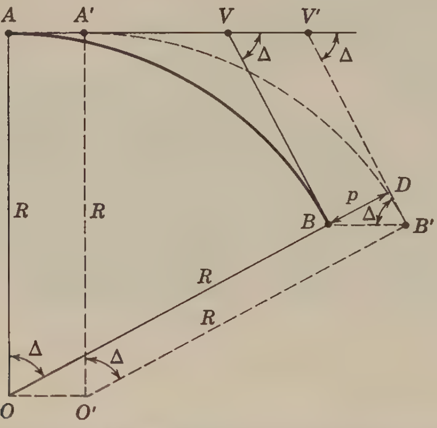


FIG. 31. Forward tangent shifted outward (same radius).

Δ = angle $BB'D$. Then

$$AA' = VV' = BB' = \frac{p}{\sin \Delta}. \qquad (a)$$

If the forward tangent had been shifted *inward*, Eq. (a) would still hold true, but the distance AA' would be *subtracted* from the stationing of the PC.

Case 2. Given a simple curve joining two tangents. If the forward tangent is shifted outward parallel to itself a distance p (Fig. 32), without changing the PC, it is required to calculate the new radius R'.

Solution. Let AB be the old curve of radius R and AB' the new curve of radius R', where, in this case, R' is greater than R.

Also, let $p = BD =$ perpendicular distance between the tangents.

Draw the chord AB' and note that the angle at $B' = \frac{1}{2}\Delta$. Then

$$BB' = \frac{p}{\sin \frac{1}{2}\Delta}. \tag{b}$$

With a transit set at A (or B), point B' can be located by measuring BB' in the direction AB. From Fig. 32, we have

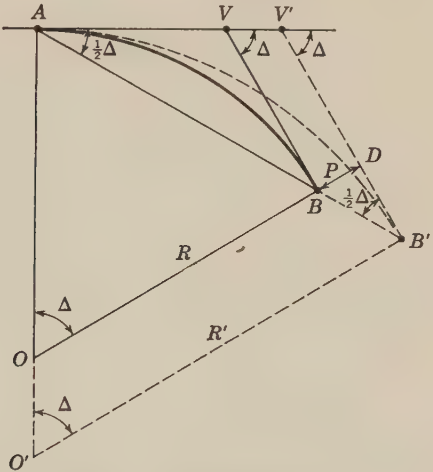


FIG. 32. Forward tangent shifted outward (new radius).

$$VV' = \frac{p}{\sin \Delta} \tag{c}$$

and

$$T' = T + \frac{p}{\sin \Delta}. \tag{d}$$

Substituting $T' = R' \tan \frac{1}{2}\Delta$ and $T = R \tan \frac{1}{2}\Delta$ and noting that

$$\tan \tfrac{1}{2}\Delta = \frac{1 - \cos \Delta}{\sin \Delta} = \frac{\text{vers } \Delta}{\sin \Delta},$$

we get $$R' = R + \frac{p}{\text{vers}\,\Delta}. \tag{e}$$

Case 3. Referring to Fig. 33, let $d = AB = 300$ ft, $\alpha = 25°$, and $\beta = 38°$. Required: a curve a radius R that will be tangent to the lines $A'A$, AB, and BB'.

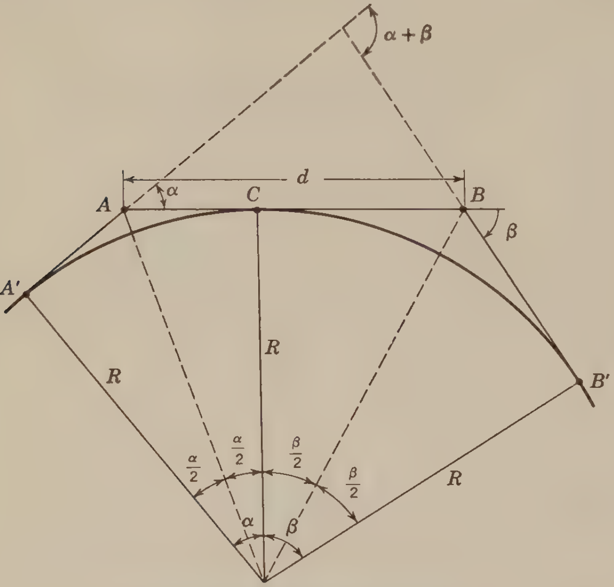


FIG. 33. Determining radius of curve.

Solution. Letting A', C, and B' be the points of tangency (Fig. 33), we have

$$A'A = AC = R \tan \tfrac{1}{2}\alpha, \tag{a}$$
$$CB = BB' = R \tan \tfrac{1}{2}\beta. \tag{b}$$

Adding (*a*) and (*b*), we get

$$d = AC + CB = R(\tan \tfrac{1}{2}\alpha + \tan \tfrac{1}{2}\beta), \text{ from which} \tag{c}$$
$$R = \frac{d}{\tan \frac{1}{2}\alpha + \tan \frac{1}{2}\beta} = \frac{300}{0.2216947 + 0.3443276} = 530.01 \text{ ft.}$$

Then $$D \text{ (arc def.)} = \frac{5{,}729.578}{530.01} = 10.8103°.$$

For convenience, let $D = 10°40' = 10.6667°$; then

$$R = 537.148 \text{ ft.}$$

If α and β are unchanged,

$$\begin{aligned} AC &= R \tan \tfrac{1}{2}\alpha = 119.083, \\ CB &= R \tan \tfrac{1}{2}\beta = \underline{184.955} \\ & \qquad\qquad\quad\;\; 304.038 = d \text{ (revised).} \end{aligned}$$

With the position of A' (the new PC) *fixed*, the forward tangent would be shifted to an outward parallel position, as indicated in Fig. 32.

Case 4. Changing the direction of the forward tangent: (*a*) PT moved ahead; (*b*) PT unchanged.

a. In Fig. 34*a*, the new tangent $V'B'$ makes an angle θ with the old tangent VB while A (the PC) remains fixed. Required: T' and R' for the new curve when Δ, T, and R of the old curve are known.

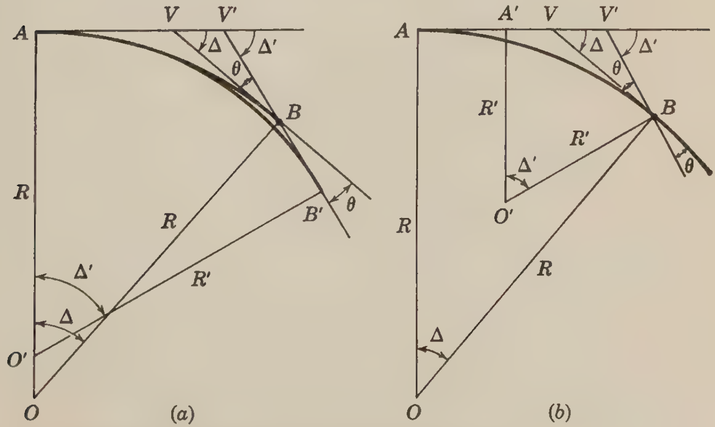


FIG. 34. Forward tangent VB changed to position $V'B'$ in (*a*) and $V'B$ in (*b*).

Solution. $\Delta' = \Delta + \theta$. By the law of sines,

$$VV' = VB \frac{\sin \theta}{\sin \Delta'}. \tag{a}$$

$$T = AV = VB; \qquad T' = AV' = T + T \frac{\sin \theta}{\sin \Delta'}$$

$$= T\left(1 + \frac{\sin \theta}{\sin \Delta'}\right). \tag{b}$$

Also, $$R' = T' \cot \Delta'; \qquad D' = \frac{5{,}729.58}{R'}.$$

b. In Fig. 34*b*, the new tangent $V'B$ makes an angle θ with the old tangent VB, but B (the PT) remains fixed, while the PC is shifted to A'. Required: Δ', T', and R' of the new curve; also AA', the distance A is moved forward.

Solution

$$\Delta' = \Delta + \theta. \tag{a}$$

By the law of sines,

$$V'B = VB \frac{\sin \Delta}{\sin \Delta'}. \tag{b}$$

$$T = AV = VB; \qquad T' = A'V' = V'B;$$

hence $$T' = T \frac{\sin \Delta}{\sin \Delta'}. \tag{c}$$

$$R' = T' \cot \Delta'. \tag{d}$$

By the law of sines,

$$VV' = T \frac{\sin \theta}{\sin \Delta'}. \tag{e}$$

Since $AA' + A'V = T$, we get

$$AA' = T - T' + VV',$$

which with (*e*) becomes

$$AA' = T\left(1 + \frac{\sin \theta}{\sin \Delta'}\right) - T'. \tag{f}$$

52. Tying Property Line to Circular Curve

Case 1. Given line NQ (Fig. 35), α (the angle NQ makes with the curve tangent AQ), AQ ($= a$), and the curve data. Required: θ and QP, where P is the intersection of line NQ with curve AB.

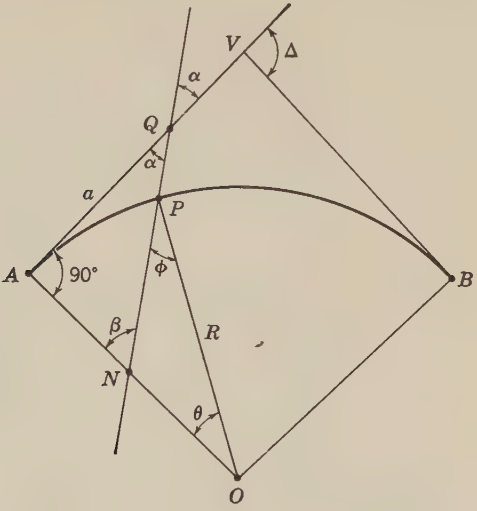


FIG. 35. Tying line NQ to curve AB.

Solution

Step 1. $\sin \phi = \cos \alpha - \frac{a}{R} \sin \alpha.$

Step 2. $\theta = 90 - (\alpha + \phi).$

Step 3. $QP = \dfrac{a - R \sin \theta}{\cos \alpha}.$

Proof

$$NA = a \tan \alpha; \qquad NQ = \frac{a}{\cos \alpha}; \qquad ON = R - NA = R - a \tan \alpha;$$

$$\beta = 90 - \alpha; \qquad \sin \beta = \cos \alpha.$$

In triangle ONP,

$$\frac{\sin \phi}{\sin \beta} = \frac{\sin \phi}{\cos \alpha} = \frac{ON}{R} = \frac{R - a \tan \alpha}{R} = 1 - \frac{a}{R} \tan \alpha;$$

then $$\sin \phi = \cos \alpha - \frac{a}{R} \sin \alpha,$$

from which ϕ is determined. Next, $\theta = \beta - \phi = 90 - (\alpha + \phi)$. In triangle ONP,

$$NP = \frac{\sin\theta}{\cos\alpha} R.$$

Then $$QP = NQ - NP = \frac{a}{\cos\alpha} - \frac{\sin\theta}{\cos\alpha} R = \frac{a - R\sin\theta}{\cos\alpha};$$

$$\text{arc } AP = 100\left(\frac{\theta}{D}\right).$$

Example. Given $\Delta = 60°$, $R = 200$ ft, $a = AQ = 100$ ft, and $\alpha = 40°$; it follows that $\theta = 26°24'03''$, $\phi = 23°35'57''$ ($= 23.59917°$), $NP = 104.520$, arc $AP = 100 \times (23.59917/28.6479) = 82.377$ ft, chord $AP = 81.796$ ft.

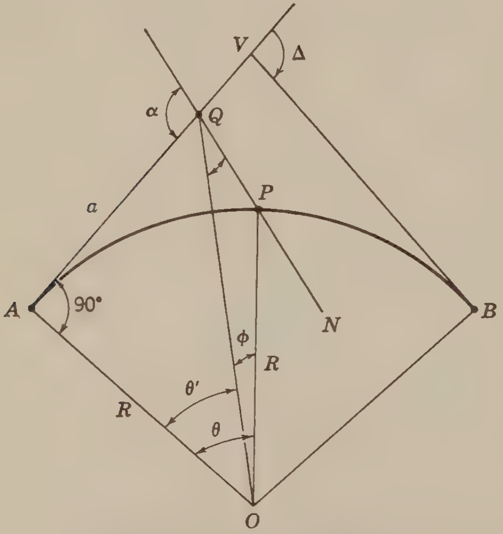


FIG. 36. Tying line NQ to curve AB.

Case 2. Given line NQ (Fig. 36), α, AQ ($= a$), and the curve data. Required: θ and QP, where P is the intersection of line NQ with the curve AB.

Solution. $\tan\theta' = a/R$, from which θ' is found. $OQ = R/\cos\theta'$.

In triangle OPQ,

$$OQP = 180 - (90 - \theta') - \alpha = 90 - (\alpha - \theta'),$$

$$\frac{\sin OPQ}{\sin OQP} = \frac{\sin OPQ}{\cos(\alpha - \theta')} = \frac{OQ}{OP} = \frac{R}{\cos\theta'}\,\frac{1}{R} = \frac{1}{\cos\theta'};$$

then

$$\sin OPQ = \frac{\cos(\alpha - \theta')}{\cos\theta'},$$

from which OPQ (or its supplement) is found. Next, $\phi = 180 - (OPQ + OQP)$; $\theta = \theta' + \phi$.

$$QP = R\,\frac{\sin\phi}{\cos(\alpha - \theta')}.$$

$$\text{arc } AP = 100\left(\frac{\theta}{D}\right).$$

If $a = 80$ ft, $\alpha = 90°$, and $R = 200$ ft, then $\theta = 23°34'41''$, $QP = 16.696$ ft, and arc $AP = 82.303$ ft.

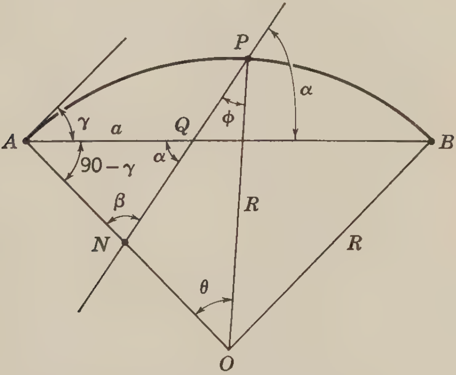


FIG. 37. Tying line NQ to curve AB.

Case 3. Given the line NQ (Fig. 37), α (angle line NQ makes with chord AB), AQ ($= a$), and the curve data. Required: θ and QP, where P is the intersection of line NQ with the curve AB.

Solution. $\gamma = \frac{1}{2}(AOB)$, $\sin\gamma = \dfrac{\text{chord } AB}{2R}$. $\beta = 180 - (\alpha + 90 - \gamma) = 90 - (\alpha - \gamma)$.

In triangle NAQ,

$$NA = a \times \frac{\sin\alpha}{\sin\beta}; \qquad NQ = a \times \frac{\cos\gamma}{\sin\beta}.$$

$$ON = R - NA = R - \frac{a\sin\alpha}{\sin\beta}.$$

In triangle ONP,

$$\frac{\sin\phi}{\sin\beta} = \frac{ON}{R};$$

$$\sin\phi = \frac{\sin\beta}{R}\left(R - \frac{a\sin\alpha}{\sin\beta}\right) = \sin\beta - \frac{a}{R} \times \sin\alpha. \qquad \theta = \beta - \phi.$$

$$NP = R \times \frac{\sin\theta}{\sin\beta}.$$

$$QP = NP - NQ = R \times \frac{\sin\theta}{\sin\beta} - a \times \frac{\cos\gamma}{\sin\beta} = \frac{R\sin\theta - a\cos\gamma}{\sin\beta}.$$

$$\text{arc } AP = 100\left(\frac{\theta}{D}\right).$$

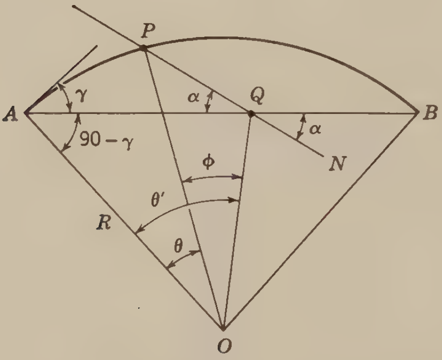


Fig. 38. Tying line NQ to curve AB.

Case 4. Given line NQ (Fig. 38), α, AQ ($= a$), and the curve data. Required: θ and QP, where P is the intersection of line NQ with the curve AB.

Solution. $\gamma = \frac{1}{2}(AOB)$, $\sin\gamma = \dfrac{\text{chord } AB}{2R}$.

$$OQ = \sqrt{R^2 + a^2 - 2aR\sin\gamma}.$$

In triangle AOQ,

$$\sin \theta' = \frac{a \cos \gamma}{OQ}$$

from which θ' is found.

$$OQA = 180 - (\theta' + 90 - \gamma) = 90 - (\theta' - \gamma).$$

In triangle OQP,

$$OQP = OQA + \alpha; \qquad \frac{\sin OPQ}{\sin OQP} = \frac{OQ}{OP}; \qquad \sin OPQ = \frac{OQ}{R} \times \sin OQP.$$

$$\phi = 180 - (OQP + OPQ). \qquad \theta = \theta' - \phi.$$

$$QP = R \times \frac{\sin \phi}{\sin OQP}. \qquad \text{arc } AP = 100\left(\frac{\theta}{D}\right).$$

Example. Given $R = 200$ ft, $\Delta = 60°$, $a = 120$ ft, $\alpha = 38°$; it follows that $OQ = 174.356$ ft, $OQA = 82°24'48''$, $OPQ = 47°44'54''$, $\theta = 25°44'54''$ ($= 25.7483°$), arc $AP = 89.879$ ft, chord $AP = 89.124$ ft. As a check, $R \times$ (θ radians) $= 89.879$ ft.

Case 5. Given the coordinates of N and Q (Fig. 39) of line NQ, and also the coordinates of O, the center of circle, and A (the PC). Required: QP and arc AP, where A (PC) is known and P is the intersection of line NQ with curve.

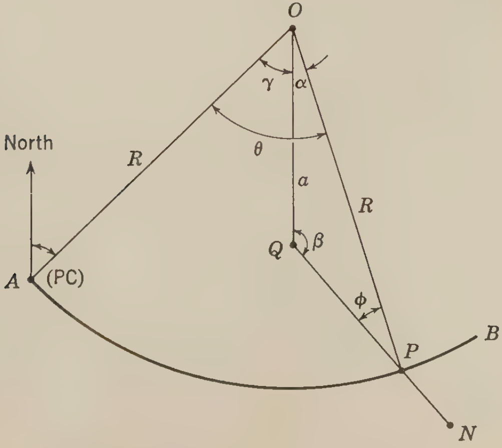


FIG. 39. Tying line NQ to curve AB.

Solution

1. With the coordinates of Q and O given, the bearing and length of QO ($= a$) are found.
2. Knowing bearing of NQ and QO giveș angle PQO ($= \beta$).
3. In triangle QOP, $\sin \phi = \frac{a \sin \beta}{R}$; QOP $(= \alpha) = 180 - (\beta + \phi)$.
4. $QP = \frac{R \sin \alpha}{\sin \beta}$.
5. $NP = NQ - PQ$.
6. Knowing bearing of AO and QO gives angle AOQ ($= \gamma$).
7. $\theta = \alpha + \gamma$. Arc $AP = 100\left(\frac{\theta}{D}\right)$. Station of $PC + AP =$ station of P.

Case 6. Given the coordinates of N and Q (Fig. 40) of line NQ, also the coordinates of points O, A_1, and A_2, where O is the center of the circle and A_1 (or A_2) is the PC of the inner and outer concentric curve. Required: QP_1, QP_2, P_1P_2, and arcs A_1P_1 and A_2P_2, where P_1 and P_2 are points of intersection of line NQ with the inner and outer curve.

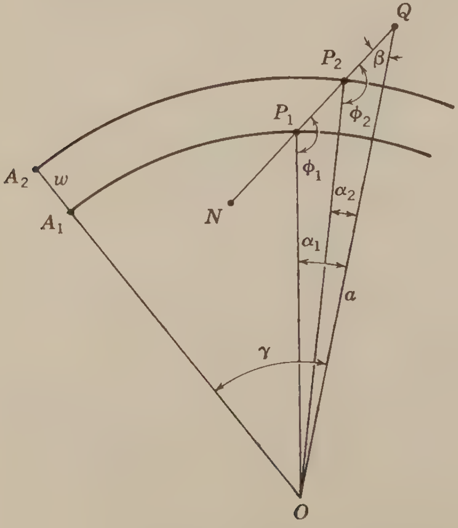


FIG. 40. Tying line NQ to concentric curves.

Solution

1. With the coordinates of Q and O given, the bearing and length of OQ $(= a)$ are found.

2. Knowing bearing of NQ and OQ gives angle NQO $(= \beta)$.

3. In triangle QOP_1, $\sin \phi_1 = \dfrac{a \sin \beta}{R_1}$; also in triangle QOP_2, $\sin \phi_2 = \dfrac{a \sin \beta}{R_2}$. $\alpha_1 = 180 - (\beta + \phi_1)$; $\alpha_2 = 180 - (\beta + \phi_2)$.

4. $QP_1 = \dfrac{R_1 \sin \alpha_1}{\sin \beta}$; $QP_2 = \dfrac{R_2 \sin \alpha_2}{\sin \beta}$.

5. $NP_1 = NQ - QP_1$; $NP_2 = NQ - QP_2$; $P_1P_2 = NP_2 - NP_1$.

6. Knowing the bearing of OA_1 (or OA_2) and OQ gives A_1OQ $(= \gamma)$.

7. A_1OP_1 $(= \theta_1) = \gamma - \alpha_1$; A_2OP_2 $(= \theta_2) = \gamma - \alpha_2$.

8. arc $A_1P_1 = 100\left(\dfrac{\theta_1}{D_1}\right)$; arc $A_2P_2 = 100\left(\dfrac{\theta_2}{D_2}\right)$, where D_1, D_2 = degree of curve corresponding to radii R_1, R_2. $R_2 - R_1 = w$ = radial distance between concentric curves.

Example 1. Referring to *Case* 5, let a $(= QO) = 110$ ft, $R = 200$ ft, $NQ = 140$ ft, bearing of QO = N2°W, bearing of QN = S62°E, and bearing of OA = S40°W. It is required to find QP and arc AP.

Solution. $\beta = 2° + 90° + 62° = 154°$. $\sin \phi = 110 \sin 26°/200 = 0.2411041$, from which $\phi = 13°57'06''$. Then $\alpha = 180° - 167°57'06'' = 12°02'54''$.

$QP = R \sin \alpha / \sin \beta = 95.23289$ ft; $NP = NQ - PQ = 44.767109$ ft. $\theta = \alpha + \gamma = 54°02'54'' = 54.04833°$. Arc $AP = 100\ \theta/D = 188.6643$ ft. As a check, arc $AP = R$ (θ radians) $= 200(0.9433214) = 188.664$ ft.

Chord $AP = 2R \sin \tfrac{1}{2}\theta = 2 \times 200\ (\sin 27°01'27'') = 181.75$ ft.

Example 2. Referring to *Case* 6, let $w = 40$ ft, $R_2 = 200$ ft, $R_1 = 160$ ft, $OQ = 235$ ft, $NQ = 160$ ft, bearing of OQ = N12°E, bearing of NQ = N42°E, and bearing of OA_2 = N50°W. It is required to find QP_1, QP_2, NP_1, NP_2, P_1P_2, arc A_1P_1, and arc A_2P_2.

Solution. $\sin \theta_1 = a \sin \beta / R_1 = 0.7343750$, from which $\phi_1 = 180° - 47°15'16'' = 132°44'44''$. Next, $\sin \phi_2 = a \sin \beta / R_2 = 0.5875000$, from which $\phi_2 = 180° - 35°58'47'' = 144°01'13''$. Then $\alpha_1 = 180° - (30° + 132°44'44'') = 17°15'16''$, $\alpha_2 = 180° - (30° + 144°01'13'') = 5°58'17''$.

$QP_1 = R_1 \sin \alpha_1 / \sin \beta = 94.9170$ ft; $QP_2 = R_2 \sin \alpha_2 / \sin B = 41.6706$ ft.

Then $NP_1 = NQ - QP_1 = 65.0830$ ft; $NP_2 = NQ - QP_2 = 118.3294$ ft; $P_1P_2 = 53.246$ ft; $\gamma = 12° + 50° = 62°$; A_1OP_1 $(= \theta_1)$

$= \gamma - \alpha_1 = 44°44'44'' = 44.74555°$; A_2OP_2 $(= \theta_2) = \gamma - \alpha_2 = 56°01'13'' = 56.020278°$; arc $A_1P_1 = R_1$ (θ_1 radians) $= 160(0.7809573) = 124.953$ ft; arc $A_2P_2 = R_2$ (θ_2 radians) $= 200(0.9777381) = 195.548$ ft. Chord $A_1P_1 = 2 \times 160 \times \sin 22°22'22'' = 121.80$ ft; chord $A_2P_2 = 2 \times 200 \times \sin 28°00'36.5'' = 187.79$ ft.

Review Problems

1. Given $\Delta = 43°05'$ and $E = 172$ ft (approx). What is the degree of curve? *Ans.* $D = 2°30'$.

2. If $\Delta = 30°12'$ and $D = 2°48'$, find T and L. *Ans.* $T = 552.13$ ft, $L = 1{,}078.57$ ft.

3. Using the data of Prob. 2 and assuming the PI to be at Sta. 42 + 16.6, what is the correct number for the PT? *Ans.* Sta. 47 + 43.04.

4. If the radius of a curve is 4,000 ft, calculate the deflection angle for a 40-ft arc and check the result by Table 14. *Ans.* 0°17.2'.

5. What is the middle ordinate M for an arc of 100 ft, which subtends a central angle of 8°00'? Solve by formula and check by Table 1. *Ans.* $M = 1.745$ ft.

6. Given $D = 6°0''$ (arc definition) and the PC at Sta. 21 + 00. With reference to the PC and the initial tangent, what are the coordinates x and y locating Sta. 23 + 00? *Ans.* $x = 198.54$ ft, $y = 20.87$ ft.

7. Given $\Delta = 22°36'$ and $D = 4°$. With the PC at Sta. 40 + 22.4, calculate the deflection angles for the circular curve and check by seeing that the last deflection (to the PT) $= \frac{1}{2}\Delta$.

8. Given $D = 1°15'$ and PC at Sta. 32, what is the "deflection distance" for setting Sta. 33? *Ans.* 2.18 ft.

9. Given $D = 1°15'$ and the PC at Sta. 32 + 20, what is the "deflection distance" for setting Sta. 33? *Ans.* 1.396 ft.

10. Given $\Delta = 47°12'$, PI = Sta. 58 + 22. Find T, E, and L for $D = 6°00'$ (arc definition); also stationing of the PC and PT.

11. Same as Prob. 10, except for chord definition.

12. Given $\Delta = 24°20'$, $R = 5{,}000$ ft. Locate the PC and PT, if PI = Sta. 36 + 42.32. *Hint:* Determine T by Table 1 and L by Table 13. *Ans.* PC = Sta. 25 + 64.32; PT = Sta. 46 + 87.80.

13. Given a metric curve with $\Delta = 38°10'$ and $D_m = 6°$ (arc definition). Locate the PC and PT if the PI = Sta. 12 + 35.4. *Ans.* PC = Sta. 12 + 02.36; PT = Sta. 12 + 65.97.

14. In the layout of a 15° curve, if two equal chords are used instead of one 100-ft chord, what should be their length? *Ans.* 50.11 ft.

15. In the layout of a 15° curve, if two equal chords are used to establish a 100-ft arc, what should be their length? *Ans.* 49.96 ft.

Q.7 + Q.10 [illegible]

16. Given a curve with PI at Sta. 80 and $\Delta = 62°40'$. Using the arc definition of D, find (*a*) the tangent T_1 and length of curve L_1 for a 4° curve; (*b*) the tangent T_2 and length of curve L_2 for a 3°36′ curve.

Ans. (*a*) $T_1 = 872.1$ ft; $L_1 = 1{,}566.7$ ft; (*b*) $T_2 = 969.0$ ft; $L_2 = 1{,}740.7$ ft.

17. Given $\Delta = 22°36'$ and $T = 817.77$ ft. Required: D and L.
Ans. $D = 1°24'$; $L = 1{,}614.29$ ft.

18. Find the length of a curve whose radius is 5,000 ft and central angle $\Delta = 32°10'$. *Ans.* $L = 2{,}807.07$ ft.

19. Using the data of Prob. 18, tabulate the deflections to full stations along the curve when a transit is set at Sta. 47 + 24 (PC), vernier reading 0°00′ on initial tangent.

20. If the PI is inaccessible give the steps for determining Δ and the positions of the PC and PT.

21. What is the procedure for passing a curve through, or near, a fixed point?

22. If part of a curve runs into an obstruction, such as a stream or building, how may the balance of the curve be set out and checked?

23. As a field exercise, establish a curve by offsets and check by deflections.

24. Given a curve that has been established tentatively with $\Delta = 58°12'$, $D = 6°$, PC at Sta. 47 + 10.5, and PT at Sta. 56 + 80.5. Required: new stationing if the forward tangent is moved outward parallel to itself a distance of 60 ft without changing the radius; also write down the distance BB' (Fig. 31) from the old to the new PT.

Ans. PC = 47 + 81.1; PT = 57 + 51.1; distance = 70.60 ft.

25. Same data as in Prob. 24, except that the forward tangent is moved *inward.*

Ans. PC = 46 + 39.9; PT = 56 + 09.9; distance = 70.60 ft.

26. Given a curve that has been established tentatively with $\Delta = 38°12'$, $D = 3°20'$, PC at Sta. 74 + 25.4, and PT at Sta. 85 + 75.4. Required: the new degree of curve and the new PT if the forward tangent is moved outward parallel to itself a distance of 100 ft (Fig. 32) without changing the position of the PC.

Ans. $D = 2°37.27'$; PT = Sta. 88 + 82.74.

27. Same data as in Prob. 26, except that the forward tangent is moved *inward.* *Ans.* $D = 4°34.602'$; PT = Sta. 82 + 60.1.

Miscellaneous Problems

1. List several important factors that should be considered in the location of (*a*) a railroad, (*b*) a highway, or (*c*) an airport.

2. Using a U.S. Geological Survey topographic map of mountainous terrain, trace the location of a route connecting points on opposite sides of a ridge. When is a tunnel advisable? If the route crosses a river at a specified elevation, what length of bridge is required, and what area of watershed exists at this crossing?

3. Knowing the scale of a local map, indicate how an existing cross-country road with bad alignment can be improved so that no curve has a radius of less than 1,000 ft.

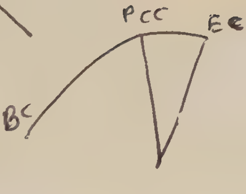
4. Project a route on an aerial map to satisfy specified alignment and grade requirements.

5. What are important advantages of aerial surveys in projecting route location?

6. List six of the most common items that appear on the plan for a new highway.

7. As a field problem, make a reconnaissance for a bypass route around a town.

Credit is due to Roy T. Brown for critical reviews of curve problems in this and other chapters.



CHAPTER 4

COMPOUND AND REVERSE CURVES

53. Definitions. Two consecutive circular curves constitute a *compound curve* if they join at a point of tangency where both curves are on the same side of the common tangent. The radii of the two curves are different but in the same direction at their junction. This point of tangency is called PCC, meaning *point of compound curvature.*

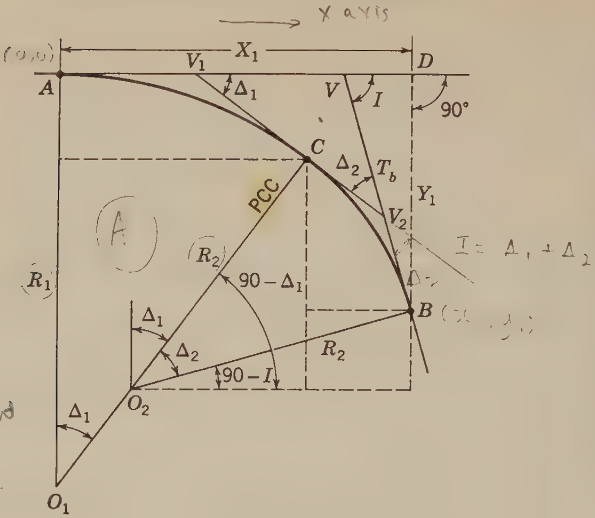
When two such curves have their centers on opposite sides of the common tangent, the combination is known as a *reverse curve.*

Compound curves are advantageous in placing the road to fit the ground. This is particularly true in mountainous country where two, three, or more simple curves of different radii may be needed. In such cases, the compound-curve system would be called two-centered, three-centered, four-centered, etc. The appearance of a kink in the alignment at the PCC is avoided when spirals are inserted between the curves of different radii (see Chap. 7).

TWO-CENTERED COMPOUND CURVES

54. Notation. Figures 41 and 42 show identical compound curves where Fig. 42 is merely Fig. 41 in reverse. The two simple curves AC and CB are laid out in the field as separate circular curves; the PC of the second curve coincides with the PT of the first curve.

For the flatter curve AC, let R_1, T_1, and Δ_1 represent the radius, tangent distance (AV_1), and intersection angle, respectively, while R_2, T_2, and Δ_2 are the corresponding parts of the sharper curve BC. Also, let $I = \Delta_1 + \Delta_2$. Further, let X_1, Y_1 = coordinates of point B (Fig. 41) with reference to A as origin and AV as X axis, while X_2, Y_2 = coordinates of point A (Fig. 42) with


x axis
(0,0)
X_1
V_1
D
A
Δ_1
V
I
90°
C
Δ_2
T_b
Y_1
V_2
PCC
(A)
R_2
$90-\Delta_1$
$I = \Delta_1 + \Delta_2$
R_1
Δ_1
Δ_2
B (x1, y1)
R_2
$90-I$
O_2
Δ_1
O_1

AV = long tangent

VB = short tangent.

FIG. 41. Two-centered compound curve.

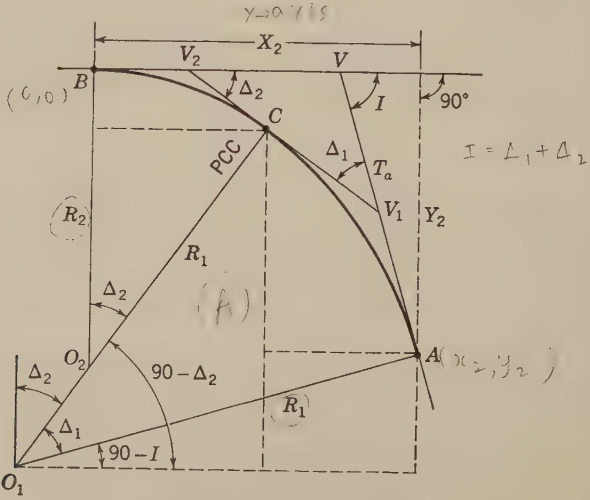

y axis
X_2
V_2
V
B
(0,0)
Δ_2
I
90°
C
PCC
Δ_1
T_a
$I = \Delta_1 + \Delta_2$
V_1
Y_2
R_2
R_1
Δ_2
(A)
O_2
Δ_2
$90-\Delta_2$
A (x2, y2)
Δ_1
R_1
$90-I$
O_1

FIG. 42. Same as Fig. 41 in reverse.

reference to B as origin and BV as X axis. Also, let

$$T_a = AV = \textit{long tangent};\ T_b = VB = \textit{short tangent}.$$

There are seven parts of the compound curve, I, R_1, T_a, Δ_1, R_2, T_b, and Δ_2. In all cases, $I = \Delta_1 + \Delta_2$. When four of these, including an angle, are given or assumed, the other three may usually be determined.

55. Derivation of Formulas. By Fig. 41, we get

$$X_1 = R_1 \sin \Delta_1 + R_2 \sin I - R_2 \sin \Delta_1, \text{ or}$$
$$X_1 = R_2 \sin I + (R_1 - R_2) \sin \Delta_1; \qquad (a)$$
$$Y_1 = R_1 \operatorname{vers} \Delta_1 + R_2 \cos \Delta_1 - R_2 \cos I, \text{ or}$$
$$Y_1 = R_1 - R_2 \cos I - (R_1 - R_2) \cos \Delta_1. \qquad (b)$$

Also, since $T_b = Y_1/\sin I$, we have

$$T_b = \frac{R_1 - R_2 \cos I - (R_1 - R_2) \cos \Delta_1}{\sin I}. \qquad (36)$$

By Fig. 42, we get

$$X_2 = R_2 \sin \Delta_2 + R_1 \sin I - R_1 \sin \Delta_2, \text{ or}$$
$$X_2 = R_1 \sin I - (R_1 - R_2) \sin \Delta_2; \qquad (c)$$
$$Y_2 = R_2 \operatorname{vers} \Delta_2 + R_1 \cos \Delta_2 - R_1 \cos I, \text{ or}$$
$$Y_2 = R_2 - R_1 \cos I + (R_1 - R_2) \cos \Delta_2. \qquad (d)$$

Also, since $T_a = Y_2/\sin I$, we have

$$T_a = \frac{R_2 - R_1 \cos I + (R_1 - R_2) \cos \Delta_2}{\sin I} \qquad (37)$$

From (36), we get

$$\operatorname{vers} \Delta_1 = \frac{T_b \sin I - R_2 \operatorname{vers} I}{R_1 - R_2}. \qquad (38)$$

From 37, we get

$$\operatorname{vers} \Delta_2 = \frac{R_1 \operatorname{vers} I - T_a \sin I}{R_1 - R_2}. \qquad (39)$$

From (a), we get

$$\sin \Delta_1 = \frac{X_1 - R_2 \sin I}{R_1 - R_2} = \frac{T_a + T_b \cos I - R_2 \sin I}{R_1 - R_2}. \qquad (40)$$

From (c), we get

$$\sin \Delta_2 = \frac{R_1 \sin I - X_2}{R_1 - R_2} = \frac{R_1 \sin I - T_a \cos I - T_b}{R_1 - R_2}. \qquad (41)$$

Dividing (38) by (40), and noting that vers $\Delta_1/\sin \Delta_1 = \tan \frac{1}{2}\Delta_1$, we get

$$\tan \tfrac{1}{2}\,\Delta_1 = \frac{T_b \sin I - R_2 \operatorname{vers} I}{T_a + T_b \cos I - R_2 \sin I}. \tag{42}$$

Likewise, dividing (39) by (41), we get

$$\tan \tfrac{1}{2}\,\Delta_2 = \frac{R_1 \operatorname{vers} I - T_a \sin I}{R_1 \sin I - T_a \cos I - T_b}. \tag{43}$$

From (*b*), we get

$$R_1 = R_2 + \frac{Y_1 - R_2 \operatorname{vers} I}{\operatorname{vers} \Delta_1} = R_2 + \frac{T_b \sin I - R_2 \operatorname{vers} I}{\operatorname{vers} \Delta_1}. \tag{44}$$

Likewise, from (*d*), we get

$$R_2 = R_1 - \frac{R_1 \operatorname{vers} I - T_a \sin I}{\operatorname{vers} \Delta_2}. \tag{45}$$

56. Check Solution by Traverse. As an independent check on the expressions for X_1 and Y_1, Fig. 41 will be treated as a closed traverse, where O_1A is assumed north (zero azimuth). Noting that latitude = *length × cosine of azimuth* and departure = *length × sine of azimuth*, we have (Fig. 41)

		Azimuth	*Latitude*	*Departure*
O_1A	R_1	0°	R_1	0
AD	X_1	90	0	X_1
DB	Y_1	180	$-Y_1$	0
BO_2	R_2	$180 + I$	$-R_2 \cos I$	$-R_2 \sin I$
O_2O_1	$R_1 - R_2$	$180 + \Delta_1$	$-(R_1 - R_2) \cos \Delta_1$	$-(R_1 - R_2) \sin \Delta_1$
		Sum =	0	0

From Σ departures = 0 and Σ latitudes = 0, we have

$$X_1 = R_2 \sin I + (R_1 - R_2) \sin \Delta_1 \text{ (check)};$$
$$Y_1 = R_1 - R_2 \cos I - (R_1 - R_2) \cos \Delta_1 \text{ (check)}.$$

Note: As a further check, $X_1 = x_1 + x_2$; $Y_1 = y_1 + y_2$ (see Appendix F).

57. Typical Problems

Case 1. Given R_1, R_2, Δ_1, Δ_2. Required: I, T_a, T_b.

Solution. $I = \Delta_1 + \Delta_2$. Use (37) for T_a; (36) for T_b.

Case 2. Given I, T_a, T_b, R_1 (or R_2). Required: R_2 (or R_1), Δ_2, Δ_1.

Solution. Use (43) for Δ_2; (45) for R_2; $\Delta_1 = I - \Delta_2$. Given R_2, use (42) for Δ_1; (44) for R_1; $\Delta_2 = I - \Delta_1$.

Case 3. Given R_1, R_2, I, T_a (or T_b). Required: Δ_1, Δ_2, T_b (or T_a).

Solution. Use (39) for Δ_2; (36) for T_b. $\Delta_1 = I - \Delta_2$. Given T_b, use (38) for Δ_1; (37) for T_a. $\Delta_2 = I - \Delta_1$.

Case 4. Given I, Δ_1, R_1, T_a. Required: Δ_2, R_2, T_b.

Solution. $\Delta_2 = I - \Delta_1$. Use (45) for R_2; (36) for T_b.

Case 5. Given I, Δ_2, R_2, T_b. Required: Δ_1, R_1, T_a.

Solution. $\Delta_1 = I - \Delta_2$. Use (44) for R_1; (37) for T_a.

Case 6. Given AB, angles VAB and VBA, R_1. Required: T_a, T_b, Δ_2, R_2.

Solution. $I = VAB + VBA$. Solve triangle AVB for T_a and T_b. Use (43) for Δ_2; (45) for R_2. $\Delta_1 = I - \Delta_2$.

Case 7. Given AB, angles VAB and VBA, R_2. Required: T_a, T_b, Δ_1, R_1.

Solution. $I = VAB + VBA$. Solve triangle AVB for T_a and T_b. Use (42) for Δ_1; (44) for R_1. $\Delta_2 = I - \Delta_1$.

58. Compound Curve Ending in a Parallel Tangent. It is required to shift the PCC of a compound curve so that the second branch of the curve will end in a parallel tangent. In all cases considered here, the radius remains unchanged in the shifting process; and p = the known perpendicular offset from old to new parallel tangent.

With the usual notation for the parts of the original curve, the corresponding parts of the new curves will be designated by primed symbols. Thus Δ_1' and Δ_2' are the central angles of the new curves, B is the original PT, while B' is the new PT on a parallel tangent. Also,

$$I = \Delta_1 + \Delta_2 = \Delta_1' + \Delta_2'.$$

Case 1. The second (and sharper) branch ends in a parallel tangent, which is *inside*, as shown in Fig. 43. ACB is the old curve; $AC'B'$ is the new curve.

Case 2. The second (and sharper) branch ends in a parallel tangent, which is *outside*, as shown in Fig. 44 ACB is the old curve; $AC'B'$ is the new curve.

Formulas for Case 1 *and Case* 2. In Figs. 43 and 44, project O_1O_2 and O_1O_2' (each $= R_1 - R_2$) on a line perpendicular to the forward tangent. Then, in Fig. 43, $p = O_1H - O_1M = O_1O_2 \cos \Delta_2 - O_1O_2' \cos \Delta_2'$, or $p = (R_1 - R_2)(\cos \Delta_2 - \cos \Delta_2')$, from which

$$\cos \Delta_2' = \cos \Delta_2 - \frac{p}{R_1 - R_2}. \qquad (a)$$

Also, $\Delta_1' = I - \Delta_2'$.

Likewise, in Fig. 44, $p = O_1H - O_1M = (R_1 - R_2)(\cos \Delta_2' - \cos \Delta_2)$, from which

$$\cos \Delta_2' = \cos \Delta_2 + \frac{p}{R_1 - R_2}. \tag{b}$$

Also, $\Delta_1' = I - \Delta_2'$.

Case 3. The second (and flatter) branch ends in a parallel tangent, which is *outside*, as shown in Fig. 45. BCA is the old curve; $BC'A'$ is the new curve.

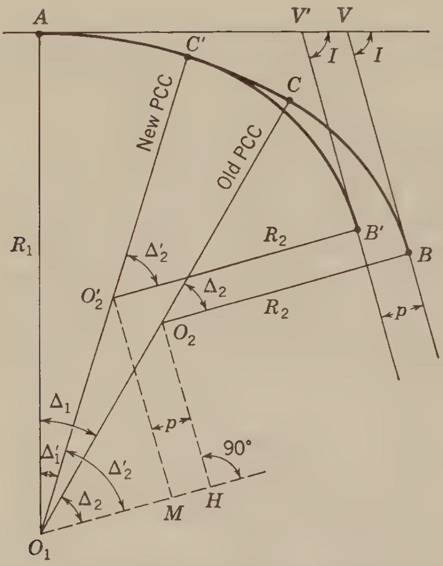

A
V'
V
C'
I
I
C
New PCC
Old PCC
R_1
B'
R_2
B
Δ_2'
O_2'
Δ_2
R_2
p
O_2
Δ_1
p
90°
Δ_1'
Δ_2'
Δ_2
M
H
O_1

FIG. 43. Two-centered curve where sharper branch is shifted inside.

Case 4. The second (and flatter) branch ends in a parallel tangent, which is inside, as shown in Fig. 46. BCA is the old curve: $BC'A'$ is the new curve.

Formulas for Case 3 *and Case* 4. In Figs. 45 and 46, project O_1O_2 (each $= R_1 - R_2$) on a line perpendicular to the forward tangent Then, in Fig. 45, $p = O_1H - O_1'M = (R_1 - R_2)(\cos \Delta_1 - \cos \Delta_1')$ from which

$$\cos \Delta_1' = \cos \Delta_1 - \frac{p}{R_1 - R_2}. \tag{c}$$

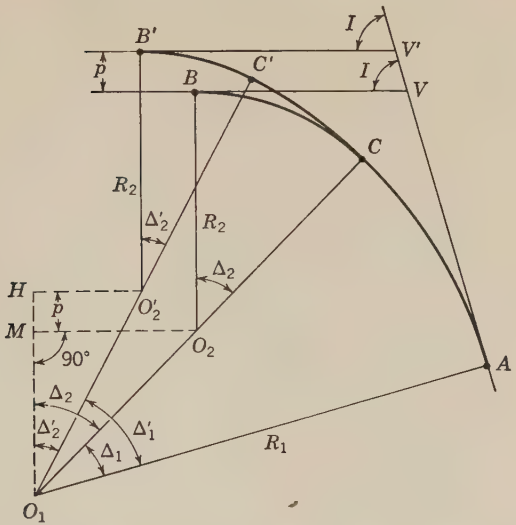

I
B'
V'
p
C'
B
I
V
C
R_2
Δ'_2
R_2
Δ_2
H
p
O'_2
M
O_2
90°
A
Δ_2
Δ'_2
Δ'_1
Δ_1
R_1
O_1

FIG. 44. Two-centered curve where sharper branch is shifted outside.

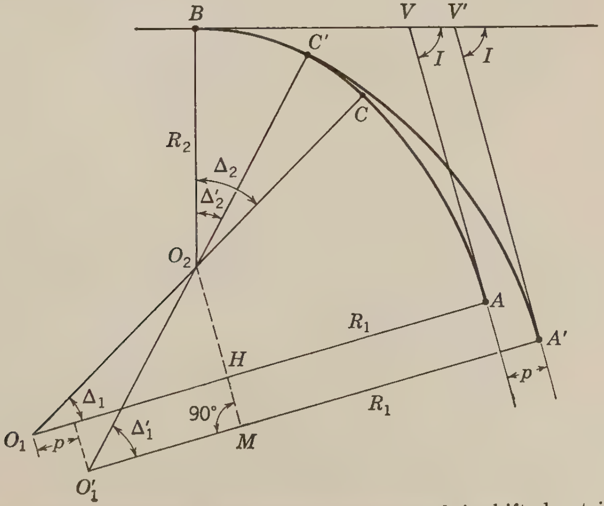

B
V
V'
C'
I
I
C
R_2
Δ_2
Δ'_2
O_2
A
R_1
A'
H
p
Δ_1
90°
R_1
O_1
p
Δ'_1
M
O'_1

FIG. 45. Two-centered curve where flatter branch is shifted outside.

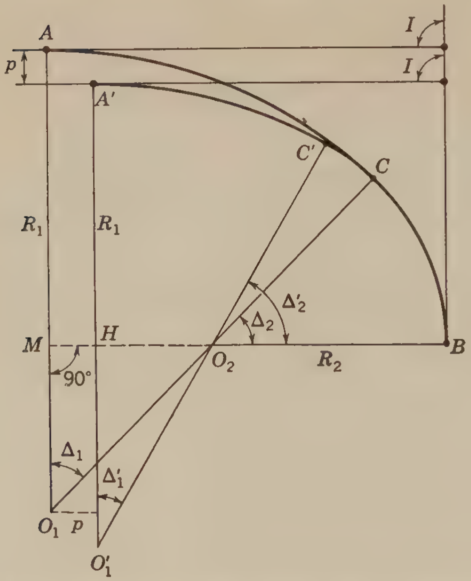


FIG. 46. Two-centered curve where flatter branch is shifted inside.

Also, $\Delta_2' = I - \Delta_1'$. Likewise, in Fig. 46, $p = O_1'H - O_1M = O_1'O_2 \cos \Delta_1' - O_1O_2 \cos \Delta_1$, from which

$$\cos \Delta_1' = \cos \Delta_1 + \frac{p}{R_1 - R_2}. \tag{d}$$

Also, $\Delta_2' = I - \Delta_1'$.

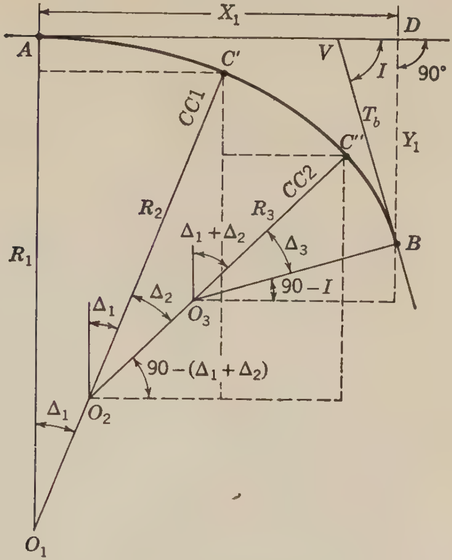
THREE-CENTERED COMPOUND CURVES

59. Formulas for Long and Short Tangents. In Fig. 47, the three-centered compound curve has centers at O_1, O_2, O_3, with central angles equal to Δ_1, Δ_2, Δ_3.

$$I = \text{total central angle} = \Delta_1 + \Delta_2 + \Delta_3.$$

Proceeding from flatter to sharper curve, the radii are R_1, R_2, R_3. The first PCC is at C', the second at C''.

Let $T_a = AV =$ *long tangent;* $T_b = VB =$ *short tangent.* Also, let X_1, Y_1 equal the coordinates of point B with reference to A as



X_1
D
A
C′
V
I
90°
CC1
T_b
Y_1
C″
CC2
R_2
R_3
$\Delta_1+\Delta_2$
Δ_3
B
R_1
$90-I$
Δ_2
Δ_1
O_3
$90-(\Delta_1+\Delta_2)$
Δ_1
O_2
O_1

FIG. 47. Three-centered compound curve.

origin and AV as X axis, where $X_1 = AD$, $Y_1 = DB$. Then

$$Y_1 = T_b \times \sin I; \qquad X_1 = T_a + T_b \times \cos I; \tag{a}$$

$$T_b = \frac{Y_1}{\sin I}; \qquad T_a = X_1 - T_b \times \cos I. \tag{b}$$

In Fig. 47,

$$X_1 = R_1 \sin \Delta_1 + R_2 \sin (\Delta_1 + \Delta_2) - R_2 \sin \Delta_1 + R_3 \sin I - R_3 \sin (\Delta_1 + \Delta_2),$$

or

$$X_1 = (R_1 - R_2) \sin \Delta_1 + (R_2 - R_3) \sin (\Delta_1 + \Delta_2) + R_3 \sin I; \tag{46}$$

$$Y_1 = R_1 \operatorname{vers} \Delta_1 + R_2 \cos \Delta_1 - R_2 \cos (\Delta_1 + \Delta_2) + R_3 \cos (\Delta_1 + \Delta_2) - R_3 \cos I,$$

or

$$Y_1 = R_1 - R_3 \cos I - (R_1 - R_2) \cos \Delta_1 - (R_2 - R_3) \cos (\Delta_1 + \Delta_2). \tag{47}$$

Then, since $T_b = Y_1/\sin I$, we have

$$T_b = \frac{R_1 - R_3 \cos I - (R_1 - R_2)\cos \Delta_1 - (R_2 - R_3)\cos(\Delta_1 + \Delta_2)}{\sin I}. \tag{48}$$

T_a may be found from substitutions in (*b*). Also,

$$T_a = \frac{Y_2}{\sin I},$$

where Y_2 will now be derived by the traverse method.

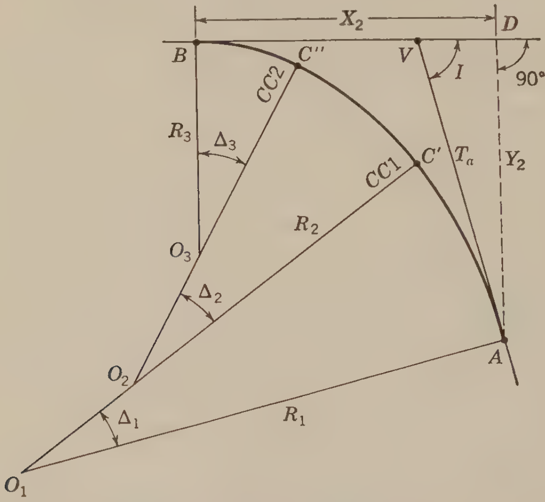


FIG. 48. Same as Fig. 47 in reverse.

60. Check Solution by Traverse. In Fig. 48, let O_3B be assumed north (zero azimuth). An expression for Y_2 is found by setting Σ latitudes = 0. The following results are self-explanatory.

Line	*Length*	*Azimuth*	*Latitude*
O_3B	R_3	0°	R_3
BD	X_2	90	0
DA	Y_2	180	$-Y_2$
AO_1	R_1	$180 + I$	$-R_1 \cos I$
O_1O_2	$R_1 - R_2$	$\Delta_2 + \Delta_3$	$(R_1 - R_2)(\cos(\Delta_2 + \Delta_3)$
O_2O_3	$R_2 - R_3$	Δ_3	$(R_2 - R_3)\cos \Delta_3$
		Sum =	0

Then it follows that

$$Y_2 = R_3 - R_1 \cos I + (R_1 - R_2) \cos (\Delta_2 + \Delta_3) + (R_2 - R_3) \cos \Delta_3; \quad (49)$$

$$T_a = \frac{R_3 - R_1 \cos I + (R_1 - R_2) \cos (\Delta_2 + \Delta_3) + (R_2 - R_3) \cos \Delta_3}{\sin I}. \quad (50)$$

Note: As a further check, see Appendix F.

61. Broken-back Curve Displacement. When two curves deflecting in the same direction are joined by a short tangent, the system is called a *broken-back curve.* Such curves are displeasing—particularly along highways—when exposed to full view in easy terrain.

It is usually possible to replace the system with a three-centered compound curve; or preferably, for better appearance and riding quality, by just one circular curve of longer radius (see Case 4).

Next, three cases will be presented depending on the relative magnitudes of the given radii and the connecting tangents. Let R_1, R_2, with centers at O_1, O_2, equal the radii of flatter and sharper curves AB and CD subtending central angles of θ_1 and θ_2, respectively; R, with center at O, is the radius of the substitute curve, and BC $(=d)$ is the length of connecting tangent. Additional notation will appear later.

Case 1. $O_2O = O_2O_1$ (Fig. 49). Given R_1, R_2, and d. Required: R, θ_1, θ_2, and the lateral distance GE.

Solution. In the right triangle FO_1O_2, $FO_1 = R_1 - R_2$, $FO_2 = d$; then $\tan \alpha = (R_1 - R_2)/d$, from which α is found.

With $O_2O = O_2O_1$, it follows that $R - R_2 = \sqrt{d^2 + (R_1 - R_2)^2}$, the solution of which gives R. Also, $\sin \frac{1}{2}\phi = \frac{1}{2}(OO_1/O_2O) = \frac{1}{2}[(R - R_1)/(R - R_2)]$, from which ϕ is found.

Then $\Delta = \frac{1}{2}(180 - \phi) = 90 - \frac{1}{2}\phi$. Also, $\theta_2 = 90 - (\alpha + \phi)$; $\theta_1 = \Delta - \theta_2$.

In Fig. 49, GE = distance the center line is moved laterally $= R - R_1 - (R - R_1) \cos \theta_1 = (R - R_1) \text{ vers } \theta_1$. Similarly, $GE = (R - R_2) \text{ vers } \theta_2$.

Example 1. Given $R_1 = 400$ ft, $R_2 = 300$ ft, and $d = 300$ ft.

Solution. $R = 300 + 100 \sqrt{9 + 1} = 616.23$ ft; $\tan \alpha = \frac{100}{300}$, from which $\alpha = 18°26'06''$.

$$\sin \tfrac{1}{2}\phi = \frac{108.115}{316.23} = 0.3418872; \quad \tfrac{1}{2}\phi = 19°59'31''; \quad \phi = 39°59'02''.$$

Then $\Delta = 70°00'29''$; $\theta_2 = 31°34'52''$; $\theta_1 = 38°25'37''$.

Checks

$$
\begin{aligned}
GE &= (R - R_1) \text{ vers } \theta_1 = 46.84 \text{ ft};\ (R - R_1) \sin \theta_1 = 134.39 \text{ ft}\\
GE &= (R - R_2) \text{ vers } \theta_2 = 46.84 \text{ ft};\ (R - R_2) \sin \theta_2 = \underline{165.61} \text{ ft}\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad 300.00 = d
\end{aligned}
$$

Example 2. Given $D_1 = 4°$ curve ($R_1 = 1432.3945$ ft), $D_2 = 5°$ curve ($R_2 = 1145.9156$ ft), and $a = 200$ ft.

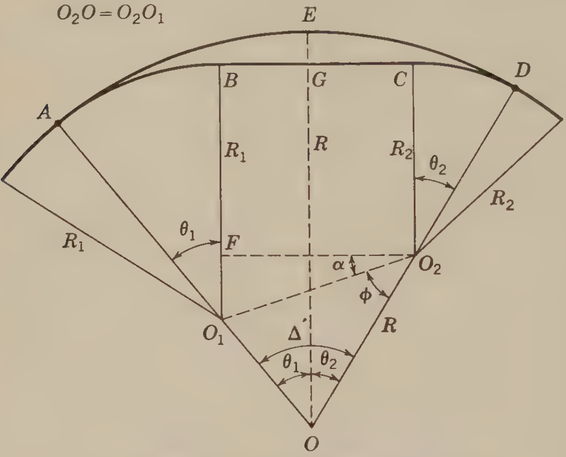


FIG. 49. Broken-back curve where $O_2O = O_2O_1$.

Solution. $\tan \alpha = R_1 - R_2/200 = 286.4789/200 = 1.4323945$, from which $\alpha = 55°04'47,6''$. From the formula $R - R_2 = \sqrt{d^2 + (R_1 - R_2)^2}$, we get $R - R_2 = 100 \sqrt{(2)^2 + (2.864789)^2} = 100 \sqrt{12.20701601} = 349.3854$. Next, $\sin \frac{1}{2}\phi = \frac{1}{2}(0.1800490)$, from which $\phi = 10°19'48''$. Next, it follows that $\theta_2 = 24°35'24.4''$ and $\theta_1 = \Delta - \theta_2 = 60°14'41.6''$. Then $GE = (R - R_2) \text{ vers } \theta_2 = 31.686$; $GE = (R - R_1) \text{ vers } \theta_1 = 31.686$ (check).

Case 2. $O_2O > O_2O_1$ (Fig. 50). Given R_1, R_2, and d.

Solution. Assume a value for R, but note that to be consistent with this case, $R > R_2 + \sqrt{d^2 + (R_1 - R_2)^2}$. The balance of the solution now follows.

Step 1. In the right triangle FO_1O_2, $O_1F = R_1 - R_2$, $FO_2 = d$. Calculate O_1O_2 and the angle α.

Step 2. With the sides of triangle OO_1O_2 known, find the angle ϕ, using the law of cosines.

Step 3. Draw O_1Z perpendicular to OO_2. Then, since $O_1Z = O_1O \sin \Delta = O_1O_2 \sin \phi$, the angle Δ can be found.

Step 4. $\theta_2 = 90 - (\alpha + \phi)$; $\theta_1 = \Delta - \theta_2$.

Step 5. $GE = (R - R_1) \text{ vers } \theta_1$; $GE = (R - R_2) \text{ vers } \theta_2$.

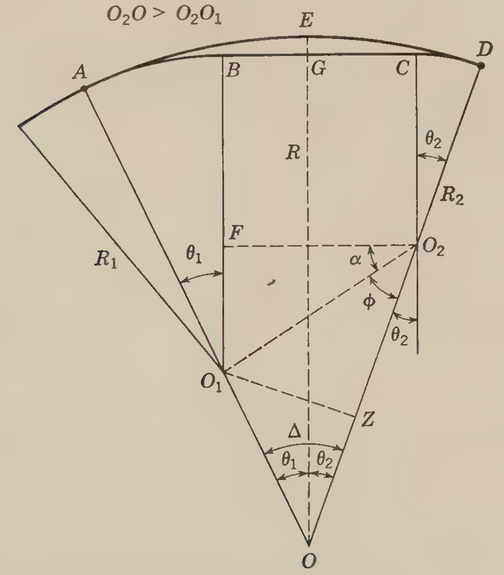

$O_2O > O_2O_1$
E
D
A
B
G
C
θ_2
R
R_2
F
O_2
R_1
θ_1
α
ϕ
θ_2
O_1
Z
Δ
θ_1
θ_2
O

FIG. 50. Broken-back curve where $O_2O > O_2O_1$.

Example 3. Given $R_1 = 500$ ft, $R_2 = 300$ ft, $R = 800$ ft (assumed), and $d = 300$ ft.

Solution. $\tan \alpha = \frac{200}{300} = 0.6666667$; $\alpha = 33°41'24''$. $O_1O_2 = \sqrt{(300)^2 + (200)^2} = 100\sqrt{13} = 360.555$ ft. With the sides of triangle OO_1O_2 known, $\phi = 36°27'22''$.

$$\sin \Delta = \frac{360.555 \sin \phi}{(R - R_1)} = 0.7141478; \Delta = 45°34'24''.$$

$$\theta_2 = 90 - (\alpha + \phi) = 19°51'14''; \theta_1 = \Delta - \theta_2 = 25°43'10''.$$

Checks

$$
\begin{aligned}
GE &= 300 \text{ vers } \theta_1 = 29.72 \text{ ft}; \; 300 \sin \theta_1 = 130.19 \text{ ft} \\
GE &= 500 \text{ vers } \theta_2 = 29.72 \text{ ft}; \; 500 \sin \theta_2 = \underline{169.81} \text{ ft} \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad 300.00 = d
\end{aligned}
$$

Case 3. $O_2O < O_2O_1$; or $R - R_2 < \sqrt{d^2 + (R_1 - R_2)^2}$ (Fig. 51).

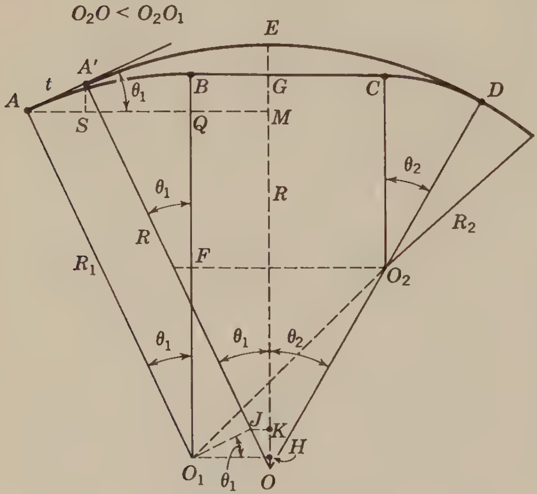

$O_2O < O_2O_1$
E
A'
t
A
θ_1
B
G
C
D
S
Q
M
θ_2
θ_1
R
R
R_2
R_1
F
O_2
θ_1
θ_1
θ_2
J
K
H
O_1
θ_1
O

FIG. 51. Broken-back curve where $O_2O < O_2O_1$.

Given θ_1, θ_2, R_1, R_2, and d. Required: R, t, and GE, R being the radius of new curve beginning at A', where A' lies on a tangent to the flatter curve at a distance AA' ($= t$) to be determined.

Solution. In Fig. 51,

$$AM = AQ + QM = R_1 \sin \theta_1 + d - (R - R_2) \sin \theta_2. \qquad (a)$$

Also,

$$AM = AS + SM = t \cos \theta_1 + R \sin \theta_1. \qquad (b)$$

Equating (a) and (b) and reducing, we get

$$t = \frac{d - (R - R_1) \sin \theta_1 - (R - R_2) \sin \theta_2}{\cos \theta_1}. \qquad (51)$$

Again

$$GE = (R - R_2) \text{ vers } \theta_2. \qquad (c)$$

Draw O_1J perpendicular to OA' and note that $JA' = O_1A = R_1$; $O_1J = t$; $HK = t \sin \theta_1$. Then

$$GE = R - R_1 - OH = R - R_1 - (OK - HK) = R - R_1 - [(R - R_1)\cos\theta_1 - t\sin\theta_1];$$

that is,

$$GE = (R - R_1) \operatorname{vers} \theta_1 + t \sin \theta_1. \qquad (d)$$

Equating (c) and (d) and reducing, we get

$$R = \frac{R_1 \operatorname{vers} \theta_1 - R_2 \operatorname{vers} \theta_2 - t \sin \theta_1}{\operatorname{vers} \theta_1 - \operatorname{vers} \theta_2}. \qquad (52)$$

Substituting Eq. (51) for t in Eq. (52), factoring, and reducing,

$$R = \frac{R_2 [\operatorname{vers} (\theta_1 + \theta_2) - \operatorname{vers} \theta_1] + R_1 \operatorname{vers} \theta_1 + d \sin \theta_1}{\operatorname{vers} (\theta_1 + \theta_2)}. \qquad (53)$$

Example 4. Given $R_1 = 600$ ft, $R_2 = 300$ ft, $d = 300$ ft, $\theta_1 = 25°$, $\theta_2 = 30°$.

Solution. By Eq. (53) $R = 663.238$ ft. By Eq. (52), $t = 101.131$ ft.

$GE = (R - R_2) \operatorname{vers} \theta_2 = 48.66$ ft.

$GE = (R - R_1) \operatorname{vers} \theta_1 + t \sin \theta_1 = 5.925 + 42.740 = 48.66$ ft (*check*).

$(R - R_2) \sin \theta_2 = 181.619$

$(R - R_1) \sin \theta_1 = 26.725$

$t \cos \theta_1 = 91.658$

$300.000 = d$ (*check*).

If R is rounded off to $R = 660$ ft, then t, by Eq. (51), becomes $t = 104.427$ ft.

Case 4 (the general case). It is required to replace two nearby circular curves (a broken-back arrangement) with one circular curve bounded by the initial and final tangents.

Given Δ_1, D_1, d, Δ_2, D_2 (see Fig. 52), let points A and H be the PC and PT respectively of the single curve of radius R and central angle $(\Delta_1 + \Delta_2)$, while points A and F are the corresponding PC and PT of the original circular curves. Also, let AG and GH be the tangents of this curve, where $AG = GH = T_3$.

For the closed traverse, where OA = north, we have

Line	*Length*	*Azimuth*	*Latitude*	*Departure*
OA	R	0°	R	0
AG	T_3	90°	0	T_3
GH	T_3	$90 + (\Delta_1 + \Delta_2)$	$-T_3 \sin (\Delta_1 + \Delta_2)$	$T_3 \cos (\Delta_1 + \Delta_2)$
HO	R	$180 + (\Delta_1 + \Delta_2)$	$-R \cos (\Delta_1 + \Delta_2)$	$-R \sin (\Delta_1 + \Delta_2)$
		Sum	0	0

$$R = \frac{T_3(1 + \cos (\Delta_1 + \Delta_2))}{\sin (\Delta_1 + \Delta_2)} \quad (a); \qquad R = \frac{T_3 \sin (\Delta_1 + \Delta_2)}{1 - \cos (\Delta_1 + \Delta_2)}. \quad (b)$$

In the triangle BEG, $BE = T_1 + d + T_2$; $BG/BE = \sin \Delta_2/\sin (\Delta_1 + \Delta_2)$; $GE/BE = \sin \Delta_1/\sin (\Delta_1 + \Delta_2)$. To locate point H, $EH = GH - (GE + T_2)$.

$$GE = (T_1 + d + T_2) \times \frac{\sin (\Delta_1)}{\sin (\Delta_1 + \Delta_2)};$$

$$BG = (T_1 + d + T_2) \times \frac{\sin \Delta_2}{\sin (\Delta_1 + \Delta_2)}.$$

Example 5. Given $\Delta_1 = 20°$, $D_1 = 20°$, $\Delta_2 = 60°$, $D_2 = 18°$, $d = 100$ ft. It is required to find the radius R of the circular curve replacing the system (see Fig. 52).

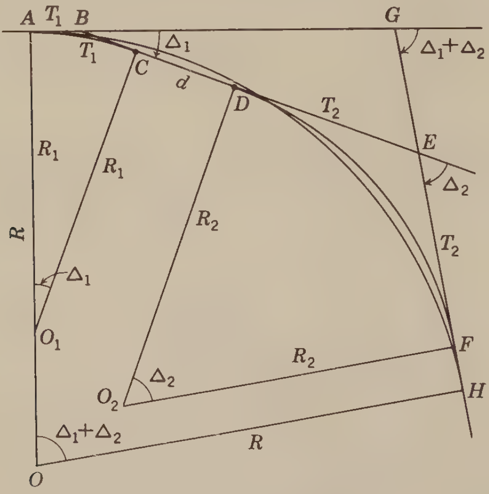

A
T_1
B
G
T_1
Δ_1
$\Delta_1+\Delta_2$
C
d
D
T_2
E
R_1
R_1
Δ_2
R
R_2
T_2
Δ_1
O_1
F
R_2
Δ_2
O_2
H
$\Delta_1+\Delta_2$
R
O

FIG. 52. One curve AH of radius R replacing two nearby circular curves.

Solution. $T_1 = (5729.578)/20 \times \tan 10° = 50.514$. $T_2 = (5729.578)/18 \times \tan 30° = 183.7763$. $BE = T_1 + d + T_2 = 334.2903$. $BG = BE \times (\sin 60/\sin 80) = 293.9699$; $GE = BE \times (\sin 20/\sin 80) = 116.0978$. $AG = T_1 + BG = 344.484$; $GF = GE + T_2 = 299.874$; $GH = AG = 344.484 = T_3$.
$R = 344.484 \times (1.1736482)/0.9848078 = 410.54$.
$R = 344.484 \times (0.9848078)/(1 - 0.1736482) = 410.54$ (check).
Also, $D = 5729.578/R = 13.956°$; $FH = GH - GF = 44.610$.

Example 6. Given $\Delta_1 = 30°$, $R_1 = 3{,}700$ ft, $\Delta_2 = 20°$, $d = 1{,}900$ ft. Required: radius R of the circular curve replacing the system.

Solution. $T_1 = 3{,}700 \times \tan \frac{1}{2}\Delta_1 = 991.411$; $T_2 = 4{,}700 \times \tan \frac{1}{2}\Delta_2 = 828.737$; $BE = T_1 + d + T_2 = 3720.148$; $BG = BE \times \sin 20/\sin 50 = 1660.955$; $GE = BE \times \sin 30/\sin 50 = 2428.1543$; $AG = T_1 + BG = 2652.366$; $GF = GE + T_2 = 3256.8913$. $AG = GH = T_3 = 2652.366$.
$R = 2652.366 \times (1 + \cos 50)/\sin 50 = 5688.02$; $R = 2652.366 \times \sin 50/(1 - \cos 50) = 5688.02$ (check). Also $D = 5729.578/R = 1.0073°$; $FH = GH - GF = -604.525$.

Example 7. Given $\Delta_1 = 32°$, $D_1 = 3°$, $\Delta_2 = 28°$, $D_2 = 4°$, $d = 600$ ft. Required: Radius R of the circular curve replacing the system.

Solution. $T_1 = (5729.578/3) \times \tan \frac{1}{2}\Delta_1 = 547.6434$; $T_2 = (5729.578/4) \times \tan \frac{1}{2}\Delta_2 = 357.13605$; $BE = T_1 + d + T_2 = 1504.7794$; $BG = BE \times (4694716)/0.8660254 = 815.7396$; $GE = BE \times (0.5299193)/0.8660254 = 920.7717$; $AG = T_1 + BG = 1363.383$; $GF = GE + T_2 = 1277.9077$; $AG = GH = T_3 = 1363.383$.
$R = (1363.383)(1.5)/0.8660254 = 2361.45$; $R = (1363.383)(0.8660254)/(1 - 0.5) = 2361.45$ (check). Also $D = (5729.578)/R = 2.4263°$; $FH = GH - GF = 85.475$.

FOUR-CENTERED COMPOUND CURVES

62. Formulas for Long and Short Tangents. In showing a 4-centered compound curve, Fig. 53 and the same in a reverse direction are identical if symbols are interchanged.

This case is an extension of the 3-centered curve. The expressions for T_a and T_b, the long and short tangents, are the same except for another term taking into account R_4 and Δ_4.

$$T_b = \frac{Y_1}{\sin I} \qquad \text{and} \qquad T_a = \frac{Y_2}{\sin I},$$

where (by traverse)

$$Y_1 = R_1 - R_4 \cos I - (R_1 - R_2) \cos \Delta_1 - (R_2 - R_3) \cos (\Delta_1 + \Delta_3) - (R_3 - R_4) \cos (\Delta_1 + \Delta_2 + \Delta_3). \quad (54)$$

$$Y_2 = R_4 - R_1 \cos I + (R_1 - R_2) \cos (\Delta_2 + \Delta_3 + \Delta_4) + (R_2 - R_3) \cos (\Delta_3 + \Delta_4) + (R_3 - R_4) \cos \Delta_4. \quad (55)$$

REVERSE CURVES

63. Introduction. Two circular curves AC and CB (Fig. 54) on opposite sides of a common tangent constitute a *reverse curve.*

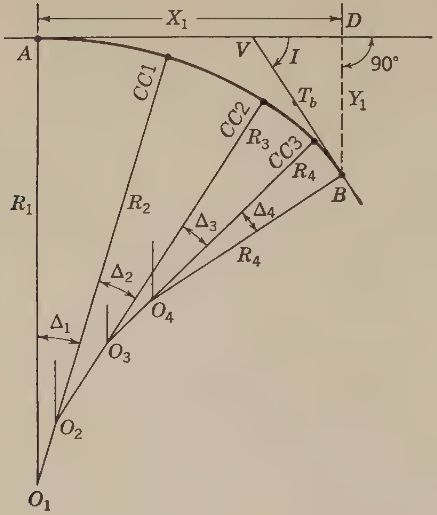

X1
D
A
V
I
90°
CC1
Tb
Y1
CC2
R3
CC3
R4
B
R1
R2
Δ3
Δ4
R4
Δ2
O4
Δ1
O3
O2
O1

Fig. 53. Four-centered compound curve.

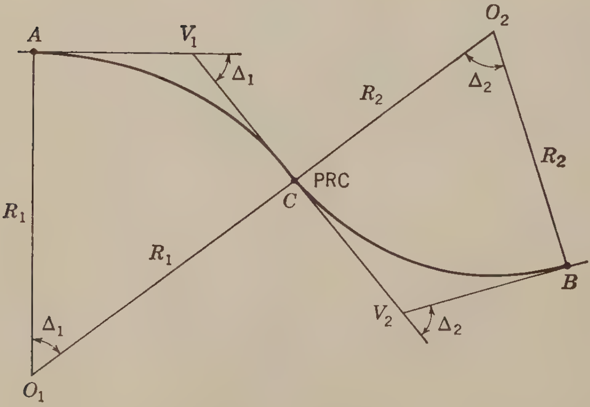

O2
A
V1
Δ1
Δ2
R2
R2
PRC
C
R1
R1
B
V2
Δ2
Δ1
O1

Fig. 54. Reverse curve.

The common point of reverse (or opposite) curvature is called PRC.

Reverse curves are objectionable and should be avoided on major high-speed highways and railroads, as there is no opportunity to elevate the outer bank at the PRC. Nevertheless they are frequently encountered in mountainous country, in cities, and in the layout of railway spur tracks and crossovers.

It is desirable that the two curves be separated by an intermediate tangent distance of sufficient length to expedite the superelevation approach and runoff.

Example 1. Given $\Delta_1 = 40°20'$, $V_1C = 312$ ft, $CV_2 = 260$ ft, and $\Delta_2 = 34°$. Required: D_1 and D_2.

Solution. $T_1 = V_1C = 312$ ft, and $T_2 = CV_2 = 260$ ft, but these values may be revised slightly in order to get a convenient value for D_1 (or D_2). It is assumed that a small amount of flexibility in distance and direction is permissible.

Using Table 2, we have

$$D_1 = \frac{2{,}104.29}{312} = 6.7445°, \qquad D_2 = \frac{1{,}751.71}{260} = 6.7373°.$$

Let $D_1 = D_2 = 6.75° = 6°45'$; then

$$T_1 = \frac{2{,}104.29}{6.75} = 311.75 \text{ ft} \qquad \text{and} \qquad T_2 = \frac{1{,}751.71}{6.75} = 259.51 \text{ ft}.$$

Example 2. Given $V_1V_2 = 1{,}200$ ft, $\Delta_1 = 40°$, and $\Delta_2 = 50°$. Required: T_1, T_2, D_1, and D_2.

Solution. Assuming an intermediate tangent distance of 300 ft, there remains 900 ft $= T_1 + T_2$. Let $T_1 = 500$; then $T_2 = 400$. Using Table 2, we have

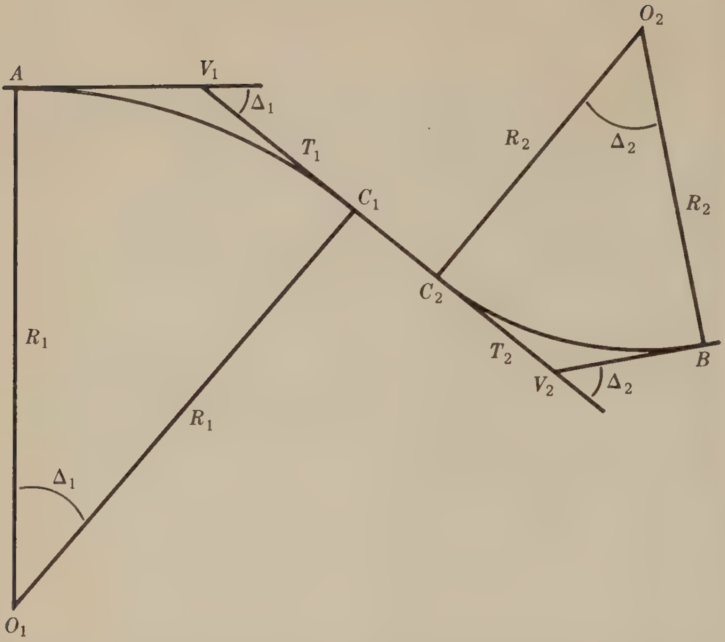
$$D_1 = \frac{2{,}085.4}{500} = 4.17°, \qquad D_2 = \frac{2{,}671\ 75}{400} = 6.68°.$$

For final values, let $D_1 = 4°$ and $D_2 = 6°40'$; then

$$T_1 = \frac{2{,}085.4}{4} = 521.35 \text{ ft} \qquad \text{and} \qquad T_2 = \frac{2{,}671.75}{6.667} = 400.75 \text{ ft}.$$

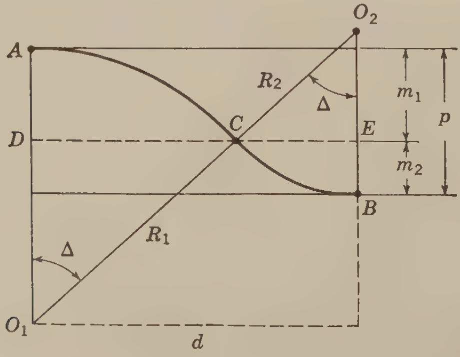
The revised intermediate tangent distance is 277.89 ft.

64. Reverse Curve between Parallel Tangents. In Fig. 56 we have reverse curve ACB between parallel tangents such as would occur in connection with railroad turnouts.



A
V_1
Δ_1
T_1
C_1
O_2
R_2
Δ_2
R_2
C_2
R_1
T_2
B
V_2
Δ_2
R_1
Δ_1
O_1

FIG. 55. Reverse curve with intermediate tangent distance (see Example 2, p. 141).



O_2
A
R_2
m_1
Δ
p
C
E
D
m_2
B
R_1
Δ
O_1
d

FIG. 56. Reverse curve between parallel tangents.

Let $d = DE = DC + CE$;

$p = m_1 + m_2 =$ distance between parallel tracks.

The radii R_1 and R_2 may be equal or unequal; *if equal*, $R_1 = R_2 = R$, $DC = CE = \frac{1}{2}d$, $m_1 = m_2 = \frac{1}{2}p$.

The five quantities d, R_1, R_2, Δ, and p are related as follows (Fig. 56).

$$d = R_1 \sin \Delta + R_2 \sin \Delta = (R_1 + R_2) \sin \Delta, \tag{56}$$

$$p = R_1 \operatorname{vers} \Delta + R_2 \operatorname{vers} \Delta = (R_1 + R_2) \operatorname{vers} \Delta. \tag{57}$$

If any three are given, the other two may be found.

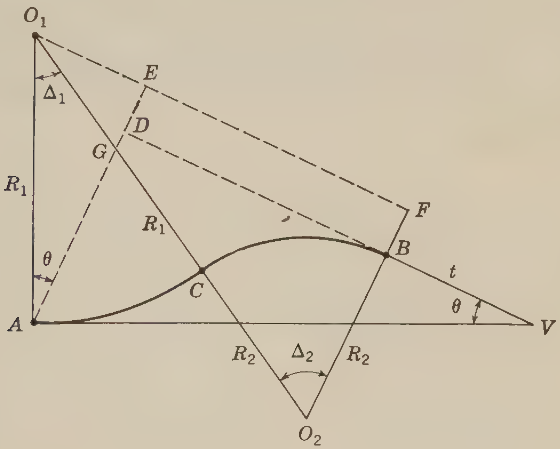


FIG. 57. Reverse curve between nonparallel tangents.

When the radii are *equal*, $d = 2R \sin \Delta$, $p = 2R \operatorname{vers} \Delta$, or

$$\operatorname{vers} \Delta = \frac{p}{2R}, \quad \text{from which } \cos \Delta = \frac{2R - p}{2R}.$$

Also, if R and p are known, $d = 2\sqrt{p(R - p/4)}$.

65. Reverse Curve between Nonparallel Tangents. Given point A (PC) lying on a given tangent of a reverse curve ACB (Fig. 57); the radii R_1 and R_2; point V, the intersection of the initial and final tangents; and the angle θ (at V) between the tangents VA and VB. Required: the central angles Δ_1 and Δ_2 and the position B (PT).

Let $t =$ distance BV, which, together with Δ_1 and Δ_2, is to be determined.

Extend VB to meet $\perp$ from A to D; also extend AD and O_2B to meet $\perp$ through O_1 at points E and F.

By construction, angle $O_1AE = \theta$. Let $d = AV$, which is known. Hence $AD = d \sin \theta$ and $AE = R_1 \cos \theta$. Also

$$\cos \Delta_2 = \frac{O_2F}{O_1O_2} = \frac{R_2 + DE}{R_1 + R_2} = \frac{R_2 + R_1 \cos \theta - d \sin \theta}{R_1 + R_2}, \qquad (a)$$

from which Δ_2 may be found.

Since angle $AGC = \Delta_2$, we have

$$\Delta_2 = \Delta_1 + \theta, \qquad \text{or} \qquad \Delta_1 = \Delta_2 - \theta. \qquad (b)$$

Now

$$t + BD = d \cos \theta, \qquad (c)$$

in which

$$BD = FE = FO_1 - EO_1 = (R_1 + R_2) \sin \Delta_2 - R_1 \sin \theta. \qquad (d)$$

Substituting (d) in (c) and solving, we get

$$t = d \cos \theta + R_1 \sin \theta - (R_1 + R_2) \sin \Delta_2. \qquad (58)$$

Review Example. (*Case* 2). Given a broken-back curve with the PC (point A, Fig. 50) at Sta. 50 + 00. $R_1 = 2{,}000$ ft, $R_2 = 1{,}200$ ft, and $d = 600$ ft. Required: the station number of point D.

Solution. Assuming $R = 3{,}000$ ft, we have $R - R_1 = 1{,}000$ ft, $R - R_2 = 1{,}800$ ft. Then $\tan \alpha = (R_1 - R_2)/d = \frac{800}{600} = 1.3333333$, from which $\alpha = 52°07'42''$. Next, $O_1O_2 = 600/\sin \alpha = 1{,}013.444$ ft;

$$\cos \phi = \frac{O_1O_2 + OO_2 - OO_1}{2(O_1O_2 \times OO_2)} = \frac{(10.13444)^2 + (18)^2 - (10)^2}{2(10.13444)(18)} = 0.8954802,$$

from which $\phi = 26°25'47''$. Then $\sin \Delta = O_1O_2 \sin \phi / O_1O = 0.45108269$, from which $\Delta = 26°48'48''$. Arc $AD = R \times \Delta$ (Δ in radians) $= 3{,}000(0.4679809) = 1{,}403.94$ ft. Then point D is at Sta. $50 + 00 + 14.0394 = 64.039$.

66. Review Problems

1. Given a 2-centered compound curve with $\Delta_2 = 20°$, $D_2 = 3°24'$, $\Delta_1 = 25°$, $D_1 = 2°$. Calculate T_a and T_b, the long and short tangents.

Ans. $T_b = 854.32$ ft; $T_a = 1{,}086.03$ ft.

2. Given a 2-centered compound curve with the PCC at Sta. 40 + 20, $\Delta_1 = 20°$, $D_1 = 4°$, $\Delta_2 = 40°$, and $D_2 = 6°$. If the forward tangent is shifted 50 ft ($p = 50$) to a parallel position, which is *outside*, find the station number of point B', the new PT.

Ans. Sta. 47 + 74.6.

Interstate 81, Austin, Tex.; a six-lane expressway. (Photo courtesy Texas Highway Department.)

3. Given a 2-centered compound curve with AB = 1,200 ft, angle VAB = 27°21′, VBA = 32°20′, D_1 = 2°40′. Calculate T_a, T_b, Δ_1, Δ_2, and D_2.

Ans. T_a = 743.49 ft; T_b = 638.65 ft; Δ_1 = 6°21′27″; Δ_2 = 53°19′33″; D_2 = 5.2076°.

CHAPTER 5.

PARABOLIC CURVES

PART I: GENERAL TOPICS

67. Adaptability. Parabolic curves, owing to their easement qualities and adaptability to the method of offsets, are used exclusively for vertical curves connecting highway and railroad grade profile tangents. Also they are used widely in designing crowns, or cross profiles, for pavements.

For horizontal curves, however, the parabola fails to possess the advantages of the circle or spiral, since it cannot be laid out readily by deflections from a tangent, nor is its radius known at all points along the curve. But in laying out comparatively short curves, such as paths, without the aid of a transit, the parabola is easily adaptable and is sometimes used in lieu of a circular curve.

Horizontal Curves

68. Useful Properties. With a proper choice of axes, the equation of the ordinary parabola is $y = kx^2$. In the most general case, the coordinate axes are oblique; the X axis coincides with a tangent to the curve (as AV, in Fig. 58) and the Y axis is parallel to a so-called diameter, which is a line from V to the middle of chord AB.

For any ordinate LR (Fig. 58) parallel to the diameter VM, we have

$$\frac{LR}{VC} = \left(\frac{AL}{AV}\right)^2 \qquad \text{or} \qquad LR = \left(\frac{AL}{AV}\right)^2 \times VC.$$

That is, if $AL = \frac{3}{4}AV$, then $LR = (\frac{3}{4})^2 \times VC = \frac{9}{16}e$.

The foregoing illustrates a useful property of the parabola, namely, *the offsets from a tangent to the curve vary as the square of the distance along the tangent from the point of tangency.*

Observing that the tangents AV and VB may be unequal and that a diameter extends from V (at the intersection of the tangents) to point M at the middle of any chord AB (Fig. 58), another useful property of a parabola is the following: *The curve bisects a line joining the mid-point of a chord and the intersection of the tangents at the ends of the chord.*

69. Staking Out a Horizontal Curve by Tape Alone. Let it be required to locate a parabolic horizontal curve joining the tangents AV and VB at A and B (Fig. 58), the tangent lengths being not necessarily equal.

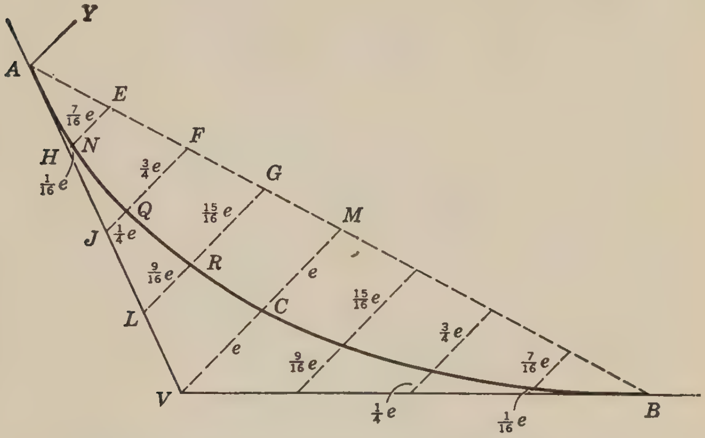

Y
A
E
$\frac{7}{16}e$
F
N
H
$\frac{1}{16}e$
$\frac{3}{4}e$
G
Q
$\frac{15}{16}e$
M
J
$\frac{1}{4}e$
R
$\frac{9}{16}e$
e
C
$\frac{15}{16}e$
L
$\frac{3}{4}e$
e
$\frac{9}{16}e$
$\frac{7}{16}e$
V
$\frac{1}{4}e$
$\frac{1}{16}e$
B

FIG. 58. Locating horizontal parabola with tape alone.

Procedure. Referring to Fig. 58, bisect chord AB at M; then bisect VM at C, giving a point on the parabola. As many intermediate points as are desired may be obtained as follows:

Divide MA and VA each into the same number of equal parts (four parts, for example). Let distance $VC = CM = e$.

$$\text{Ordinate } HN \text{ from tangent to curve} = \left(\frac{AH}{AV}\right)^2 \times VC = \tfrac{1}{16}e.$$

$$\text{Ordinate } JQ \text{ from tangent to curve} = \left(\frac{AJ}{AV}\right)^2 \times VC = \tfrac{1}{4}e, \text{ etc.}$$

Similarly, points defining portion CB of the parabola may be established.

Vertical Curves

70. Foreword. Vertical curves joining the intersecting grade lines of railroads, highways, or other routes tend to smooth out the changes in vertical motion. In fact, for railroads and highways, they contribute to the safety, comfort, and appearance in just as vital a manner as do horizontal curves. Only when the algebraic difference of the intersecting grades is less than say one-half of 1 percent do vertical curves become unnecessary.

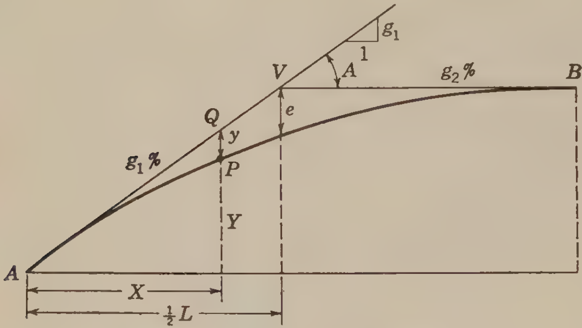


FIG. 59. Vertical curve (parabola).

All distances along vertical curves are measured *horizontally*, and all offsets from the tangents to the curve are measured *vertically*. Accordingly, the length of a vertical curve is its horizontal projection. The error resulting from this assumption is negligible in practice since the curve is quite flat except for special situations such as on ramps. But for clearness of presentation, all the vertical curves shown here are greatly exaggerated. Unless otherwise defined, vertical curves are symmetrical in the sense that the tangents are equal in length.

71. Vertical Offsets from Tangent to Curve. In Fig. 59, the vertical curve AB is referred to rectangular axes, where X (or x) and Y, with origin at A (the PC), represent the horizontal and vertical coordinates of any point P on the curve. The distance X (or x) is measured in stations, while all offsets are in feet.

Let g_1 and g_2 = percent grades of tangents AV and VB, respectively, using *plus* for grades ascending forward and *minus* for grades descending. Also, let $A = g_2 - g_1$ = algebraic difference in grades (percent) of the grade tangents, or their rate of *divergence*. The other notation follows.

L = total length of curve, in stations;

e = vertical offset (ft) from the vertex V (called PI) to the middle of curve.

Mathematically speaking, the slopes at ends A and B of the parabola are represented by g_1 and g_2, respectively. Also, since the rate of change in slope of all parabolas is constant, we have

$$\frac{d^2Y}{dX^2} = r = \text{a constant.} \tag{59}$$

Integrating, $$\frac{dY}{dX} = rX + C. \tag{a}$$

$$\text{When } X = 0,\ \frac{dY}{dX} = g_1; \quad \text{when } X = L,\ \frac{dY}{dX} = g_2.$$

and $$g_1 = 0 + C, \quad g_2 = rL + C; \quad \text{or} \quad r = \frac{g_2 - g_1}{L}. \tag{b}$$

Hence, $$\frac{dY}{dX} = \left(\frac{g_2 - g_1}{L}\right) X + g_1. \tag{c}$$

Integrating again, $$Y = \left(\frac{g_2 - g_1}{L}\right) \frac{X^2}{2} + g_1X + C'. \tag{d}$$

But $C' = 0$, since $Y = 0$ when $X = 0$.

From similar triangles, $(Y + y)/X = g_1/1$. Substituting this value of Y in (d), and reducing, we get

$$y = -\frac{1}{2}\left(\frac{g_2 - g_1}{L}\right) X^2.$$

Since y is measured *downward* (from Q) instead of *upward*, like Y, the above sign will be changed from minus to plus. Also, x will replace X. Then

$$y = \frac{1}{2}\left(\frac{g_2 - g_1}{L}\right) x^2. \tag{60}$$

Hence, the vertical offsets from a tangent to the vertical curve vary as the square of the distance from the PC.

At V, $y = e$ and $x = \frac{1}{2}L$; then

$$e = \left(\frac{g_2 - g_1}{8}\right) L. = \frac{1}{8}\,\text{AL}. \tag{61}$$

Strict attention must be paid to the algebraic signs of g_1 and g_2 in applying Eq. (61). Thus, if $g_1 = -3$, $g_2 = +5$, and $L = 600$ ft (6 stations), $e = \dfrac{5 - (-3)}{8} \times 6 = +6$ ft; if $g_1 = -1$, $g_2 = -5$, and $L = 10$ stations, $e = \dfrac{-5 - (-1)}{8} \times 10 = -5$ ft.

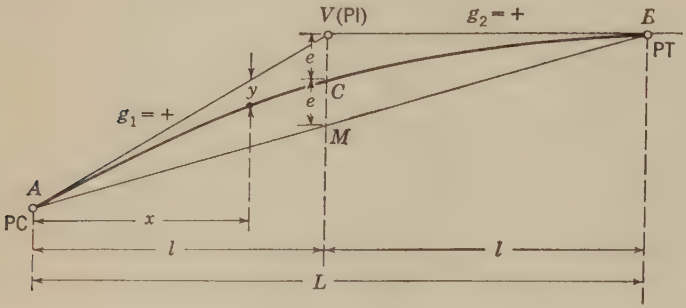
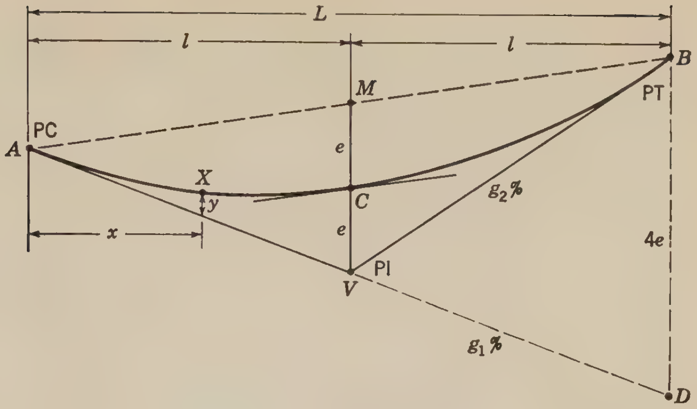


FIG. 60. Crest vertical curve.

Since e is the vertical distance from the vertex to the middle of the curve, it is measured downward in a *negative* direction for all "crest" curves (Fig. 60) and upward in a *positive* direction for "sag" curves (Fig. 61). Hence, the algebraic sign of e indicates whether elevations along the curve are to be obtained by adding or subtracting offsets from the corresponding tangent elevations.

Figure 62 shows in a much exaggerated manner several types of crest and sag curves representing six possible combinations of g_1 and g_2.

Another Proof for e. Since M bisects chord AB (Figs. 60 and 61) and since the mid-point C of the parabola is halfway between V and M, $e = \frac{1}{2}VM$.


L
l
l
B
M
PT
PC
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y
C
g_2%
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x
4e
PI
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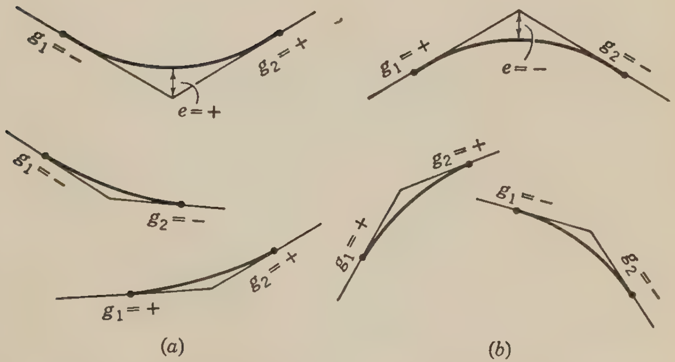
D

FIG. 61. Sag vertical curve.


$g_1 = -$
$g_2 = +$
$e = +$
$g_1 = +$
$e = -$
$g_2 = -$
$g_1 = -$
$g_2 = -$
$g_2 = +$
$g_1 = +$
$g_1 = -$
$g_1 = +$
$g_2 = +$
$g_2 = -$

(a) (b)

FIG. 62. Types of vertical curves. (a) Sag curves. (b) Crest curves.

The vertical divergence of the two grade tangents VD and VB (Fig. 61) is $(g_2 - g_1)$ ft per station; hence for l stations (where $l = \frac{1}{2}L$), we have

$$DB = (g_2 - g_1)l. \qquad (a)$$

From similar triangles, $VM = \frac{1}{2}DB = \frac{1}{2}(g_2 - g_1)l$; hence

$$e = VC = \tfrac{1}{2}VM = \tfrac{1}{4}(g_2 - g_1)l = \tfrac{1}{8}(g_2 - g_1)L. \qquad (b)$$

Also, $$DB = 4e = \tfrac{1}{2}(g_2 - g_1)L. \tag{62}$$

72. Elevations along Vertical Curves. Dividing Eq. (60) by Eq. (61) and reducing, we get

$$y = \left(\frac{x}{L}\right)^2 \times 4e \quad \text{or} \quad y = \left(\frac{x}{l}\right)^2 \times e. \tag{63}$$

Also, Eq. (60) may be written in the form

$$y = \tfrac{1}{2}rx^2, \tag{64}$$

where r [$= (g_2 - g_1)/L$] is defined as the *rate of change of grade*.

Use this ↓

TABLE A*

Station	x (stations)	x^2	Elev. on tangent	Offsets $(+0.2)x^2$	Elev. on curve	Differences	
48 + 50 PC	0	0	426.34	0	426.34		
49	0.5	0.25	425.84	+0.05	425.89		
49-49.5						> −0.60	
50	1.5	2.25	424.84	+0.45	425.29		> 0.40
						> −0.20	
51	2.5	6.25	423.84	+1.25	425.09		> 0.40
						> +0.20	
52	3.5	12.25	422.84	+2.45	425.29		> 0.40
						> +0.60	
53	4.5	20.25	421.84	+4.05	425.89		> 0.40
						> +1.00	
54	5.5	30.25	420.84	+6.05	426.89		> 0.40
						> +1.40	
55	6.5	42.25	419.84	+8.45	428.29		> 0.40
						> +1.80	
56	7.5	56.25	418.84	+11.25	430.09		
56 + 50 PT	8.0	64.00	418.34	+12.80	431.14		

rt with

f for setting

every 50 ft stn.

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* See Example 1 on next page.

Note: For points equally spaced, the second differences of the elevations must be constant. This is an important check applying to all vertical parabolic curves.

Both Eqs. (63) and (64) indicate simple ways of finding tangent offsets, which when combined successively with the corresponding elevations along the tangent will give the required curve elevations.

The entire curve may be established by offsets from the initial tangent (say AD in Fig. 61) or the first half may be referred to AV and the second half to BV, the only advantage being that the same set of offsets would apply to the second half of the curve. This implies the usual case of equal tangents and also symmetry as to stationing; however, the latter condition is not always realized. Referring the entire curve to only one tangent is recommended.

Two general methods each with certain variations in procedure will be described in detail by examples that follow. These are *Method A*, based on the *rule of offsets*, and *Method B*, based on the *rate of change of grade.*

Method A

Example 1. A −1.0 percent grade meets a +2.2 percent grade at Sta. 52 + 50 of elevation 422.34 ft. A vertical curve of length L = 800 ft (8 stations) will be used. For each station it is required to calculate (1) elevation on tangent, (2) offset from tangent to curve, and (3) elevation on curve. See Table A on the preceding page.

Solution

	Station	*Elevation (ft)*
PI	52 + 50	422.34
PC	48 + 50 = 422.34 + 4.00 =	426.34
PT	56 + 50 = 422.34 + 8.80 =	431.14
		2)857.48
		428.74
		422.34
		6.40 = 2 × e

Then $4 \times e = 12.80$ ft.

Also, by Eq. (62), $4e = \frac{1}{2}[+2.2 - (-1.0)] \times 8 = 12.80$ ft. $g_1 = -1.0$; $L = 8$; $y = (x/L)^2 \times 4e = (x^2/64) \times 12.80 = (+0.2)x^2$.

Example 2. Given the same data as in Example 1; that is, $g_1 = -1.0$, $g_2 = +2.2$, $L = 8$ stations, and the PI at Sta. 52 + 50 of elevation 422.34 ft. Required: elevations along the curve by using $y =$

$C \times e$, where the coefficient C is taken directly from Table 31. The results are given in Table B that follows.

$g_1 = -1.0$; $L = 800$ ft; $y = C \times e = C \times 3.2$

Use this ↓

TABLE B ✓ Table 31

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Station	x (ft)	Elev. on AV and BV	C (Table 31)	Offset $C \times 3.2$	Elev. on curve
48 + 50 PC	0	426.34	0	0	426.34
49	50	425.84	0.0156	0.05	425.89
50	150	424.84	0.1406	0.45	425.29
51	250	423.84	0.3906	1.25	425.09
52	350	422.84	0.7656	2.45	425.29
52 + 50 PI	400	422.34	1	3.20	425.54
53	350	423.44	0.7656	2.45	425.89
54	250	425.64	0.3906	1.25	426.89
55	150	427.84	0.1406	0.45	428.29
56	50	430.04	0.0156	0.05	430.09
56 + 50 PT	0	431.14	0	0	431.14

Method B. By Eq. (64), $y = \frac{1}{2}rx^2$, where $r = (g_2 - g_1)/L$ = rate of change of grade.

The elevation E_x of any point on the curve x stations ahead of the PC is given by the expression

$$E_x = E_a + g_1x + y, \tag{65}$$

where E_a = elevation of point A (the PC) and $y = \frac{1}{2}rx^2$.

Then $$E_x = E_a + g_1x + \tfrac{1}{2}rx^2 = E_a + (g_1 + \tfrac{1}{2}rx)(x). \tag{66}$$

Equation (66) provides the simplest method for obtaining elevations (E_x) along the curve. Since the factor $(g_1 + \frac{1}{2}rx)$ is linear, its value is found easily by setting g_1 in the calculator and continuously adding (or subtracting) constant increments from station to station. This avoids squaring quantities and dealing with elevations along the tangent. Merely add (algebraically) in succession the products of the two factors to E_a (the elevation of the PC) and the result is E_x, the desired curve elevation. This method will now be applied to the case treated previously.

Example 3. Given the same data as in Examples 1 and 2; that is, $g_1 = -1.0$, $g_2 = +2.2$, $L = 8$ stations, and the PI at Sta. 52 + 50 of elevation 422.34 ft.

Solution. $r = (g_2 - g_1)/L = [2.2 - (-1.0)]/8 = 0.4$; $\frac{1}{2}r = 0.2$. By Eq. (66),

$$E_x = 426.34 + (-1.0 + 0.2x)(x). \tag{67}$$

See Table C.

TABLE C

Station	x (stations)		$(-1.0 + 0.2x)$		Product (ft)	Elev. on curve
48 + 50 PC	0	×	−1.0	=	0	426.34
49	0.5	×	−0.9	=	−0.45	425.89
50	1.5	×	−0.7	=	−1.05	425.29
51	2.5	×	−0.5	=	−1.25	425.09
52	3.5	×	−0.3	=	−1.05	425.29
53	4.5	×	−0.1	=	−0.45	425.89
54	5.5	×	+0.1	=	+0.55	426.89
55	6.5	×	+0.3	=	+1.95	428.29
56	7.5	×	+0.5	=	+3.75	430.09
56 + 50 PT	8.0	×	+0.6	=	+4.80	431.14

73. Factors Governing the Length of Vertical Curves. In selecting the length of vertical curves over crests or into sags, the algebraic difference of gradients enters into the calculations, for both highways and railroads. For the former the controlling factors are safe visibility, comfort, and appearance; for the latter, it is rate of *change of grade.*

From the standpoint of operating efficiency, smooth riding, and pleasing appearance, vertical curves should have sufficient length to hold the rate of change of grade to a minimum. The AREA recommends for high-speed main tracks "rates of change of grade not exceeding 0.10 on summits or 0.05 in sags." Thus if a +1.00 percent grade meets a −0.20 percent grade (producing a summit), the total change of grade is −0.20 − 1.00 = −1.20;

hence to satisfy the AREA requirement, $1.20/L = 0.10$, or $L = 12$ stations (1,200 ft).

Also, on highways considerations of comfort require that a rate of change of grade be kept within tolerable limits. This is most important in sag curves where gravitational and centrifugal forces act in the same direction. A long curve has a more pleasing appearance than a short one; a smooth grade line with gradual changes is preferred to a line with numerous breaks and short lengths of grade. A broken-back grade line should be avoided.

To satisfy the requirements of minimum stopping sight distance, comfort, and appearance, the AASHO specifies a value of L not shorter than KA, where A is the algebraic difference of grades in percent, and the values of K are as follows:

Design speed, mph	30	40	50	60	70
Minimum K value for:					
Crest vertical curves	28	50	80	150	240
Sag vertical curves	35	50	70	100	140

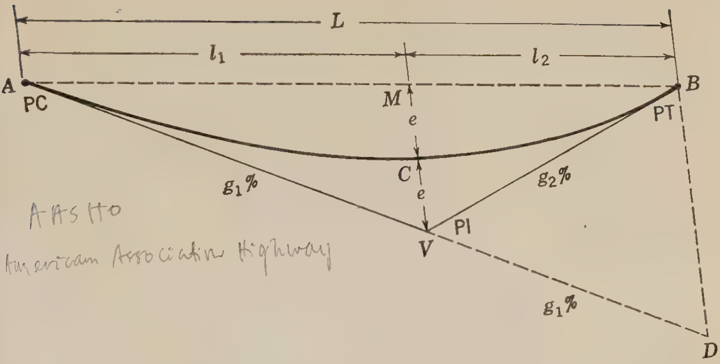
74. Unsymmetrical Curves. Figure 63 shows an unsymmetrical parabolic vertical curve connecting the tangents AV and VB, which are unequal. It is possible that an unsymmetrical curve will fit more closely certain imposed requirements than the usual symmetrical equal-tangent curve.

Let a vertical line through the vertex V intersect the curve at C and the chord AB at M. Points C and M are not mid-points of the curve and chord, nor is C the lowest point of the curve, but C bisects the vertical VM, as is proved in Appendix B.

a. The Offset e from V (the PI) to the Curve. It is required to derive a formula for the offset e from V to the curve.

As before, $A = g_2 - g_1$ = algebraic difference in the grades of tangents AV and VB, and e is the vertical distance from V to the curve. Also, let l_1 and l_2 = lengths (in stations) of curves AC and CB, respectively, and $L = l_1 + l_2$ = total length of curve, assumed to be the same as its horizontal projection.

The vertical divergence of the two grade tangents VD and VB



AASHTO
American Association Highway

FIG. 63. Unsymmetrical vertical curve.

(Fig. 63) is $(g_2 - g_1)$ ft per station; hence for l_2 stations, we have

$$DB = (g_2 - g_1)l_2. \tag{a}$$

From similar triangles, $DB/VM = L/l_1$, or

$$DB = \frac{L}{l_1} VM = \frac{L}{l_1} \times 2e. \tag{b}$$

Equating (*a*) and (*b*) and reducing, we have

$$e = \frac{g_2 - g_1}{2L} l_1 l_2. \tag{68}$$

Letting $l_2 = l_1 = \frac{1}{2}L$, Eq. (68) reduces to Eq. (61).

b. Elevations in Terms of e. The elevation E_x for any point on the curve is as follows: For the left branch *AC*,

$$E_x = E_a + g_1 x + \left(\frac{x}{l_1}\right)^2 e, \tag{69}$$

where E_a is the elevation of the PC.

For the right branch *BC*,

$$E_x = E_b - g_2 x + \left(\frac{x}{l_2}\right)^2 e, \tag{70}$$

where E_b is the elevation of the PT and a negative sign precedes the term $g_2 x$ since the direction of measurement is reversed, x being referred to point *B*, the PT.

c. Elevations in Terms of "Rate of Change of Grade." If r_1 and r_2 = *rate of change of grade* of the tangents AV and VB, respectively, it follows (see Appendix B) that

$$r_1 = \frac{g_2 - g_1}{L}\frac{l_2}{l_1} \quad \text{and} \quad r_2 = \frac{g_2 - g_1}{L}\frac{l_1}{l_2}. \tag{a}$$

Now

$$e = \frac{g_2 - g_1}{2L} l_1 l_2, \tag{b}$$

as in Eq. (68).

Dividing (*b*) by (*a*), we get

$$\frac{e}{r_1} = \tfrac{1}{2}l_1^2 \quad \text{and} \quad \frac{e}{r_2} = \tfrac{1}{2}l_2^2,$$

from which

$$e = \tfrac{1}{2}r_1 l_1^2 \quad \text{and} \quad e = \tfrac{1}{2}r_2 l_2^2. \tag{c}$$

Substituting (*c*) in turn in Eqs. (69) and (70), we have

$$E_x = E_a + g_1 x + (\tfrac{1}{2}r_1)x^2; \tag{71}$$
$$E_x = E_b - g_2 x + (\tfrac{1}{2}r_2)x^2. \tag{72}$$

Equations (71) and (72) give elevations of points along the left and right branches of an unsymmetrical curve. As a check, the elevation of point C opposite the vertex should be the same by both formulas.

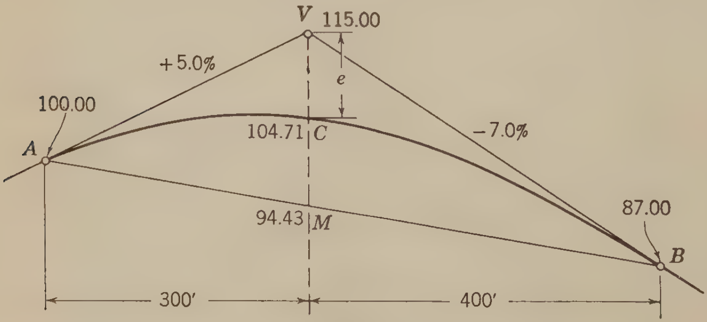


FIG. 64

Example. A +5.0 percent grade meets a −7.0 percent grade (Fig. 64) at Sta. 32 + 00 of elevation 115.00 ft. The tangents are unequal; l_1 = 300 ft (3 stations), l_2 = 400 ft (4 stations), and the length of curve L is 700 ft. Required: curve elevations.

Solution

$$r_1 = \frac{g_2 - g_1}{L}\frac{l_2}{l_1} = \frac{-12}{7} \times \frac{4}{3} = -2.2857; \qquad \tfrac{1}{2}r_1 = -1.1428.$$

$$r_2 = \frac{g_2 - g_1}{L}\frac{l_1}{l_2} = \frac{-12}{7} \times \frac{3}{4} = -1.2857; \qquad \tfrac{1}{2}r_2 = -0.6428.$$

$$E_x = E_a + g_1x + \tfrac{1}{2}r_1x^2; \qquad (g_1 + \tfrac{1}{2}r_1x) = 5.0 - 1.1428x.$$

$$E_x = E_b - g_2x + \tfrac{1}{2}r_2x^2; \qquad (-g_2 + \tfrac{1}{2}r_2x) = 7.0 - 0.6428x.$$

Station	x (stations)	$5.0 - 1.1428x$	Product	Elev. on curve	Differences	
29 + 00 PC	0	× 5.0	= 0	100.00		
					> +3.86	
30	1	× 3.8572	= + 3.86	103.86		> 2.29
					> +1.57	
31	2	× 2.7144	= + 5.43	105.43		> 2.29
					> −0.72	
32	3	× 1.5716	= + 4.71	104.71		
		$7.0 - 0.6428x$				
32	4	× 4.4288	= +17.71	104.71		
					> −2.50	
33	3	× 5.0716	= +15.21	102.21		> 1.28
					> −3.78	
34	2	× 5.7144	= +11.43	98.43		> 1.29
					> −5.07	
35	1	× 6.3572	= + 6.36	93.36		> 1.29
					> −6.36	
36 + 00 PT	0	× 7.0	= 0.0	87.00		

Note: The second differences are constant for each branch of the curve, being −2.29 along *AC* and −1.29 along *CB*.

75. Turning Point on a Vertical Curve. The turning point is the highest (or lowest) point on the curve profile. This point is usually not vertically below or above the PI, but on the right or left branch of the curve where its slope is horizontal, as at *H* in Fig. 65. It remains to determine not only the *position* but its actual *elevation* as well.

The low points are useful in connection with drainage problems and the overhead clearance to structures. It should be noted that the rates of change of grade for sags are positive.

Treating first the *equal-tangent* parabola, expressions will be derived which, in a similar form, apply also to the curve with *unequal* tangents.

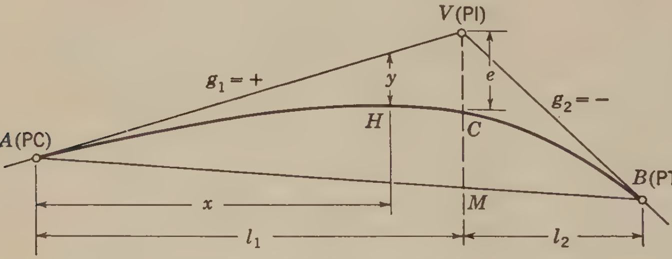


FIG. 65. High point on vertical curve.

a. Position of Turning Point. 1. *Symmetrical curves.* The position and elevation of any point on the curves are given in Eq. (66), that is,

$$E_x = E_a + g_1 x + (\tfrac{1}{2}r)x^2. \tag{73}$$

For maximum (or minimum), $dE_x/dx = 0$, giving $g_1 + rx = 0$; hence, solving for x, we obtain the distance x_t (in stations) from the PC to the turning point,

$$x_t = -\frac{g_1}{r} = \frac{g_1 L}{g_1 - g_2}. \tag{74}$$

For the usual summit curve, g_1 is positive and r is negative; for sag curves, g_1 is negative and r positive. Hence the distance x_t will be positive in all cases.

Example 1. A −5 percent grade meets a +4 percent grade at Sta. 27 + 00. A 600-ft vertical curve is used. Required: position of the low point of the curve.

Solution

$$x_t = \frac{g_1 L}{g_1 - g_2} = \frac{-5 \times 6}{-5 - 4} = \frac{-30}{-9} = 3.333 \text{ stations} = 333.3 \text{ ft}$$

from the PC.

2 *Unsymmetrical curves.* Letting $dE_x/dx = 0$ in Eqs. (71) and (72) and reducing, we get

$$x_1 = -\frac{g_1}{r_1} = \frac{g_1 L}{g_1 - g_2}\frac{l_1}{l_2} \tag{75}$$

and

$$x_2 = \frac{g_2}{r_2} = \frac{g_2 L}{g_2 - g_1}\frac{l_2}{l_1}, \tag{76}$$

where x_1 = distance from the PC to turning point on the left branch and x_2 = distance from the PT to turning point on right branch (if it occurs here). The values of x_1 and x_2 (in stations) do not exceed l_1 and l_2, respectively.

b. Elevation of Turning Point. 1. *Symmetrical curves.* Substituting Eq. (74) in Eq. (73) and reducing, we get

$$E_m = E_a - \frac{1}{2}\frac{g_1^2}{r} = E_a - \frac{1}{2}\frac{Lg_1^2}{g_2 - g_1}, \tag{77}$$

where E_m = elevation of turning point.

Example 2. A -5 percent grade meets a $+4$ percent grade at Sta. 27 + 00. A 600-ft vertical curve is to be used with elevation of PC = 100.00 ft. Required: elevation of the low point.

Solution

$$E_m = 100.00 - \frac{1}{2}\frac{6(-5)^2}{4 - (-5)} = 100.00 - 8.33 = 91.67 \text{ ft.}$$

2. *Unsymmetrical curves.* Substituting Eqs. (75) and (76) in Eqs. (71) and (72) and reducing, we get

$$E_1 = E_a - \frac{1}{2}\frac{g_1^2}{r_1} = E_a - \frac{1}{2}\frac{Lg_1^2}{g_2 - g_1}\frac{l_1}{l_2}, \tag{78}$$

$$E_2 = E_b - \frac{1}{2}\frac{g_2^2}{r_2} = E_b - \frac{1}{2}\frac{Lg_2^2}{g_2 - g_1}\frac{l_2}{l_1}, \tag{79}$$

where E_1 and E_2 = elevation of turning point on left (or right) branch as the case may be.

PART II: SPECIAL TOPICS

76. Passing a Curve through Points of Fixed Elevation. If the curve is *symmetrical* (equal tangent), it may be established readily so as to pass through a point of given position and elevation by substituting the known data in Eq. (66) and solving for L, the length of curve, as will be shown in the examples that follow.

Example 1. Given $g_1 = -4.0$ percent; $g_2 = +3.6$ percent grade, $E_a = 375.62$ ft at Sta. 82 + 40. It is required to find the length of a vertical curve that will pass through Sta. 90 + 00 of elevation 368.08 ft.

Solution

$$x = 90.0 - 82.4 = 7.6 \text{ stations}; \qquad r = \frac{g_2 - g_1}{L} = \frac{7.6}{L}; \qquad \tfrac{1}{2}r = \frac{3.8}{L}.$$

By Eq. (66), we have

$$E_x = E_a + g_1 x + \tfrac{1}{2}rx^2;$$

that is,

$$368.08 = 375.62 - 4(7.6) + \left(\frac{3.8}{L}\right)(7.6)^2,$$

from which

$$\frac{219.488}{L} = 22.86 \qquad \text{or} \qquad L = \frac{219.488}{22.86} = 9.60 \text{ stations} = 960 \text{ ft.}$$

Example 2. A -1.2 percent grade meets a $+1.8$ percent grade at Sta. 23 + 40 of elevation 209.04 ft. Required: length of curve that will pass through Sta. 24 + 00 of elevation 211.56 ft.

Solution

$$r = \frac{g_2 - g_1}{L} = \frac{1.8 - (-1.2)}{L} = \frac{3.0}{L}. \qquad (a)$$

$$E_a = E_v - g_1 \times \tfrac{1}{2}L = 209.04 + 0.6L. \qquad (b)$$

$$\text{Stationing of PC} = 23.40 - \tfrac{1}{2}L. \qquad (c)$$

$$x = \text{distance (stations) from PC to Sta. } 24 + 00 = 0.6 + \tfrac{1}{2}L. \qquad (d)$$

$$g_1 x = -0.72 - 0.6L. \qquad (e)$$

$$(\tfrac{1}{2}r)x^2 = \frac{1.5}{L}(0.6 + \tfrac{1}{2}L)^2. \qquad (f)$$

Substituting (*b*), (*e*), and (*f*) in Eq. (66), combining terms, and noting that $E_x = 211.56$, we get

$$\tfrac{3}{8}L - 2.34 + \frac{0.54}{L} = 0; \qquad \text{or} \qquad L^2 - 6.24L + 1.44 = 0;$$

from which $L = 6$ stations $= 600$ ft.

77. Changing Position of Forward Tangent. The forward grade tangent is to be steepened or flattened while the PC and the initial grade tangent remain unchanged. Let g_1, g_2, L, and r refer to the old curve; g_1, g'_2, L', and r' refer to the new curve.

Example. Given $g_1 = 3.0$ percent, $g_2 = -5.0$ percent, $L = 6$ stations. The forward tangent is to be flattened so that $g'_2 = -3.4$ percent. Assume the PI to be moved forward from Sta. 27 + 00 (elevation 601.70) to Sta. 28 + 00 (elevation 604.70) so that $L' = 8$ stations, while the PC remains fixed (elevation 592.70 ft). Required: elevations along the new curve.

Solution

E'_b (the new PT) $= E'_v + g'_2 \times \frac{1}{2}L' = 604.70 - 3.4 \times 4 = 591.10$ ft.

$$r' = \frac{g'_2 - g_1}{L'} = \frac{-3.4 - 3.0}{8} = \frac{-6.4}{8} = -0.8; \quad \tfrac{1}{2}r' = -0.4.$$

By Eq. (67),

$$E'_x = E_a + (g_1 + \tfrac{1}{2}r'x) \times (x); \qquad g_1 + \tfrac{1}{2}r'x = 3.0 - 0.4x.$$

Station	x (stations)	$3.0 - 0.4x$	Product	Elev. on curve	Differences	
24 + 00 PC	0	× 3.0	= 0	592.70		
					> 2.60	
25	1	× 2.6	= +2.6	595.30		> 0.80
					> 1.80	
26	2	× 2.2	= +4.4	597.10		> 0.80
					> 1.00	
27	3	× 1.8	= +5.4	598.10		> 0.80
					> 0.20	
28	4	× 1.4	= +5.6	598.30		> 0.80
					> −0.60	
29	5	× 1.0	= +5.0	597.70		> 0.80
					> −1.40	
30	6	× 0.6	= +3.6	596.30		> 0.80
					> −2.20	
31	7	× 0.2	= +1.4	594.10		> 0.80
					> −3.00	
32 PT	8	× −0.2	= −1.6	591.10		

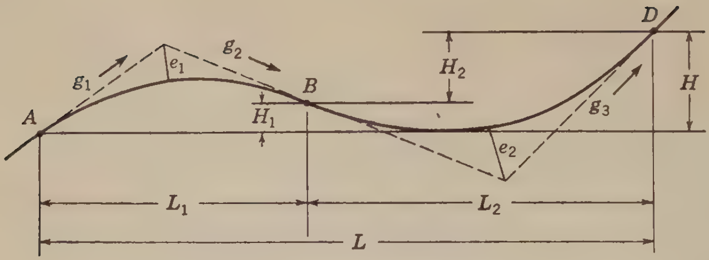


FIG. 66. Reverse vertical curve.

78. Reverse Vertical Curves.* The two vertical curves AB and BD (Fig. 66), having a common tangent at B, may represent the vertical alignment for an interchange ramp between two roadways. Formulas giving relations between the quantities involved follow.

Let g_1 and g_3 = percent grades of approaching and receding tangents while g_2 = grade of common tangent.

L_1 and L_2 = lengths of first and second curves; $L = L_1 + L_2$.

E_a, E_b, E_d = elevations of points A, B, D, respectively.

$H_1 = E_b - E_a$ = difference in elevation of B and A.

$H_2 = E_d - E_b$ = difference in elevation of D and B.

$H = E_d - E_a$ = difference in elevation of D and A.

r_1, r_2 = rate of change of grade of the first and second curves, respectively.

Then

$$r_1 = \frac{g_2 - g_1}{L_1}, \quad \text{or} \quad L_1 = \frac{g_2 - g_1}{r_1}; \tag{a}$$

$$r_2 = \frac{g_3 - g_2}{L_2}, \quad \text{or} \quad L_2 = \frac{g_3 - g_2}{r_2}. \tag{b}$$

Adding (*a*) and (*b*),

$$L = \frac{g_2 - g_1}{r_1} + \frac{g_3 - g_2}{r_2}. \tag{c}$$

Solving (*c*) for g_2, we get

$$g_2 = \frac{r_1(r_2L - g_3) + r_2g_1}{r_2 - r_1}. \tag{d}$$

With the value of g_2 known from (*d*), the corresponding values of L_1 and L_2 can be found from Eqs. (*a*) and (*b*).

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* See Clarence J. Brownell, Rate of Change of Grade per Station, *Proc. ASCE*, no. 121, March, 1952.

Next, we have

$$\text{Elev. } B = \text{elev. } A + g_1 \times \tfrac{1}{2}L_1 + g_2 \times \tfrac{1}{2}L_1,$$

that is,

$$E_b = E_a + (g_1 + g_2) \times \tfrac{1}{2}L_1.$$

Hence,

$$H_1 = E_b - E_a = (g_1 + g_2) \times \tfrac{1}{2}L_1. \qquad (e)$$

Likewise,

$$\text{Elev. } D = \text{elev. } B + g_2 \times \tfrac{1}{2}L_2 + g_3 \times \tfrac{1}{2}L_2,$$

that is,

$$E_d = E_b + (g_2 + g_3) \times \tfrac{1}{2}L_2.$$

Hence,

$$H_2 = E_d - E_b = (g_2 + g_3) \times \tfrac{1}{2}L_2, \qquad (f)$$

$$H = H_1 + H_2. \qquad (g)$$

Substituting the given data in Eqs. (*a*) to (*g*), the other data may be found by solving the resulting equations simultaneously.

The usual sign notation must be followed in applying these equations: *plus* for grades ascending forward, *minus* for grades descending. Accordingly, r_1 (or r_2) is positive for sags and negative for crests.

Example 1. Given $g_1 = 4$ percent, $g_3 = 6$ percent, and $L = 4$ stations. Required: the values of g_2, L_1, L_2, H_1, H_2, and H.

Solution. Assume $r_1 = -2$, $r_2 = 3$. Substituting in Eq. (*d*), we get

$$g_2 = \frac{(-2)(12 - 6) + 3(4)}{5} = \frac{-12 + 12}{5} = 0.$$

Then

$$L_1 = \frac{0 - 4}{-2} = 2; \qquad L_2 = \frac{6 - 0}{3} = 2 \text{ stations.}$$

$$H_1 = (g_1 + g_2) \times \tfrac{1}{2}L_1 = 4 \text{ ft}; \qquad H_2 = (g_2 + g_3) \times \tfrac{1}{2}L_2 = 6 \text{ ft};$$

$$H = H_1 + H_2 = 4 + 6 = 10 \text{ ft.}$$

Example 2. Given $g_1 = 2$ percent, $g_3 = 4$ percent, and $H = 18$ ft. Required: g_2, L_1, L_2, and L.

Solution. Assume $r_1 = -0.5$, $r_2 = 0.5$. These and the given data will be substituted in expressions that follow. From Eq. (*g*), we have

$$H_1 + H_2 = H = 18. \qquad (h)$$

Now

$$H_1 = (g_1 + g_2) \times \tfrac{1}{2}L_1 = \frac{g_2^2 - g_1^2}{2r_1} = \frac{g_2^2 - 4.0}{2(-0.5)}. \qquad (i)$$

Also,

$$H_2 = (g_2 + g_3) \times \tfrac{1}{2}L_2 = \frac{g_3^2 - g_2^2}{2r_2} = \frac{16 - g_2^2}{2(0.5)}. \qquad (j)$$

Substituting (*i*) and (*j*) in (*h*) and solving, we get $g_2 = 1.0$ percent. Then,

$$L_1 = \frac{1.0 - 2.0}{-0.5} = 2.0; \qquad L_2 = \frac{4.0 - 1.0}{0.5} = 6.0;$$

$$L = 2.0 + 6.0 = 8 \text{ stations.}$$

79. Review Problems

1. Two intersecting profile grade lines consist of a +5 percent grade meeting a −4 percent grade at Sta. 92 + 00, and the length of vertical curve is 800 ft (8 stations).

(*a*) What is the rate of change of grade? *Ans.* −1.125.

(*b*) What is the offset (*e*) from PI to the middle of the curve?
we 50ft sta to calculate *Ans.* −9.00 ft.

(*c*) What is the tangent offset locating Sta. 90 + 00 on the curve?
Ans. −2.25 ft.

(*d*) Calculate elevations to the nearest hundredth for the full stations along the curve.

2. Two intersecting profile grades consist of a −3 percent grade meeting a +5 percent grade at Sta. 112 + 00 whose elevation is 674.50 ft. The length of curve is 6 stations.

(*a*) What is the rate of change of grade? What is the offset from the PI to the curve? *Ans.* $r = 1.333$; $e = 6.00$ ft.

(*b*) What is the elevation of Sta. 111 + 00 on the curve?
Ans. 680.17 ft.

(*c*) Calculate elevations to the nearest hundredth for the full stations along the curve.

3. A +6 percent grade meets a +2 percent grade at Sta. 49 + 00 whose elevation is 842.20 ft, and the length of curve is 4 stations.

(*a*) What is the rate of change of grade? What is the offset from the PI to the curve? *Ans.* $r = -1.00$; $e = -2.00$ ft.

(*b*) What is the elevation of Sta. 50 + 00 on the curve?
Ans. 843.70 ft.

(*c*) Calculate elevations to the nearest hundredth for the full stations along the curve.

4. Two intersecting railroad profile grades consist of a −0.286 percent grade meeting a +0.308 percent grade at Sta. 80 + 00 whose elevation is 846.52 ft, and the length of vertical curve is 12 stations.

What is the rate of change of grade? Is the AREA recommendation satisfied? *Ans.* $r = 0.0495$; yes.

5. A −6 percent grade meets a +4 percent grade at Sta. 160 + 00 whose elevation is 400.20 ft, and the length of curve is 8 stations.

(*a*) What is the elevation of the middle point of curve?
Ans. 410.20 ft.

(*b*) What are the position and the elevation of the low point of curve? *Ans.* Sta. 160 + 80; 409.80 ft.

6. A +5 percent grade meets a −5 percent grade at Sta. 20 + 00 whose elevation is 100.00 ft. An unsymmetrical vertical curve of length 10 stations is used, where $l_1 = 6$ and $l_2 = 4$ stations.

(*a*) What are the rates of change of grade in the left and right branches of the curve? What is the offset (e) from the PI to the curve? *Ans.* $r_1 = -0.667$; $r_2 = -1.50$; $e = -12.00$ ft.

(*b*) What is the elevation of Sta. 17 + 00? *Ans.* 82.00 ft.

(*c*) What is the elevation of Sta. 22 + 00? *Ans.* 87.00 ft.

(*d*) Tabulate the elevations to the nearest hundredth for all full stations along the curve.

7. The PC of a vertical curve is at Sta. 80, elevation 100.00. For any one of the six curves listed below determine the elevations of stations along the curve.

Curve number	g_1, %	g_2, %	L (ft)
1	+3	+5	1,000
2	+3	+2	600
3	+3	−5	2,000
4	−4	−5	600
5	−4	−2	1,000
6	−4	+5	2,000

Mt. Cheam Trans-Canada Highway. (Photo courtesy British Columbia Government.)

CHAPTER 6

TRANSITION SPIRALS

80. Introduction. Transition spirals for railroads and highways are curves which provide a gradual change in curvature from a *straight* to a *circular* path. Such *easement* curves have long been found necessary on high-speed railroads from the standpoint of comfortable operation and of gradually bringing about the full superelevation of the outer rail on curves.

The use of transition spirals in the design of highways possesses the same general advantages as for railroads, with the added "factor of safety," namely, a *transitional path is provided,* thus reducing the tendency to deviate from the logical traffic lane.

When a car traveling on a straight stretch of highway reaches a circular path, the steering wheel must be set at a new angle depending upon the radius of the curve. This movement cannot be done instantly but in a measurable time interval, thus creating a demand for a transition curve, the length of which equals *speed* × *time.*

81. Notation and Formulas

TS = point of change from tangent to spiral;
SC = point of change from spiral to circle;
CS = point of change from circle to spiral;
ST = point of change from spiral to tangent;
SS = point of change from one spiral to another;
l = spiral arc from the TS to any point on spiral;
l_s = total length of spiral from TS to SC;
θ = central angle of spiral arc l;
θ_s = central angle of spiral arc l_s, called "spiral angle";
ϕ = spiral deflection angle at the TS from initial tangent to any point on spiral;
D = degree of curve of the spiral at any point; R = radius;
D_c = degree of curve of the shifted circle to which the spiral

becomes tangent at the SC. See Fig. 67. R_c = its radius;

$K = D_c/l_s$ = rate of change of degree of curvature per foot of spiral;

Δ = total central angle of the circular curve;

Δ_c = central angle of circular arc of length L_c extending from SC to CS;

y = tangent offset of any point on spiral with reference to TS and initial tangent; y_s = tangent offset at the SC;

x = tangent distance of any point on spiral with reference to TS and initial tangent; x_s = tangent distance for the SC;

p = offset from the initial tangent to the PC of the shifted circle; $p = y_s' - R_c \text{ vers } \theta_s$. See Table 15;

k = abscissa of the shifted PC referred to the TS;

$k = x_s - R_c \sin \theta_s$. See Table 15;

a = distance from PC to TS of the spiral which is applied to an existing (unshifted) circular curve. Special case, see Table 26;

T_s = total tangent distance = distance from PI to TS, or from PI to ST; $T_s = (R_c + p) \tan \frac{1}{2}\Delta + k$. See Table 18;

E_s = total external distance = $(R_c + p) \text{ exsec } \frac{1}{2}\Delta + p$. See Table 19;

LC = long chord; LT = long tangent; ST = short tangent. See Fig. 70;

$$l_s = \frac{200\theta_s}{D_c};\ D_c = \frac{200\theta_s}{l_s};\ \theta_s = \frac{l_s D_c}{200};$$

$$\theta = \left(\frac{l}{l_s}\right)^2 \theta_s;\ \phi = \frac{\theta}{3} - C_s = \frac{\theta_s}{3}\left(\frac{l}{l_s}\right)^2 - C_s, \quad \text{(see Table 20).}$$

82. Development of Spiral Theory. In Fig. 67, the curve APC is a transition spiral connecting the tangent AX with point C (SC) on a circle, called *osculating circle*. At point A on the tangent where the spiral begins (the TS), its radius is infinite; at the point of tangency with the circular arc, its radius is R_c and its degree of curve is D_c, the same as that of the circle; at any intermediate point of the spiral arc (point P), its radius is R and its degree of curve is D, where $D:D_c$ = arc AP:arc AC.

∴ intermediate pt.

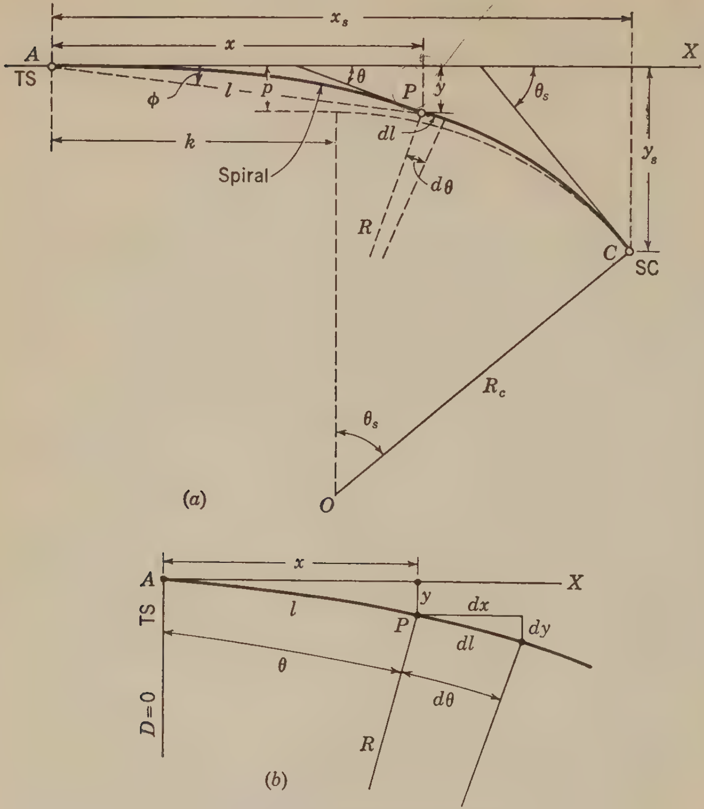
x_s
x
A
TS
X
ϕ
$l - p$
θ
P
y
θ_s
y_s
k
dl
$d\theta$
Spiral
R
C
SC
R_c
θ_s
(a)
O
x
A
X
TS
y
dx
l
P
dy
dl
θ
$d\theta$
D=0
R
(b)

FIG. 67. Elements of the spiral.

In Fig. 67, let

R = variable radius at any point P of spiral ($R = R_c$ at the SC);

l = length of spiral arc AP ($l = l_s$ at the SC);

θ = central angle of arc AP ($\theta = \theta_s$ at the SC);

x and y = coordinates of point P with reference to the TS and axis AX (x and y become x_s and y_s at the SC).

Since the degree of curve varies directly as the distance l from the TS, its radius must vary *inversely* as the distance; hence

$$R = \frac{K}{l}, \tag{a}$$

in which K is a constant.

At point C (SC), the end of the spiral, the above equation becomes

$$R_c = \frac{K}{l_s}. \tag{b}$$

Eliminating K by dividing (a) by (b), we have

$$R = \frac{R_c l_s}{l}. \tag{80}$$

From the differential sector at point P, we have $d\theta = dl/R$. Substituting the value of R from Eq. (80) gives

$$d\theta = \frac{l}{R_c l_s}\,dl.$$

Integrating,
$$\theta = \frac{l^2}{2R_c l_s} \quad (\theta \text{ in radians}). \tag{81}$$

Since R_c and l_s are constants, it is seen that θ varies as the *square of the distance* from the TS.

At point C (the SC) $\theta = \theta_s$, which is called the *spiral angle*. Letting $\theta = \theta_s$, then $l = l_s$; hence Eq. (81) becomes

$$\theta_s = \frac{l_s^2}{2R_c l_s} = \frac{l_s}{2R_c}, \qquad \text{or} \qquad l_s = 2R_c\theta_s, \tag{82}$$

in which θ_s is measured in radians.

Substituting $5{,}729.58/D_c$ for R_c and expressing θ_s in degrees, Eq. (82) becomes

$$\theta_s = \frac{l_s}{200} D_c, \tag{83}$$

or

$$l_s = \frac{200\theta_s}{D_c}. \tag{84}$$

Hence the total length of spiral is twice the circular arc of central angle θ_s and radius R_c (and degree of curve, D_c).

Comparing Eqs. (81) and (82), we have $\theta:\theta_s = l^2:l_s^2$, or

$$\theta = \left(\frac{l}{l_s}\right)^2 \theta_s. \tag{85}$$

Equation (85) is the most convenient formula for finding θ at any point along the spiral. Since l/l_s is an abstract ratio, θ and θ_s may be expressed in either radians or degrees, the latter being preferable in practice.

From the differential triangle at P (shown to an enlarged scale) in Fig. 67*b*, we have $dy = dl \sin \theta$ and $dx = dl \cos \theta$.

Expanding the sine and cosine functions, integrating, and reducing, we get *approximately* (see Appendix C for full solution and more accurate formulas):

$$y = \frac{l}{100}[0.58\theta - \cdot \cdot \cdot], \tag{86}$$

$$x = \frac{l}{100}[100 - 0.3046\theta^2(10)^{-2} + 0.4296\theta^4(10)^{-7} - \cdot \cdot \cdot] \tag{87}$$

If $l = 250$ ft, $\theta = 12°$, y(Table 15) $= 0.069595 \times 250 = 17.40$ ft.

Replacing θ in Eq. (86) with the value given by Eq. (81), we get approximately

$$y = \frac{l^3}{6R_cl_s}, \tag{88}$$

showing that the transition spiral is approximately a *cubic parabola;* hence the tangent offsets are closely proportional to the cube of the distance from the TS.

83. Functions of the Spiral and Circle. In Fig. 68, the two spiral arcs extending from the TS to the SC and from the CS to the ST are connected by an intermediate circular arc subtending a central angle of $\Delta_c = (\Delta - 2\theta_s)$ deg. To make room for the spiral, the circular arc has been shifted radially inward to the position FF', where $HF = H'F' = p$. The corresponding radial shift opposite the PI, called the "throw," equals $p \sec \frac{1}{2}\Delta$.

Important parts, or functions, of the spiraled circular curve are (see Figs. 67*a* and 68):

$$p = HF = y_s - R_c \text{ vers } \theta_s \qquad \text{and} \qquad k = x_s - R_c \sin \theta_s,$$

where x_s, y_s, R_c, and θ_s have already been defined.

It is a useful fact that the offset from H to the spiral is very closely $\frac{1}{2}(HF)$ or $\frac{1}{2}p$. Also, line HF approximately bisects the spiral.

Referring to Fig. 69, we have

$$E_s = \text{total external distance} = (R_c + p) \sec \tfrac{1}{2}\Delta - R_c$$
$$= (R_c + p) \text{ exsec } \tfrac{1}{2}\Delta + p, \quad (89)$$
$$T_s = \text{total tangent distance} = (R_c + p) \tan \tfrac{1}{2}\Delta + k. \quad (90)$$

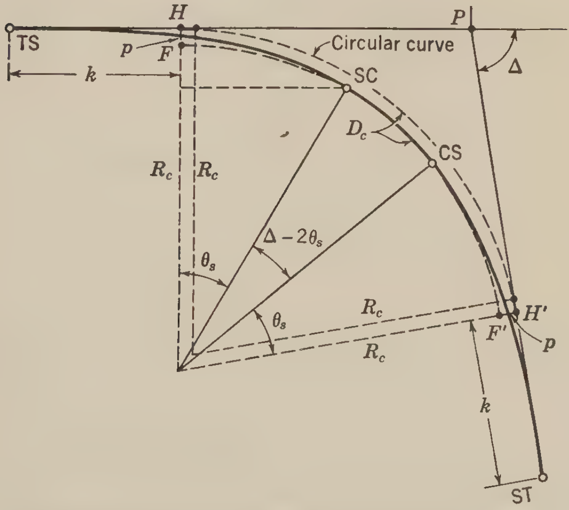

FIG. 68. The spiraled as related to the unspiraled curve.

It follows that $E_s = E + p \sec \frac{1}{2}\Delta$ and $T_s = T + p \tan \frac{1}{2}\Delta + k$, where E and T are the external and tangent distances for the unshifted circular curve.

If Δ, D_c, and l_s are known, T_s and E_s may be determined by two methods: by Eqs. (89) and (90) or by interpolation in Tables 18 and 19.

1. In applying Eqs. (89) and (90), the quantities p and k (for $l_s = 1$ station) are taken directly from Table 15 and multiplied by the actual length of spiral, as will be shown in Example 1.
2. Interpolating in Tables 18 and 19 over ranges of 1° in both Δ and θ_s gives results closely to the nearest hundredth of a foot.

Since Tables 18 and 19 are based on the 1° curve, they are applicable to both the *arc* and the *chord* definition.

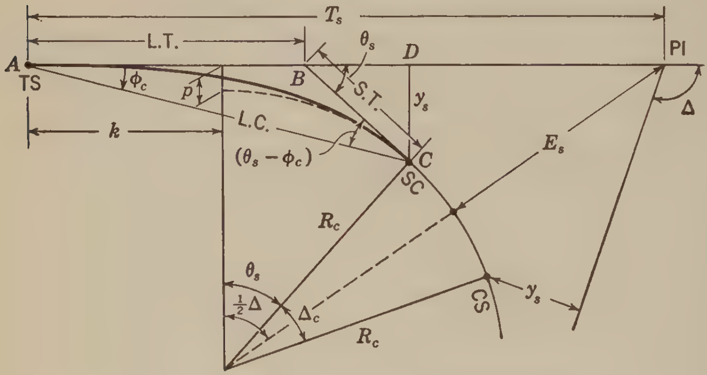

FIG. 69. Parts of the spiral and osculating circle.

Example 1. Given $\Delta = 40°20'$, $D_c = 6°$, $l_s = 400$ ft. Required: T_s and E_s (2 solutions), where $l_s = 400 = 200\theta_s/6$, from which $\theta_s = 12°$.

Solution. 1. Using Table 15, the values of p and k are $p = 1.743 \times 4 = 6.97$; $k = 49.927 \times 4 = 199.71$. Since $D_c = 6°$, R_c may be found in Table 5 or from the formula $R_c = 5{,}729.58/6 = 954.93$. Then noting that the natural tangents and external secants are found in Tables 40 and 41, we have, from Eqs. (89) and (90),

$$T_s = 961.90 \times 0.3672680 + 199.71 = 552.99 \text{ ft},$$
$$E_s = 961.90 \times 0.0653102 + 6.97 = 69.8 \text{ ft}.$$

2. Entering Table 18 with $\Delta = 40°20'$ and $\theta_s = 12°$, we get

$$\begin{array}{r} 3{,}298.87 \\ 19.07 \\ \hline T_s = 3{,}317.94 \end{array} \text{ (for 1° curve);} \qquad T_s \text{ (6° curve)} = \frac{3{,}317.94}{6} = 552.99.$$

Similarly, from Table 19, we get

$$\begin{array}{rl} & 412.2 \\ & \underline{6.6} \\ E_s = & 418.8 \text{ (for 1° curve);} \end{array} \qquad E_s \text{ (6° curve)} = \frac{418.8}{6} = 69.8 \text{ ft.}$$

Example 2. Given $\Delta = 50°12'$, $D_c = 8.4°$, and $l_s = 250$ ft. Required: total tangent distance T_s.

Solution. From the formula $l_s = 200\theta_s/D_c$, we have $250 = 200\theta_s/8.4$, from which $\theta_s = 10.5°$. Using Table 18, interpolation is made vertically and then horizontally, as indicated below.

		θ_s		
Δ		10°	11°	
50°00′		3,684.28	3,786.79	
	$(61.43 \times \frac{12}{60})$	12.29	12.30	$(61.49 \times \frac{12}{60})$
50°12′		3,696.57	3,799.09	
	$(102.52 \times \frac{1}{2})$	51.26		
		3,747.83 (for 1° curve)		

$$T_s \text{ (for 8.4° curve)} = \frac{3{,}747.83}{8.4} = 446.17 \text{ ft.}$$

Note: Double interpolation would be avoided by assigning integral values for l_s and θ_s, while D_c becomes fractional.

In Fig. 69, the *long tangent*

$$(\text{LT, or } U) = AB = AD - BD = x_c - y_c \cot \theta_s;$$

also the *short tangent* (ST, or V) $= BC = y_c/\sin \theta_s$. Instead of using these formulas, the long and short tangents are much more easily determined from Table 15.

Point B, the intersection of the spiral tangents AB and BC, is called PI_s, as is the corresponding point on the forward tangent.

84. Deflection Angles. In Fig. 67*a*, ϕ = angle PAX = deflection angle from the initial to any point on the spiral. In Appendix C, it is shown that

$$\phi = \tfrac{1}{3}\theta - C_s; \qquad \text{or} \qquad \phi = \frac{1}{3}\left(\frac{l}{l_s}\right)^2 \theta_s - C_s; \tag{91}$$

where C_s (in seconds) is a small correction given in Table 20, or by the formula C_s (in seconds) $= 0.0031\theta_s^3 + 0.0023\theta_s^5(10)^{-5}$.

At the SC, where $l/l_s = 1$,

$$\phi_c = \tfrac{1}{3}\theta_s - C_s. \tag{92}$$

For values of θ less than 15°, C_s is negligible.

Example. Given $D_s = 7°15'$, $l_s = 400$ ft, $\theta_s = 15°$, and the TS at Sta. 37 + 10.2. Required: location of the spiral by deflections from the TS and the initial tangent.

Note: Since the correction C_s for the final deflection angle is only 13 seconds, it will be neglected.

Method 1. Using the formula $\phi = \frac{1}{3}\theta = \frac{1}{3}(l/l_s)^2\theta_s$, it is best to express $\frac{1}{3}\theta_s$ in *minutes;* thus $\frac{1}{3}(15°) = 300$ minutes. Then $\phi = (l/l_s)^2 \times 300$ minutes, as given in Table A.

TABLE A

Station	l	$\dfrac{l}{l_s}$	$\left(\dfrac{l}{l_s}\right)^2$	Deflection
37 + 10.2 TS	0	0	0	0
+ 50	39.8	0.100	0.010	0°03′
38 + 00	89.8	0.224	0.050	0°15′
+ 50	139.8	0.350	0.122	0°37′
39 + 00	189.8	0.474	0.225	1°07.5′
+ 50	239.8	0.600	0.360	1°48′
40 + 00	289.8	0.724	0.524	2°37′
+ 50	339.8	0.850	0.722	3°37′
41 + 00	389.8	0.975	0.951	4°45′
41 + 10.2 ST	400.0	1.000	1.000	5°00′

Method 2. If the spiral is divided into 10 equal parts of 40 ft each, the deflections to the points of equal division are given directly in Table 23 for $\theta_s = 15°$. This is the simplest procedure, but *even stations* are *not* located, as would usually be desired.

85. General Procedure for Laying Out Spiral and Circle. By means of tables given herewith, the fieldwork of laying out a spiraled circular curve is greatly simplified. Let it be assumed that the PI, Δ, and D_c are known. The following steps are typical (Fig. 69).

1. Select the length of spiral, then the spiral angle follows from $\theta_s = l_s D_c/200$.

2. Determine the total tangent distance T_s from Table 18, or from the formula $T_s = (R_c + p) \tan \frac{1}{2}\Delta + k$ (p and k from Table 15). With T_s known, the TS (or ST) may be located by measurements from the PI. Also, by the distance $(T_s - U)$ from the PI (where U = LT, from Table 15), point B, the PI_s, is established. Then the SC may be located in advance (as a check) as follows: Set up the transit at B, sight on the PI, turn the angle θ_s, and measure the distance V, where V = ST from Table 15.

3. With transit set up at the TS, vernier reading 0°00′ on the tangent, the deflection angles vary as the square of the distance along the spiral. The total deflection to the SC is $\frac{1}{3}\theta_s$ minus a correction that is usually negligible.

4. Next, with transit set up at the SC, and oriented by a back-

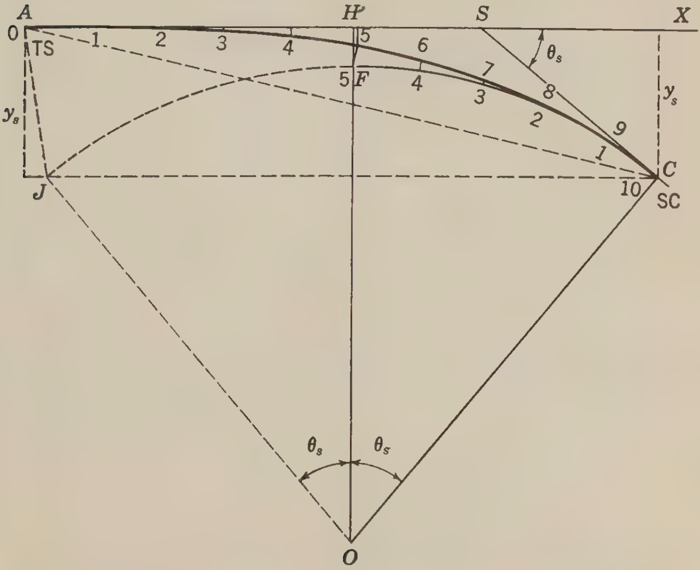

FIG. 70. Angular and linear divergence property.

sight on either the TS or the PI_s, the circular curve is located by deflections in the usual way.

Further details on the layout of spirals are given in Chap. 13.

86. Angular and Linear Divergence Property. Since a straight line and a circle each have *constant* curvature and the transition spiral is a curve of *uniformly changing* curvature, it is true that the *spiral diverges in angle and offset from the circular curve, for a given distance, at the same rate as from the initial tangent.* This is a useful property of the spiral.

Figure 70 shows a spiral diverging from the straight line AH with the same offsets as from the circle CF. If the spiral is divided in 10 equal parts, the offsets from points 1, 2, 3, etc., on the circle to corresponding points 9, 8, 7, etc., on the spiral are the *same* practically as the perpendicular offsets from the initial tangent to spiral points 1, 2, 3, etc., respectively.

With a transit at the TS (Fig. 70), vernier reading 0°00′ on initial tangent, the deflections to points 1, 2, 3, etc., are ϕ_1, ϕ_2, ϕ_3, etc., where the ϕ's vary as the square of the distance from the TS. If a transit is set at the SC, with vernier reading 0°00′ on the auxiliary tangent, the deflections to points 9, 8, 7, etc., are $\delta_1 - \phi_1$, $\delta_2 - \phi_2$, $\delta_3 - \phi_3$, etc., where the ϕ's vary as the square of the distance from the ST, and the δ's are those for a circle.

Example. Using the same data as in Example 1 on page 174, where $\theta_s = 12°$, $l_s = 400$ ft, $D_c = 6°$, it is required to calculate the deflections for the layout of the second spiral from the CS (see Table A).

Transit at the CS, vernier reading 0°00′ *on tangent.*

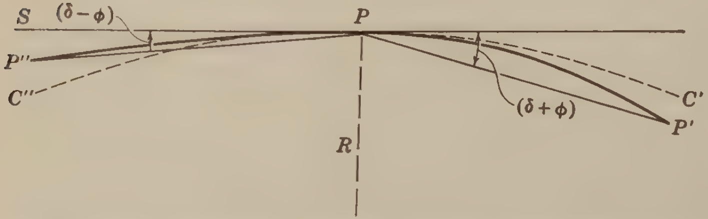

FIG. 71. Showing forward and backward deflection angles from any point P on spiral.

TABLE A

Station	Deflections				
	For 6° circular curve		For spiral as if from tangent at the TS		For spiral referred to tangent at the CS
38 + 00 (CS)	0°00′	−	0°00′	=	0°00′
+ 50	1°30′	−	0°04′	=	1°26′
39	3°00′	−	0°15′	=	2°45′
+ 50	4°30′	−	0°34′	=	3°56′
40	6°00′	−	1°00′	=	5°00′
+ 50	7°30′	−	1°34′	=	5°56′
41	9°00′	−	2°15′	=	6°45′
+ 50	10°30′	−	3°04′	=	7°26′
42 + 00 (ST)	12°00′	−	4°00′	=	8°00′

87. Deflections from Intermediate Point on Spiral. The fact that "the spiral diverges from a circle at the same rate as from the initial tangent" provides a method for locating forward or backward stations from any point.

Thus, if the transit is set at point P (Fig. 71), vernier reading 0°00′ on auxiliary tangent PS, the deflection to the *forward* point P' is $\delta + \phi$ = deflection for circular arc PC' ($= PP'$) of radius R *plus* deflection for spiral arc PP', as if point P were the TS.

Also, the deflection from P to any *backward* point P'' is $\delta - \phi$.

Example. Given spiral angle $\theta_s = 16°$, $D_c = 4°$, and the TS at Sta. 31 + 12.4. Assuming that 32 to 36 have been established by deflections from the TS, it is required to locate the balance of the spiral by deflections from Sta. 36 + 00.

Solution. $l_s = 200 \times \frac{16}{4} = 800$ ft.

$$\begin{aligned} \text{TS} &= \text{Sta. } 31 + 12.4 \\ l_s &= \phantom{\text{Sta. }3}8 + 00.0 \\ \hline \text{SC} &= \text{Sta. } 39 + 12.4 \end{aligned}$$

Letting D_s = degree of curve of spiral at any point (as Sta. 36), we have $D_s : D_c = l : l_s$, where l is the distance from the TS to Sta. 36 (= 487.6 ft).

Hence $D_s = D_c \times l/l_s = 4 \times 487.6/800 = 2.438° = 2°26.28'$.

Then the circular curve deflection for Sta. 37, 100 ft ahead, is $\frac{1}{2}D_s = 1°13.14' = 73.14'$.

Using the formula $\phi = \left(\frac{l}{l_s}\right)^2 \times 320'$, the spiral deflection for a distance of 100 ft is

$$\phi = (\tfrac{100}{800})^2 \times 320' = 5 \text{ minutes}.$$

Therefore, with transit at Sta. 36, vernier reading 0°00′ on auxiliary tangent, the deflections are as given in Table A.

TABLE A

Station	Distance (ft)	Deflections $\delta + \phi$
37	100	$1°13.14' + 0°05.0' = 1°18.1'$
38	200	$2°26.28' + 0°20.0' = 2°46.3'$
39	300	$3°39.42' + 0°45.0' = 4°24.4'$
39 + 12.4 (SC)	312.4	$3°48.50' + 0°48.8' = 4°37.3'$

Note: It will be observed that the circular curve deflections δ vary *directly* as the distance from the point of tangency, while the spiral deflections ϕ vary as the *square* of this distance.

In order to orient the instrument at Sta. 36, it is necessary to know the total spiral deflection from the TS to Sta. 36, which is

$$\left(\frac{487.6}{800}\right)^2 \times \frac{16}{3} = 1.981° = 1°59'.$$

Figure 72 shows that the backsight angle from any point P to the TS is $\theta - \phi = 2\phi$ (since $\phi = \frac{1}{3}\theta$). Hence the backsight angle at Sta. 36 is 3°58′; and when the vernier is turned to 0°00′, the telescope is on the auxiliary tangent.

Alternative Method. A general formula devised by Kofoid and sponsored by the Public Roads Administration (PRA) is given below.

$$\phi = (l_1 + l_2 + l_3)(l_3 - l_1)\frac{\theta_s}{3l_s^2}, \qquad (86)$$

in which θ_s is the spiral angle and ϕ (for a setup at any point on spiral) is the *deflection angle from any backsight to any foresight* as indicated in Fig. 73.

When the setup is on TS, $l_1 = l_2 = 0$, and the backsight is on the tangent.

When the setup is on SC, $l_2 = l_3 = l_s$, and the foresight is on the local tangent.

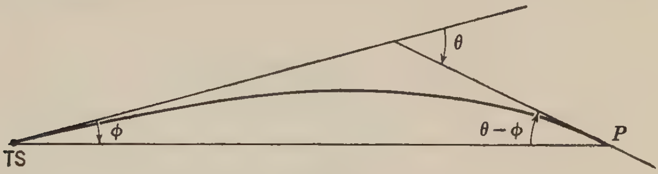

FIG. 72

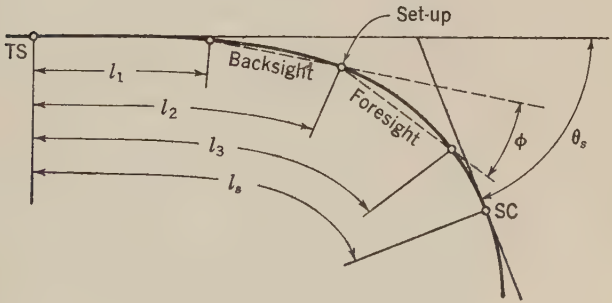

FIG. 73

When the setup is on any point, the deflection angle from the backsight to the local tangent can be computed by making $l_3 = l_2$, and from the foresight by making $l_1 = l_2$.

The results are reasonably accurate for all practical purposes except where C corrections for sharp transitions are appreciable. But when TS and SC are located directly and not by carrying transition stations from TS to SC, errors are not cumulative and C corrections rarely are necessary.

Special Case: 10-chord Spiral. If the spiral is divided in 10 equal parts, the deflections to each of the 10 points are given directly in Table 20 for various values of the spiral angle θ_s.

88. Double-spiral Curve. If there is no intermediate circular arc, that is, if $\Delta_c = 0$, we have a *double-spiral* curve as shown in Fig. 74, where $\theta_s = \frac{1}{2}\Delta$. Such a curve is transitional throughout and theoretically would require changing superelevation throughout, which is objectionable. However, the superelevation could be made constant around the middle portion of the curve since the actual deviation of the spiral from a circle is here very small.

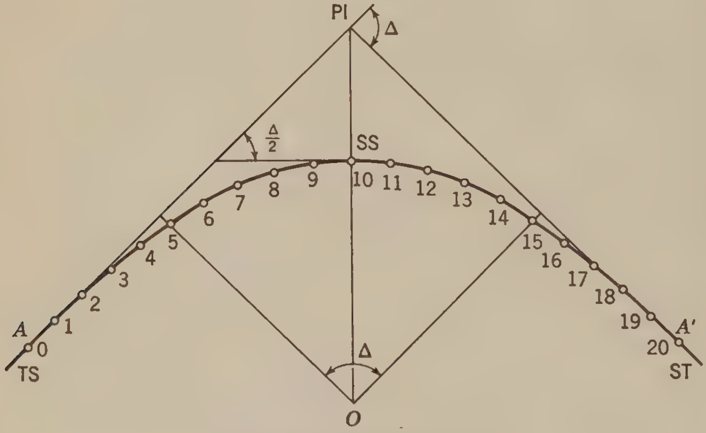

FIG. 74. Double-spiral curve.

Thus, if $\Delta = 20°$, $\theta_s = 10°$, $l_s = 500$ ft, $y_s = 29$ ft, and the total length of curve = 1,000 ft, the offset from the tangent to point 2 (100 ft from the SS) $= (\frac{1}{5})^3 \times 29 = 0.23$ ft.

Hence the middle fifth of the curve (of length 200 ft) is very nearly an arc of a circle, since it deviates no more than 0.23 ft therefrom.

Table 25 giving tangents and externals to the unit double-spiral curve is similar to Table 1 for circular curves. In the example which follows, full stations along the curve are located by deflections from the TS for the first spiral, and from the ST for the second spiral.

Example. Given $\Delta = 32°10'$, unlimited tangent distance, but the external distance is limited to about 48 ft. The PI is at Sta. 87 + 12.2. Required: the curve data for a *double spiral.*

Solution. Table 25 gives E_s (for 100-ft spiral) $= 9.683$; hence *ratio* $48/9.683 = 4.99 = 5.00$, say. Then the length of each spiral $= 500$ ft; more exactly, $E_s = 9.683 \times 5 = 48.4$ ft; also $T_s = 101.897 \times 5 = 509.5$ ft.

The spiral angle $= \frac{1}{2}\Delta = 16°05' = 965'$. Using Eq. (92) and noting that the correction $C_s = 0$, we have

$$\phi \text{ (minutes)} = \left(\frac{l}{l_s}\right)^2 \times 321.7 \text{ for the 1st spiral (from the TS),}$$

$$\phi \text{ (minutes)} = \left(\frac{l_s - l}{l_s}\right)^2 \times 321.7 \text{ for the 2d spiral (from the ST).}$$

It will be seen that all the deflection angles in Table A are rounded off to the nearest minute.

TABLE A

Station	$\frac{l}{l_s}$	Deflection
82 + 02.7 (TS)	0	0°00′
83	0.195	0°12′
84	0.395	0°50′
85	0.595	1°54′
86	0.795	3°23′
87	0.995	5°18′
87 + 02.7 (SS)	1	5°22′
88	0.805	3°28′
89	0.605	1°58′
90	0.405	0°53′
91	0.205	0°14′
92	0.005	0°00′
92 + 02.7 (ST)	0	0°00′

General Conclusions. When E_s is *limited* and T_s is *unlimited,* a favorable alignment approaches, or equals, a spiral which is transitional throughout; when E_s is *unlimited* and T_s is *limited,* a favorable alignment approaches, or equals, a circular curve throughout.

89. Spiral Applied to Existing Circular Curve Where Central Portion Remains Unshifted. In Fig. 75, it is assumed that the circular curve of radius R and degree of curve D is already established, and the central portion is fixed in alignment. But the end portions may be sharpened so as to become osculating circles for spirals that fit the same tangents and compound with the existing curve at points called the SC, or the CS.

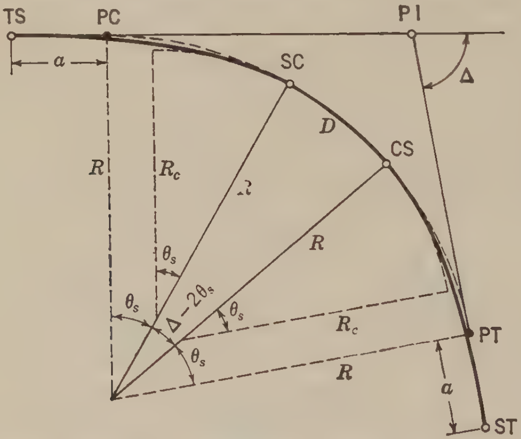

FIG. 75. Spiral compounded with unshifted circular arc.

Let a = distance from the original PC to the TS. The quantities l_s, D_c, R_c, y_s, LT, and ST have the usual meaning except that they apply to a spiral of length about three-fourths that of the nominal distance from the TS to the SC.

Since y_s (Fig. 76) is the same for the spiral as for the original circular curve, it is possible to find the length and other functions of the spiral which will satisfy the foregoing requirements. Appendix D gives the theoretical background for Table 26 containing values of l_s, a, D_c, etc., which simplify the layout of the spiral. These quantities are calculated on the basis of a 1° circular curve and various values of the spiral angle θ_s; hence they are applicable to both the *chord* and the *arc* definition.

It will be seen that a is about $\frac{1}{3}l_s$; exact values are given in Table 26.

Example. Given an existing circular curve: $\Delta = 42°12.5'$, $D = 5°$, and the PC at Sta. 81 + 11.40. It is assumed that the original PC and the middle part of the curve must remain unshifted. Required: curve data for equal spirals compounded with the central circular curve.

Solution. If the spiral length, l_s, is to be 300 ft, l_s (for 1° curve) = $300 \times 5 = 1{,}500$ ft. Then by Table 26, we have $\theta_s = 10°00.2'$, $a = 500.16/5 = 100.03$ ft, $D_c = 1.3338 \times 5 = 6.669° = 6°40'$, $y_s = 87.11/5 = 17.42$ ft, LT = 1,001.61/5 = 200.32 ft, and ST = 501.45/5 = 100.29 ft.

The last four quantities may or may not be of use in laying out the curve. D_c is needed when there is an intermediate setup on the spiral.

With $\theta_s = 10°00.2'$, there remains $\Delta_c = \Delta - 2\theta_s = 22°12.1'$ for central circular arc of length $100 \times \Delta_c/D = 100 \times 22.2017/5 = 444.03$ ft.

Subtracting the distance a (= 100.03 ft) from the stationing of the PC gives the TS. Thus 81 + 11.43 − (1 + 00.03) = 80 + 11.40. With transit set at Sta. 80 + 11.40, the TS, vernier reading 0°00′ on tangent, the deflections to full stations are given by the formula

$$\frac{\theta_s}{3}\left(\frac{l}{l_s}\right)^2 = \left(\frac{l}{l_s}\right)^2 \times 200', \text{ where } \theta_s = 10° = 600'.$$

Note: In the foregoing example where $\Delta = 42°12.5'$ and $D = 5°$, E (by Table 2) = 82.4 ft; suppose it is permissible for the surveyed center line to be shifted further inward opposite the PI, so that E is about 90 ft. Then D (revised value) $= \dfrac{E \text{ (for 1° curve)}}{90} = 4.577°$. Now, for simplicity of calculations, let $D = 4.5°$ (final value); then the slightly adjusted value of E becomes $\dfrac{E \text{ (for 1° curve)}}{4.5} = \dfrac{411.925}{4.5} = 91.54$ ft.

If $l_s = 300$ ft, then l_s (for 1° curve) $= 4.5 \times 300 = 1{,}350$ ft. By Table 26, we have $\theta_s = 9°00'$, $a = \dfrac{a \text{ (for 1° curve)}}{4.5} = \dfrac{450.11}{4.5} = 100.02$ ft, $D_c = 1.3337 \times 4.5 = 6.002°$, $y_s = \dfrac{70.58}{4.5} = 15.68$, etc.

90. Unequal Spirals. For spirals applied to existing circular curves, as in Fig. 75 or 76, the two end spirals may be of unequal length without introducing complications. The intermediate circular arc remains fixed in position; hence it fits in with either spiral.

Let θ_{s1} and l_{s1} equal the spiral angle and length respectively of the first spiral, while θ_{s2} and l_{s2} are corresponding parts of the second spiral. Also let the subscripts 1 and 2 apply respectively to the other parts of the first and second spiral.

The degree of curve D remains the same for both spirals, while D_c (called D_{c1} and D_{c2}) will be different.

Let $\Delta = 42°12.5'$ and $D = 5°$, as in the example on page 185, the data of which will now be assumed applicable to the first branch of the two unequal spirals. If $l_{s2} = 400$ ft, then l_{s2} (for 1° curve) $= 400 \times 5 = 2{,}000$ ft. Accordingly, by Table 26, the corresponding parts of the second spiral are $\theta_{s2} = 13°20.52'$, $a = 667.02/5 = 133.40$ ft, $D_{c2} = 1.3342 \times 5 = 6.6710°$, $x_{s2} = 1{,}989.2/5 = 397.84$ ft, $y_{s2} = 154.64/5 = 30.93$ ft, etc.

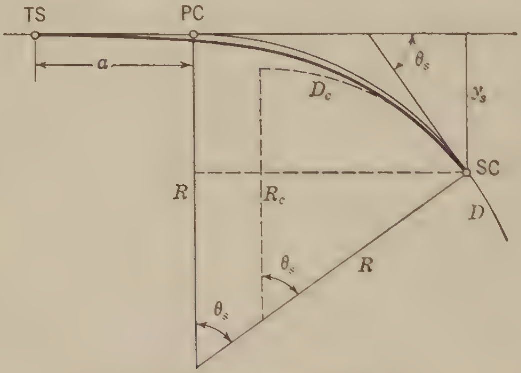

FIG. 76. Showing part of Fig. 75.

Noting that the central angle of the intermediate circular curve $= \Delta_c = \Delta - (\theta_{s1} + \theta_{s2})$, we have $L_c = 100\Delta_c/D = 100 \times 18.8633/5 = 377.27$ ft.

91. Location of Spiral by Offsets. *Offsets from Tangent.* Knowing the central angle θ at any point of a spiral from the formula $\theta = (l/l_s)^2 \times \theta_s$, the corresponding coordinates x and y may be determined by multiplying the unit values (from Table 15) by l. When chord measurements are made along the curve, the x distances are not necessary. The offsets y may be checked approximately by assuming the spiral to be a cubic parabola.

Example. Given the TS at Sta. 81 + 11.4, θ_s (spiral angle) $= 10°$, $l_s = 300$ ft $= 3$ stations. Required: coordinates x and y for points along the spiral 50 ft apart beyond Sta. 81.

Solution. Calculate θ for each point, then enter Table 15 for unit values of x and y which are to be multiplied in turn by l, as shown in Table A below.

TABLE A

Station	l (stations)	$\left(\frac{l}{l_s}\right)^2$	θ	x (ft)	y (ft)
80 + 11.4 TS	0	0	0	0	0
81	0.886	$(0.2953)^2 = 0.0872$	0°52.3′	88.6	0.45
81 + 50	1.386	$(0.4620)^2 = 0.2134$	2°08.0′	138.6	1.72
82	1.886	$(0.6287)^2 = 0.3953$	3°57.2′	188.5	4.34
82 + 50	2.386	$(0.7953)^2 = 0.6325$	6°19.5′	238.3	8.77
83	2.886	$(0.9620)^2 = 0.9254$	9°15.2′	287.8	15.51
83 + 11.4 SC	3.000	1	10°00.0′	299.1	17.42

92. Spiraled Reverse Curve. In Fig. 77, the two circular curves HB and BF', shown by dashed lines, are reversed at B (PRC).

Let Δ_1, Δ_2 = external angle of the first and second curves, respectively, while other corresponding parts are D_1, D_2 (degree of curve); T_1, T_2 (tangent); L_1, L_2 (length of curve); etc. It is required to improve the alignment in every possible way without changing radically the existing layout.

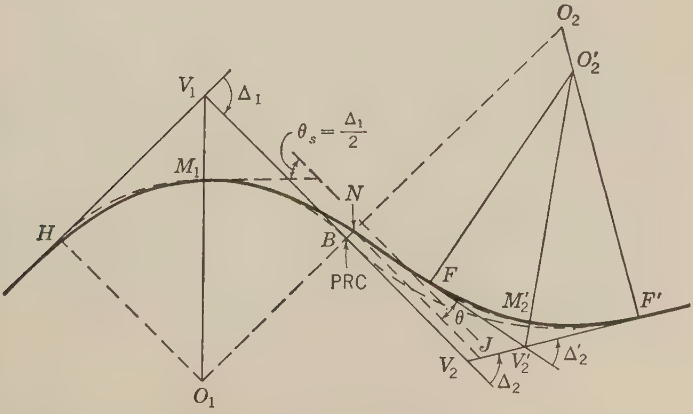

FIG. 77. Spiraled reverse curve.

The method that follows will introduce an intervening tangent. A similar procedure may be used to extend intervening tangents that are initially of insufficient length for superelevation runoff.

Assume the middle point M_1 opposite PI of the first curve (the sharper) to be fixed in position, but that the second curve may be shifted slightly toward its center.

Procedure

1. At point B (PRC) (Fig. 77) offset at right angles to tangent V_1BV_2 the distance BN ($= p - \frac{1}{8}y_s$), where p and y_s are determined from a "spiral angle" ($= \frac{1}{2}\Delta_1$) and spiral arc ($= 2 \times$ arc BM_1).
2. Run in the half spiral NM_1 by offsets from the arc BM_1 or by deflections (the right half of this spiral is omitted).
3. Let the line through N parallel to V_1V_2 intersect the terminal tangent at J.
4. With a transit at N, fix the line of sight on a tangent to the half spiral NM_1 at N. This is done by turning the angle JNV_2' ($= \theta$) from the line NJ, where $\theta = (\frac{1}{2})^2 \times \theta_s = \frac{1}{8}\Delta_1$, since such angles vary as the square of the distance from the beginning point of the spiral.
5. Point V_2' (the new PI) may be established in the field or by computations as follows: Solve triangle NJV_2' for JV_2' and NV_2', the known parts being the angles at N and J, and the side $NJ = BV_2 + BN \times \tan (90 - \Delta_2)$.
6. Calculate the new external angle Δ_2', noting that $\Delta_2' = \Delta_2 - \theta$.
7. Run in the circular curve FF' terminating at, or near, F', the original PT.

It is seen that the length of intervening tangent is NF and that the new alignment is improved as well as shortened.

Example. Given reverse-curve data: $\Delta_1 = 90°$, $D_1 = 30°$, $L_1 = 300$ ft, $T_1 = 191.0$ ft; $\Delta_2 = 60°$, $D_2 = 20°$, $L_2 = 300$ ft, $T_2 = 165.4$ ft. Let it be assumed that there is initially no intervening tangent (as in Fig. 77).

Solution. If $\theta_s = \frac{1}{2}\Delta_1 = 45°$, p (Table 15, for spiral length $=$ 300 ft) $= 6.403 \times 3 = 19.21$ ft.

Since the offsets y from the initial tangent at the TS (not shown in Fig. 77) vary closely at the cube of the distance from the TS, we get y (to point N) $= (\frac{1}{2})^3 \times y_s = \frac{1}{8} \times 75.147 = 9.39$ ft, where y_s is determined from Table 15, for $\theta_s = 45°$ and $l_s = 300$ ft. Then $BN = 19.21 - 9.39 = 9.82$ ft.

$$\text{Angle } JNV_2' = \theta = \tfrac{1}{4}\theta_s = 11°15',$$
$$\Delta_2' = \Delta_2 - \theta = 60° - 11°15' = 48°45',$$

$$
\begin{aligned}
NJ &= BV_2 + 9.82 \times \tan(90 - \Delta_2) = T_2 + 9.82 \times \tan 30^\circ \\
&= 165.40 + 5.67 = 171.07 \text{ ft}, \\
JV_2' &= \frac{171.07}{\sin 48^\circ 45'} \times \sin 11^\circ 15' = 44.39 \text{ ft}, \\
NV_2' &= \frac{171.07}{\sin 48^\circ 45'} \times \sin 120^\circ = 197.05 \text{ ft}, \\
V_2V_2' &= 44.39 + 9.82 \times \sec 30^\circ = 55.73 \text{ ft}.
\end{aligned}
$$

Assuming PT of second curve unchanged, $V_2F' = T_2 - V_2V_2' = 165.40 - 55.73 \text{ ft} = T_2'$ = tangent length of revised curve.

NF = length of intervening tangent $= NV_2' - FV_2' = 197.05 - T_2' = 197.05 - 109.67 = 87.38$ ft.

$$\text{Length of revised curve } FF' = 100 \times \frac{T_2'}{T_0} = 100 \times \frac{109.67}{53.252} = 205.95$$

ft, where T_0 = tangent length (Table 1) for curve of length 100 ft and $\Delta_2' = 48^\circ 45'$.

$$\text{Degree of curve of } FF' = 100 \times \frac{48.75}{205.95} = 23.671^\circ.$$

It follows that the distance by way of the tangent NF and arc FF' is 6.67 ft shorter than the circular arc BF'; also a superelevation runoff of 87.38 ft is provided when none existed in the initial layout.

93. Parallel Spirals. In Fig. 78, let R = variable radius of the center-line spiral at a distance l from the TS; R_o = variable radius of the outer parallel spiral; R_i = variable radius of the inner parallel spiral. At the SC, $R = R_c$, $R_o = R_c + w$, and $R_i = R_c - w$, where R_c = radius of the center-line circular curve and w = radial distance from the center-line spiral to the outer (or inner) parallel spiral.

For any differential arc of length dl subtending a central angle $d\theta$ (Fig. 78*b*), the corresponding outer and inner concentric arcs are $dl_o = (R + w)\, d\theta$ and $dl_i = (R - w)\, d\theta$, respectively, while $dl = R\, d\theta$.

Noting that $\theta = (l/l_s)^2 \times \theta_s$, from which $d\theta = 2l/l_s^2 \times \theta_s$,

we have $$dl_o = R\, d\theta + w\, d\theta = dl + \frac{2l}{l_s^2} w\theta_s, \qquad (a)$$

where $$R\, d\theta = dl.$$

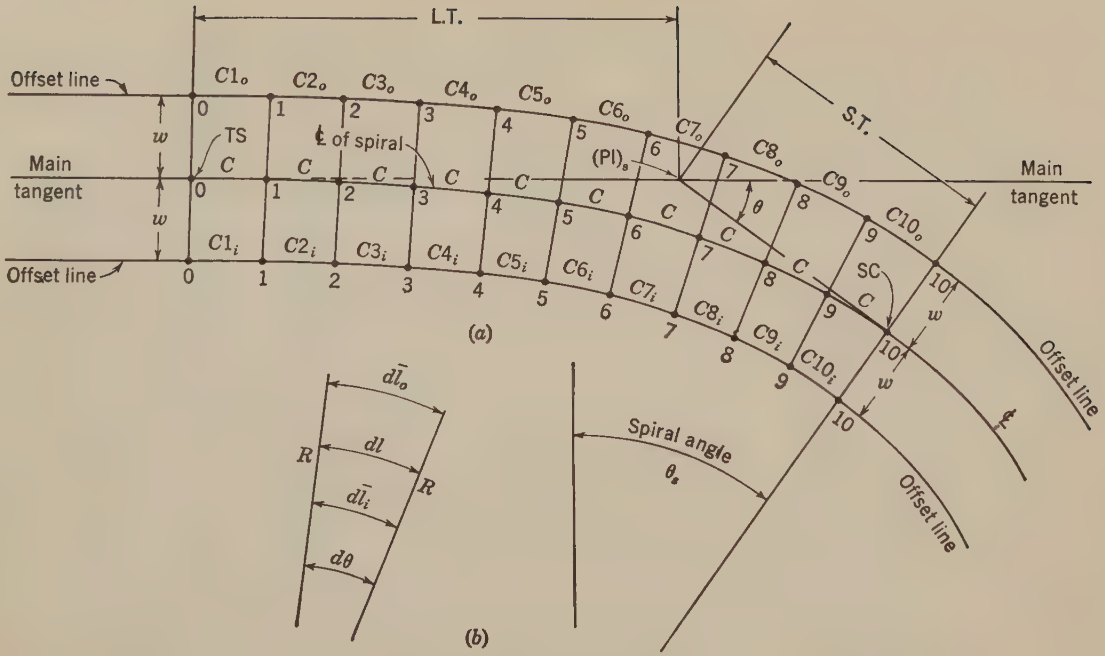
L.T.
S.T.
Offset line
Main tangent
Offset line
Main tangent
Offset line
Offset line
w
w
w
w
TS
SC
(PI)$_s$
℄ of spiral
℄
C
θ
$C1_o$ $C2_o$ $C3_o$ $C4_o$ $C5_o$ $C6_o$ $C7_o$ $C8_o$ $C9_o$ $C10_o$
$C1_i$ $C2_i$ $C3_i$ $C4_i$ $C5_i$ $C6_i$ $C7_i$ $C8_i$ $C9_i$ $C10_i$
0 1 2 3 4 5 6 7 8 9 10
(a)
$d\bar{l}_o$
dl
$d\bar{l}_i$
$d\theta$
R
R
Spiral angle
θ_s
(b)

FIG. 78. Parallel spirals.

Integrating, we get

$$l_o = l + \left(\frac{l}{l_s}\right)^2 w\theta_s \ (\theta_s \text{ in radians});$$

or

$$l_o = l + (0.017453)\left(\frac{l}{l_s}\right)^2 w\theta_s \ (\theta_s \text{ in degrees}). \qquad (b)$$

Similarly, we have

$$l_i = l - (0.017453)\left(\frac{l}{l_s}\right)^2 w\theta_s \ (\theta_s \text{ in degrees}). \qquad (c)$$

At the SC, $l = l_s$; then the lengths of outside and inside spirals become

$$L_o = l_s + (0.017453)w\theta_s; \qquad (d)$$
$$L_i = l_s - (0.017453)w\theta_s. \qquad (e)$$

Location by Deflection Angles. Knowing the length of curves and the spiral angle θ_s, which remains the same as for the center-line spiral, the outer and inner parallel spirals are located in the usual way by deflection angles to any desired point along the curve.

Since the outer and inner parallel curves must be radially opposite the center-line spiral, both at the TS and at the SC, the spiral characteristics of these curves are slightly modified, thus causing a slight (but inappreciable) compounding at the SC.

Location by Offsets. Two procedures may be used for staking the outer and inner parallel curves: (1) when the center-line curve is divided into equal parts; (2) when points on the parallel curves are located opposite full stations on the center-line curve.

1. *Assume the center-line curve divided into n equal parts.* With the center-line curve divided into n equal parts, Eq. (*b*) shows that the length of the first chord (called $c1_o$) is

$$c1_o = \frac{l_s}{n} + (0.017453)\frac{1}{n^2} w\theta_s.$$

The next chords, in succession, increase by *constant* increments that involve $2/n^2$, the *second* differences of $1/n^2$, $4/n^2$, $9/n^2$, $16/n^2$, etc. Hence, in general, the constant increment (called CI) is

$$\mathrm{CI} = (0.034906)\,\frac{w\theta_s}{n^2}. \tag{94}$$

Equation (94) gives corrections to be added to (or subtracted from) each successive chord.

It should be noted that the *first* outside chord (called $c1_o$) always equals l_s/n plus one-half of the increment CI.

The offset stakes are set by laying off successively the corrected chords at the desired radial distance w from the center-line curve points. An optional method of establishing the inner line of stakes is as follows: After setting the outer line of stakes by the method described above, measure twice the offset distance from the outside stake, passing the tape directly over the pin marking the center-line chord point.

Example 1. Given $w = 15$ ft, $l_s = 300$ ft, $D_c = 6°$, from which data $\theta_s = 9°$. Required: the successive outer and inner chords when the center-line curve is divided into six equal parts of 50 ft.

Solution. With $n = 6$, Eq. (94) gives

$$\mathrm{CI} = (0.034906) \times 15 \times \tfrac{9}{36} = 0.131 \text{ ft.}$$

$$\begin{array}{rrlrl}
\text{The chords are } & c1_o = & 50.065; & c1_i = & 49.935 \\
 & c2_o = & 50.196; & c2_i = & 49.804 \\
 & c3_o = & 50.327; & c3_i = & 49.673 \\
 & c4_o = & 50.458; & c4_i = & 49.542 \\
 & c5_o = & 50.589; & c5_i = & 49.411 \\
 & c6_o = & \underline{50.720}; & c6_i = & \underline{49.280} \\
 & & 302.355 & & 297.645
\end{array}$$

2. *Points on the parallel spirals opposite even stations*

Example 2. Given $w = 15$ ft, $l_s = 300$ ft, $D_c = 6°$, $\theta_s = 9°$ (same data as in Example 1). Also, assume the PC at Sta. 51 + 60. Required: the successive outer and inner chords.

Solution. By Eq. (*b*), $l_o = l + (0.017453)(l/l_s)^2 \times 15 \times 9 = l + (2.35616)(l/l_s)^2 = l + \text{curve increment}$.

The tabulated results that follow are self-explanatory.

Station	l	$\frac{l}{l_s}$	$\left(\frac{l}{l_s}\right)^2$	Curve inc.	l_o	c_o	c_i
51 + 60	0	0	0	0	0		
						> 40.042	39.958
52	40	0.1333	0.0178	0.042	40.042		
						> 50.170	49.830
52 + 50	90	0.3	0.09	0.212	90.212		
						> 50.301	49.699
53	140	0.4667	0.2178	0.513	140.513		
						> 50.432	49.568
53 + 50	190	0.6333	0.4011	0.945	190.945		
						> 50.563	49.437
54	240	0.8	0.64	1.508	241.508		
						> 60.848	59.152
54 + 60	300	1	1	2.356	302.356		

94. Nonparallel Outer and Inner Spirals where the Radial Distances Are Variable. In Fig. 79, let R = variable radius of the center-line spiral at any point distant l from the TS; R_o = variable radius of the outer curve; R_i = variable radius of the inner curve (these outer and inner nonparallel curves are not strictly true spirals, but they represent satisfactory reference lines); R_c = radius of the circular curve at the SC. At the TS, $R_o = R + w_2$, $R_i = R - w_1$; at the SC, $R_o = R_c + w_2'$, $R_i = R_c - w_1'$.

Let Δw equal the constant or variable radial increment. If the center-line curve is divided into n equal parts, $\Delta w = (w_2' - w_2)/n$ on the outside, and $\Delta w = (w_1' - w_1)/n$ on the inside. When the center-line curve begins or ends with a subchord, the procedure is slightly modified, as will be shown later.

$$\begin{aligned} dl_o \text{ (see Fig. 79}b\text{)} &= R\,d\theta + (w_2 + \Delta w)d\theta \\ &= R\,d\theta + w_2\,d\theta + \Delta w\,d\theta. \qquad (a) \end{aligned}$$

Substituting in Eq. (a) $R\,d\theta = dl$, $d\theta = (2l/l_s^2)\theta_s$, $\Delta w = (l/l_s)(w_2' - w_2)$, we get

$$dl_o = dl + w_2\left(\frac{2l}{l_s^2}\right)\theta_s + \frac{2l^2}{l_s^3}(w_2' - w_2)\ \theta_s, \qquad (b)$$

where θ_s = spiral angle in radians (multiply by 0.0174533 if in degrees).

Integrating Eq. (*b*), we get

$$l_o = l + w_2\left(\frac{l}{l_s}\right)^2(\theta_s) + \frac{2}{3}\left(\frac{l}{l_s}\right)^3(w_2' - w_1)(\theta_s). \tag{95}$$

Likewise $$l_i = l - w_1\left(\frac{l}{l_s}\right)^2(\theta_s) + \frac{2}{3}\left(\frac{l}{l_s}\right)^3(w_1' - w_1)(\theta_s). \tag{96}$$

Use the same notation as in Art. 93, where $C1_o$, $C2_o$, $C3_o$, etc., equal successive outer chords, while $C1_i$, $C2_i$, $C3_i$, etc., are corresponding inner chords or arcs, as shown in Fig. 79.

Let $l1_o$, $l2_o$, $l3_o$, etc., equal arc distances along the outer curve corresponding to the arc distances l_1, l_2, l_3, etc., along the center-line curve. Then $C1_o = l1_o$, $C2_o = l2_o - l1_o$, $C3_o = l3_o - l2_o$, etc., while $C1_i = l1_i$, $C2_i = l2_i - l1_i$, $C3_i = l3_i - l2_i$.

Applying Eq. (95), we have

$$C1_o = l1_o = l_1 + w_2\left(\frac{l_1}{l_s}\right)^2(\theta_s) + \tfrac{2}{3}(w_2' - w_2)\left(\frac{l_1}{l_s}\right)^3(\theta_s), \tag{c}$$

$$\begin{aligned} C2_o = l_2 - l_1 + w_2&\left[\left(\frac{l_2}{l_s}\right)^2 - \left(\frac{l_1}{l_s}\right)^2\right](\theta_s) \\ &+ \tfrac{2}{3}(w_2' - w_2)\left[\left(\frac{l_2}{l_s}\right)^3 - \left(\frac{l_1}{l_s}\right)^3\right](\theta_s), \end{aligned} \tag{d}$$

$$\begin{aligned} C3_o = l_3 - l_2 + w_2&\left[\left(\frac{l_3}{l_s}\right)^2 - \left(\frac{l_2}{l_s}\right)^2\right](\theta_s) \\ &+ \tfrac{2}{3}(w_2' - w_2)\left[\left(\frac{l_3}{l_s}\right)^3 - \left(\frac{l_2}{l_s}\right)\right]^3(\theta_s), \text{ etc.} \end{aligned} \tag{e}$$

Likewise

$$C1_i = l1_i = l_1 - w_1\left(\frac{l_1}{l_s}\right)^2(\theta_s) - \tfrac{2}{3}(w_1' - w_1)\left(\frac{l_1}{l_s}\right)^3(\theta_s), \tag{f}$$

$$\begin{aligned} C2_i = (l_2 - l_1) - w_1&\left[\left(\frac{l_2}{l_s}\right)^2 - \left(\frac{l_1}{l_s}\right)^2\right](\theta_s) \\ &- \tfrac{2}{3}(w_1' - w_1)\left[\left(\frac{l_2}{l_s}\right)^3 - \left(\frac{l_1}{l_s}\right)^3\right](\theta_s), \end{aligned} \tag{g}$$

$$\begin{aligned} C3_i = l_3 - l_2 - w_1&\left[\left(\frac{l_3}{l_s}\right)^2 - \left(\frac{l_2}{l_s}\right)^2\right](\theta_s) \\ &- \tfrac{2}{3}(w_1' - w_1)\left[\left(\frac{l_3}{l_s}\right)^3 - \left(\frac{l_2}{l_s}\right)^3\right](\theta_s), \text{ etc.} \end{aligned} \tag{h}$$

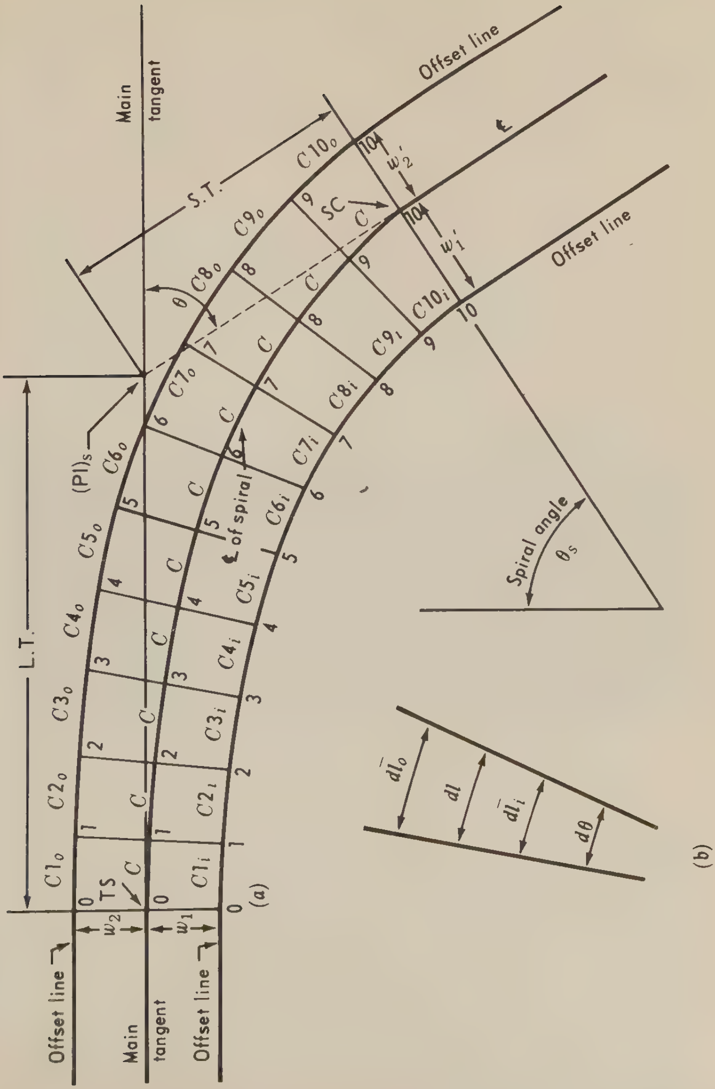
Main tangent
Offset line
Offset line
℄
S.T.
L.T.
(PI)s
TS
SC
θ
w_1
w_2
w_1'
w_2'
℄ of spiral
Spiral angle
θ_s
$C1_o$
$C2_o$
$C3_o$
$C4_o$
$C5_o$
$C6_o$
$C7_o$
$C8_o$
$C9_o$
$C10_o$
$C1_i$
$C2_i$
$C3_i$
$C4_i$
$C5_i$
$C6_i$
$C7_i$
$C8_i$
$C9_i$
$C10_i$
C
0
1
2
3
4
5
6
7
8
9
10
(a)
$\overline{dl_o}$
dl
$\overline{dl_i}$
$d\theta$
(b)

FIG. 79. Nonparallel outer and inner spirals where the radial distances are variable.

If the center-line curve is divided into n equal divisions, $l_1 = l_2 - l_1 = l_3 - l_2 = l_s/n$, etc. If it begins with a subchord, l_1 is less than l_s/n. With each center-line division $= l_s/n$, it follows that $C1_o + C2_o + C3_o + \ldots Cn_o$ becomes

$$\Sigma C_n = l_s + w_2 \times 1.0 \times \theta_s + \tfrac{2}{3}(w_2' - w_2) \times \theta_s; \qquad (i)$$

also $C1_i + C2_i + C3_i + \ldots Cn_i$

$$= l_s - w_1 \times 1.0 \times \theta_s - \tfrac{2}{3}(w_1' - w_1) \times \theta_s. \qquad (j)$$

Equations (i) and (j) serve as useful checks.

When the center-line curve begins and ends with subchords, a similar check can be readily devised.

If the radial distances to the outer curve remain equal, then $(w_2' - w_2) = 0$; if w_2' is less than w_2, $(w_2' - w_2)$ is negative.

U.S. 40, a four-lane highway between Salt Lake City and Gorgoza, Utah.

Example. Given $l_s = 300$ ft, $D_c = 6°$ (from which θ_s = spiral angle $= l_sD_c/200 = 9°$), $n = 6$, $w_2 = 20$ ft, $w_2' = 23$ ft, $w_1 = 21$ ft, and $w_1' = 27$ ft. Required: the successive outer and inner chords or arcs when the center-line curve is divided into six equal parts of 50 ft.

Solution. Δw for the outer curve equals $\dfrac{23 - 20}{6} = 0.5$ ft; Δw for the inner curve equals $\dfrac{27 - 21}{6} = 1.0$ ft.

By Eqs. (b), (c), (d), etc., $C1_o = 50.088721$, $C2_o = 50.291980$, $C3_o = 50.463967$, $C4_o = 50.6646798$, $C5_o = 50.874119$, $C6_o = 51.09228$. The total equals 303.45575.

By Eq. (i), the total equals $300 + 20 \times \theta_s + \frac{2}{3} \times 3.0 \times \theta_s = 300 + 3.1416 + 0.3142 = 303.4558$ (check).

By Eqs. (e), (f), (g), etc., $C1_i = 49.905461$, $C2_i = 49.704748$, $C3_i = 49.486583$, $C4_i = 49.250962$, $C5_i = 48.997886$, $C6_i = 48.727367$. The total equals 296.073007.

By Eq. (j), the total equals $300 - 21 \times \theta_s - \frac{2}{3} \times 6 \times \theta_s = 300 - 3.2987 - 0.6283 = 296.0730$ (check).

Review Problems

1. Given $\Delta = 40°12'$, $D_c = 4°00'$ (arc definition), and $l_s = 400$ ft. Find T_s by Table 18 and check by use of Eq. (90).
Ans. $\theta_s = 8°00'$; $T_s = 725.76$ ft.

2. Same as Prob. 1 except for $D_c = 5°$.
Ans. $\theta_s = 10°00'$; $T_s = 621.28$ ft.

3. Given the data of Prob. 1, find the deflection from the TS (assumed at Sta. 88 + 19.5) to Sta. 89 + 00.
Ans. (for either definition of D) 0°06.5′.

4. Given spiral angle $\theta_s = 8°00'$. Find deflections from the TS to points 6, 8, and 10 (SC) of a 10-chord spiral. *Procedure:* Use Table 23 and check by Eq. (91). *Ans.* 0°57.6′; 1°42.4′; 2°40.0′.

5. A transit is set at point 7 of a 10-chord spiral, vernier reading 0°00′ on auxiliary tangent, and spiral angle $\theta_s = 6°00'$. Using Table 24, calculate the deflection angles to points 8, 9, and 10 (SC).

6. The PC of an existing circular curve ($D = 6°00'$) = Sta. 85 + 32.48, $\Delta = 36°36'$, and $L = 610$ ft. The middle third of this curve is fixed in position, but the end thirds (each with central angles of about 12°00′) may be sharpened to allow approach and runoff spirals. If the spiral angle $\theta_s = 12°00'$, locate the TS and SC. *Hint:* Use Table 26 as if $\theta_s = 12°00.39'$.
Ans. Sta. 84 + 32.44; Sta. 87 + 32.44.

7. Given PI = Sta. 64 + 32, $\Delta = 27°00'$, and $D_c = 6°00'$. If a double spiral is used, so that $\theta_s = \frac{1}{2}\Delta = 13.5°$ and $l_s = 450$ ft, locate the TS and SS by Table 25.
Ans. TS = Sta. 59 + 76.04; SS = Sta. 64 + 26.04.

8. Referring to Prob. 7, show that the central part of the double spiral of length 150 ft does not deviate more than 0.2 ft from the osculating circle; hence the superelevation (see Chap. 8) may be considered constant and fully developed around this part of the curve.

9. Given a spiraled curve where $\Delta = 44°10'$, $D_c = 5°$, $\theta_s = 10°$, and $l_s = 400$ ft. Find the tangent distance T_s and long tangent (LT). (Use Tables 18 and 15.) *Ans.* $T_s = 667.08$ ft; LT = 267.09 ft.

10. Using pertinent data of Prob. 9, calculate deflections from the TS to stations along the spiral, circle, and spiral, if the TS is at Sta. 84 + 12.5.

11. A spiral is applied to an existing circular curve whose central portion is fixed in position, but the end portions may be sharpened so that spirals can be inserted. Given $\Delta = 46°24'$, $D = 5°$, and PC at Sta. 84 + 12.4. If $l_s = 300$ ft, find θ_s and stationing of the TS, SC, and CS. *Hint:* Use Table 26.

Ans. $\theta_s = 10°$; TS = 83 + 12.4; SC = 86 + 12.4; CS = 91 + 40.4.

CHAPTER 7

SPIRALED COMPOUND CURVES

95. Foreword. At the point of tangency (the PCC), where two curves with different radii are joined together, there is an abruptness of curvature that may be eased by the insertion of a transition spiral. The function of the spiral is to provide a gradual change in curvature from the beginning of one curve to the end of the other.

As in the case of a spiral connecting a tangent with a circular curve, a small radial shift of one of the component parts is necessary in order to allow proper clearance for the spiral.

This shift, or separation, of two branches of the compound curve is effected by moving either the sharper branch radially inward or the flatter branch radially outward. Thus in Fig. 80, curve CB (the sharper) is shifted inward at C to the position C_1B_1, where it assumes tangency with the spiral at B_1.

Actually a spiral connecting two circular arcs is the latter part of a spiral (not shown) running from a tangent. Thus, a 200-ft spiral connecting a 4° to a 6° curve might be called the last third of a 600-ft spiral running from a tangent to a 6° curve.

96. Two-centered Compound Curve with One Connecting Spiral. In Fig. 80, let D_1, R_1 equal, respectively, the degree of curve and radius of the flatter curve AC with central angle Δ_1; let D_2, R_2 apply to the sharper curve CB with central angle Δ_2; and let $I = \Delta_1 + \Delta_2$.

L_s = length of combining spiral AB_1.
l = length of any arc AP.
θ = central angle of arc AP.
D = degree of curve of the spiral at any point (as P).
θ_s = nominal spiral angle.
p = radial shift, that is, the distance one branch is moved inward or outward at C (the PCC).

X, Y = the coordinates of point B_1 with reference to A as origin and AV as X axis.

T_1, T_2 = tangent distances AV and VB_1, respectively.

ϕ = deflection angle VAP to any point of the spiral.

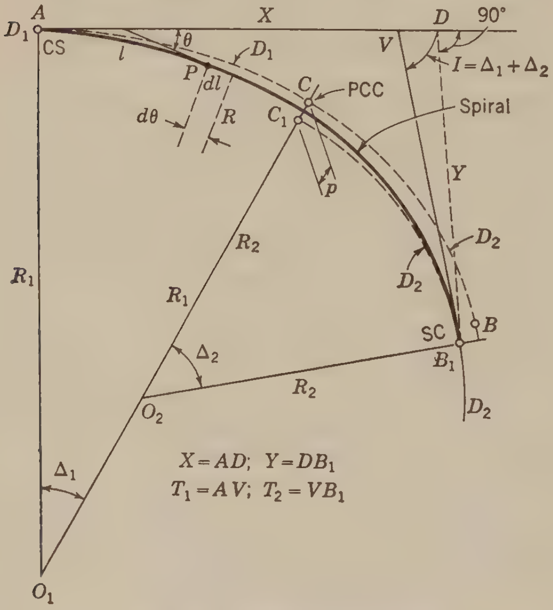
A
D1
CS
X
θ
l
D1
D
90°
V
I=Δ1+Δ2
P
dl
C
PCC
Spiral
dθ
R
C1
p
Y
R2
D2
R1
R1
D2
SC
B
B1
Δ2
R2
O2
D2
X=AD; Y=DB1
T1=AV; T2=VB1
Δ1
O1

FIG. 80. Combining spiral AB_1.

Case I: Sharper Curve Shifted Inward (*Fig.* 80). Since the degree of curve of the spiral increases as the distance from point A, we have

$$D = D_1 + Kl, \tag{97}$$

where K is a constant.

At point B_1, $D = D_2$ and $l = L_s$; hence Eq. (97) becomes $D_2 = D_1 + KL_s$, from which $K = (D_2 - D_1)/L_s$. Substituting this value of K in Eq. (97), we have

$$D = D_1 + \frac{l}{L_s}(D_2 - D_1). \tag{98}$$

From the differential sector at point P, we have

$$d\theta = \frac{dl}{R} = \frac{D}{100}\,dl, \tag{a}$$

from which

$$D = 100 \times \frac{d\theta}{dl}\ \text{(radians)}. \tag{b}$$

Substituting the above value of D in Eq. (98) and reducing, we have

$$d\theta = \frac{1}{100}\left[D_1 + (D_2 - D_1)\frac{l}{L_s}\right]dl. \tag{c}$$

Integrating and factoring, we get

$$\theta = \frac{l}{100}\left[D_1 + \frac{1}{2}(D_2 - D_1)\frac{l}{L_s}\right] \tag{99}$$

At point B_1, where $l = L_s$, θ becomes I, that is,

$$I = \Delta_1 + \Delta_2 = \frac{L_s}{200} \times (D_1 + D_2), \tag{100}$$

or

$$L_s = 200 \times \frac{I}{D_1 + D_2}. \tag{101}$$

Equation (102) is satisfied if

$$100 \times \frac{\Delta_1}{D_1} = 100 \times \frac{\Delta_2}{D_2} = \frac{1}{2}L_s = \text{arc } AC = \text{arc } CB \tag{d}$$

Also,

$$\Delta_1 = \frac{L_sD_1}{200} \quad \text{and} \quad \Delta_2 = \frac{L_sD_2}{200} \tag{102}$$

Equation (83), giving the "spiral angle" θ_s, when a spiral connects a tangent with a circular arc, may be written

$$\theta_s = \frac{l_s}{200}(D_c - 0),$$

in which $(D_c - 0)$ is the total change in degree of curvature. Likewise, the nominal spiral angle for a spiral connecting two curves of different radii is

$$\theta_s = \frac{L_s}{200} \times (D_2 - D_1) = \Delta_2 - \Delta_1. \tag{103}$$

Equation (99) may be written

$$\theta = D_1 \times \frac{l}{100} + \theta_s \times \left(\frac{l}{L_s}\right)^2, \tag{104}$$

where θ_s is given by Eq. (103).

If $D_1 = 0$ (straight line), Eq. (104) reduces to Eq. (85). Equation (104) is useful for orientation when the transit is moved to some point on the spiral. The backsight angle between a tangent at any point and the chord to the CS is $\theta - \phi$, where ϕ is the forward deflection angle to the point.

The radial shift p (ref. Art 81 for comparison) may be found by entering Table 15 with values of θ_s and L_s as arguments.

Long and short tangents. The values of T_1 (long tangent) and T_2 (short tangent) are determined by two methods, each leading to

$$T_2 = \frac{Y}{\sin I} \qquad \text{and} \qquad T_1 = X - Y \cot I \tag{105}$$

Attention is called to the fact that cot I and cos I are negative if $I > 90°$.

Method I. The expressions for X and Y, given in Table 22, are based on formulas derived in Appendix E. Knowing X and Y,

$$T_2 = \frac{Y}{\sin I}; \qquad T_1 = X - Y \cot I. \tag{106}$$

Method II. Solve for X and Y by treating Fig. 80 as a closed traverse. Let O_1A = north (zero azimuth) and note that $O_2O_1 = R_1 - R_2 - p$. Then proceeding clockwise, we get from Σ latitude = 0 and Σ longitude = 0:

$$Y = R_1 - R_2 \cos I - (R_1 - R_2 - p) \cos \Delta_1; \tag{a}$$
$$X = R_2 \sin I + (R_1 - R_2 - p) \sin \Delta_1. \tag{b}$$

By Eq. (105), we have

$$T_2 = \frac{Y}{\sin I} = \frac{R_1 - R_2 \cos I - (R_1 - R_2 - p) \cos \Delta_1}{\sin I} \tag{107}$$

and $T_1 = X - Y \cot I$

$$= \frac{R_2 - R_1 \cos I + (R_1 - R_2 - p) \cos \Delta_2}{\sin I}. \tag{108}$$

Table A gives values of T_1 and T_2 for a few combinations of D_1 and D_2 with spiral lengths of 200, 300, 400, and 500 ft. See Table 22 for other cases.

Table A. Long and Short Tangents for Spiraled Compound Curves

Degree of curve		200-ft spiral		300-ft spiral		400-ft spiral		500-ft spiral		Degree of curve	
D_1	D_2	T_1	T_2	T_1	T_2	T_1	T_2	T_1	T_2	D_1	D_2
1°	3°	116.71	83.40	175.13	125.13	233.65	166.97	292.28	208.93	1°	3°
1	4	120.06	80.06	180.21	120.19	240.49	160.46	300.96	200.89	1	4
1	5	122.31	77.86	183.63	116.94	245.14	156.20	306.92	195.70	1	5
1	6	123.93	76.30	186.11	114.65	248.56	153.24	311.36	192.17	1	6
2	4	111.20	88.98	166.97	133.63	222.95	178.48	279.20	223.60	2	4
2	5	114.41	85.83	171.85	128.97	229.56	172.37	287.66	216.12	2	5
2	6	116.83	83.48	175.54	125.51	234.62	167.87	294.18	210.70	2	6
2	7	118.72	81.67	178.45	122.85	238.65	164.47	299.46	206.66	2	7
2	8	120.24	80.23	180.69	120.73	241.98	161.84	303.89	203.62	2	8
3	5	108.50	91.82	163.05	138.03	217.98	184.61	273.41	231.66	3	5
3	6	111.32	89.09	167.36	134.00	223.88	179.37	281.04	225.34	3	6
3	7	113.59	86.91	170.86	130.81	228.71	175.27	287.80	220.48	3	7
3	8	115.46	85.13	173.76	128.24	232.77	172.02	292.75	216.70	3	8
3	9	117.03	83.67	176.22	126.15	236.25	169.41	297.42	213.74	3	9
4	6	106.92	93.58	160.87	140.84	215.40	188.68	270.73	237.29	4	6
4	7	109.40	91.21	164.68	137.38	220.69	184.23	277.65	232.03	4	7
4	8	111.48	89.24	167.91	134.53	225.20	180.63	283.64	227.84	4	8
4	9	113.25	87.59	170.69	132.16	229.12	177.68	288.94	224.51	4	9
4	10	114.28	86.18	173.11	130.17	232.61	175.25	293.70	221.86	4	10
5	7	105.92	94.81	159.59	142.89	214.12	191.80	269.79	241.90	5	7
5	8	108.13	92.73	163.01	139.89	218.89	188.03	276.19	237.52	5	8
5	9	110.03	90.96	165.99	137.36	223.13	184.89	281.90	233.99	5	9
5	10	111.69	89.44	168.62	135.21	226.91	182.27	287.09	231.15	5	10
5	11	113.15	88.12	170.97	133.37	230.34	180.09	291.87	228.88	5	11
6	12	111.94	89.68	169.49	136.07	229.06	184.34	291.46	235.34	6	12

Deflection angles. With a transit set at point A (Fig. 80), vernier reading 0°00′ on auxiliary tangent, the deflection angle ϕ to any forward point on the spiral equals the sum of the deflections (1) *for circular curve of degree* D_1 and (2) *the spiral deflection for the same distance as if measured from the TS.*

Also, with a transit set at point B_1 (Fig. 80), vernier reading 0°00′ on auxiliary tangent at B_1, the deflection angle ϕ to any *backward* point on the spiral equals the *difference* of the deflection angles (1) *for circular curve of degree* D_2 and (2) *the spiral deflection for the same distance as if measured from the TS.*

Corrections. Certain *corrections** apply to Δ_1, Δ_2, p, and ϕ. For flat spirals these corrections are negligible, but with increasing sharpness they become appreciable.

Δ_2 should be slightly larger and Δ_1 smaller by the same amount. (The sum of the corrected Δ's $= I$.) The correction to be added to Δ_2 is $C_b = 0.03655\Delta_2\Delta_1(\Delta_2 - \Delta_1)$ (approx.), where the Δ's are in degrees and C_b is in seconds.

The value of the shift p (found in Table 15) is to be decreased by C_p, where

$$C_p = 0.222L_s\Delta_2\Delta_1(\Delta_2 - \Delta_1) \times 10^{-7}, \tag{109}$$

in which C_p and L_s are expressed in feet and the Δ's are in degrees.

Proceeding in the direction of increasing curvature,

$$\phi = \frac{D_1 l}{200} + \frac{1}{3}\theta_s\left(\frac{l}{L_s}\right)^2 - C; \tag{110}$$

proceeding in the direction of decreasing curvature (Fig. 81),

$$\phi = \frac{D_2 l}{200} - \frac{1}{3}\theta_s\left(\frac{l}{L_s}\right)^2 + C, \tag{111}$$

in which $C = C_s + C_a$ (see Tables 20 and 21).

At the end of the spiral, Eq. (110) becomes

$$\phi_1 = \frac{D_1 L_s}{200} + \frac{1}{3}\theta_s - C$$

and Eq. (111) becomes $\phi_2 = D_2L_s/200 - \frac{1}{3}\theta_s + C$.

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* See paper by Paul Hartman, *Trans. ASCE*, vol. 122, p. 389, 1957.

Note: No corrections for Δ_2, Δ_1, and p are needed in connection with Method I; in fact, the quantity p is not used.

Case II: Flatter Curve Shifted Outward. This case is shown by Fig. 81, which is the same as Fig. 80 in reverse.

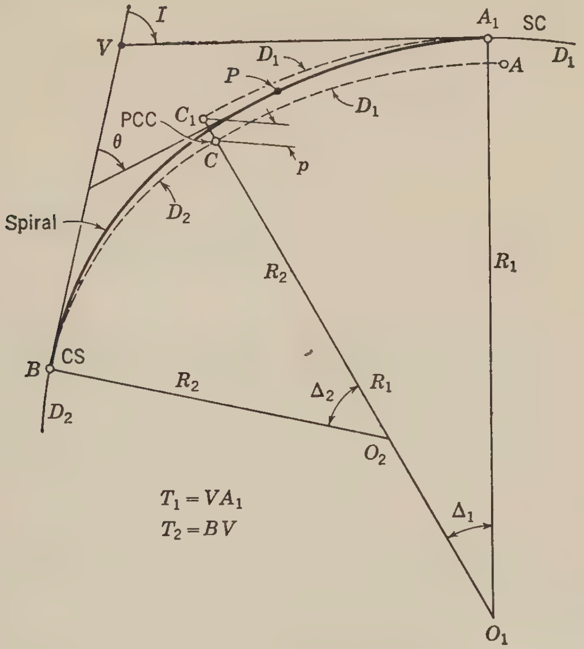

I
V
A₁
SC
D₁
D₁
A
P
D₁
PCC
C₁
θ
C
p
Spiral
D₂
R₂
R₁
CS
B
R₂
Δ₂
R₁
D₂
O₂
T₁ = VA₁
T₂ = BV
Δ₁
O₁

FIG. 81. Combining spiral BA_1.

Hence Methods I and II for finding long and short tangents apply also to Case II.

In this case the degree of curve for any point on the spiral decreases as the distance from B, the CS. Hence

$$D = D_2 - \frac{l}{L_s}(D_2 - D_1). \qquad (a)$$

Proceeding as in Case I, we get

$$\theta = D_2 \times \frac{l}{100} - \theta_s \times \left(\frac{l}{L_s}\right)^2. \tag{112}$$

Equation (112) is useful for orientation when the transit is moved to some point on the spiral.

Table A gives values of T_1 and T_2, the long and short tangents, for a few combinations of D_1, D_2, and L_s.

Example. Given $D_1 = 4°$, $D_2 = 10°$, $L_s = 300$ ft. Required: long and short tangents.

Solution. Using Method I (see Eq. (106),

$$\Delta_1 = \frac{300 \times 4}{200} = 6°; \qquad \Delta_2 = \frac{300 \times 10}{200} = 15°; \qquad I = 21°;$$

$$\theta_s = \tfrac{300}{200}(10 - 4) = 9°; \qquad N = \frac{2D_1}{D_2 - D_1} = \tfrac{4}{3} = 1\tfrac{1}{3}.$$

By 'Table 22,

$$\begin{aligned} Y &= 3[9(1.745329) - (0.9)^3(0.218758) + (0.9)^5(0.001168)] \\ &= 3 \times (15.707961 - 0.159469 + 0.000690) = 3 \times (15.549182) \\ &= 46.6475; \\ X &= 3 \times [(100 - (0.9)^2(2.22258) + (0.9)^4(0.017452) \\ &\qquad - (0.9)^6(0.000067)] \\ &= 3(100 - 1.80029 + 0.01145 - 0.00004) = 3(98.21112) \\ &= 294.6334. \end{aligned}$$

The deflection angle ϕ_1 is calculated in two ways that agree:

(1) $\tan \phi_1 = Y/X = 0.1583240$, from which $\phi_1 = 8°59'48''$.

(2) $\phi_1 = \frac{300}{200} \times 4 + \frac{9}{3} - C = 6° + 3° - 12'' = 8°59'48''$, where C (by Tables 20 and 21) $= 2.3 + 9.8 = 12$ sec.

Also, $$T_2 = \frac{Y}{\sin I} = 130.166 \text{ ft},$$

and $T_1 = X - Y \cot I = 294.6334 - 121.5210 = 173.112$ ft.

97. Parallel Spirals. Proceeding as for the simple spiral (Fig. 78*b*), let dl = length of differential arc subtending central angle $d\theta$, where the corresponding outer and inner concentric arcs are $dl_o = (R + w)\,d\theta = dl + w\,d\theta$ and $dl_i = (R - w)\,d\theta = dl - w\,d\theta$.

Substituting $d\theta = \frac{1}{100}[D_1 + (D_2 - D_1)l/L_s]\,dl$ and integrating, we get

$$l_o = l + \frac{w}{100}\left[D_1 l + (D_2 - D_1)\frac{l^2}{2L_s}\right]$$

$$= l + \frac{w}{100} D_1 l + w\left(\frac{l}{L_s}\right)^2 \theta_s \text{ (angles in radians);}$$

or

$$l_o = l + (0.017453)w\left[\frac{D_1 l}{100} + \left(\frac{l}{L_s}\right)^2 \theta_s\right] \text{ (angles in degrees).} \qquad (a)$$

At the SC, $l = L_s$; then Eq. (a) becomes

$$L_o = L_s + (0.017453)w\left(\frac{D_1 L_s}{100} + \theta_s\right). \qquad (b)$$

Likewise, the length of inside spiral is

$$L_i = L_s - (0.017453)w\left(\frac{D_1 L_s}{100} + \theta_s\right) \qquad (c)$$

Location by Deflection Angles. With L_o and L_i, the lengths of outside and inside curves, known, they may be located in the usual way by deflection angles to any desired point of division, at a distance l_o (or l_i) from the TS. The spiral angle θ_s remains the same as for the center-line combining spiral, while the degree of the flatter curve is D_1 (outside curve) $= 200 \times \theta_s/L_o$ and D_1 (inside curve) $= 200 \times \theta_s/L_i$.

Location by Offsets. Two procedures may be used for staking the outside and inside parallel curves: (1) when the center-line curve is divided into equal parts; (2) when points on the parallel curves are located opposite full stations along the center-line curve.

1. *Center-line curve divided into n equal parts.* With the center-line curve divided into n equal parts, Eq. (a) shows that the length of the first outer chord (called $c1_o$) is

$$c_{1o} = \frac{L_s}{n}[1 + (0.00017453)wD_1] + (0.017453)(w\theta_s)\left(\frac{1}{n^2}\right).$$

As for the simple spiral (see page 192), the second differences in the lengths of the successive chords involve $2/n^2$. Hence the constant increment (called CI) is

$$\text{CI} = (0.034906)\left(\frac{w\theta_s}{n^2}\right). \qquad (d)$$

Equation (d) gives corrections to be added to (or subtracted from) each successive chord. Also, the *first* outside chord equals

$$\frac{L_s}{n}[1 + (0.00017453)wD_1] + \tfrac{1}{2}(\mathrm{CI}).$$

Example 1. Given $w = 20$ ft, $L_s = 200$ ft, $D_1 = 4°$, $D_2 = 8°$, from which data $\theta_s = 4°$. Required: the outer and inner chords when the center-line spiral is divided into five equal parts.

Solution. With $n = 5$, $\mathrm{CI} = (0.034906)(20 \times 4/25) = 0.112$ ft.
Also, $40(1 + 0.00017453 \times 20 \times 4) = 40(1.01396) = 40.558$ ft.
Then $c1_o = 40.558 + 0.056 = 40.614$; $c1_i = 39.386$
$c2_o = 40.614 + 0.112 = 40.726$; $c2_i = 39.274$
$c3_o = 40.726 + 0.112 = 40.838$; $c3_i = 39.162$
$c4_o = 40.838 + 0.112 = 40.950$; $c4_i = 39.050$
$c5_o = 40.950 + 0.112 = 41.062$; $c5_i = 38.938$
204.190 195.810

By Eq. (b), $L_o = 204.189$ ft; by Eq. (c), $L_i = 195.811$ ft.

2. *Points on the parallel spirals opposite even stations*

Example 2. Assume the PC at Sta. 60 + 40, while the curve data are the same as in Example 1, that is, $w = 20$ ft, $L_s = 200$ ft, $D_1 = 4°$, $D_2 = 8°$, $\theta_s = 4°$. Required: outer and inner chords.

Solution. Substituting the values of w, D_1, and θ_s in Eq. (a), we get $l_o = l + (0.34906)[4 \times l/100 + (l/L_s)^2 \times 4] = l +$ curve increment.

The tabulated results that follow are self-explanatory.

Station	l	$\frac{l}{L_s}$	$\frac{4 \times l}{100}$	$\left(\frac{l}{L_s}\right)^2 \times 4$	Sum	Curve increment	l_o	c_o	c_i
60 + 40	0	0	0	0	0	0	0		
								> 60.963	59.037
61	60	0.3	2.40	0.36	2.76	0.963	60.963		
								> 50.995	49.005
61 + 50	110	0.55	4.40	1.21	5.61	1.958	111.958		
								> 51.170	48.830
62	160	0.8	6.40	2.56	8.96	3.128	163.128		
								> 41.061	38.939
62 + 40	200	1	8.00	4.00	12.00	4.189	204.189	204.189	195.811

Note: It is seen that the values of L_o and L_i agree with those in Example 1.

Decreasing Curvature. For the case of decreasing curvature, where $D = D_2 - (l/L_s)(D_2 - D_1)$, it follows that

$$l_o = l + (0.017453)w\left[\frac{D_2 l}{100} - \left(\frac{l}{L_s}\right)^2 \theta_s\right]$$

and

$$l_i = l - (0.017453)w\left[\frac{D_2 l}{100} - \left(\frac{l}{L_s}\right)^2 \theta_s\right], \qquad (e)$$

where θ_s (in degrees) $= (D_2 - D_1)(L_s/200)$.

98. Nonparallel Outer and Inner Connecting Spirals Where the Radial Distances Are Variable. Referring to the notations of Arts. 96 and 97, where D_1 = degree of curve at the CS, D_2 = sharper degree of curve at the SC, and l = length of any arc AB ($l_s = L_s$ = total length of spiral), we find (Fig. 82)

$$d\theta = \tfrac{1}{100}\left[D_1 + (D_2 - D_1)\left(\frac{l}{L_s}\right)\right]dl \qquad (a)$$

and

$$dl_o = R\,d\theta + (w_2 + \Delta w)\,dl = dl + w_2\,d\theta + \Delta w\,d\theta. \qquad (b)$$

Substituting (a) in (b) and letting $\Delta w = (l/l_s)(w_2' - w_2)$, we get

$$dl_o = dl + w_2\left[\frac{D_1}{100} + \left(\frac{D_2 - D_1}{100}\right)\frac{l}{l_s}\right]dl + (w_2' - w_2)\left[\frac{D_1 l}{100 l_s} + \frac{(D_2 - D_1)l^2}{100 l_s^2}\right]dl. \qquad (c)$$

Integrating Eq. (c), we get

$$l_o = l + w_2\left[\frac{D_1 l}{100} + \frac{(D_2 - D_1)l^2}{200 l_s}\right] + (w_2' - w_2)\left[\frac{D_1 l^2}{200 l_s} + \frac{(D_2 - D_1)l^3}{300 l_s^3}\right]. \qquad (d)$$

Replacing $D_2 - D_1$ with $200\theta_s/l_s$ [see Eq. (103)], Eq. (d) becomes

$$l_o = l + w_2\left[\frac{D_1 l}{100} + \left(\frac{l}{l_s}\right)^2 \theta_s\right] + (w_2' - w_2)\left[\frac{D_1 l^2}{200 l_s} + \frac{2}{3}\left(\frac{l}{l_s}\right)^3 \theta_s\right], \qquad (e)$$

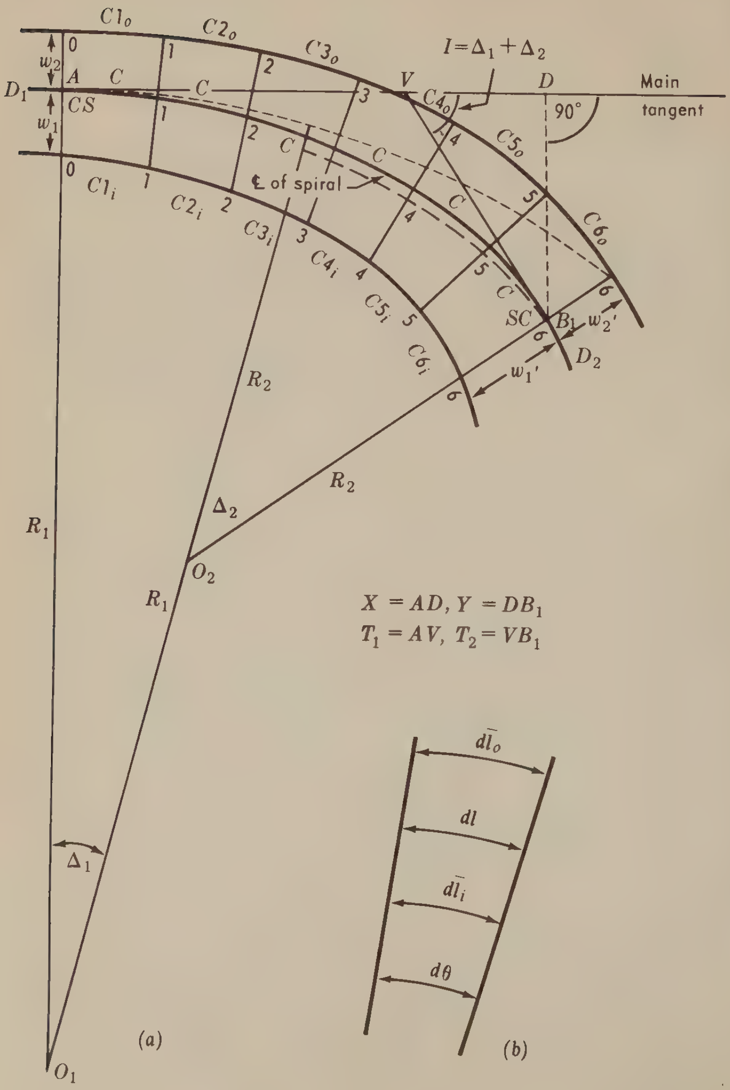

$C1_o$
$C2_o$
$C3_o$
$I = \Delta_1 + \Delta_2$
V
D
Main tangent
90°
$C4_o$
$C5_o$
$C6_o$
w_2
A
C
D_1
CS
w_1
0
1
2
3
4
5
6
$C1_i$
$C2_i$
$C3_i$
$C4_i$
$C5_i$
$C6_i$
℄ of spiral
SC
B_1
w_2'
D_2
w_1'
R_2
R_2
Δ_2
O_2
R_1
R_1
Δ_1
O_1
$X = AD, Y = DB_1$
$T_1 = AV, T_2 = VB_1$
$d\bar{l}_o$
dl
$d\bar{l}_i$
$d\theta$
(a)
(b)

FIG. 82

where the angles are in radians (multiply by 0.0174533 if in degrees).

Assume for the moment that the center-line curve is divided into n equal parts. Then

$$C1_o = l1_o = \frac{L_s}{n} + w_2\left[\frac{D_1 l_1}{100} + \left(\frac{l_1}{l_s}\right)^2 \theta_s\right] + (w_2' - w_2)\left[\frac{D_1 l_1^2}{200 l_s} + \frac{2}{3}\left(\frac{l_1}{l_s}\right)^3 \theta_s\right]; \tag{f}$$

$$\begin{aligned} C2_o = 2\left(\frac{L_s}{n}\right) &+ w_2\left\{\frac{D_1}{100}(l_2 - l_1) + \left[\left(\frac{l_2}{L_s}\right)^2 - \left(\frac{l_1}{L_s}\right)^2\right] \times \theta_s\right\} \\ &+ (w_2' - w_2)\left\{\frac{D_1}{200 l_s}(l_2^2 - l_1^2) + \frac{2}{3}\left[\left(\frac{l_2}{L_s}\right)^3 - \left(\frac{l_1}{L_s}\right)^3\right] \times \theta_s\right\}; \end{aligned} \tag{g}$$

$$\begin{aligned} C3_o = 3\left(\frac{L_s}{n}\right) &+ w_2\left\{\frac{D_1}{100}(l_3 - l_2) + \left[\left(\frac{l_3}{L_s}\right)^2 - \left(\frac{l_2}{L_s}\right)^2\right] \times \theta_s\right\} \\ &+ (w_2' - w_2)\left\{\frac{D_1}{200 l_s}(l_3^2 - l_2^2) + \frac{2}{3}\left[\left(\frac{l_3}{L_s}\right)^3 - \left(\frac{l_2}{L_s}\right)^3\right] \times \theta_s\right\}; \text{ etc.} \end{aligned} \tag{h}$$

Letting $\Sigma C_o = C1_o + C2_o + C3_o + \cdots$, we get (when $n = 5$)

$$\Sigma C_o = L_s + w_2\left[\frac{D_1 L_s}{100} + \theta_s\right] + (w_2' - w_2)\left[\frac{D_1 L_s}{200} + \frac{2}{3}\theta_s\right]. \tag{i}$$

Likewise, for the inner curve,

$$l_i = l - w_1\left[\frac{D_1 l}{100} + \left(\frac{l}{l_s}\right)^2 \theta_s\right] - (w_1' - w_1)\left[\frac{D_1 l^2}{200 l_s} + \frac{2}{3}\left(\frac{l}{l_s}\right)^3 \theta_s\right], \tag{j}$$

where the angles are in radians (multiply by 0.0174533 if in degrees).

Assume for the moment that the center-line curve is divided into n equal parts. Then

$$C1_i = \frac{L_s}{n} - w_1 \left[\frac{D_1 l_1}{100} + \left(\frac{l_1}{l_s} \right)^2 \theta_s \right] - (w_1' - w_1) \left[\frac{D_1 l_1^2}{200 l_s} + \frac{2}{3} \left(\frac{l_1}{l_s} \right)^3 \theta_s \right]; \quad (k)$$

$$C2_i = \frac{2L_s}{n} - w_1 \left\{ \frac{D_1}{100} (l_2 - l_1) + \left[\left(\frac{l_2}{l_s} \right)^2 - \left(\frac{l_1}{l_s} \right)^2 \right] \theta_s \right\} - (w_1' - w_1) \left\{ \frac{D_1}{200 l_s} (l_2^2 - l_1^2) + \frac{2}{3} \left[\left(\frac{l_2}{l_s} \right)^3 - \left(\frac{l_1}{l_s} \right)^3 \right] \times \theta_s \right\}; \text{ etc.} \quad (l)$$

Letting $\Sigma C_i = C1_i + C2_i + C3_i + \cdots$, we get (when $n = 5$)

$$\Sigma C_i = L_s - w_1 \left[\frac{D_1 L_s}{100} + \theta_s \right] - (w_1' - w_1) \left[\frac{D_1 L_s}{200} + \frac{2}{3} \theta_s \right]. \quad (m)$$

Example. Given $D_1 = 4°$, $D_2 = 8°$, $l_s (= L_s) = 200$ ft, $w_2 = 20$ ft, $w_2' = 22.5$ ft, $w_1 = 21$ ft, $w_1' = 26$ ft, and $n = 5$. Required: the successive outer and inner chords or arcs when the center-line connecting spiral is divided into five equal parts.

Solution. $\theta_s = L_s/200(D_2 - D_1) = 4°$; Δw for the outer curve $= \frac{2.5}{5} = 0.5$ ft; and Δw for the inner curve $= \frac{5.0}{5} = 1.0$ ft.

By Eqs. (k), (l), etc.,

$$\begin{array}{rcrcrcr}
C1_i & = & 39.354926 & - & 0.015824 & = & 39.339102 \\
C2_i & = & 39.237640 & - & 0.054920 & = & 39.182720 \\
C3_i & = & 39.120354 & - & 0.105185 & = & 39.015169 \\
C4_i & = & 39.003068 & - & 0.166621 & = & 38.836447 \\
C5_i & = & \underline{38.885781} & - & \underline{0.239227} & = & \underline{38.646554} \\
 & & 195.601769 & & 0.581777 & & 195.019991
\end{array}$$

By Eq. (m), $\Sigma C_i = 200 - 252 \times 0.0174533 - \frac{100}{3} \times 0.017453 = 200 - 4.398232 - 0.581777 = 195.01999$ (check).

By Eqs. (*f*), (*g*), (*h*), etc.,

$$\begin{array}{lrcrcr}
C1_o = & 40.614356 & + & 0.007912 & = & 40.622268 \\
C2_o = & 40.726057 & + & 0.027460 & = & 40.753517 \\
C3_o = & 40.837758 & + & 0.052593 & = & 40.890351 \\
C4_o = & 40.949460 & + & 0.083310 & = & 41.032770 \\
C5_o = & \underline{40.061161} & + & \underline{0.119613} & = & \underline{41.180774} \\
 & 204.188792 & & 0.290888 & & 204.479680
\end{array}$$

By Eq. (*i*), $\Sigma C_o = 200 + 240 \times 0.0174533 + 16.666666 \times 0.0174533 = 200 + 4.188792 + 0.290888 = 204.479680$ (check).

99. Two-centered Compound Curve with Three Spirals. Proceeding from flatter to sharper curve (Fig. 83), let

$$I_1 = \theta + \theta' + \Delta_1 = \text{total central angle up to the PCC;}$$
$$I_2 = \Delta_2 + \theta'' + \theta_2; I = \text{total external deflection angle} = I_1 + I_2;$$

in which θ_1 and θ_2 are spiral angles applying to end spirals from the initial and forward tangents; θ' and θ'' are central angles of possible additional circular arcs; and $(\Delta_1 + \Delta_2)$ is the central angle of the combining spiral. Then, if L_1, L_s, and L_2, taken in order, are the spiral lengths, we have

$$\theta_1 = \frac{L_1}{200} \times D_1; \qquad \Delta_1 = \frac{L_s}{200} \times D_1; \qquad \Delta_2 = \frac{L_s}{200} \times D_2;$$
$$\theta_2 = \frac{L_2}{200} \times D_2.$$

This case is an extension of the unspiraled 2-centered compound curve where the long and short tangents are T_a and T_b (see page 125).

Figure 84 shows the relative positions of the tangents to the spiraled and unspiraled curves. Let $T_{s1} = T_a + A_t + k_1$, $T_{s2} = T_b + B_t + k_2$, where A_t and B_t are displacements along VA and VB due to the combined effect of the three offsets or "throws" and the k's are defined on page 168 (if $p_1 = 0$, then $k_1 = 0$; if $p_2 = 0$, then $k_2 = 0$).

For convenience, let $t_{s1} = T_a + A_t$, $t_{s2} = T_b + B_t$; then

$$T_{s1} = t_{s1} + k_1; \qquad T_{s2} = t_{s2} + k_2.$$

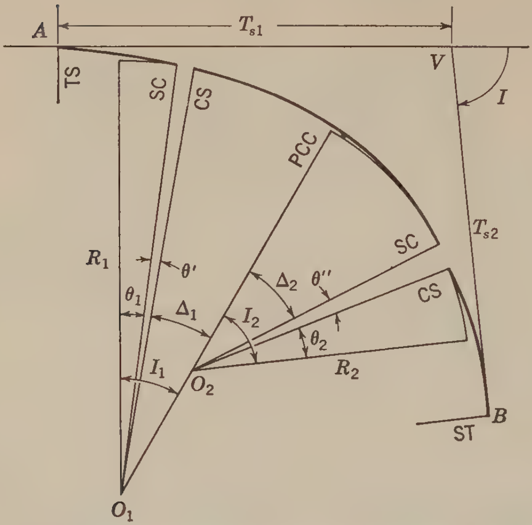

A
T_{s1}
V
I
TS
SC
CS
PCC
T_{s2}
R_1
θ'
θ_1
Δ_1
Δ_2
θ''
SC
CS
I_2
θ_2
I_1
O_2
R_2
B
ST
O_1

FIG. 83. Two-centered compound curve completely spiraled.

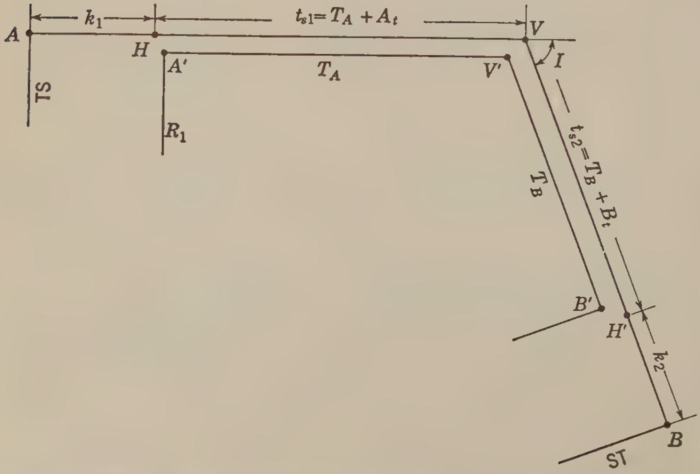

A
k_1
H
$t_{s1} = T_A + A_t$
V
I
TS
A'
T_A
V'
R_1
T_B
$t_{s2} = T_B + B_t$
B'
H'
k_2
B
ST

FIG. 84. Relative positions of the tangents to the spiraled and unspiraled curves.

Formulas for A_t and B_t. In Fig. 85, let $A_t = a_1 + a + a_2$, where a_1, a, and a_2, treated separately, are the displacements in the direction of the tangent VA resulting from the offsets p_1, p, and p_2. Also, let $B_t = b_1 + b + b_2$, where b_1, b, and b_2, treated separately, are the displacements in the direction of the tangent VB resulting from the offsets p_1, p, and p_2. The a's and b's are found by resolving the offsets into oblique components along VA and VB. The algebraic signs conform to conventional trigonometry. Accordingly, cos I and cot I are negative if $I > 90°$. Then

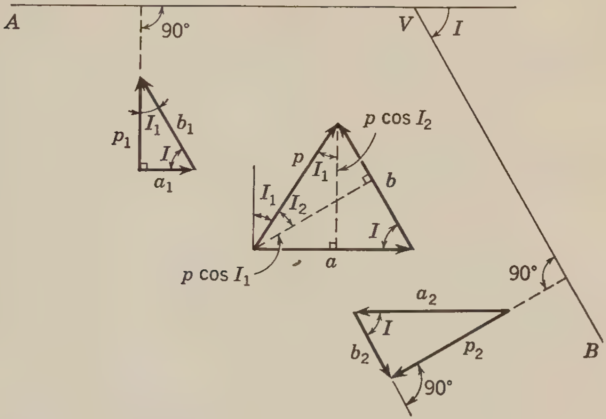


FIG. 85. Displacements along VA and VB due to the offsets p_1, p, and p_2.

$$a_2 = \frac{p_2}{\sin I}, \qquad a_1 = p_1 \cot I = \frac{p_1 \cos I}{\sin I}, \qquad a = \frac{p \cos I_2}{\sin I}.$$

Combining the a's and noting that the arrows pointing in the directions VA and VB are positive, in the reverse direction, negative, we have

$$A_t = \frac{p_2 - p \cos I_2 - p_1 \cos I}{\sin I}. \tag{115}$$

Likewise, combining the b's and noting that the arrows of Fig.

85 are reversed in the counterclockwise circuit, we get

$$B_t = \frac{p_1 + p \cos I_1 - p_2 \cos I}{\sin I}, \tag{116}$$

where $I_1 = I - I_2$.

Formulas for t_{s1} and t_{s2}. Adding the expressions for A_t and B_t to those for T_a and T_b (see page 125), we get

$$t_{s1} = \frac{(R_2 + p_2) - (R_1 + p_1) \cos I + (R_1 - R_2 - p) \cos I_2}{\sin I}; \tag{117}$$

$$t_{s2} = \frac{(R_1 + p_1) - (R_2 + p_2) \cos I - (R_1 - R_2 - p) \cos I_1}{\sin I}. \tag{118}$$

If the end spirals are omitted, that is, if $p_1 = p_2 = 0$, Eqs. (117) and (118) reduce to Eqs. (107) and (108), in which $\Delta_1 = I_1$ and $\Delta_2 = I_2$.

100. Three-centered Compound Curve with Four Spirals. This case is an extension of the 2-centered curve. Let L_s and L_s', taken in order, equal the lengths of the intermediate combining spirals, while p and p' are the corresponding radial shifts.

Let $I_1 = \theta_1 + \theta' + \Delta_1$ = total central angle up to the first PCC; $I_2 = I_1 + \Delta_2 + \Delta_3$ = total central angle up to the second PCC; I = entire central angle $= I_2 + \Delta_4 + \theta' + \theta_2$; $I_3 = \Delta_4 + \theta' + \theta_2$.

As before, let $t_{s1} = T_a + A_t$, $t_{s2} = T_b + B_t$, where T_a and T_b are the long and short tangents for the unspiraled 3-centered compound curve.

Except for an additional term including p' (Fig. 86), the expressions for A_t and B_t are like those for the 2-centered curve. Hence by analogy, we have

$$A_t = \frac{p_2 - p_1 \cos I - p \cos I_2 - p' \cos I_3}{\sin I}; \tag{119}$$

$$B_t = \frac{p_1 - p_2 \cos I + p \cos I_1 + p' \cos I_2}{\sin I}. \tag{120}$$

Combining A_t and B_t with Eqs. (50) and (48) giving values of

T_a and T_b for the unspiraled 3-centered curve, we have

$$t_{s1} = \frac{(R_3 + p_2) - (R_1 + p_1) \cos I + (R_1 - R_2 - p) \cos I_2 + (R_2 - R_3 - p') \cos I_3}{\sin I};$$

$$t_{s2} = \frac{(R_1 + p_1) - (R_3 + p_2) \cos I - (R_1 - R_2 - p) \cos I_1 - (R_2 - R_3 - p') \cos I_2}{\sin I}. \tag{121, 122}$$

Note: The radial offsets p and p' are assumed inward as if running from a flatter to a sharper curve; however, if the third circular arc is *flatter* than the preceding arc, that is, if $R_3 > R_2$, as is the usual case in practice, then the sign of p' is to be *reversed.*

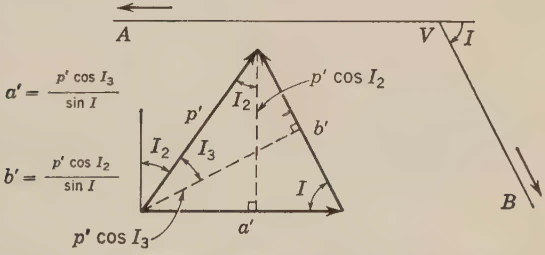

A
V
I
$a' = \frac{p' \cos I_3}{\sin I}$
$b' = \frac{p' \cos I_2}{\sin I}$
$p' \cos I_2$
I_2
p'
b'
I_2
I_3
I
B
a'
$p' \cos I_3$

FIG. 86. Displacements a' and b' due to the radial shift p'.

101. Four-centered Compound Curve with Five Spirals. This case is an extension of the 3-centered curve to include another circular arc of radius R_4 and a third radial shift p''.

Let I_1 = central angle up to the first PCC; I_2 = central angle up to the second PCC, I_3 = central angle up to the third PCC; I = total central angle; and $I_4 = I - I_3$.

It follows that

$$t_{s1} = \frac{(R_4 + p_2) - (R_1 + p_1) \cos I + (R_1 - R_2 - p) \cos I_2 + (R_2 - R_3 - p') \cos I_3 + (R_3 - R_4 - p'') \cos I_4}{\sin I}; \tag{123}$$

$$t_{s2} = \frac{\begin{array}{c}(R_1 + p_1) - (R_4 + p_2)\cos I - (R_1 - R_2 - p)\cos I_1 \\ - (R_2 - R_3 - p')\cos I_2 - (R_3 - R_4 - p'')\cos I_3\end{array}}{\sin I}. \tag{124}$$

Note: The three radial shifts p, p', and p'' are assumed inward as if running from a flatter to a sharper curve; however, if any arc is flatter than the preceding, the corresponding shift is to be *reversed* in sign.

Note: To get T_{s1} and T_{s2}, add the values of t_{s1} and t_{s2} to the corresponding k's.

CHAPTER 8

ELEMENTS OF HIGHWAY SAFETY AND DESIGN

102. Foreword. The topics involving highway safety and design will here be limited to (*a*) curve superelevation, (*b*) widening on curves, and (*c*) sight distance.

Other safety features, including the functional design of intersections and interchanges (see Chapter 9 for typical cases), are based on the characteristics and volumes of traffic flow, the details of which are outside the scope of this volume.

A. CURVE SUPERELEVATION

103. Centrifugal Force. The outward reactive force, called *centrifugal force,* which is exerted when bodies are moved rapidly around curved paths is equivalent to the product of the *mass* times the *acceleration* toward the center of curvature; that is,

$$\text{Centrifugal force} = \frac{W}{g}\frac{v^2}{r},$$

where W = weight of body, lb;
v = velocity, ft per sec;
r = radius of curve, ft;
g = acceleration due to gravity, ft per sec^2.

On highway pavements around curves, this centrifugal force tends to cause overturning or skidding outward from the center of curvature; on curved railroad track, it may cause flange pressure against the rails. To resist these outward forces (which are taken to be horizontal), it is customary to incline the highway surface *inward,* or *raise* the outer railroad rail, so that a slight banking effect is produced wherein the neutralizing forces of gravity are automatically exerted.

104. Banking Highway Curves. In Fig. 87 are given (*a*) the plan of a small portion of a two-lane highway pavement on curve and (*b*), to an enlarged scale, a cross section showing superelevation and the forces acting on a car.

Let α = angle which the inclined surface AB makes with the horizontal, $S = \tan \alpha$ = superelevation slope.

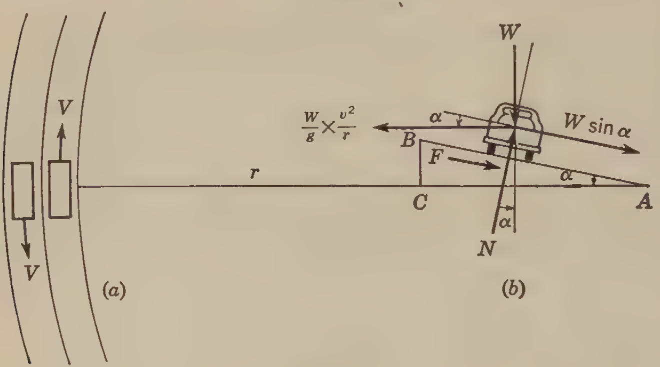


Fig. 87

Noting that the forces acting on the car are in equilibrium and considering the rectangular components parallel to AB, we have

$$W \sin \alpha + F = \frac{W}{g}\frac{v^2}{r} \cos \alpha, \qquad (125)$$

in which F is the adjustable and variable frictional force having a maximum value of fW, f being the allowable side friction factor.

Putting $F = fN = fW \cos \alpha$ in Eq. (125), then dividing by $W \cos \alpha$, we get

$$\tan \alpha + f = \frac{v^2}{gr}, \qquad \text{or} \qquad \tan \alpha = \frac{v^2}{gr} - f. \qquad (126)$$

Replacing v in feet per second by V in miles per hour and letting g = 32.2 ft per sec², Eq. (126) becomes

$$S = \textit{transverse slope} = \frac{V^2}{15r} - f. \qquad (127)$$

The values of f vary greatly with speed, condition of the car tires, and the roadway surface, whether dry, wet, smooth, or coated with ice. Tests by Moyer and others show that the side friction at which skidding is impending, for new tires on dry concrete surfaces, may reach a value equal to 0.5 at low speeds with a reduction to 0.35 at high speeds. Tests on the Pennsylvania Turnpike showed maximum values of 0.30 at very high speeds (up to 105 mph).

On the basis of nearly 900 driving tests and with a consideration of safety, Barnett (U.S. Bureau of Public Roads) recommends f values of 0.16 for speeds of 30 to 60 mph, and 0.14 for a speed of 70 mph.

For slow speeds around curves, the outward centrifugal force is relatively small, and hence sliding inward would take place except for friction. On icy pavements, the friction factor is reduced to about 0.10. Therefore, the limiting superelevation ratio S is about the same as f, or 0.10.

Taking into account the prevalence of both slow-moving and fast traffic under variable weather conditions, the ideal superelevation rate cannot be realized in practice. In regions free of ice and snow, a maximum rate for skid-resistant surfaces of $1\frac{1}{2}$ in. per ft (0.125) may be used. If occasional icy pavements are likely, maximum S values of about $1\frac{1}{4}$ in. per ft (0.10) are widely used; however, still flatter maximum slopes have been recommended for regions subject to severe winter conditions.

Highways on tangents are normally given a slight convex crown to expedite drainage. If this crown were continued around the curve, the inner half of a two-lane pavement would have a partial banking effect, but the outer lane for traffic in the reverse direction would have an unsafe adverse banking. Therefore, good practice demands that all curves, say those sharper than 1°, are superelevated so that the surface of the middle portion becomes a plane inclined toward the center of curvature.

In approaching a curve, the change from a convex to a plane surface must be done *gradually*. The following steps illustrate a typical procedure (Fig. 88): (1) The outer lane is warped gradually from a half crown to a plane level section at, or near, the TS, while the inner lane continues with the normal crown; (2) the inner lane is gradually flattened until it becomes a straight inclined surface, both lanes being now in one plane sloping inward

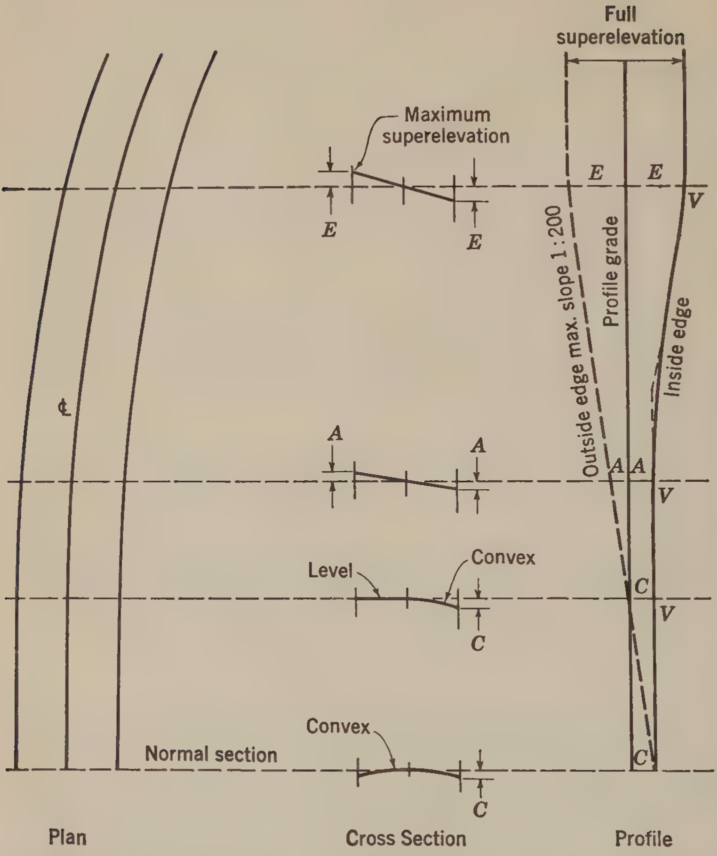

Full
superelevation
Maximum
superelevation
E
E
E
E
V
Outside edge max. slope 1 : 200
Profile grade
Inside edge
℄
A
A
A
A
V
Level
Convex
C
C
V
Convex
Normal section
C
C
Plan
Cross Section
Profile

FIG. 88. Typical method of attaining superelevation, where pavement is revolved about center line. *C* equals crown and *E* equals maximum superelevation. Short vertical curves by eye adjustment of stakes or forms in the field may be inserted, if desired, at points marked *V*.

at about one-half the final transverse rate of superelevation; (3) beyond this point the entire two-lane pavement is rotated about construction center line, the outer and inner lanes remaining in the same sloping plane, until full superelevation is accomplished at, or near, the SC.

In situations where lowering the inner edge introduces drainage difficulties, as in deep cuts, superelevation may be effected by rotation about the *inner edge,* or a line near the inner edge and parallel to the center line.

The banked sections between the SC and the CS remain constant. The runoff warped sections are like the approach sections but in reverse order.

The transition spiral with its constant rate of change of curvature provides the logical and least hazardous path along which superelevation is effected.

In obtaining superelevation, it is desirable that the slope of the outer edge of pavement relative to the profile of the center line should not be greater than 1 in 200.

On unspiraled curves, motorists make their own transition in proceeding from a straight path to the circular path. At low speeds or on widened pavements, this transition may be made within the traffic lane; for greater speeds, the driver on the outer lane tends to encroach on the inner lane, thus creating a hazard.

105. Length of Highway Spiral. The change from a crown section on tangent to a fully banked section on curve requires a measurable *time* (t sec, say) during which a car traveling at constant speed will traverse a certain distance, which here equals the required length of transition spiral. That is,

$$t = \frac{l_s}{v} \tag{128}$$

Rearranging Eq. (125) and letting $\cos \alpha = 1$, $\sin \alpha = S$, $W/g = M$ (mass of car), and $F = Ma$ (mass × acceleration), we get

$$Ma = M\frac{v^2}{r} - MgS, \tag{129}$$

or

$$a = \frac{v^2}{r} - gS, \tag{130}$$

where a, v^2/r, and gS are accelerations proportional respectively to frictional, centrifugal, and gravitational forces.

Let t $(= l_s/v)$ = time for traveling at a constant speed the v

distance l_s; t_1 ($= v^2/c_1r$) = time for acquiring the radial acceleration proportional to the centrifugal force; and t_2 ($= gS/c_2$) = time for rotating the car through an angle α, in which c_1 and c_2 are corresponding variable rates of change in acceleration.

Since these time intervals occur simultaneously, they are *equal;* that is, $t = t_1 = t_2$.

But if we replace c_1 and c_2 with C_1 and C_2, the latter values being *maximum* rates of change in these accelerations that can take place without discomfort, then the corresponding t_2 is usually less than t_1.

Assuming $t = t_1$ and reducing, we get

$$l_s = \frac{v^3}{C_1 r}. \tag{131}$$

Likewise on the basis of $t = t_2$, we get

$$l_s = \frac{vgS}{C_2}. \tag{132}$$

Replacing v in feet per second by V in miles per hour, and letting $g = 32.2$ ft/sec², Eqs. (131) and (132) become

$$l_s = \frac{3.155V^3}{C_1 r} \tag{133}$$

and

$$l_s = \frac{47.23VS}{C_2}. \tag{134}$$

Many years ago, Shortt of England, on the basis of railway superelevation studies, proposed a formula like Eq. (133), which, with an assumed $C_1 = 2$, has been widely used by highway engineers in this country, in the form

$$l_s = \frac{1.58V^3}{r}, \tag{135}$$

where $C = 2$ = maximum rate of change of acceleration that can take place without discomfort.

Leeming of England was the first to point out the fact that Eq. (135) is fallacious, since it disregards superelevation. However, it deviates on the side of safety, because a spiral of considerable length provides an easier approach and better appearance than one which is too short.

The *rotational effect* similar to railroad practice has been advocated by Haile and others as a means of determining the length of spiral. This method was applied to the Pennsylvania Turnpike on the basis of a rotational change of 0.02 ft per ft per sec. Thus if $S = 0.10$, then $t = 0.10/0.02 = 5$ sec. Accordingly for a speed of 60 mph (88 ft per sec), the minimum length of spiral is $5 \times 88 =$ about 440 ft.

This rotational method is in effect an adaptation of Eq. (134). Thus if $C_2 = 0.644$ (conforming to a rotational change of 0.02 ft per ft per sec), we have

$$l_s = 73.3VS, \tag{136}$$

where l_s = recommended spiral length, V = design speed (mph), and S = maximum transverse slope of pavement.

All the foregoing formulas give *minimun* lengths of spirals.

106. Elevation of Outer Railroad Rail on Curves. In Fig. 87, let BC = elevation of outer rail at the gauge line, where gauge = 4 ft $8\frac{1}{2}$ in. = 56.5 in.

Neglecting friction (that is, assuming $f = 0$), Eq. (127) gives

$$\tan \alpha = \frac{BC}{AC} = \frac{V^2}{15r}. \tag{137}$$

Letting $AC = 56.5$ in. and substituting $r = 5{,}730/D$, BC in Eq. (137) becomes

$$BC = e = 0.00066DV^2, \tag{138}$$

where e = total elevation of outer rail, in.,
D = degree of curve,
V = speed, mph.

To accommodate both slow freight and fast passenger traffic, the theoretical e is modified (reduced) in practice. Many railroads use a flat reduction of 3 in., called *unbalanced superelevation.*

The inner rail is kept at grade throughout the curve and the outer rail is elevated gradually to the full amount e, which is never over 8 in.; when the track carries both fast and slow traffic, it usually does not exceed 6 in.

107. Length of Railroad Spiral. The length of the railroad spiral is generally determined by using a uniform rate of rotational change in transverse slope to develop the maximum superelevation. This rate is governed by what can be attained without discomfort to passengers. After years of experience and experiment, the AREA has established the rate of $1\frac{1}{4}$ in. per sec, which is equivalent to a rotational change of 2 percent of cross slope per second at the assumed speed.

Accordingly, if $e = 5$ in. and the speed is 70 mph, the minimum length of spiral is

$$\frac{5}{1.25} \times 88 \times \frac{70}{60} = 410 \text{ ft.}$$

B. Widening on Highway Curves

108. Purpose. It is advisable to widen pavements on curves for two reasons: (1) The rear wheels of a motor vehicle when rounding curves follow a path of shorter radius than the front wheels, thus requiring a wider space laterally than on tangents. Hence to provide the *same clearance* between vehicles on curves as on tangents, pavements on curves must have an extra width. (2) There is a psychological demand for more clearance to divert a fast-moving car safely around the turn.

Taking into account the foregoing requirements, Voshell of the U.S. Bureau of Public Roads has proposed the following formula for two-lane highways:

$$f = 2(R - \sqrt{R^2 - B^2}) + \frac{35}{\sqrt{R}}, \tag{139}$$

in which f = full extra widening in feet, B = length of wheel base (20 ft recommended), and R = radius of curve in feet.

The *proof* of the first part of Eq. (139) follows. From Fig. 89, we have

$$R_1 + d = \sqrt{R_2^2 - B^2}, \tag{a}$$

in which R_1 = radius of inner rear-wheel curved track in feet, R_2 = radius of outer front-wheel track in feet, and d = the car gauge in feet.

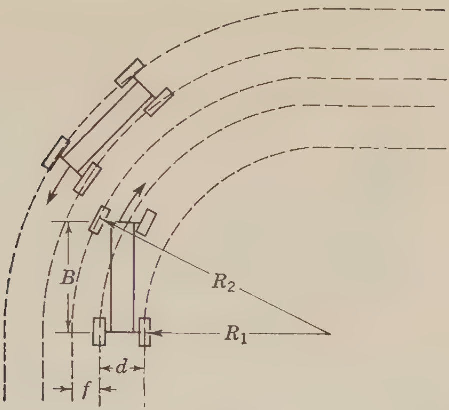


FIG. 89

Substituting $R_2 - f$ for $R_1 + d$ in Eq. (a) and rearranging terms, we get

$$f = R_2 - \sqrt{R_2^2 - B^2}. \tag{140}$$

Putting R for R_2 and multiplying by 2 to allow for two lanes, we get Eq. (139), which may be transformed into a more convenient but slightly approximate form as follows:

Let

$$\sqrt{R_2^2 - B^2} = (R^2 - 400)^{\frac{1}{2}} = R\left(1 - \frac{400}{R^2}\right)^{\frac{1}{2}} = R\left(1 - \frac{200}{R^2}\right).$$

Then Eq. (139) reduces to

$$f = \frac{400}{R} + \frac{35}{\sqrt{R}}. \tag{141}$$

Substituting $R = 5{,}729.58/D$, Eq. (141) becomes

$$f = 0.07D + 0.462\sqrt{D}, \tag{142}$$

in which D is the degree of curve.

The Voshell formula was designed for a speed of 35 mph, the second term being empirical. As a modification, Barnett proposes

$$f = n(R - \sqrt{R^2 - B^2}) + \frac{V}{\sqrt{R}}, \tag{143}$$

in which n = number of lanes and V = assumed design speed in mph.

Equation (143) may be written in a form similar to Eq. (142) as follows:

$$f = n(0.035D) + 0.0132V\sqrt{D}. \tag{144}$$

The author has proposed the formula

$$f = 1 + \frac{D}{10}. \tag{145}$$

Practice is not uniform as to the amount of widening. Ordinarily it varies from 1.5 to 6.0 ft, depending upon the degree of curvature and other factors. A vast mileage of secondary roads and streets needs extra width at the turns.

109. Methods of Widening. The extra width for central widening having been determined, two cases will be considered here: (1) *when the surveyed center line is a spiral* and (2) *when the surveyed center line is a circular curve.*

In Case 1, the inner edge is a modified spiral, and widening takes place only on the inside with perhaps a slight adjustment, by eye, at the SC. However, it would be feasible to divide the widening so that half takes place on the inner and half on the outer edge.

In Case 2, the inner edge is a compound circular curve, and widening occurs only on the inside.

Case 1. *Spiraled Curve*

110. Length of Inner Edge of Widened Pavement. The inner edge of a widened pavement is assumed to be a curve whose radial distance from the center-line spiral (Fig. 90) increases gradually from $\frac{1}{2}w$ at the TS to $\frac{1}{2}w + f$ at the SC, or some preceding section. The widening, therefore, varies directly as the distance l along the surveyed center line and according to the formula

$$\bar{R} = R' - \frac{l}{l_s}f = R - \frac{1}{2}w - \frac{l}{l_s}f,$$

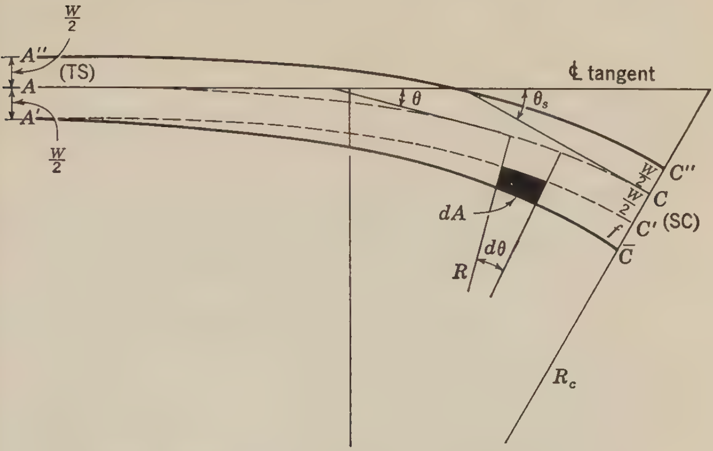


FIG. 90. Widening where surveyed center line is a spiral.

where R = variable radius of surveyed center line of pavement;
R' = variable radius of unwidened inner edge of pavement;
$\bar{R}$ = variable radius of widened inner edge of pavement.

In Fig. 90, let $d\bar{l}$ = differential part of inner curve; then

$$d\bar{l} = \bar{R}\,d\theta = \left(R - \frac{1}{2}w - \frac{l}{l_s}f\right)d\theta = R\,d\theta - \frac{1}{2}w\,d\theta - f\left(\frac{l}{l_s}\right)d\theta.$$

Noting that $R\,d\theta = dl$ and $l/l_s = \sqrt{\theta/\theta_s}$ (see equation (85)); then integrating, we get

$$\bar{l} = l - \tfrac{1}{2}w\theta - \tfrac{2}{3}f\sqrt{\frac{\theta}{\theta_s}} \times \theta. \tag{146}$$

At the SC, $\theta = \theta_s$; hence Eq. (146) becomes

$$\bar{l}_s = l_s - (\tfrac{1}{2}w + \tfrac{2}{3}f)\theta_s, \tag{147}$$

which equals the length of the inner edge of the widened pavement from a point opposite the TS to a radial line at the SC.

The angle θ_s is measured in radians (which equal *degrees* divided by 57.29578).

For the unwidened pavement, $l'' + l' = 2l_s$.

Example. Given $l_s = 400$ ft, $w = 20$ ft, $\theta_s = 15° = 0.2618$ radian, and $f = 3.0$ ft. It is required to find $\bar{l}$ and l'', the inner and outer edges, respectively, of the pavement extending from the TS to the radial lines at the SC.

Solution. $\bar{l} = 400 - (10 + 2)(0.2618) = 396.9$ ft, and $l'' = 400 + (10)(0.2618) = 402.6$ ft.

111. Extra Area Due to Widening Pavement

a. Spiral Sections on Each Side. In Fig. 90, it is required to find a formula for the extra widened area A_s between the curves $A'C'$ and $A'\bar{C}$.

As stated in Art. 110, the widening is assumed to begin at the TS and increase gradually up to f ft at the SC; that is, the widening varies directly as the distance from TS to SC.

Using the notation of Art. 110, the *differential area* (blackened area) $= dA_s =$ difference between two sectors of radii R' and $\bar{R}$ is

$$\tfrac{1}{2}R'^2\,d\theta - \tfrac{1}{2}\bar{R}^2\,d\theta = \tfrac{1}{2}(R' + \bar{R})(R' - \bar{R})\,d\theta$$
$$= \tfrac{1}{2}\left(2R - w - \frac{l}{l_s}f\right)\left(\frac{l}{l_s}f\right)d\theta.$$

Expressing l in terms of θ ($l = l_s\sqrt{\theta/\theta_s}$), integrating between the limits 0 and θ_s, and reducing, we get

$$A_s = \tfrac{1}{2}l_s f - (\tfrac{1}{3}fw + \tfrac{1}{4}f^2)\theta_s. \tag{148}$$

Multiplying by 2 to include both spiral sections, dividing by 9 to give results in square yards, and expressing the spiral angle θ_s in degrees, the formula for area of *both spiral sections* (see Fig. 91) $= 2A_s$ is

$$\frac{l_s f}{9} - \left(\frac{2}{27}fw + \frac{1}{18}f^2\right)\frac{\pi}{180}\,\theta_s° \text{ (sq yd)}. \tag{149}$$

Finally,
$$2A_s = \frac{l_s f}{9} - B, \tag{150}$$

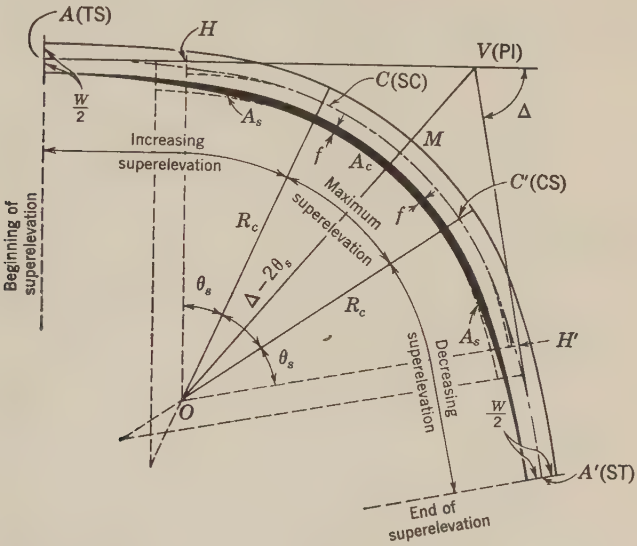
where B is given in Table 28 for various values of θ_s and f and for $w = 16$, 18, 20, and 22 ft.

Example 1. Given $l_s = 400$ ft, $\theta_s = 20°$, $f = 2$ ft, and $w = 20$ ft. It is required to find the extra area due to widening.

Solution. Using Eq. (149) and referring to Table 28 (part 3), the extra area due to widening is

$$\frac{400 \times 2}{9} - 1.11 = 87.77 \text{ sq yd.}$$

b. Central Circular Section. In Fig. 91, the area A_c of the circular section due to the constant extra width of f ft equals the difference between two sectors of central angle $(\Delta - 2\theta_s)$ and radii R' and $(R' - f)$, where $R' = R_c - \frac{1}{2}w$ (R_c being the center-line radius); or, more simply,



A(TS)
H
V(PI)
C(SC)
Δ
W/2
A_s
M
Increasing
superelevation
f
A_c
C'(CS)
Maximum
superelevation
f
Beginning of
superelevation
R_c
Δ−2θ_s
θ_s
R_c
A_s
H'
θ_s
superelevation
Decreasing
W/2
O
A'(ST)
End of
superelevation

FIG. 91

$$\begin{aligned}
\text{Area} &= (\text{length of arc through middle of strip}) \times f \\
&= (R' - \tfrac{1}{2}f)(\Delta - 2\theta_s)(f) \\
&= (R_c - \tfrac{1}{2}w - \tfrac{1}{2}f)(f)(\Delta - 2\theta_s) \\
&= L_c f - \left(\frac{\Delta - 2\theta_s}{2}\right)(wf + f^2) \text{ sq ft,}
\end{aligned}$$

where L_c is length of nominal center-line curve from SC to CS, and the angle is expressed in radians.

Area per *foot* of center-line curve is

$$f - \frac{\Delta - 2\theta_s}{2L_c}(wf + f^2) = f - \frac{1}{2R_c}(wf + f^2)$$

$$= f - \frac{D_c}{200}(wf + f^2)(\text{sq ft}), \quad (151)$$

where D_c is expressed in radians.

Dividing by 9 and expressing D_c in degrees, we have

Area per center-line linear foot

$$= \frac{f}{9}\left[1 - \frac{D_c^\circ}{200}(w + f)\frac{\pi}{180}\right] \text{sq yd.} \quad (152)$$

Table 29 is based on Eq. (152).

Example 2. Referring to Example 1, suppose $\Delta = 52°12'$, $\theta_s = 20°$, $D_c = 10°$, $f = 2$ ft, and $w = 20$ ft. What is the extra area around the central circular arc?

Solution. From Table 29, the extra area per foot of center-line curve = 0.218 sq yd for 20-ft pavement.

Since $L_c = \frac{100(\Delta - 2\theta_s)}{D_c} = \frac{100 \times 12.2}{10} = 122.0$ ft, the total extra area $= 0.218 \times 122.0 = 26.6$ sq yd.

112. Laying Out Inner Edge of Widened Pavement. The inner edge of a widened pavement is located very easily by means of radial offsets from the surveyed center-line spiral. These offsets vary directly with the center-spiral-arc distance from the TS. Thus, if the widening at the SC is f ft, that at the mid-point is $\frac{1}{2}f$ ft, etc.

Case 2. Unspiraled Curve

113. Inner Edge of Widened Pavement. The tangent approach distance being limited, the curve is widened at the inside, while both the center line and the outer edge remain concentric circular curves, as shown in Fig. 92. The widened inner edge of pavement is a compound circular curve. The inner circular curve provides this flexibility of alignment and serves as a sort of transition approach and runoff, from full curvature.

Referring to Fig. 92,

w = normal width of pavement on tangent;
f = constant extra width around central portion of curve;
$f' = A'F$ = perpendicular offset from A' (PC) to widened inner edge;
$R = OA$ = radius of center-line curve AC;
$R' = OA' = R - \frac{1}{2}w$ = radius of concentric circle $A'C'$;
$\bar{R} = \bar{O}\bar{A}$ = radius of inner circle $\bar{A}\bar{B}$ which is compounded at $\bar{B}$ with the central concentric inner circle of radius $= R'' = R' - f = R - \frac{1}{2}w - f$;
$g = A'\bar{A}$ = distance from A' to the beginning of inner circle;
θ_c = central angle intercepting arcs AB, $A'B'$, and $\bar{A}\bar{B}$;
θ' = central angle intercepting arc $\bar{A}F$ of radius $\bar{R}$.

Through points B' and $\bar{B}$ draw lines $B'J$ and $\bar{B}K$ perpendicular to $\bar{O}\bar{A}$ and OA. Draw $\bar{B}N$ perpendicular to $B'J$ and note that the angle $N\bar{B}B'$ is θ_c. Then

$$g = K\bar{B} - JN = K\bar{B} - (JB' - NB') = \bar{R}\sin\theta_c - R'\sin\theta_c + f\sin\theta_c = (\bar{R} - R' + f)\sin\theta_c. \tag{153}$$

But
$$\bar{R} - R' = \bar{O}\bar{A} - OA' = \bar{O}M = g\cot\theta_c. \tag{154}$$

Substituting Eq. (154) in Eq. (153) and reducing, we have

$$g = g\cos\theta_c + f\sin\theta_c,$$

from which
$$g = \frac{f\sin\theta_c}{1 - \cos\theta_c} = f\cot\tfrac{1}{2}\theta_c. \tag{155}$$

From Eq. (154) we get

$$\bar{R} = R' + g\cot\theta_c. \tag{156}$$

If FQ is drawn perpendicular to OA, it follows that

$$f' = \bar{R}\operatorname{vers}\theta', \tag{157}$$

in which $\sin\theta' = g/\bar{R}$.

Now

$$\operatorname{vers}\theta' = 1 - \cos\theta' = 1 - \sqrt{1 - \sin^2\theta'} = 1 - (1 - \sin^2\theta')^{\frac{1}{2}} = \tfrac{1}{2}\sin^2\theta' \text{ (closely).}$$

The foregoing result is sufficiently exact since $\sin^2\theta'$ is very small in comparison with unity.

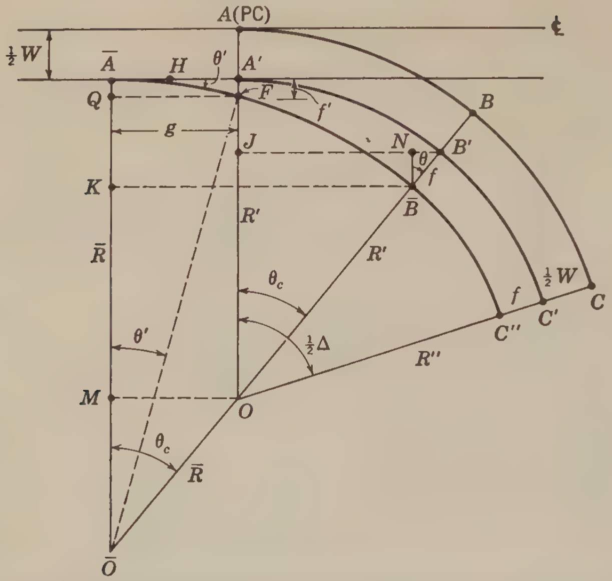


FIG. 92. Widening where surveyed center line is a circular curve.

Substituting in Eq. (157) and reducing, we have

$$f' = \frac{1}{2}\frac{g^2}{\bar{R}}. \tag{158}$$

The length of the inner arc $\bar{A}\bar{B}$ equals $100\theta_c/\bar{D}$, where

$$\bar{D} = \frac{5{,}729.58}{\bar{R}}.$$

With g and $\bar{R}$ (or $\bar{D}$) known, this inner curve of the widened area can be laid out. The offset f' from the PC to the curve serves as a check.

A line drawn tangent to the inner curve at F intersects $A'\bar{A}$ at H (Fig. 92). Then, approximately,

$$A'H = f' \cot \theta' = \frac{f'\bar{R}}{g}. \tag{159}$$

The above approximation is sufficiently exact for small values of θ', where $\sin\theta' = g/\bar{R}$, and $\cot\theta' = \bar{R}/g$ (closely).

Example 1. Given $\Delta = 42°30'$, $w = 18$ ft, $D = 10°$, $f = 2$ ft. It will be assumed that *full* widening begins at a section 50 ft ahead of the PC and extends around the middle portion of the curve to a section 50 ft from the PT. Required: data for laying out the inner curve (a circular arc).

Solution. Since arc AB (Fig. 92) = 50 ft, the corresponding central angle $\theta_c = 50D/100 = (50 \times 10)/100 = 5°$. Then $g = f \cot \frac{1}{2}\theta_c = 2(22.904) = 45.81$ ft. $R = 5{,}730/D = 573$ ft. $R' = 564$ ft. $\bar{R} = R' + g \cot\theta_c = 564 + (45.81)(11.430) = 1{,}087.6$ ft. $\bar{D} = 5{,}730/\bar{R} = 5.268° = 5°16'$. $f' = \frac{1}{2}(g^2/\bar{R}) = 0.96$ ft. $A'H = f'\bar{R}/g = (0.96)(1{,}087.6)/45.81 = 22.8$ ft.

Special case ($\theta_c = \frac{1}{2}\Delta$). It is often advisable to widen a curved pavement gradually all the way around to the mid-point of the turn (Fig. 93). This procedure is particularly advantageous for the sharp turns of city streets.

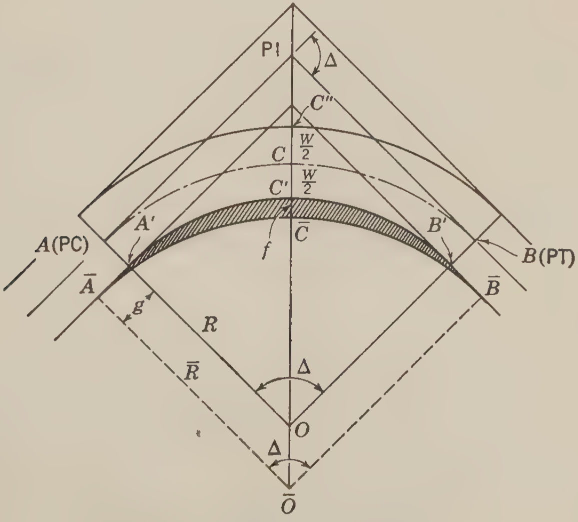


FIG. 93. Special case of Fig. 92.

Example 2. Given $\Delta = 100°20'$, $w = 30$ ft, $D = 50°$, $f = 6.0$ ft. It is assumed that widening will take place gradually all the way around to the mid-point of the turn, as shown in Fig. 93. Required: data for laying out the inner curve (a circular arc).

Solution. $\theta_c = \frac{1}{2}\Delta = 50°10'$; $g = f \cot \frac{1}{4}\Delta = 6(2.13639) = 12.82$ ft; $R = 5{,}729.58/50 = 114.59$ ft; $R' = 99.59$ ft; $\bar{R} = R' + g \cot \frac{1}{2}\Delta = 99.59 + (12.82)(0.8342) = 110.28$ ft; $\bar{D} = 5{,}729.58/110.28 = 51.95° = 51°57.0'$; $f' = \frac{1}{2}g^2/\bar{R} = \frac{1}{2}(12.82)^2/110.28 = 0.745$ ft.

114. Extra Area Due to Widening. The extra widened area exclusive of the central portion of constant widening (Fig. 92) = trapezoid $\bar{O}\bar{A}A'O$ + sector $OA'B'$ − sector $\bar{O}\bar{A}\bar{B}$ =

$$\tfrac{1}{2}(\bar{R} + R')g + \tfrac{1}{2}(R')^2\theta_c - \tfrac{1}{2}(\bar{R})^2\theta_c = \tfrac{1}{2}(\bar{R} + R')[g + \theta_c(R' - \bar{R})], \tag{160}$$

where the angle θ_c is measured in radians.

Using Eqs. (155) and (156) for g and $\bar{R}$ and noting that

$$R' = R - \tfrac{1}{2}w,$$

Eq. (160) for *area* becomes

$$\tfrac{1}{2}[2R - w + f(\cot \tfrac{1}{2}\theta_c)(\cot \theta_c)][f(\cot \tfrac{1}{2}\theta_c) - \theta_c f(\cot \tfrac{1}{2}\theta_c)(\cot \theta_c)]. \tag{161}$$

Multiplying by 2 to include both sides of the turn, dividing by 9 to express the result in square yards, and factoring, we have

$$\text{Area} = \frac{f}{9}[2R - w + f(\cot \tfrac{1}{2}\theta_c)(\cot \theta_c)][(\cot \tfrac{1}{2}\theta_c)(1 - \theta_c \cot \theta_c)]. \tag{162}$$

Letting $K = \frac{1}{9}(\cot \frac{1}{2}\theta_c)(1 - \theta_c \cot \theta_c)$ and noting that

$$R = \frac{5{,}729.578}{D},$$

Eq. (162) may be written as follows:

$$\text{Area} = f\left(\frac{A}{D} - Bw + Cf\right), \tag{163}$$

in which $A = 11{,}459.156K$, $B = K$, and $C = K(\cot \frac{1}{2}\theta_c)(\cot \theta_c)$.

Applying Eq. (158) with various values of θ_c, the factors A, B, and C have been calculated and recorded in Table 30.

With D (the degree of curve of the center-line curve), w (the normal width of pavement), f (the extra width), and θ_c (the central angle subtended by arc AB) (Fig. 92) known, the extra widened area (in square yards) may be determined readily.

To the foregoing area must be added the widened area around the central portion of the curve, where the extra width f is constant and the subtended central angle is $\Delta - 2\theta_c$. Equation (150) or Table 29 will give the desired result.

Example 1. Referring to Example 1 (page **235**), we have given $\Delta = 42°30'$, $w = 18$ ft, $D = 10°$, and $f = 2$ ft. Since $\theta_c = 5°$, the central angle of the portion of constant width is 32°30′, and the intercepted arc is $(100 \times 32.5)/10 = 325$ ft. Required: the total extra *area* due to widening.

Solution. *Area* of the two end sections of variable width $= f\left(\frac{A}{D} - Bw + Cf\right)$, where Table 30 gives $A = 74.05$, $B = 0.0065$, and $C = 1.692$. Then the area of the end sections is

$$2\left[\frac{74.05}{10} - (0.0065)(18) + (1.692)(2)\right] = 21.34 \text{ sq yd.}$$

By Table 29, the area of central section is $(0.218)(325) = 70.85$ sq yd.

Total *area* due to widening $= 21.34 + 70.85 = 92.19$ sq yd.

Example 2. Referring to Example 2 (page **236**), we have given $\Delta = 100°20'$, $w = 30$ ft, $D = 50.0$ ft, $f = 6.0$ ft. Required: total extra *area* due to the widening which takes place gradually all the way around to the mid-point of the turn.

Solution. Referring to Table 30 and noting that $\theta_c = \frac{1}{2}\Delta = 50°10'$, we get $A = 733.44$, $B = 0.0640$, $C = 0.1143$. Total area due to widening is

$$6.0\left[\frac{733.44}{50} - (0.0640)(30) + (0.1141)(6.0)\right] = 80.61 \text{ sq yd.}$$

C. SIGHT DISTANCE

115. Introduction. It is widely recognized that the ability to see ahead is vitally necessary for safe and efficient highway transportation. Among the publications by the AASHO for the purpose of building safety into the design of highways to the extent possible is a pamphlet entitled "A Policy on Sight Distance for Highways," which has been widely accepted in this country as standard practice.

Sight distance is the length of roadway ahead visible to the driver. For purposes of design and operation, it is termed stopping sight distance and passing sight distance (see Fig. 94).

116. Stopping Sight Distance. Stopping sight distance is the total distance traveled during three time intervals: (1) the time for the driver to perceive the hazard, (2) the time to react, (3) the time to stop the vehicle after the brakes are applied. During the first two intervals, the vehicle proceeds at full speed; during the third interval, its speed is reduced to zero, and this must happen before hitting an obstruction or a vehicle ahead.

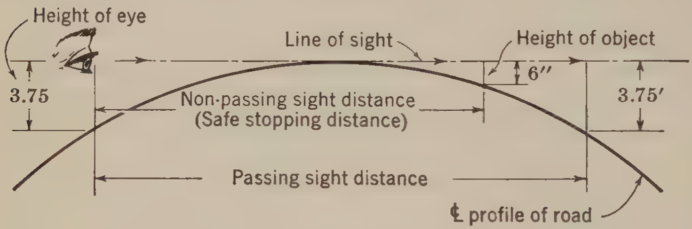


FIG. 94

Latest word from the National Safety Council (1965) calls for an average driver reaction time of ¾ sec. During this interval of ¾ sec, a car going 40 mph would travel 44 ft.

The braking distance (called d) depends upon the speed and the type of pavement. By a principle in mechanics: *force times distance equals change in kinetic energy*, or

$$Wf \times d = \frac{1}{2}\frac{W}{g}v^2 = \frac{Wv^2}{64.4}, \tag{a}$$

from which

$$d = \frac{v^2}{64.4f} = \frac{V^2}{30f}, \tag{164}$$

where V is the speed of the vehicle (mph) at the instant the brakes are applied and f is the coefficient of friction of the tires against the pavement. If t (sec) is the perception-reaction time, approximately 2.5 sec., then

$$\text{Stopping distance} = \frac{44}{30}Vt + \frac{V^2}{30f}. \tag{165}$$

Values of the friction factor (assumed constant) for normal, clean, wet pavements that are free of mud, snow, or ice vary from 0.36 for speeds of 30 mph to 0.29 for speeds of 70 mph.

At no point on the highway should the sight distance be less than the minimum stopping distance. The *nonpassing* sight distance is based upon stopping distance. The AASHO specifies a line of sight from the driver's eye $3\frac{3}{4}$ ft above the pavement surface to an object on the roadway 6 in. high, on the assumption that any object lower than 6 in. will not be a serious obstruction if hit.

117. Passing Sight Distance. Passing sight distance is the shortest distance sufficient for a vehicle to turn out of a traffic lane, pass another vehicle, and then turn back to the same lane safely and comfortably without interfering with the overtaken vehicle or an oncoming vehicle traveling at the design speed should it come into view after the passing maneuver is started (see Figs. 94, 95 and 96).

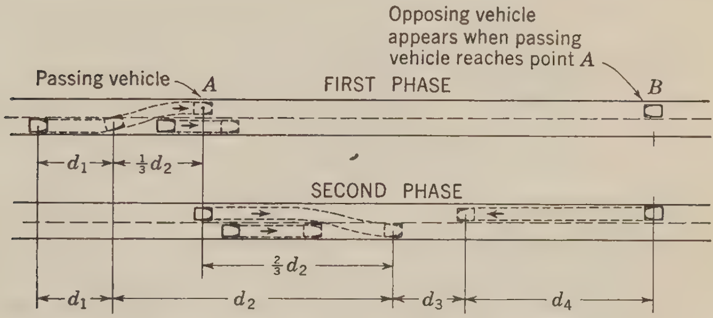


FIG. 95. Elements of passing sight distance.

118. Sight Distance along Horizontal Curves. The visibility around horizontal curves may be limited on account of obstructions such as a building or road bank. Two cases will be considered: (1) when the sight distance S is less than the length of curve L, that is, when $S < L$ (Fig. 97), (2) when $S > L$ (Fig. 98).

Let M = clear distance, ft, from the center line of roadway to the obstruction;
S = sight distance, ft, along the center line of roadway;
R = radius of center-line curve, ft;
D = corresponding degree of curvature.

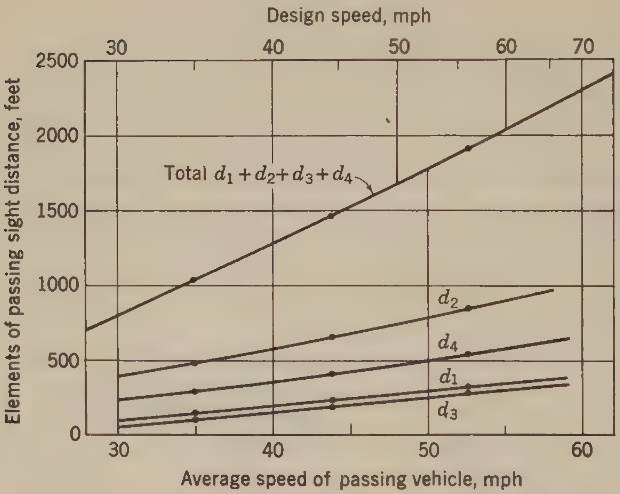


FIG. 96. Total passing sight distance for two-lane highways.

The nominal sight distance is not the chord AB; it is the arc subtended by this chord representing the actual path traversed by the car, since this is the stopping distance that can be utilized. The radius of this path is about 6 ft shorter than the center-line radius used in the formulas which follow. Deductions of 6 ft from the calculated values of M may be made for greater accuracy. However, the assumption made below that chord $AC = \frac{1}{2}S$ tends to compensate for this difference.

Case 1. $S < L$.

From Fig. 97,

$$\overline{AC}^2 = \overline{AD}^2 + M^2, \tag{a}$$

and

$$\overline{AD}^2 = R^2 - (R - M)^2. \tag{b}$$

Substituting (b) in (a), letting $AC = \frac{1}{2}S$, and reducing, we get

$$R = \frac{S^2}{8M} \tag{166}$$

and

$$S^2 = 8R \times M \tag{167}$$

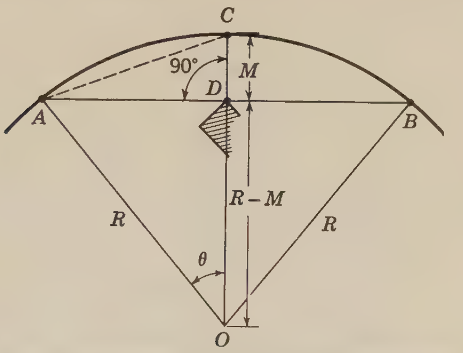


FIG. 97. Sight distance around horizontal curve where $S < L$.

A more exact value of M may be found. Thus, since

$$\cos\theta = \frac{R - M}{R},$$

we have $$M = R(1 - \cos\theta) = R \text{ vers } \theta, \qquad (168)$$

in which $\theta = (S/200) \times D$.

Example 1. The clearance to an obstruction is 30 ft and the desirable sight distance when rounding a horizontal curve is 600 ft. Required: the minimum radius of curve.

Solution. Using Eq. (166),

$$R = \frac{600 \times 600}{8 \times 30} = 1{,}500 \text{ ft}$$

Example 2. Given $D = 4°$ and $S = 1{,}000$ ft. Required: the clearance to an obstruction.

Solution. (*a*) Using Eq. (167), $M = \dfrac{1{,}000 \times 1{,}000}{8 \times 1{,}432} = 87$ ft.

(*b*) Using Eq. (168), $M = 1{,}432 \times \text{vers } 20° = 86$ ft.

Case 2. $S > L$. In Fig. 98, the sight distance overlaps the curve on to the tangents a distance of d ft beyond the points of curvature, so that $S = L + 2d$, or $d = \frac{1}{2}(S - L)$. Of the right triangles ACD, ADO, and AEO,

$$\overline{AC}^2 = \overline{AD}^2 + M^2, \tag{a}$$
$$\overline{AD}^2 = \overline{AO}^2 - (R - M)^2, \tag{b}$$
$$\overline{AO}^2 = \overline{AE}^2 + R^2. \tag{c}$$

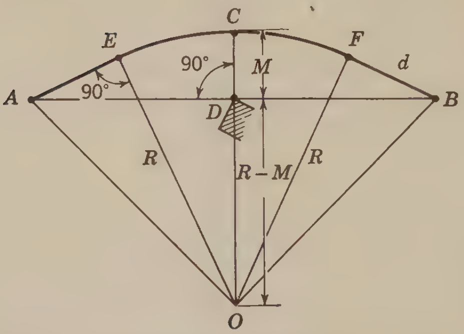


FIG. 98. Sight distance around horizontal curve where $S > L$.

Putting $AE = d = \frac{1}{2}(S - L)$ in Eq. (c) and substituting in Eq. (b), we get

$$\overline{AD}^2 = \tfrac{1}{4}(S - L)^2 + R^2 - (R - M)^2. \tag{d}$$

Substituting Eq. (d) in Eq. (a) and reducing, we get

$$\overline{AC}^2 = \tfrac{1}{4}(S - L)^2 + 2RM. \tag{e}$$

Putting $AC = \frac{1}{2}S$ in Eq. (e) and reducing, we get

$$M = \frac{L(2S - L)}{8R}. \tag{169}$$

119. Sight Distance on Vertical Curves. Formulas for sight distances on vertical curves over crests will be derived for two cases: (1) When the sight distance S is entirely on the curve ($S < L$); (2) when the sight distance overlaps the curve and extends on to the tangent ($S > L$). In Fig. 99, let

$S = (d_1 + d_2)$, ft;

h_1, h_2 = height of line of sight (ft) above the roadway, h_1 applying to the driver's eye and h_2 to the object or hazard on the traveled lane.

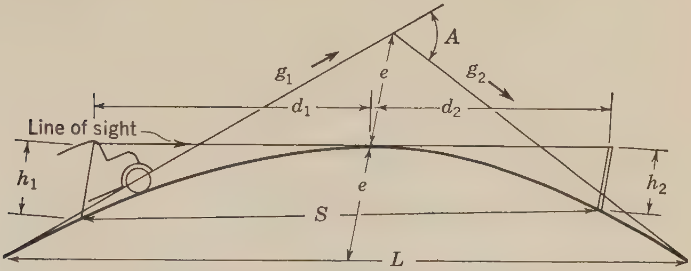


FIG. 99. Sight distance over vertical curve where $S < L$.

L, A, and e refer to the vertical curve (parabola) and have the same general meaning as in Chap. 5, except that L is measured in feet (instead of stations) and absolute values (not negative values) are used for A, where A = algebraic difference in grades in percent. Accordingly, $e = AL/800$.

Case 1. $S < L$. By the parabolic law, offsets vary as the square of the distance. Hence, from Fig. 99, we have

$$e = k(\tfrac{1}{2}L)^2; \tag{a}$$

$$h_1 = k(d_1)^2; \qquad h_2 = k(d_2)^2. \tag{b}$$

Dividing in turn (*b*) by (*a*) gives

$$\frac{4(d_1)^2}{L^2} = \frac{h_1}{e}; \qquad \frac{4(d_2)^2}{L^2} = \frac{h_2}{e}. \tag{c}$$

Noting that $e = AL/800$, solving Eq. (*c*) for d_1 and d_2, and adding, we get

$$S = d_1 + d_2 = (\sqrt{2h_1} + \sqrt{2h_2}) \times \sqrt{\frac{100L}{A}}. \tag{d}$$

Then

$$L = \frac{S^2A}{100 \times (\sqrt{2h_1} + \sqrt{2h_2})^2}, \tag{170}$$

where L is the length of vertical curve allowing a sight distance S for various values of A.

If $h_1 = h_2 = 3.75$ ft (passing conditions), ($S < L$),

$$L = \frac{S^2A}{3{,}000}. \tag{171}$$

If $h_1 = 3.75$ ft, $h_2 = 6$ in. $= \frac{1}{2}$ ft (stopping conditions), $S < L$,

$$L = \frac{S^2 A}{1{,}400}. \tag{172}$$

Case 2. $S > L$. In Fig. 100, assuming scalar values for g_2, we have

$$S = \tfrac{1}{2}L + 100 \times \left(\frac{h_1}{g_1} + \frac{h_2}{g_2}\right). \tag{a}$$

For the value of A making S a minimum, the rate of change in g_2 will be equal and opposite to the rate of change in g_1.

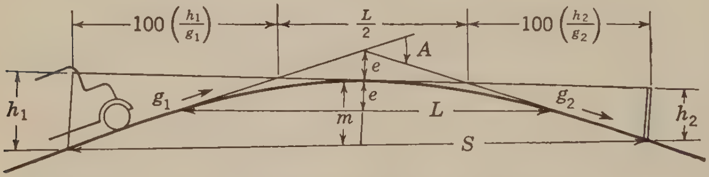

100 $\left(\frac{h_1}{g_1}\right)$
$\frac{L}{2}$
100 $\left(\frac{h_2}{g_2}\right)$
A
e
e
h_1
g_1
m
L
S
g_2
h_2

FIG. 100. Sight distance over vertical curve where $S > L$.

Setting the first derivative of $S = 0$, we get

$$\frac{h_1}{(g_1)^2} - \frac{h_2}{(g_2)^2} = 0, \qquad \text{or} \qquad g_2 = \sqrt{\frac{h_2}{h_1}} \times g_1.$$

Next,

$$A \text{ (scalar value)} = g_2 + g_1 = \left(\sqrt{\frac{h_2}{h_1}} + 1\right) \times g_1$$

$$= \frac{\sqrt{h_1} + \sqrt{h_2}}{\sqrt{h_1}} \times g_1. \tag{b}$$

Then

$$g_1 = \frac{\sqrt{h_1}}{\sqrt{h_1} + \sqrt{h_2}} \times A; \qquad g_2 = \frac{\sqrt{h_2}}{\sqrt{h_1} + \sqrt{h_2}} \times A. \tag{c}$$

Substituting (c) in (a) and reducing, we get

$$S = \tfrac{1}{2}L + \frac{100(\sqrt{h_1} + \sqrt{h_2})^2}{A}, \tag{d}$$

from which

$$L = 2S - \frac{200\,(\sqrt{h_1} + \sqrt{h_2})^2}{A}. \tag{173}$$

If $h_1 = h_2 = 3.75$ ft (passing conditions), $S > L$,

$$L = 2S - \frac{3{,}000}{A}. \tag{174}$$

If $h_1 = 3.75$ ft, $h_2 = 6$ in. $= \frac{1}{2}$ ft (stopping conditions), where $S > L$,

$$L = 2S - \frac{1{,}400}{A}. \tag{175}$$

TABLE A. SIGHT DISTANCES ON CREST VERTICAL CURVES

Stopping: $h_1 = 3.75$ ft; $h_2 = 0.50$ ft		Passing: $h_1 = 3.75$ ft; $h_2 = 4.00$ ft	
Design speed (mph)	Sight distance (ft)	Design speed (mph)	Sight distance (ft)
30	200	30	800
40	275	40	1,300
50	350	50	1,700
60	475	60	2,000
70	600	70	2,400
80	750		

120. Sag Vertical Curves. At present (AASHO policy), there is no single widely used design criterion for establishing lengths of sag vertical curves. At least four different criteria are recognized to some extent. These are (1) headlight sight distance, (2) rider comfort, (3) drainage control, and (4) a rule of thumb for general appearance.

1. *Headlight Sight Distance.* *a.* $S < L$. In Fig. 104, the headlight (with car at O) is assumed 2.0 ft above the roadway, and the light beam has an upward angle of 1° with the horizontal.

The tangents to the parabolic curve OB are OV and VB, and OVD is the horizontal axis of reference. S is the sight distance from O to the intersection of the light beam with the roadway surface (at B').

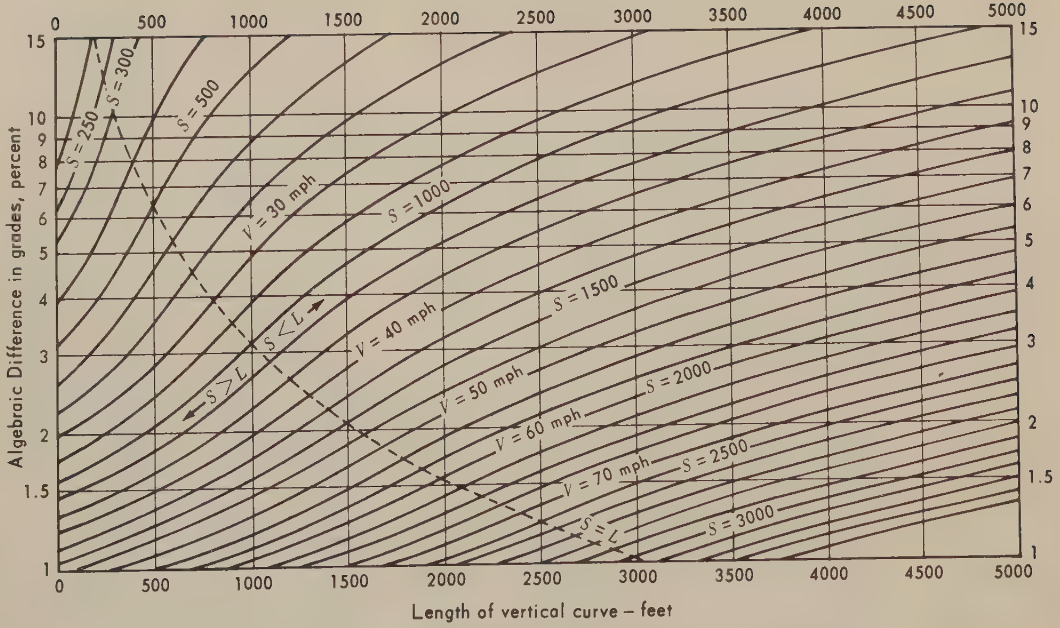


FIG. 101. Passing sight distance for crest vertical curves.

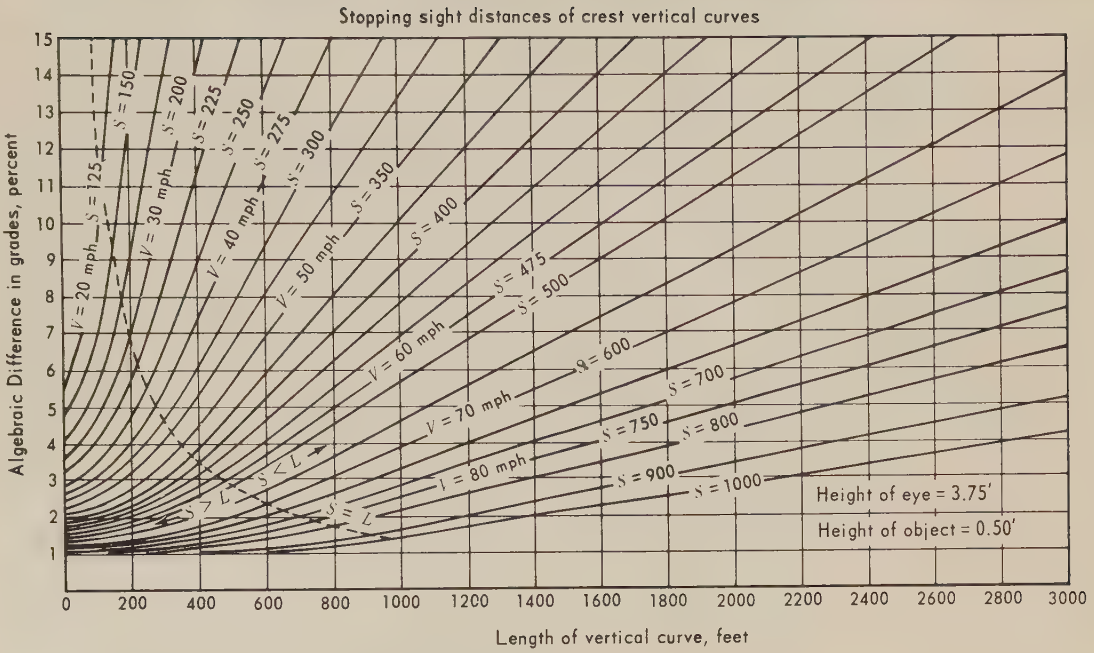


FIG. 102. Design controls for crest vertical curves.

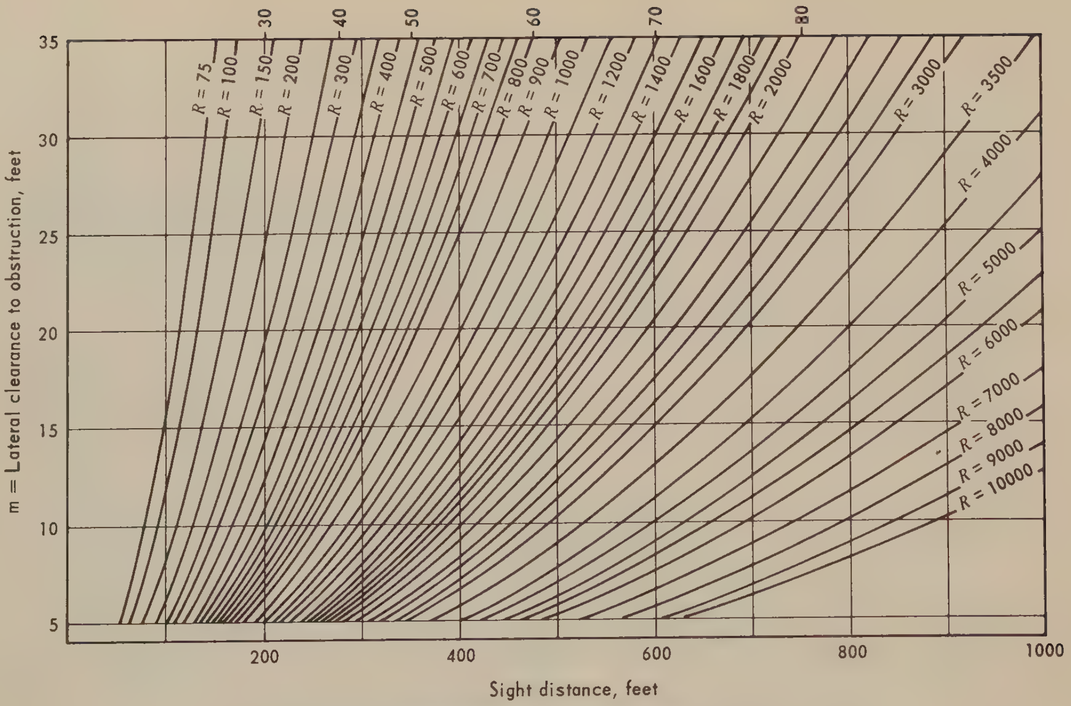


FIG. 103. Sight distance around horizontal curve where $S < L$.

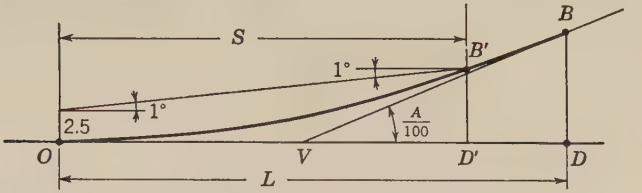


FIG. 104. Headlight beam distance where $S < L$.

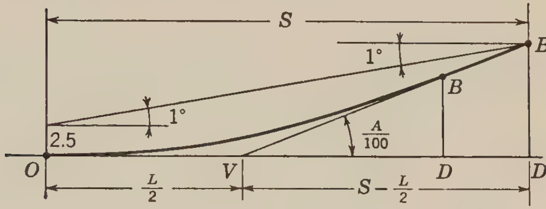


FIG. 105. Headlight beam distance where $S > L$.

Since $A/100$ = vertical divergence per foot of the grade lines VD and VB, it follows (Fig. 104) that

$$DB = \frac{A}{100}\frac{L}{2}. \tag{a}$$

Also, by the law of offsets,

$$D'B' = \left(\frac{S}{L}\right)^2 DB = \left(\frac{S}{L}\right)^2 \frac{AL}{200} = \frac{S^2A}{200L}. \tag{b}$$

Since tan 1° = 0.0175,

$$D'B' = 2.0 + 0.0175S. \tag{c}$$

Equating (*b*) and (*c*) and reducing, we get

$$L = \frac{S^2A}{400 + 3.5S}, \tag{176}$$

where L and S are in feet and $A = g_2 - g_1$ (absolute value).

b. $S > L$. In Fig. 105, OV and VB are the tangents to the parabola OB. If VB is extended to VB', where B' is the intersection of the light beam with the roadway surface, then

$$D'B' = \frac{A}{100}\left(S - \frac{L}{2}\right). \tag{a}$$

Also,

$$D'B' = 2.0 + 0.0175S. \tag{b}$$

Equating (*a*) and (*b*) and reducing, we get

$$L = 2S - \frac{400 + 3.5S}{A}. \tag{177}$$

2. The comfort effect of change in vertical direction is more pronounced in sag than on crest vertical curves because gravitational and centrifugal forces are combining rather than opposing forces. This effect is not measured readily but a broad conclusion by the AASHO states that riding is comfortable on sag curves when the *centripetal acceleration does not exceed* 1 *foot per second per second.*

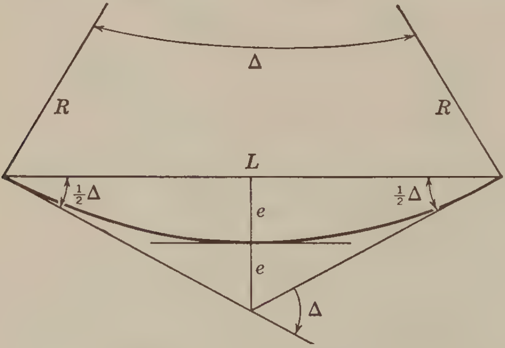


FIG. 106. Sag curve based on comfort factor.

Let V = design speed, mph, while the length of curve L and the equivalent radius R are in feet (Fig. 106).

On the basis of centripetal acceleration = 1, we have (with v = ft/sec)

$$\frac{v^2}{R} = 1 = \left(\frac{44}{30}\right)^2 \frac{V^2}{R} = 2.151\frac{V^2}{R}, \tag{a}$$

from which

$$R = 2.151V^2. \tag{b}$$

Since the parabola practically coincides with the circle of radius R,

$$L = R\Delta = 2.151V^2\Delta, \qquad (c)$$

where Δ is the central angle in radians.

Also $$\Delta = \frac{A}{100} \quad \text{(Fig. 106).} \qquad (d)$$

Substituting (d) in (c), we get

$$L = \frac{2.151V^2}{100} \times A = \frac{V^2A}{46.5}. \qquad (178)$$

GENERAL DESIGN CONTROLS (AASHO POLICY)

121. Foreword. In addition to the specific design elements for horizontal and vertical alignment discussed under previous headings, a number of general controls are recognized in practice. These are not subject to empirical or formula derivation, but they are important for the attainment of safe and smooth-flowing highways. Excessive curvature or poor combinations of curvature generate accidents, limit capacity, cause economic losses in time and operating costs, and detract from a pleasing appearance. To avoid these evidences of poor design practices, the general controls in the following paragraphs should be utilized.

122. Design Control for Horizontal Alignment

(1) Alignment should be as directional as possible, but should be consistent with the topography. A flowing line that conforms generally to the natural contours is preferable aesthetically to one with long tangents that slashes through the terrain. In general, the number of curves should be kept to a minimum. While these aesthetic qualities of curving alignment are important, passing sight distance necessitates long tangents on 2- and 3-lane highways with passing sight distance on as great a percentage of the length of highway as feasible.

(2) In alignment predicated on a given speed, use of the maximum curvature for that speed should be avoided wherever possible. The designer should attempt to use generally flat curves, retaining the maximum for the most critical conditions.

(3) Consistent alignment always should be sought. Sharp curves should not be introduced at the ends of long tangents. Sudden

changes from areas of easy curvature to areas of sharp curvature should be avoided. Where sharp curvature must be introduced it should be approached, where possible, by successively sharper curves from the generally easy curvature.

(4) Curves should be at least 500 ft long for a central angle of 5 degrees, and the minimum length should be increased 100 ft for each 1-degree decrease in the central angle.

(5) Other than flat curvature should be avoided on high, long fills. Adequately designed guard rails, maintained to a high level of visibility and strength, are necessary to alleviate the hazardous position of cars out of control on high fills.

(6) An abrupt reversal in alignment should be avoided. Such a change makes it difficult for a driver to keep within his own lane. Also, it is difficult to superelevate both curves adequately, and hazardous and erratic operation may result.

(7) To avoid the appearance of inconsistent distortion the horizontal alignment should be coordinated carefully with the profile design.

123. Design Controls for Vertical Alignment

(1) A smooth grade line with gradual changes, as consistent with the type or class of highway and the character of the terrain, should be striven for in preference to a line with numerous breaks and short lengths of grades.

(2) The "hidden-dip" type of profile is unpleasant aesthetically and hazardous during passing maneuvers. This type of profile is avoided by horizontal curvature or by more gradual grades made possible by heavier cuts and fills.

(3) Where intersections at grade occur on highway sections with moderate to steep grades, it is desirable to reduce the gradient through the intersection. Such a profile is beneficial for all vehicles making turns and serves to reduce the potential hazards.

(4) On highways carrying substantial truck traffic, provision of added lanes on upgrades for slow moving vehicles should be considered where critical lengths of grade are exceeded. This may apply on 4-lane highways where volume approaches capacity as well as on 2-lane highways.

124. Combination of Horizontal and Vertical Alignment. Horizontal and vertical alignment should not be designed independently. They complement each other and poorly designed combinations can spoil the good points and aggravate the deficiencies of each. Horizontal alignment and profile are among the more important of the

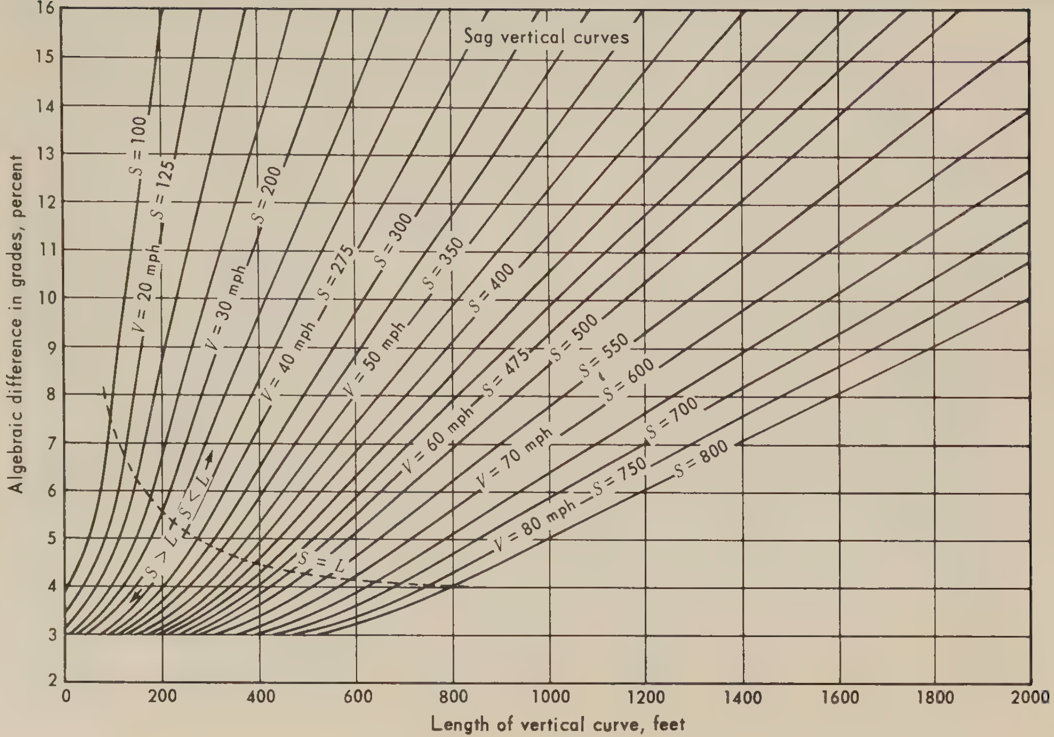


FIG. 107. Design controls for sag vertical curves (see page 150).

permanent features of the highway, for which thorough study is warranted. Excellence in their design and in the design of their combination increases utility and safety, encourages uniform speed, and improves appearance, almost always without additional cost.

Proper combination of horizontal alignment and profile is obtained by engineering study and consideration of the following controls.

(1) Curvature and grades should be in proper balance Tangent alignment or flat curvature at the expense of steep or long grades, and excessive curvature with flat grades, are both poor designs. A logical design is a compromise between the two, which offers the most in safety, capacity, ease and uniformity of operation, and pleasing appearance within the practical limits of terrain and area traversed.

(2) Sharp horizontal curvature should not be introduced at or near the top of a pronounced crest vertical curve. This condition is hazardous in that the driver cannot perceive the horizontal change in alignment, especially at night when the headlight beams go straight into space. The hazard of this arrangement is avoided if the horizontal curvature leads the vertical curvature; i.e., the horizontal curvature is made longer than the vertical curve. Also, suitable design can be made by using design values well above the minimums for the design speed.

(3) Somewhat allied to the above, sharp curvature should not be introduced at or near the low point of a pronounced sag vertical curve. Because the road ahead is foreshortened, any but flat horizontal curvature assumes an undesirable distorted appearance. Further, vehicular speeds, particularly of trucks, often are high at the bottom of grades and erratic operation may result, especially at night.

(4) Horizontal curvature and profile should be made as flat as feasible at highway intersections where sight distance along both highways is important and vehicles may have to slow down or stop.

(5) On divided highways, variation in the width of median and the use of separate profiles and horizontal alignments should be considered to derive design and operational advantages of one-way roadways. Where traffic justifies provision of 4 lanes, a superior design without additional cost generally results from the concept and logical design basis of one-way roadways.

125. Sight Distance at Underpass Structures. It is required to find the minimum length of vertical curve which will provide a specified sight distance for underpass structures. The PI of the tangents to the parabola is assumed vertically under the critical edge of the structure restricting the view ahead. Let

A = algebraic difference of grades, percent;
L = length of vertical curve, stations;
S = sight distance, stations;
C = vertical clearance at critical edge of underpass, ft;
h_1 = vertical height of driver's eye above road, ft;
h_2 = vertical height of sighted object, ft.

Two cases will be considered: (1) $S > L$, (2) $S < L$.

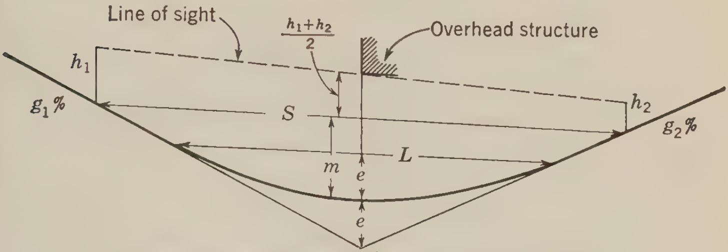


FIG. 108. Sight distance under overhead structure where $S > L$.

Case 1. $S > L$. From similar triangles (Fig. 108), we have

$$\frac{S}{L} = \frac{e+m}{2e} = \frac{1}{2} + \frac{m}{2e}, \tag{a}$$

in which $e = \frac{LA}{8}$ and $m = C - \frac{h_1 + h_2}{2}$.

If $C = 14$ ft, $h_1 = 6.0$ ft, and $h_2 = 1.5$ ft (AASHO policy), $m = 10.25$ ft. Substituting this value of m in (a), we have

$$S = \tfrac{1}{2}L + \frac{41}{A}, \tag{179}$$

from which

$$L = 2S - \frac{82}{A}. \tag{180}$$

Case 2. $S < L$ (Fig. 109). Since any flat parabola is closely a circle, let R = average radius of the vertical curve, while Δ and Δ_s are the central angles (assumed in radians) subtended by L and

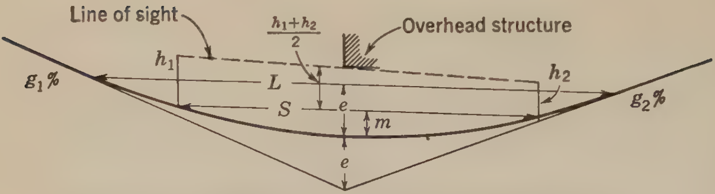


FIG. 109. Sight distance under overhead structure where $S < L$.

S, respectively. It follows that

$$e \text{ (for the parabola)} = \frac{LA}{8}. \tag{a}$$

$$e \text{ (for corresponding circle)} = R \tan\frac{\Delta}{2} \tan\frac{\Delta}{4} = \frac{R\Delta^2}{8} \text{ (approx.)}. \tag{b}$$

Equating (*a*) and (*b*) and solving, we have

$$\Delta^2 = \frac{LA}{R}. \tag{c}$$

Assuming m (for the parabola) to be equivalent to m (for the circle), we have

$$m = R \text{ vers } \frac{\Delta_s}{2} = R \sin\frac{\Delta_s}{2} \tan\frac{\Delta_s}{4} = \frac{R\Delta_s^2}{8} \text{ (closely)}. \tag{d}$$

Combining (*c*) and (*d*) and reducing, we have

$$\left(\frac{\Delta}{\Delta_s}\right)^2 = \frac{LA}{8m}. \tag{e}$$

Now $L = R\Delta$ and $S = R\Delta_s$ (approx.); hence

$$\frac{\Delta}{\Delta_s} = \frac{L}{S} \quad \text{and} \quad \left(\frac{\Delta}{\Delta_s}\right)^2 = \frac{L^2}{S^2}. \tag{f}$$

Equating (*e*) and (*f*) and reducing, we get

$$L = \frac{S^2A}{8m}. \tag{181}$$

Letting $C = 14$ ft (the required overhead clearance), $h_1 = 6$ ft, and $h_2 = 1.5$ ft, then

$$m \text{ (see Fig. 109)} = C - \frac{h_1 + h_2}{2} = 10.25 \text{ ft.}$$

Hence Eq. (181) becomes

$$L = \frac{S^2 A}{82} \tag{182}$$

and

$$S = \sqrt{\frac{82L}{A}}. \tag{183}$$

Addendum: For situations where the critical edge of the structure is not directly above the vertex of the vertical curve, Eqs. (180) and (183) are still valid, provided the edge is not more than 200 ft from the vertex. Beyond this limit, Eq. (180), applying to Case 1, is in error, but Eq. (183) is satisfactory.

126. Scenic Values in Planning and Design. For any highway, pleasing appearance is a consideration in the planning and design process. Scenic values must be considered along with the safety, utility, economy, and all other factors involved in planning and design. This is particularly true of the many portions of the state highway system situated in areas of natural beauty. In such areas, the location of the highway, its alignment and profile, the cross-section design, and other related factors should be in harmony with the setting. Economy consistent with traffic needs will always be of paramount importance, although a reasonable additional expenditure can be justified on the grounds that it enhances the beauty of the highway. Consideration should be given to the following factors to ensure the most satisfactory aesthetic results.

1. The highway should be located so that the new construction will preserve the natural environment and will lead to and unfold scenic positions.

2. Curvilinear horizontal alignment and a gently rolling profile consistent with design requirements of the highway should fit the character of the area traversed so that unsightly scars of excavation and embankment will be reduced to a minimum and a more pleasing appearance will result.

3. The destruction of valuable trees and growth should be avoided if suitable alternate locations are available at reasonable cost. If possible, timber screens should be provided to shield unsightly views, and selective clearing performed to open up scenic vistas.

4. Whenever feasible wide medians, curvilinear alignment, and independent roadways should be provided on multilane facilities to add scenic interest and relieve the monotony of parallelism.

5. Slopes should be flattened whenever practical and ground cover provided so that lines of construction are softened and beautified.

6. Structures should be located and designed to give the most pleasing appearance.

7. Drainage appurtenances should be so located that erosion, sumps, and debris collection areas are hidden from view or eliminated whenever possible.

127. Urban Design for Building and Traffic Efficiency. Roads cover from 35 to 50 percent or more of our urban space. Broad and integrated planning is especially needed in cities where bigger highways have produced larger traffic jams.

If the traffic pattern of a city is functionally efficient, the convenience, accessibility, and market value of every building, as well as the general architectural appearance of the town, are improved. A solution for the city traffic problem requires a balanced and integrated use of the entire process of urban design. City streets must be planned to expedite, not arrest, the passage of vehicles as they circulate through and around an area.

This idea is illustrated by the downtown traffic "circle," a half-mile square that has been planned for the city of Monterey, California, as a first step in solving its traffic problem. (See Fig. 110.) The plan provides 30-mph traffic arteries that carry more traffic with greater ease and safety than conventional city streets, yet avoids downtown destruction and greatly reduces the high cost of building freeways through the city.

Three Conflicting Functions. Ignoring for the moment the pedestrian and considering only the needs of the automobile, city streets have three conflicting functions: parking of automobiles; loading and unloading of people and goods, usually by means of double parking; and in the space remaining, the movement of traffic.

A first step in resolving traffic strangulation is to separate the traffic functions from the service functions of parking and loading. Some streets, preferably those that pass *around* congested neighborhoods and business centers rather than *through* them, must be designated as traffic arteries. Parking and stopping must be restricted on these arteries. Local streets going into and traversing the neighborhood or business centers must become service streets. Through traffic will then be routed around instead of through a congested area. The question arises as to whether such arteries should carry two-way or one-way traffic. To save both space and dollars, the traffic loop should be a *one-way* artery. Most city streets have adequate width for a three-lane, one-way

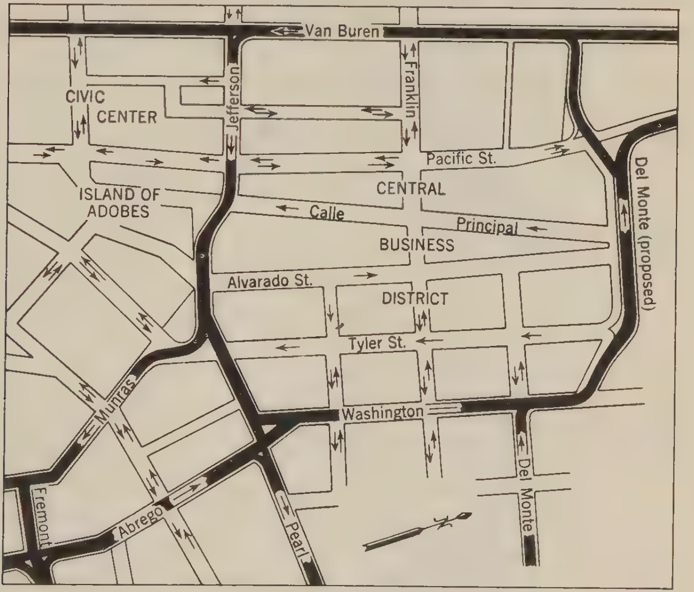

Van Buren
CIVIC
CENTER
Jefferson
Franklin
Pacific St.
ISLAND OF
ADOBES
CENTRAL
Calle
Principal
BUSINESS
Del Monte (proposed)
Alvarado St.
DISTRICT
Tyler St.
Munras
Washington
Fremont
Abrego
Pearl
Del Monte

FIG. 110. A half-mile one-way traffic "circle" that has been planned for Monterey, Calif.

traffic artery, but in order to carry a useful and wide two-way traffic artery, they would have to be widened, at an exorbitant cost for land acquisition and construction.

For a downtown area, a three-lane, one-way main street is usually preferable to a six-lane divided highway. Even the pedestrian has an advantage in narrower and safer crossings. But most important, two-way streets suffer from excessive traffic conflicts at intersections. These intersection conflicts are the main cause of downtown congestion, not insufficient lanes.

Traffic Capacities. One lane of traffic moving at 30 mph carries 1,500 cars an hour. Trucks and buses making turning movements into or out of this lane reduce this capacity by perhaps 20 percent, which reduces the lane capacity to 1,200 vehicles per hour.

A two-way traffic light set at 30 sec *red*, 25 sec *green*, and 5 sec *yellow* cuts the already reduced capacity by 60 percent, so that the lane now handles only 480 cars an hour. Finally, where there is a preponderance of left-turn movements, three-way traffic lights are required, which cuts the traffic flow of the lane further to only 300 cars an hour.

This is why left-turn movements are forbidden in many downtown areas. The figures are only approximate, since the actual traffic flow depends on the percentage of trucks and buses, pedestrian interruption, and the precise timing of the traffic lights.

One-way:Safer, Cheaper, Faster. One-way streets make it possible to simplify intersection conflicts and thus expedite traffic with greater safety. (See Fig. 111.) They are being used successfully in many cities, and hundreds of reports on them are available.

Conclusions from a lengthy survey of one-way street operation, conducted by the Institute of Traffic Engineers and the AASHO show:

1. Traffic capacity is increased from 20 to 50 percent.
2. Overall traffic speed is increased an average of 27 percent.
3. Traffic accidents are reduced 50 percent.
4. Pedestrian crossing is facilitated.

One-way streets are particularly advantageous on the fringes of a small congested area, since they tend to drain through traffic

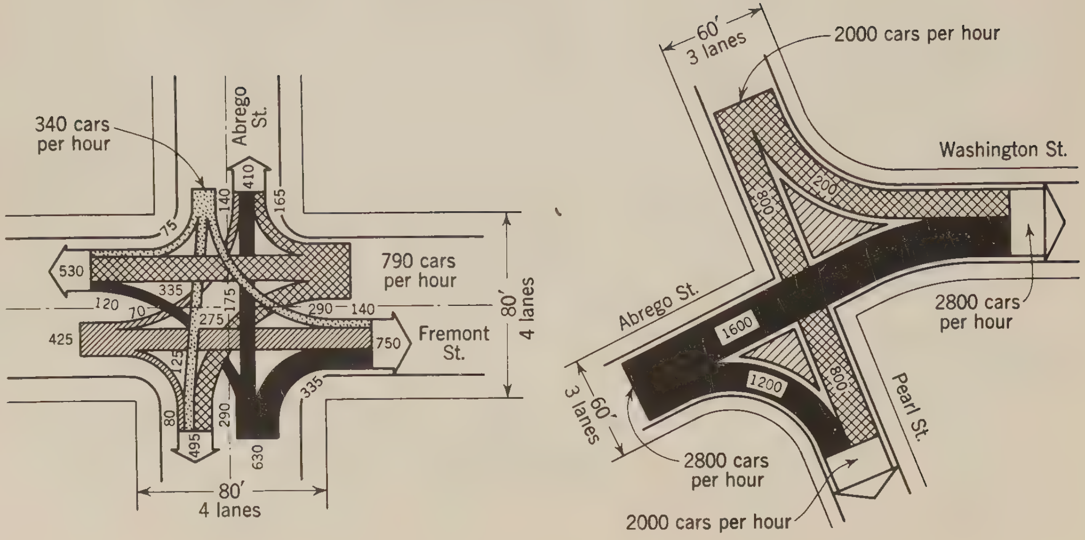

340 cars per hour
Abrego St.
410
140
165
75
530
790 cars per hour
335
120
70
175
275
290
140
425
750
Fremont St.
80′ 4 lanes
125
80
495
290
630
335
80′ 4 lanes
60′ 3 lanes
2000 cars per hour
Washington St.
200
800
Abrego St.
1600
2800 cars per hour
1200
800
Pearl St.
60′ 3 lanes
2800 cars per hour
2000 cars per hour

FIG. 111. Advantage of a one-way street operation (right-hand view).

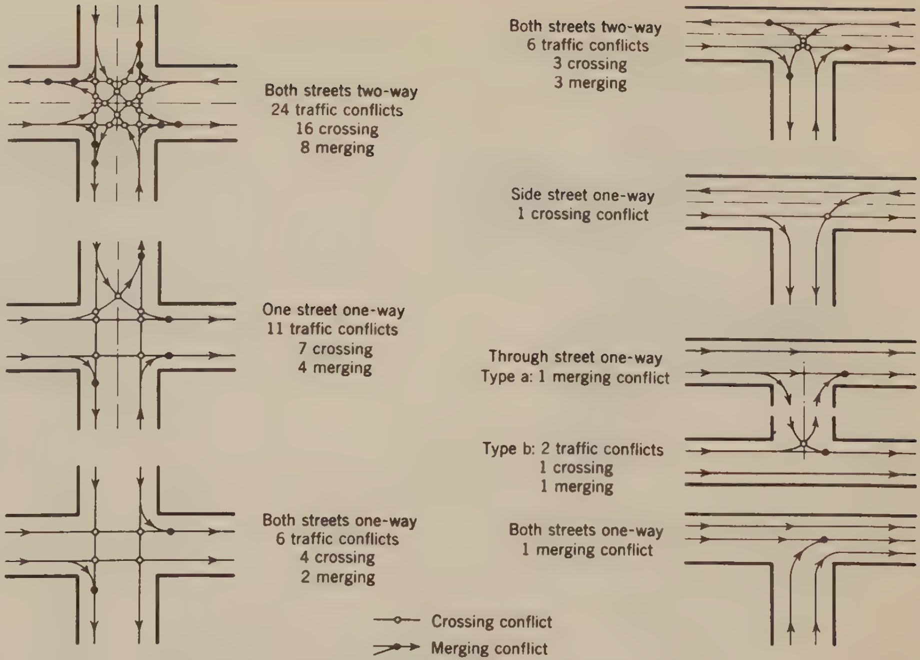

Both streets two-way
24 traffic conflicts
16 crossing
8 merging
One street one-way
11 traffic conflicts
7 crossing
4 merging
Both streets one-way
6 traffic conflicts
4 crossing
2 merging
Both streets two-way
6 traffic conflicts
3 crossing
3 merging
Side street one-way
1 crossing conflict
Through street one-way
Type a: 1 merging conflict
Type b: 2 traffic conflicts
1 crossing
1 merging
Both streets one-way
1 merging conflict
Crossing conflict
Merging conflict

FIG. 112. Examples of crossing and merging conflicts.

from that area. For full value, one-way streets must be integrated into a complete traffic circulation system.

Serious highway accidents and many fatalities are caused by cars hurtling over a median barrier into oncoming traffic. Completely independent one-way highways or freeways, separating oncoming traffic lines by several blocks rather than a few feet, would eliminate such accidents.

Night driving is much easier and safer on one-way roads. There is no disturbing glare from the headlights of oncoming cars, eliminating a cause of many fatalities at pedestrian crossings.

Potential traffic conflicts occur wherever the paths of two vehicles intersect. There are two kinds of conflicts: (1) crossing conflicts, which are the most dangerous and require the use of stop signs or traffic lights; and (2) merging conflicts, which can be safe if properly designed with adequate merging and sight distance between the two combining lanes.

Accompanying sketches, Fig. 112, show the conflicts found at typical city street intersections. For simplicity, the hazards of pedestrian conflicts are not shown.

References

1. State of California, Division of Highways, *Planning Manual*, 1965.
2. Read, Vernon, senior engineer, Holmes and Narver, Inc., Los Angeles, Calif., *Civil Eng.*, May, 1962.

CHAPTER 9

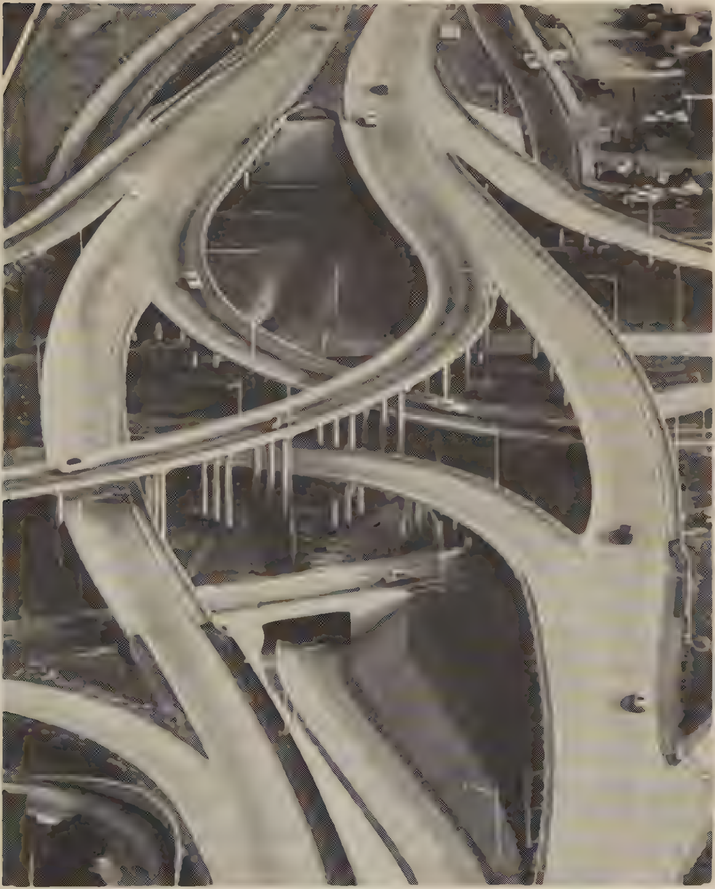
INTERSECTIONS AND INTERCHANGES

128. New Concepts for Urban Freeway Interchanges. An urban freeway is a modern highway on which traffic is able to enter with a minimum of delay, travel safely at reasonably high speed, and exit with a minimum disruption to other traffic. To provide for these factors, an urban freeway must be designed to include the following: full control of access; high-type horizontal and vertical alignment; ample cross-section elements, including total right-of-way width; grade separation of all streets and railroads crossing the freeway; planned access connections patterned to the types, volumes, and speeds of traffic; effective signs, pavement stripes, and other traffic control devices; and attention to the overall appearance. These concepts are not new; they were in the first freeways, designed before 1930. Admittedly, there has been a continual updating in their application in terms of present and predicted vehicles, volumes, and speeds, but the concept is the same.

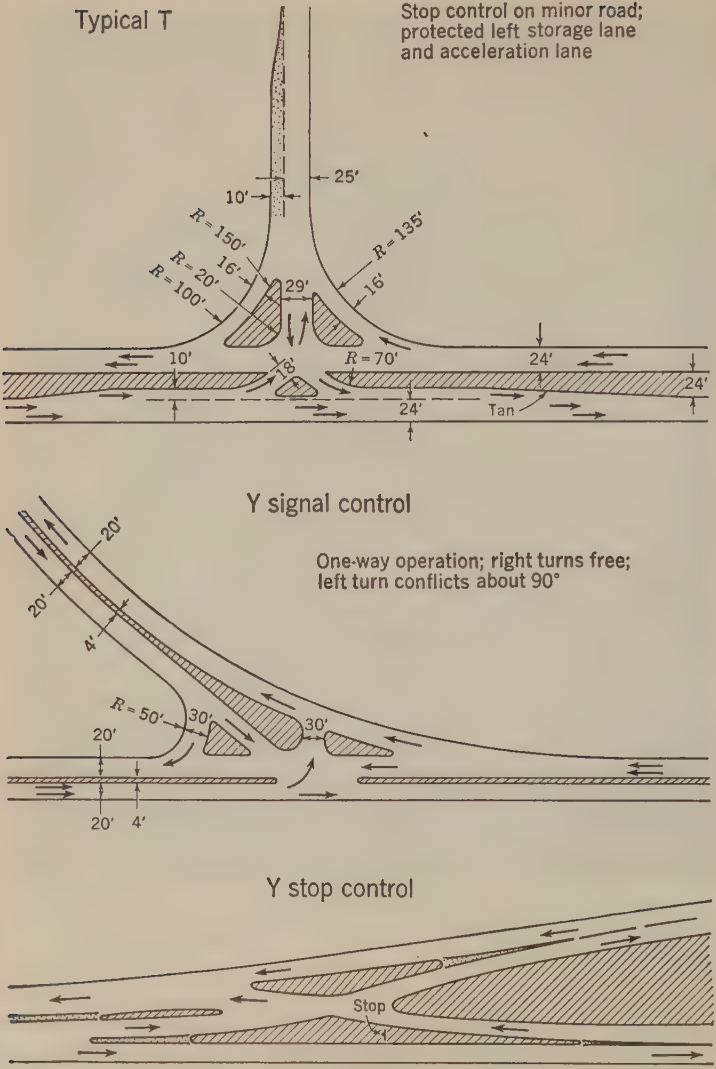
Spacing of Interchanges

When urban interchanges are spaced far apart, freeway operations are improved. There is less area devoted to merging, diverging, and lane changing. The efficiency of operation is high, but service to the local community may be low. Sizable areas of the community may be without connections to the freeway, and where there are connections, the accumulated ramp volumes are likely to exceed the capacity of the interchange crossroad, even in its ultimate development.

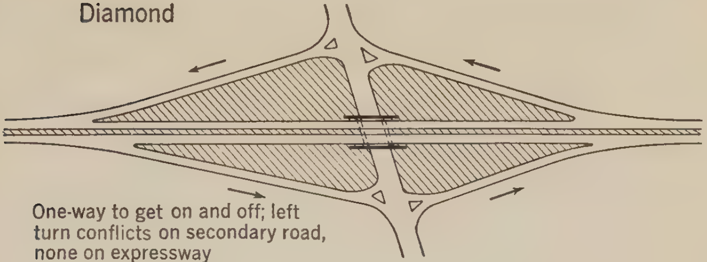
Closely spaced interchanges also have disadvantages. When interchanges are closely spaced, for example, $\frac{1}{2}$ mile to $\frac{2}{3}$ mile apart, there is a high degree of local service. Interchange volumes, being distributed, are low and probably capable of being handled by the interchange crossroads, perhaps with only minor improvements.



Four-level interchange connecting U.S. 81, State Highway 550, and the Dallas–Fort Worth Turnpike.

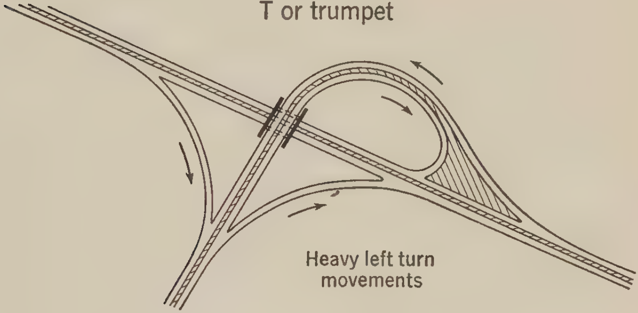

Typical T
Stop control on minor road; protected left storage lane and acceleration lane
25′
10′
R = 150′
16′
R = 20′
R = 100′
29′
R = 135′
16′
10′
18′
R = 70′
24′
24′
24′
Tan
Y signal control
One-way operation; right turns free; left turn conflicts about 90°
20′
20′
4′
R = 50′
30′
30′
20′
20′
4′
Y stop control
Stop

Existing three-leg intersections. Channelized, divided highways.



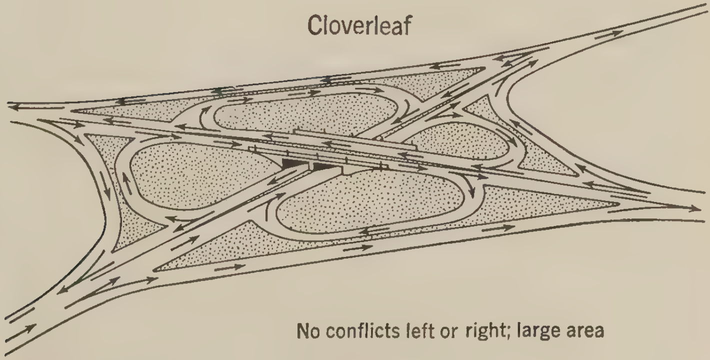
Diamond

One-way to get on and off; left turn conflicts on secondary road, none on expressway



T or trumpet

Heavy left turn movements



Cloverleaf

No conflicts left or right; large area

Types of interchange.

However, the freeway flow is subject to almost continuous impedance of weaving, exiting, and entering traffic. Furthermore, the freeway traffic includes some of the short-run local traffic that could be served just as well or better if it had used the parallel streets. Not only is the freeway efficiency low, but the safety factor drops considerably.

The spacing of urban interchanges must result from some acceptable compromise between the two extremes, which would provide reasonable freeway efficiency and safety with a fair degree of local service and suitability to crossroad abilities. Some engineers think the answer is various interchange spacing dimensions; three such approaches are considered:

1. To provide adequate advance and turnoff signing for an exit ramp, a distance along the freeway, clear of other ramps, of at least 1 mile (preferably, 2 miles) is needed.

2. The physical extension of a free-flowing entrance ramp and terminal generally extends approximately 1,500 to 2,000 ft from the interchange structure; an exit ramp is similar, but shorter. The distance along a freeway between an entrance ramp and the next exit ramp should be long enough to allow for the merging and diverging maneuvers of crossing traffic streams without adversely affecting freeway operations, capacity, and speed. These distances may vary from 500 ft to several thousand feet when the ramp volumes are high. Based on ramp volumes, interchanges must be more than 1 mile apart.

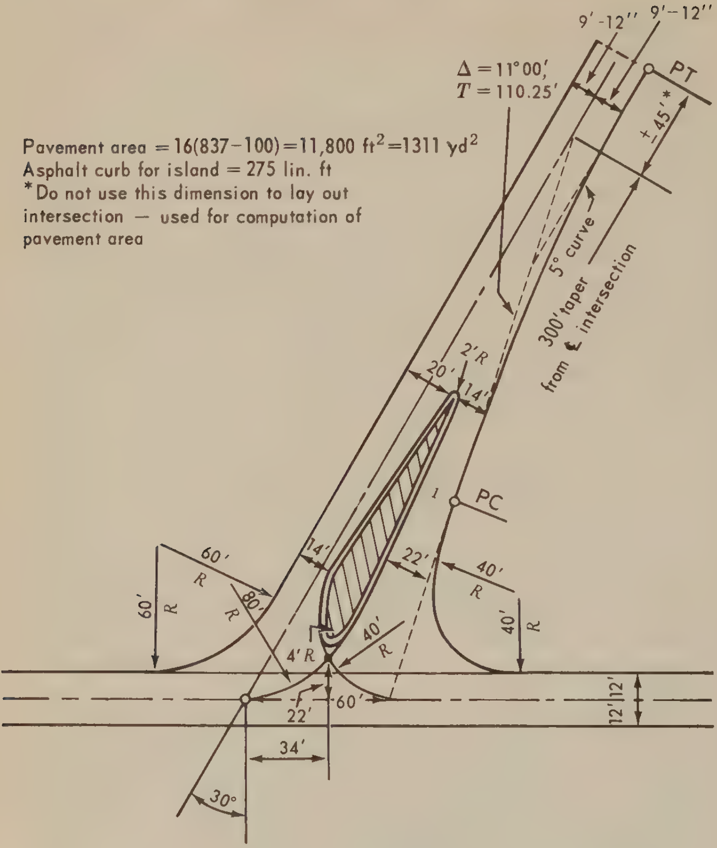
3. Drivers entering a freeway from a city street must make a significant adjustment to the frame of mind and type of operation required on the freeway. This requires from 10 to 30 sec after the driver enters the freeway. Thus, for approximately 800 to 2,200 ft beyond the entry point, the driver should be free of the diverging maneuvers of other vehicles preparing to leave at the next ramp ahead. This also requires a mile or more of spacing.

But along with the three preceding spacing dimensions, another controlling factor is the existing street-grid system. The interchange must be located at one of the more dominant cross streets, and the existing physical distance to the next similar street becomes the actual spacing. There may be some similar dominant intervening streets that are too close; they would probably be grade separated, but with no ramps provided. Thus, interchange location must be based on attention to the spacing along the freeway and the existing street system to which the interchange connects.

TYPES OF INTERCHANGE

There are three basic types of interchange *diamond, cloverleaf,* and *directional*–and numerous combinations or partial arrangements. Although each basic type has a certain field for application, it is not possible to cite a positive condition or volume that would serve as a limit for use of that type. Instead, there is a broad overlapping area in which two types may serve, which calls for detailed engineering alternate studies to decide which type to use. In determining the type of a particular urban interchange, several fundamental rules should be followed: (1) Use the simplest type that will adequately serve the traffic needs; (2) economy in first cost usually means economy in maintenance and operating costs, the total of which must be kept justifiably low; (3) urban land is irreplaceable, and the freeway should extract only what is strictly necessary; (4) the type of interchange must be realistic with regard to its type of operation and the ultimate capacity of the crossroad; and (5) consistency in the general type along a freeway means greater safety and efficiency, whereas mixed use of ramp locations or types creates confusion, interference, and hazard.

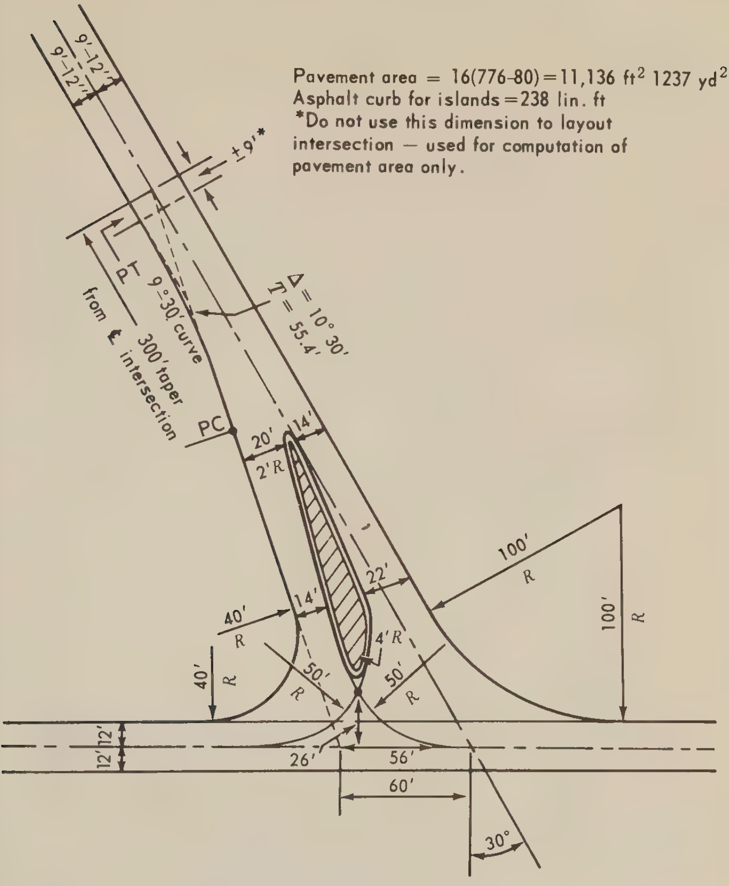
Diamond Interchange. The diamond interchange is the simplest pattern, with a single one-way ramp in each quadrant. Each ramp may be readily developed with a high-type freeway ramp terminal, and it generally involves only minor indirection for turning movements. The simple design is compact and economical in land area and construction costs. With a single freeway exit, signing is not difficult. The crossroad ramp terminal becomes another street intersection in keeping with the usual stop-and-go operations on the street. This would not be appropriate when the crossroad is an expressway. By using additional lanes on the crossroad and modern traffic-signal controls on the ramps, the ultimate capacity of the design may be developed. In this regard, the diamond pattern is highly flexible and may be economically adapted to fit almost any cross-street conditions. In addition, some of the added lanes and control devices may be


9′-12″
9′-12″
Δ = 11°00′
T = 110.25′
PT
±45′*
Pavement area = 16(837−100) = 11,800 ft² = 1311 yd²
Asphalt curb for island = 275 lin. ft
*Do not use this dimension to lay out intersection — used for computation of pavement area
5° curve
300′ taper from ℄ intersection
2′R
20′
14′
1
PC
14′
22′
40′ R
40′ R
60′ R
60′ R
80′ R
40′ R
4′ R
22′
60′
12′ 12′
34′
30°

Intersection detail at 30° right skew.

provided as later-stage work, if traffic demands make such work necessary. A diamond interchange is a simple, economical type, well adapted to urban conditions.

Full-cloverleaf Interchange. The full-cloverleaf interchange has an inner-loop ramp and an outer connection in each quadrant, a pattern that makes unnecessary any left turns at grade on the crossroad. It is a complete free-flow interchange, such as is needed between a freeway and an expressway. For assured operation as planned, the crossroad should be divided. The loop


9'–12''
9'–12''
±9'*
PT
9°30' curve
Δ = 10° 30'
T = 55.4'
300' taper
from ℄ intersection
PC
20'
2'R
14'
100' R
22'
14'
40' R
40' R
4'R
50' R
50' R
100' R
12'
12'
26'
56'
60'
30°

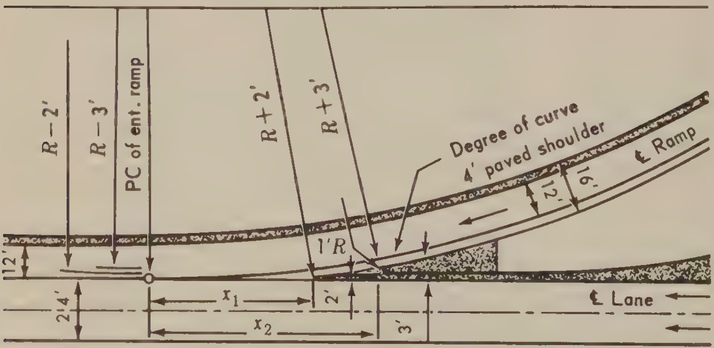
Pavement area = 16(776-80) = 11,136 ft^2 1237 yd^2
Asphalt curb for islands = 238 lin. ft
*Do not use this dimension to layout intersection — used for computation of pavement area only.

Intersection detail at 30° left skew.

ramps should be carefully laid out at the ramp terminals and around the loop. Tight radii designs (for example, less than 150 ft radius) may be operated, but it is difficult to effect the slowdown they require. However, loops of 200 to 250 ft radii may allow greater speed, but they result in a considerable amount of extra travel distance and much greater land and construction costs. The double-exit pattern, an outer connection followed by the far-side loop, requires double turnoff messages at the first exit, for which large, clear, and preferably overhead signs are needed. Along one side of the freeway, the adjacent loops result in a weaving area for their loop-turning volumes. This calls for further careful capacity and layout checks to assure a workable design, often with an added lane.

Directional Interchange. Directional interchanges are those with one or more direct or semidirect ramps necessary to accommodate high-volume, left-turning movements. Complete directional patterns are usually needed at interchanges of two high-volume freeways, but in some cases the less important movements may be handled on a loop design. Directional interchanges usually result in efficient free-flow paths with little or no extra travel distance. They require relatively large right-of-way areas and include several structures or multilevel structures, and are very costly. Each directional interchange is a separate and complex design requiring study of many alternatives in order to work out final details.

Method of locating entrance ramp island



NOT TO SCALE

Derivation

$$(X_1)^2 = (R + 2)^2 - (R - 2)^2$$
$$X_1 = \sqrt{(R + 2)^2 - (R - 2)^2}$$
$$X_2 = \sqrt{(R + 3)^2 - (R - 3)^2}$$

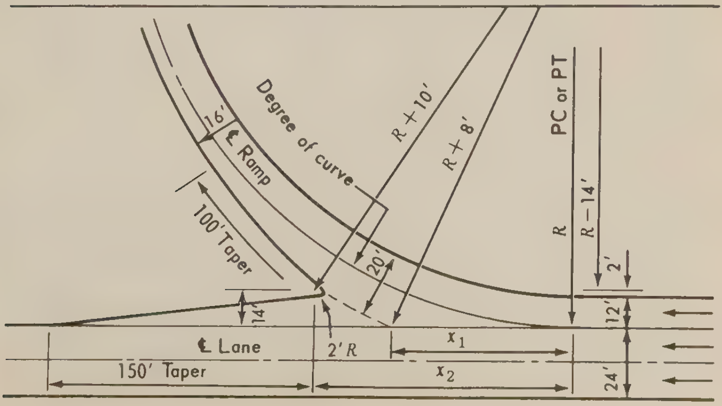
Notes:

1. Applies only for through pavements on tangent alignment.

2. Distance from PT or PC of entrance ramp to beginning of 2-ft shoulder = X_1.
3. Distance from PT or PC of entrance ramp to radius point (3 ft off edge of pavement) at island nose = X_2.

Degree of curve	X_1	X_2	Degree of curve	X_1	X_2
0°30′	302.78	370.82	8°30′	73.47	89.98
1°00′	214.09	262.21	9°00′	71.40	87.45
1°30′	174.80	214.10	9°30′	69.50	85.12
2°00′	151.39	185.42	10°00′	67.74	82.97
2°30′	135.41	165.84	10°30′	66.11	80.98
3°00′	123.61	151.39	11°00′	64.60	79.12
3°30′	114.44	140.17	12°00′	61.86	75.76
4°00′	107.05	131.11	13°00′	59.44	72.80
4°30′	100.93	123.62	14°00′	57.29	70.16
5°00′	95.76	117.28	15°00′	55.36	67.79
5°30′	91.31	111.83	16°00′	53.61	65.66
6°00′	87.42	107.07	17°00′	52.02	63.71
6°00′	83.99	102.88	18°00′	50.57	61.93
7°00′	80.94	99.14	19°00′	49.23	60.29
7°30′	78.20	95.78	20°00′	48.00	58.78
8°00′	75.73	92.74			

Method of locating exit ramp nose


16′
℄ Ramp
Degree of curve
R + 10′
R + 8′
PC or PT
100′ Taper
R
R − 14′
2′
20′
14′
12′
℄ Lane
2′ R
x_1
150′ Taper
x_2
24′

NOT TO SCALE

Derivation

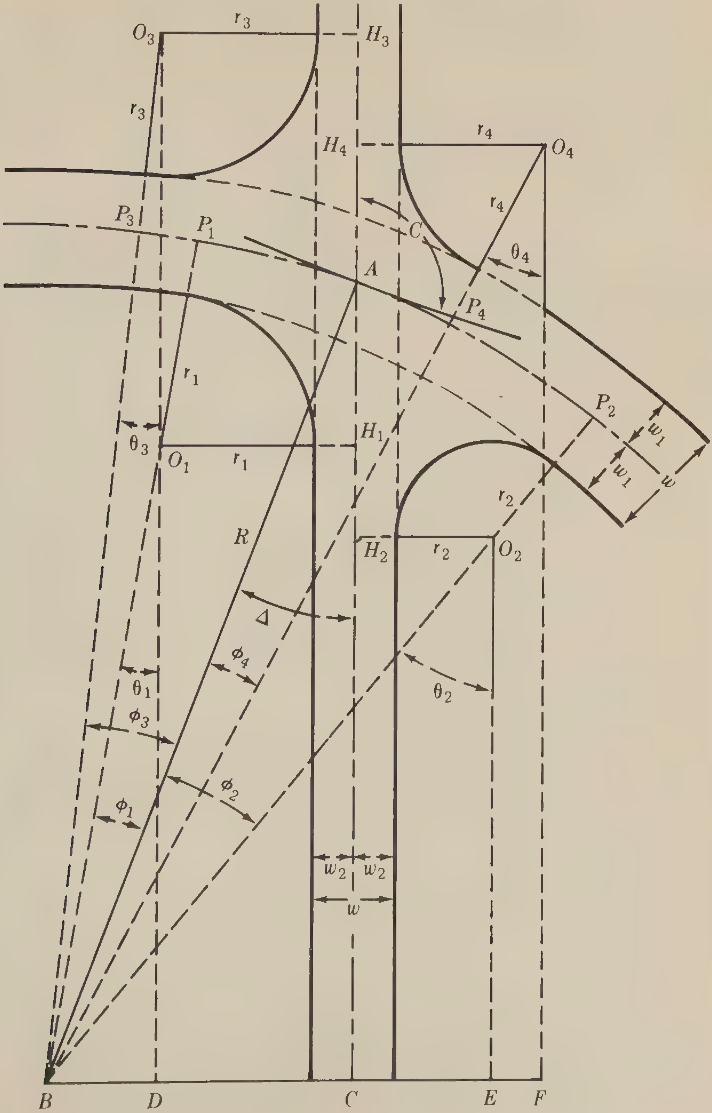
$$(X_1)^2 = (R + 8)^2 - R^2$$
$$X_1 = \sqrt{(R + 8)^2 - R^2}$$
$$X_2 = \sqrt{(R + 10)^2 - (R - 14)^2}$$

Notes:

1. Applies only for through pavements on tangent alignment.
2. Distance from PT or PC of exit ramp to edge of pavement $= X_1$.
3. Distance from PT or PC of exit ramp to radius point (14 ft off edge of pavement) at nose $= X_2$.

Degree of curve	X_1	X_2	Degree of curve	X_1	X_2
0°30′	428.26	741.58	8°30′	104.16	179.61
1°00′	302.88	524.33	9°00′	101.24	174.53
1°30′	247.34	428.08	9°30′	98.56	169.86
2°00′	214.24	370.69	10°00′	96.08	165.55
2°30′	191.66	331.53	10°30′	93.78	161.54
3°00′	174.99	302.62	11°00′	91.64	157.82
3°30′	162.04	280.15	12°00′	87.77	151.07
4°00′	151.60	262.03	13°00′	84.36	145.12
4°30′	142.95	247.02	14°00′	81.32	139.82
5°00′	135.64	234.32	15°00′	78.58	135.05
5°30′	129.35	223.40	16°00′	76.12	130.74
6°00′	123.87	213.87	17°00′	73.87	126.81
6°30′	119.03	205.46	18°00′	71.81	123.22
7°00′	114.72	197.97	19°00′	69.92	119.91
7°30′	110.85	191.24	20°00′	68.17	116.85
8°00′	107.35	185.15			

129. Intersection of a Straight and Curved Roadway. The center line of a curved roadway of width $w(= 2w_1)$ and radius R intersects at a known point A the center line of a straight roadway of width $w'(= 2w_2)$, the intersecting angle (called TC) being given. If the radii r_1, r_2, r_3, r_4 of the connecting curves with centers at O_1, O_2, O_3, O_4 are known (often some or all of the r's are equal), it is required to locate the radius points P_1, P_2, P_3, P_4, H_1, H_2, H_3, H_4, indicated in Fig. 113.



O_3
r_3
H_3
r_3
r_4
O_4
H_4
r_4
C
θ_4
P_3
P_1
A
P_4
r_1
P_2
w_1
w_1
w
θ_3
O_1
r_1
H_1
r_2
R
H_2
r_2
O_2
Δ
ϕ_4
θ_2
θ_1
ϕ_3
ϕ_2
ϕ_1
w_2
w_2
w
B
D
C
E
F

FIG. 113. Intersection of a straight and curved roadway.

Let $\Delta = \text{TC} - 90°$ (or $90° - \text{TC}$). Referring to Fig. 113, it is seen that $R \sin \Delta = BC$; also, if the vertices of angles θ_1, θ_2, θ_3, θ_4 are at O_1, O_2, O_3, O_4 respectively, it follows that

$$\sin \theta_1 = \frac{BC - O_1H_1}{O_1B}; \qquad \sin \theta_2 = \frac{BC + O_2H_2}{O_2B};$$
$$\sin \theta_3 = \frac{BC - O_3H_3}{O_3B}; \qquad \sin \theta_4 = \frac{BC + O_4H_4}{O_4B}.$$

Then the angles ϕ_1, ϕ_2, ϕ_3, ϕ_4, each with vertex at B, are found as follows: $\phi_1 = \Delta - \theta_1$; $\phi_2 = \theta_2 - \Delta$; $\phi_3 = \Delta - \theta_3$; $\phi_4 = \theta_4 - \Delta$.

Multiplying the radius R by angles ϕ (in radians) gives the arc lengths AP_1, AP_2, etc.

The distances from A to points H_1, H_2, H_3, H_4 are located by the following formulas:

$$AH_1 = R \cos \Delta - O_1B \cos \theta_1; \qquad AH_2 = R \cos \Delta - O_2B \cos \theta_2;$$
$$AH_3 = O_3B \cos \theta_3 - R \cos \Delta; \qquad AH_4 = O_4B \cos \theta_4 - R \cos \Delta.$$

Example. Given $R = 350$ ft; $\Delta = 20°$($\text{TC} = 110°$); $w = 48(w_1 = 24)$; $w' = 32(w_2 = 16)$; $r_1 = r_3 = r_4 = 60$, $r_2 = 40$.

Solution. $\theta_1 = 9°27'26.2''$; $\theta_2 = 37°54'19.6''$; $\theta_3 = 5°46'47.7''$; $\theta_4 = 26°48'14.0''$. $\phi_1 = 10°32'33.8''$; $\phi_2 = 17°54'19.6''$; $\phi_3 = 14°13'12.3''$; $\phi_4 = 6°48'14.0''$. $AP_1 = 64.402$; $AP_2 = 109.379$; $AP_3 = 86.866$; $AP_4 = 41.563$. $O_1B = 266$; $O_2B = 286$; $O_3B = 434$; $O_4B = 434$. $AH_1 = 66.508$; $AH_2 = 103.231$; $AH_3 = 102.901$; $AH_4 = 58.477$.

130. Intersection of Two Curved Roadways. Given two curved roadways of radii R_1 and R_2 with center lines intersecting at point A, where $BA = R_1$, $CA = R_2$, and the angle $BAC = \Delta$ (the TC). It is required to locate the centers O_1, O_2, O_3, and O_4 of the connecting curves with radii equal to r_1, r_2, r_3, and r_4 respectively, as shown in Fig. 114.

Solution

1. Solve the triangle BAC for side $BC(= a)$ and the angles $ABC(= B)$ and $ACB(= C)$, using the law of cosines

$$a^2 = b^2 + c^2 - 2bc\,(\cos A); \qquad \cos B = a^2 + c^2 - b^2/2ac;$$
$$\cos C = a^2 + b^2 - c^2/2ab.$$

Check: $A + B + C = 180°$.

2. To locate O_1: $O_1B = R_1 - (r_1 + w_1)$; $O_1C = R_2 - (r_2 + w_2)$.

Solve triangle O_1BC for angles $O_1BC(= B_1)$ and $O_1CB(= C_1)$. Then AP_1 is the arc with radius R_1 and central angle $(B - B_1)$; AH_1

is the arc with radius R_2 and central angle $(C - C_1)$; O_1 is the intersection of lines P_1B and H_1C.

3. To locate O_2: $O_2B = R_1 - (r_2 + w_1)$; $O_2C = R_2 + (r_2 + w_2)$. Solve the triangle O_2BC for B_2 and O_2CB for C_2. Then AP_2 is the arc with radius R_1 and central angle $(B_2 - B)$; AH_2 is the arc with radius

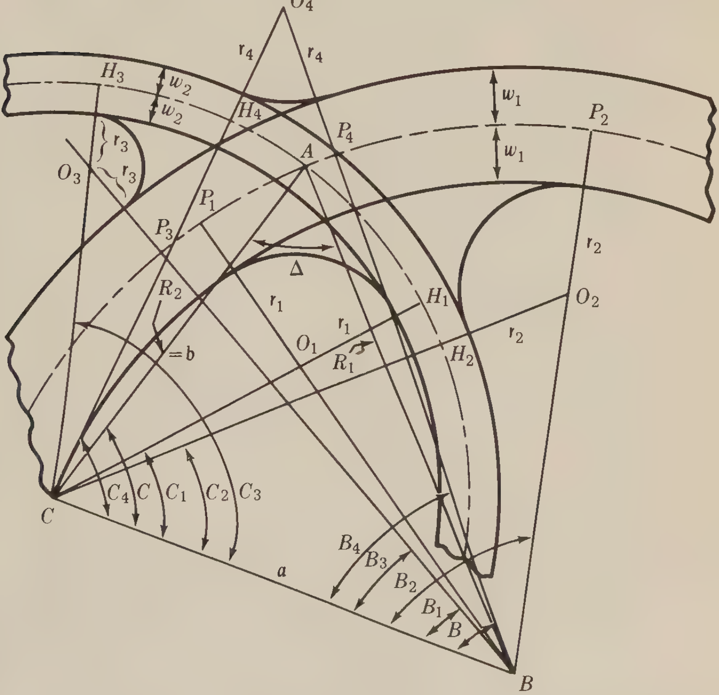

O4
r4
r4
H3
w2
w2
w1
w1
P2
H4
P4
A
r3
r3
O3
P1
P3
Δ
r2
R2
r1
H1
O2
r1
O1
R1
H2
r2
=b
C4
C
C1
C2
C3
C
a
B4
B3
B2
B1
B
B

FIG. 114. Intersection of two curved roadways.

R_2 and central angle $(C - C_2)$; O_2 is the intersection of lines P_2B and H_2C.

4. To locate O_3: $O_3B = R_1 + (r_3 + w_1)$; $O_3C = R_2 - (r_3 + w_2)$. Solve triangle O_3BC for angles $O_3BC(= B_3)$ and $O_3CB(= C_3)$. AP_3 is the arc with radius R_1 and central angle $(B - B_3)$; AH_3 is the arc with radius R_2 and central angle $(C_3 - C)$; O_3 is the intersection of lines P_3B and H_3C.

5. To locate O_4: $O_4B = R_1 + (r_4 + w_1)$; $O_4C = R_2 + (r_4 + w_2)$. Solve triangle O_4BC for angles $O_4BC(= B_4)$ and $O_4CB(= C_4)$. AP_4 is the arc with radius R_1 and central angle $(B_4 - B)$; AH_4 is the arc with radius R_2 and central angle $(C_4 - C)$; O_4 is the intersection of lines P_4B and H_4C.

131. Formulas for a Curved-road Intersection Interchange. It is required (see Fig. 115) to fit an access road extending from a known point A on curve A to an unknown point on curve B and to find the radius R of curve C and the central angle Δ of the intercepted arc AC.

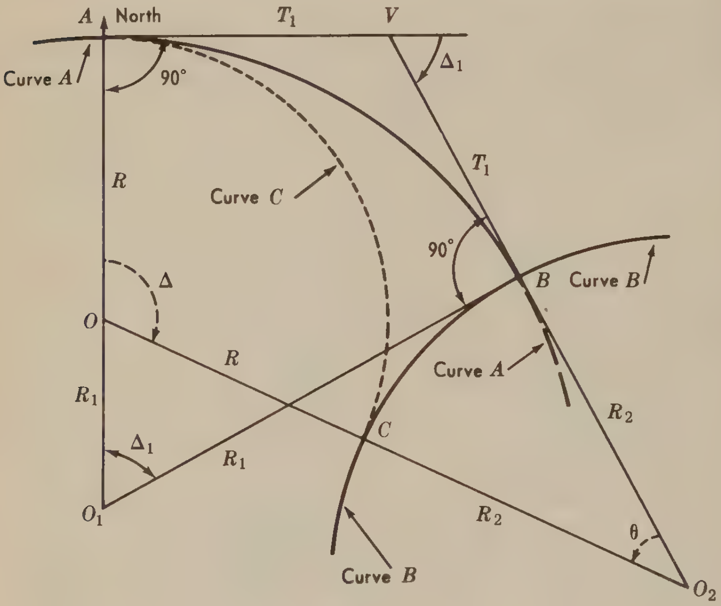

A
North
T_1
V
Curve A
90°
Δ_1
T_1
R
Curve C
90°
B
Curve B
Δ
O
R
Curve A
R_1
R_2
Δ_1
C
R_1
O_1
R_2
θ
Curve B
O_2

FIG. 115

Only the center lines of the curved roadways are indicated in Fig. 115, where the known data are points A and B of curves A and B, the corresponding radii R_1 and R_2, the angle Δ_1, and the tangents T_1 $(T_1 = R_1 \tan \frac{1}{2}\Delta_1)$.

Assuming OA to be north, the "method of Coordinates" will be applied to the closed traverse $OAVO_2O$. As in surveying and navigation, the azimuths (bearings) will be reckoned clockwise from north. Thus, if L is *length* and ϕ is *azimuth*, *latitude* = $L \cos \phi$, *departure* = $L \sin \phi$, where the same rules hold true for algebraic signs as in conventional trigonometry, *provided* the "initial line" is *north* instead of *east*.

Line	Length	Azimuth	Latitude	Departure
OA	R	0°	R	0
AV	T_1	90	0	T_1
VO_2	$(T_1 + R_2)$	$(90 + \Delta_1)$	$-(T_1 + R_2) \sin \Delta_1$	$(T_1 + R_2) \cos \Delta_1$
O_2O	$(R_2 + R)$	$(180 + \Delta)$	$-(R_2 + R) \cos \Delta$	$-(R_2 + R) \sin \Delta$
			0	0

Σ Latitudes = 0 gives:
$$\cos \Delta = \frac{R - (T_1 + R_2) \sin \Delta_1}{R_2 + R}. \tag{185}$$

Σ Departures = 0 gives:
$$\sin \Delta = \frac{T_1 + (T_1 + R_2) \cos \Delta_1}{R_2 + R}. \tag{186}$$

The *cosine* formula, Eq. (185), is preferable to the *sine*, since the algebraic sign indicates whether to select Δ or $(180 - \Delta)$.

Combining Eqs. (185) and (186) gives

$$R = \frac{T_1 + (T_1 + R_2) \cos \Delta_1}{\sin \Delta} - R_2. \tag{187}$$

But Eqs. (185) and (187) are not independent, hence the unknowns Δ and R cannot be found except by tedious trial and error.

Another independent expression for R will now be derived by a procedure involving analytic geometry. Thus, in Fig. 115, $AO = OC = OO_2 - CO_2$ that is, $AO + CO_2 = OO_2$; or

$$\begin{aligned} (R + R_2) &= OO_2; \\ (R + R_2)^2 &= (OO_2)^2. \end{aligned} \tag{a}$$

Assuming AO to be the X axis (origin at A), the coordinates (x_1,y_1) and (x_2,y_2) for points O and O_2 become: $[x_1 = R,\ y_1 = O]$; and $[x_2 = (T_1 + R_2) \sin \Delta_1,\ y_2 = T_1 + (T_1 + R_2) \cos \Delta_1]$.

Substituting $(OO_2)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ in Eq. (a),

$$(R + R_2)^2 = [(T_1 + R_2) \sin \Delta_1 - R]^2 + [T_1 + (T_1 + R_2) \cos \Delta_1 - O]^2. \tag{b}$$

Reducing, noting that $(T_1 + R_2)^2(\sin^2 \Delta_1 + \cos^2 \Delta_1) = (T_1 + R_2)^2$, factoring again, and solving, Eq. (*b*) becomes

$$R = \frac{(T_1)(T_1 + R_2)(1 + \cos \Delta_1)}{R_2 + (T_1 + R_2)(\sin \Delta_1)}. \tag{188}$$

By Eqs. (188) and (185) the unknowns R and Δ may be found, while Eq. (187) serves as a check. Then θ (see Fig. 115) $= 90 - (\Delta - \Delta_1)$. Knowing Δ, θ, and R, the arcs BC and AC may be calculated.

Example. Given $R_1 = 400$ ft, $R_2 = 300$ ft, $\Delta_1 = 60°$; to find R, Δ, θ, and the arcs BC and AC.

Solution. $T_1 = R_1 \tan \frac{1}{2}\Delta_1 = 230.94012$; by Eq. (188), $R = 242.0653$ ft; by Eq. (185), $\cos \Delta = -0.4016910$. Then $(180 - \Delta) = 66°18'57.8''$; $\Delta = 113°41'12.2''$; $\theta = 90 - (\Delta - \Delta_1) = 36°18'47.8''$; arc $BC = \dfrac{R_2\theta}{57.29578} = \dfrac{(300)(36.3132778)}{57.29578} = 190.136$ ft; chord $BC = 2R_2 \sin \frac{1}{2}\theta = 2(300)(0.3116159) = 186.97$ ft; arc $AC = \dfrac{R\,\Delta}{57.29578} = \dfrac{(242.0653)(113.6867222)}{57.29578} = 480.308$ ft; chord $AC = 2R \sin \frac{1}{2}\Delta = 2(242.0653)(0.8371784) = 405.304$ ft.

CHAPTER 10

EARTHWORK

132. Description. The term *earthwork* includes all clearing, grubbing, roadway and drainage excavation, excavation for structures, embankments, borrow, overhaul, machine grading, subgrade scarifying, rock fill, reconditioning roadway, and all the operations of preparing the subgrade foundation for highway or runway pavement or railroad ballast.

The quantity and cost of earthwork are calculated in terms of cubic yards of *excavation* in its original position on the basis of cross-section notes from field measurements. The payment usually includes full compensation for excavation, formation of embankments, trimming slopes, disposing of surplus materials, and the preparation and completion of subgrade and shoulders.

In cases where the excavation is very light, the work of finish grading is more appropriately based on the square yard. Some grading contracts provide extra payments for *overhaul*, while others make no allowance for it whatever.

On resurfacing projects, "reshaping and scarifying" to a depth of about 6 in. are often paid for by the square yard instead of by the volume quantities.

Modern grading operations are carried on by power equipment including power shovels, rooters, scrapers, bulldozers, blade graders, rollers, dragline excavators, elevating graders, motor trucks, tractors, and crawler dump wagons.

To determine the equipment most suited for a given grading job a contractor studies the plans of the project, which show the quantities to be moved, the length of haul, the character of the soil and rock formations, and a typical cross section. This is followed by a field inspection. These steps are taken before bidding on a job.

The truck hauling of material excavated with power shovels is an important operation on a large portion of highway or railroad construction jobs. On heavy work with short haul and steep grades, self-loading scrapers towed by crawler tractors are desirable. Bulldozers or pusher tractors, or both, are advantageous in open cuts and assist in scraper loading. Scarifiers should be used to loosen the material whenever it is too hard and tough to be readily loaded by pulling and pushing tractors. Elevating graders and rubber-tired tractor-wagon hauling units are quite often used in flat terrain where large quantities of easily worked soil must be hauled some distance.

133. Classification of Excavated Material. Excavated material is usually classified as (1) common excavation, (2) loose rock, or (3) solid rock. *Common excavation* is largely *earth*, or earth with detached boulders less than $\frac{1}{2}$ cu yd. *Loose rock* usually refers to rock which can be removed with pick and bar, although the use of power shovels or blasting may be advantageous. *Solid rock* comprises hard rock in place, ledge rock, and boulders that can be removed only by the use of drilling and blasting equipment.

In addition to the construction of roadways, railroads, or airport runways, there may be excavations for canals, pipelines, drainage ditches, culverts, bridges, and grade separation structures. Still another type of construction includes dredging and pumping operations for hydraulic fills, where the quantity of material would be measured and compensated for on the basis of the volume in *embankment*.

134. Shrinkage and Swell Factors. Excavated earth when placed in a fill will usually become compacted so that its final volume is less than the space originally occupied in the embankment. This difference between the original volume in a cut and the final volume in a fill may be defined as the *shrinkage*. Excavated rock when placed in a fill will occupy a larger volume; the increase is called *swell*.

The actual volume of freshly excavated earth is at first increased owing to its loose and porous condition. Then compaction and shrinkage begin to take place. The rapidity of shrinkage depends upon the character of the material, the depth of cuts and fills, the method of placing the fill, the climatic conditions, and the intensity of loads on the fill before stability has been reached. Shrinkage generally includes the slight wastage in transporting material from cut to fill and loss in material which washes beyond the toe of the slopes. A considerable amount of shrinkage usually takes place during construction. Shrinkage and swell must be taken into consideration in balancing earthwork. These vary according to the classification of the material to be excavated.

For light earth excavation in light soil and where the fills are built on swampy ground, the shrinkage may range from 20 to 40 percent or even greater. For moderately light earth excavation, the shrinkage will range from 10 to 20 percent. For very heavy earth excavation with deep cuts and fills, the range will probably be from a small swell to about 10 percent shrinkage.

Swell ranging from about 5 to 25 percent may be expected in rock excavation depending upon the proportion of solid rock and upon the size of the rock placed in the fill.

The amount of excavation required to make a given fill may be arrived at by 1.00 minus the percentage of shrinkage, or 1.00 plus the percentage of swell. Thus, if the shrinkage is 10 percent, multiply the fill quantity by 1.10; if the swell is 10 percent, divide the fill quantity by 1.10.

Settlement results from *shrinkage,* but one is not directly proportional to the other. Shrinkage should not be confused with *subsidence,* which is a settlement of the entire embankment owing to unstable foundation conditions, such as might be expected if a heavy fill is placed on swampy soil.

135. Compaction of Fills. Settlement resulting from uncompacted fills causes cracks and serious trouble with all types of road or runway surfaces. It is now the practice to spread the soil in even layers about 6 in. deep by means of bulldozers or graders.

The sheep's-foot or tamping roller is used extensively for compacting the fills. Other types of rollers are satisfactory if the required density is obtained.

136. Side Slopes. The side slopes of cuts and fills vary with the material, being naturally flatter for sand than for clay. Much steeper slopes are allowed for rock cuts.

Side slopes are defined as the ratio of horizontal to vertical. A $1\frac{1}{2}$ to 1 slope for both cuts and fills is the usual railroad practice, as it was formerly for highways. With the new construction equipment, flatter slopes are obtained more easily. These flat slopes present a pleasing appearance and are less expensive to maintain than the steeper slopes, while providing longer visibility.

Interregional standards for class A, B, and C highways specify the following slopes:

Depth of cut or fill (ft)	*Slopes*
0–5	4 to 1
5–10	3 to 1
Above 10	2 to 1

The slopes are not always made constant throughout. They may be flattened for purposes of beautification, greater sight distance, elimination of guard rail, widening for borrow material, and utilizing waste material; they may be steepened slightly to avoid excessive property damage.

To beautify and check erosion, the following are recommended: (1) the seeding of road banks, side ditches, and disposal ditches with perennial grasses and clovers, (2) the mulching of steep slopes soon after seeding to avoid washing before the newly sown cover crop has developed an adequate root system, (3) the planting of suitable and adaptable plants on road banks which are too steep for mowing.

137. Roadbed Sections. Figure 116 shows typical roadbed sections for a railroad, highway, and village street (business and residential), all of which are exaggerated vertically.

On *railroad* work, the subgrade is usually flat and not sloped on curves, superelevation being added in the ballast. However, less maintenance work will result if the subgrade is also banked.

The *highway* subgrade is usually formed with shoulders and a trenched section to receive the pavement, the finished surface being crowned to facilitate drainage. Sometimes ditches are provided on embankment sections that transfer water down the fill slopes into pipes or paved gutters for the protection of the embankment against erosion. On curves, the subgrade is banked; it is also widened for curves of 5° or sharper. The width of roadbed in cuts is wider than on fills to allow for the side ditches. The formation of the trenched subgrade and shoulders requires "fine grading," which is sometimes a separate item in the cost of "roadway grading."

The normal cross section of a *city street* has a crowned pavement with curb and gutter depressed sufficiently to receive drainage from the sidewalk and adjoining property. In residential areas, a planting space for grass and trees should intervene between the sidewalks and the curb.

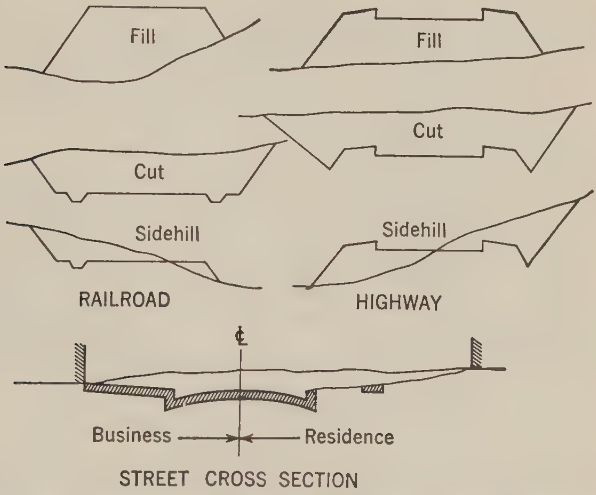


FIG. 116. Typical roadbed sections for railroad, highway, and street.

138. Slope Stakes. The final grade line having been established, slope stakes are set at points where the side slopes of the graded road will intersect the ground surface; they mark the limits of excavation and embankment. The slope stakes are driven at points of zero cut or fill, but the numbers written on them give the vertical distance with reference to the finished plane of subgrade. On the inner side of the stakes is marked in feet and tenths the "cut" or "fill" as the case may be. Thus, "C 3.2" indicates that the center-line elevation of the roadbed is to be cut 3.2 ft *below* the ground at the slope stake; and "F 4.6" indicates that the fill is to be 4.6 ft *above* the slope stake.

The stakes are driven with the tops slanting outward and with the sides upon which the cuts or fills are marked facing the roadway. The station number is marked on the other side of the stake.

The usual equipment for setting slopes consists of a level, rod, tape, notebook, stakes, and keel. The level may be an engineer's level, hand level, or "level board." On steep ground, the last two are preferable.

If a profile map has been established from previous surveys and the final grade line drawn thereon, the center-line cut or fill may be found from the map by subtracting the elevation of grade from the elevation of ground.

Also, if a cross section of the ground and finished shape of roadway have been plotted to scale, the position of the slope stakes could be scaled from the map.

Aside from the elevation of grade, the most satisfactory procedure is to determine all distances and elevations in the field. The method must necessarily be a "measure and try" process, but the work can be done rather easily.

General Method of Procedure. Let $ABEPD$ in Figs. 117 and 118 represent the cross-sectional area of highway (or railroad) in fill or cut, for which we have the following general notation applying to both excavation and embankment:

$b = AB = AC + CB$ = width of roadbed;
s = "slope ratio" for the banks AD and BE = ratio of horizontal to vertical (*plus* for cut, *minus* for fill);
$d = PC$ = depth of fill or cut at the center;
x_1 = horizontal distance from P to slope stake D;
x_2 = horizontal distance from P to slope stake E;
y = vertical distance from P to slope stake;
$h_1 = d + y_1 = FD$ = vertical distance from C to D;
$h_2 = d + y_2 = GE$ = vertical distance from C to E.

The slope stake at point D on the right is correctly established if

$$x_1 = \tfrac{1}{2}b + sh_1 = \tfrac{1}{2}b + sd + sy_1. \tag{189}$$

Likewise, point E on the left is correctly established if

$$x_2 = \tfrac{1}{2}b + sh_2 = \tfrac{1}{2}b + sd + sy_2. \tag{190}$$

In the foregoing equations, b, d, and s are known, while x and h (or y) are measured and remeasured in the field until the equations are satisfied (trial-and-error method).

If the ground is level, $y_1 = y_2 = 0$; then $x_1 = x_2 = \frac{1}{2}b + sd$.

Treating s, h, d, and y as algebraic quantities, Eqs. (189) and (190) hold true for all cases of cut or fill.

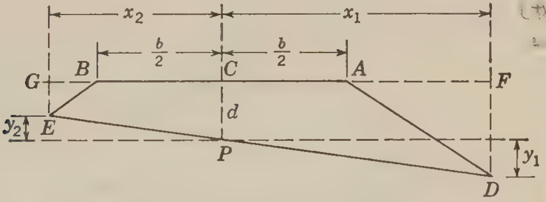


FIG. 117. Cross section on fill (slope stakes at E and D).

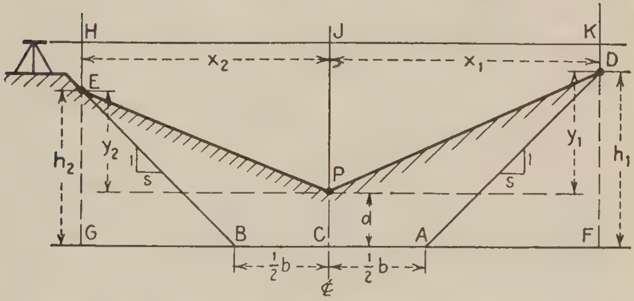


FIG. 118. Cross section in cut (slope stakes at E and D).

In Fig. 118, it is assumed that rod readings to all points within the cross profile under consideration can be taken from a single position of the level (which of course is not always possible). By means of previous differential leveling from the nearest bench mark, the HI (height of instrument or elevation of line HJK) is established.

If we imagine the bottom of the rod to be at grade at point C, the rod reading would be CJ, which is called "grade rod." That is,

$$\text{Grade rod} = \text{HI} - \text{grade elevation}. \qquad (191)$$

Negative values of grade rod would occur when the HI is below "grade."

Since the actual rod reading at the center-line stake (Fig. 118) is PJ (not CJ), the depth of cut is $CP = CJ - PJ$; that is,

$$d = \text{grade rod} - \text{ground rod}. \tag{192}$$

Negative values of d indicate *fill.*

Similarly at the slope stake D,

$$FD = FK - DK,$$

that is, $$h_1 \text{ (or } h_2) = \text{grade rod} - \text{ground rod}. \tag{193}$$

Negative values of h indicate *fill.*

The values of x_1 and h_1 (or x_2 and h_2) corresponding to the intersection of two slopes are found easily after two or three field measurements. Example 1 illustrates the procedure. Sometimes the work is expedited by using $d + y_1$ (or $d + y_2$) instead of h_1 and h_2, where y_1 (or y_2) = center-line ground rod − ground rod (at slope stake).

On sidehill construction (Fig. 119), AC is greater than CB, since there is no ditch on the embankment.

Example. For the sidehill section (Fig. 119), the slope in cut is $1\frac{1}{2}$ to 1 ($s = \frac{3}{2}$) while the slope on fill is 2 to 1 ($s = -2$). The elevation of subgrade, the grade rod, and the final trial rod readings are indicated in Fig. 119. Required: the position of the slope stakes (at D and E).

Solution. To cover the entire section, it is necessary to set up the level twice. In each position, the HI and the *grade rod* are determined by methods similar to ordinary differential leveling [see Eq. (191)].

To check the position of the slope stake at D, we have

$$x_1 = 36.1 = 16 + (\tfrac{3}{2})(13.4) = 36.1.$$

Similarly, for point E,

$$x_2 = 36.0 = 12 + (-2)(-12.0) = 36.0.$$

The position of the grade point and the center cut ($d = 1.8$) are also indicated in Fig. 119.

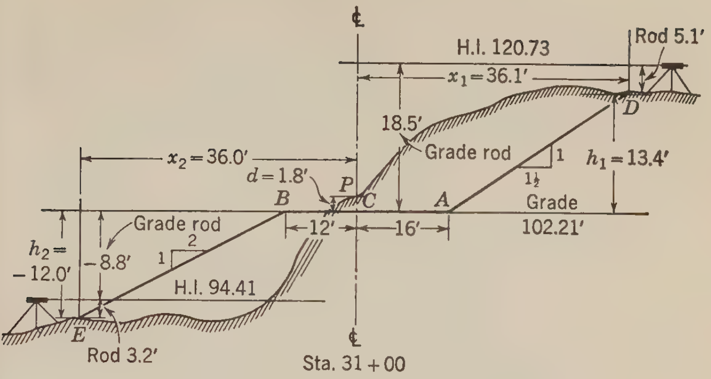

℄
H.I. 120.73
Rod 5.1′
$x_1 = 36.1′$
D
18.5′
Grade rod
1
$h_1 = 13.4′$
$1\frac{1}{2}$
Grade
102.21′
$x_2 = 36.0′$
$d = 1.8′$
P
B
C
A
Grade rod
2
1
$h_2 = -12.0′$
−8.8′
−12′−
−16′−
H.I. 94.41
E
Rod 3.2′
℄
Sta. 31 + 00

FIG. 119. Sidehill section (slope stakes at *E* and *D*).

The slope-stake notes for this station are:

Left	℄	*Right*
$\frac{\text{F } 12.0}{36.0}$	$\frac{\text{C } 1.8}{0}$	$\frac{\text{C } 13.4}{36.1}$

The data above may accompany the usual notes for profile leveling or they may be listed with elevations obtained from the profile as shown in Table A.

TABLE A

Station	Elevation		Left	℄	Right
	Ground	Grade			
31 + 00	104.0	102.21	$\frac{\text{F } 12.0}{36.0}$	$\frac{\text{C } 1.8}{0}$	$\frac{\text{C } 13.4}{36.1}$

139. Grade Points in Sidehill Sections. Grade points occur where the plane of the subgrade intersects the ground surface. These points may be found in the field by moving the rod until the "ground rod" equals the "grade rod."

In passing from cut to fill in sidehill sections, the junction rarely takes place entirely at one cross section, but along an oblique line, as shown, in plan, by Fig. 120.

Aside from full stations, three intermediate cross sections should be taken: (1) where the grade line intersects the surface at the center line (at Sta. 10 + 71), (2) where the outside of base in cut intersects the surface (at Sta. 10 + 32), (3) where the outside of base in fill intersects the surface (at Sta. 10 + 90).

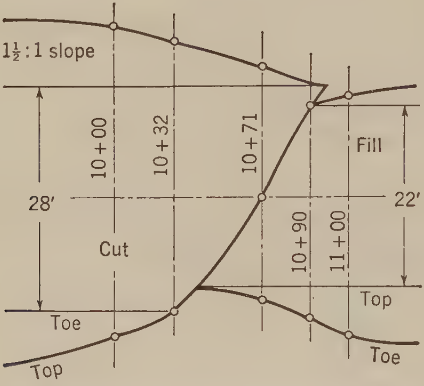


FIG. 120. Sections passing from cut to fill.

140. Area of Cross Section. From the data supplied by slope-stake or cross-section notes, the area of cross section may be calculated. If the ground is level or regular, simple geometry may be applied; for irregular ground, two general methods are used; (1) the graphical or planimeter method and (2) the coordinate method.

141. Area for Level Ground. For *level ground*, the area of cross section in cut (or fill) is merely that of a trapezoid.

In Fig. 121, b = width of base AB,
d = center cut (or fill),
s = slope of banks = $MD/AM = NE/BN$.

Hence, $$\text{Area} = \tfrac{1}{2}d(b + b + 2sd) = d(b + sd). \tag{194}$$

142. Area for Three-level Section. With three readings taken directly from slope-stake notes, one at the center and one at each slope stake, the area of cross section may be obtained. For regular ground, this is an accurate and very satisfactory method. Such a section is known as a *three-level section*, and the area may be calculated readily from field notes without plotting.

Imagine the area $ABED$ (Fig. 122) to be divided into four triangles, *two* having the common base d and altitudes x_1 and x_2, and

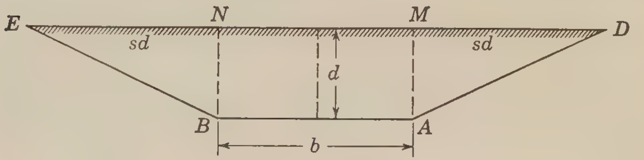


Fig. 121

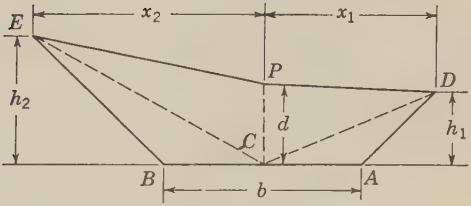


Fig. 122

two having bases $= \frac{1}{2}b$ and altitudes h_1 and h_2. Hence the area of section is

$$A = \tfrac{1}{2}[d(x_1 + x_2) + \tfrac{1}{2}b(h_1 + h_2)]. \tag{195}$$

If the slope-stake notes are not available and the center fill is unknown, or if the construction is contrary to specifications, the end area A may be found conveniently from the four field measurements indicated in Fig. 123.

Assuming uniform slope of the original ground underneath the fill,

$$A = \tfrac{1}{2}(h_1x'' + h_2x'). \tag{196}$$

Proof. Draw line EF parallel to AB; then area $A = ABED =$ trapezoid $ABEF$ + triangle DEF $= \frac{1}{2}h_2(b + b + 2sh_2) + \frac{1}{2}(h_1 - h_2)(b + 2sh_2) = \frac{1}{2}h_1(b + sh_2) + \frac{1}{2}h_2(b + sh_1)$. Hence,

$$A = \tfrac{1}{2}(h_1x'' + h_2x'). \tag{197}$$

143. Area by Graphical Methods. With the cross profile of ground surface plotted to scale and the cross section of finished roadbed and side slopes drawn thereon to the same scale and at the established center-line profile elevation for that station, the areas in cut (or fill) are obtained by means of a planimeter. Templates giving the desired shape of subgrade, shoulders, and ditches form an integral part of the cross section.

The scale of the cross sections is usually 1 in. = 10 ft or 1 in. = 5 ft, the latter predominating. For very wide cross sections, scales of 1 in. = 20 ft or 1 in. = 25 ft are used in conjunction with the coordinate or "machine method" (see Art. 144).

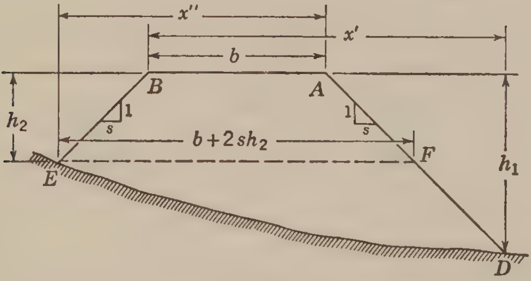


FIG. 123

144. Area by Coordinate Method. With the coordinates of all the corners of a cross section known, the end area may be computed by means of the *coordinate method.* This method is presented in texts on plane surveying, but it will be derived here and formulated with particular reference to end areas of earthwork.

Proof. Let the corners A, B, C, and D of the area $ABCD$ (Fig. 124) be located by the coordinates (x_1,y_1), (x_2,y_2), (x_3,y_3), and (x_4,y_4). Then the area is given by the algebraic sum of four trapezoids. Thus,

$$\text{Area} = ABba + BCcb - ADda - DCcd,$$

or

$$\text{Area} = \tfrac{1}{2}(x_1 + x_2)(y_2 - y_1) + \tfrac{1}{2}(x_2 + x_3)(y_3 - y_2) - \tfrac{1}{2}(x_1 + x_4)(y_4 - y_1) - \tfrac{1}{2}(x_3 + x_4)(y_3 - y_4). \quad (198)$$

Reducing and factoring, Eq. (198) becomes

$$\text{Area} = \tfrac{1}{2}[y_1(x_4 - x_2) + y_2(x_1 - x_3) + y_3(x_2 - x_4) + y_4(x_3 - x_1)]. \tag{199}$$

From Eq. (199) we may state the following rule for *area:*

Multiply each ordinate by the algebraic difference between the adjacent abscissas, find the algebraic sum of the products, then take half of this result.

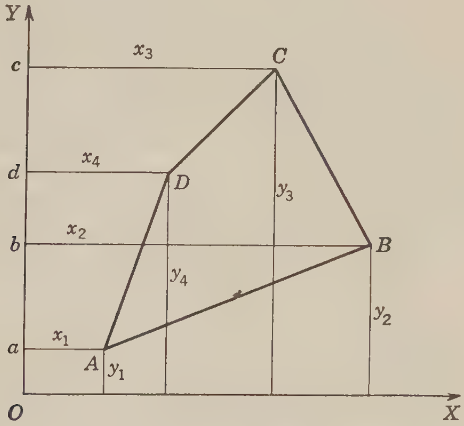


FIG. 124. Area by coordinate method.

A simpler rule for area follows if we arrange in counterclockwise order the coordinates (Fig. 124) in the form of fractions, the initial fraction (beginning at any corner) being repeated to give a closed boundary. Thus, we have

$$\frac{y_1}{x_1} \diagup \frac{y_2}{x_2} \diagup \frac{y_3}{x_3} \diagup \frac{y_4}{x_4} \diagup \frac{y_1}{x_1}.$$

Multiply along the marked diagonals and add the products (all positive); multiply along the unmarked diagonals and add the products (all negative). The difference gives the *double area.*

In using a calculating machine, the positive products are accurately accumulated. Then, without clearing the product dial, the machine is thrown into reverse and reverse-multiplied along the unmarked diagonal. When the end is reached, the remainder is divided by 2, giving the desired area of section.

Areas of cut and fill are calculated separately. For the latter, the same rule applies as for cuts, and the elevations and offsets are considered positive.

When the method is thoroughly understood, the computer might omit tabulation of fractions and calculate areas directly from cross sections by machine. For simple sections the "constant area" of excavation for foundation of highway pavement (the part below the plane of reference) is to be *added* in cuts and *subtracted* in fills. For sections entirely in cut or fill, the axes are usually the center line and the base, but for sidehill sections, it may be advantageous to shift the vertical axis to the grade point (the point of zero cut or fill).

Example 1. *Excavation Section.* The base width (Fig. 125) is 40 ft and the fractions giving coordinates of the corners of the cross section are indicated. Required: the area of the section.

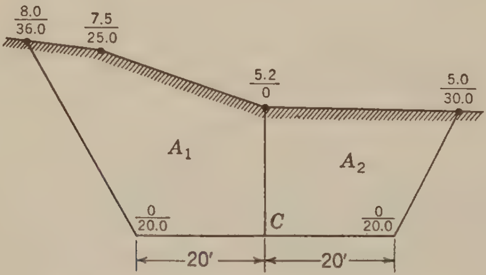


FIG. 125. Finding area of excavation section.

Solution. (1) Divide the area in two parts, A_1 and A_2, one to the left of the center line, the other to the right of it. Write the fractions in clockwise order for the left part, in counterclockwise order for the right part. (2) Consider *all* offsets and elevations (values of x and y) to be *positive*. (3) Multiply along the marked diagonals and add the products (all positive); multiply along the unmarked diagonals and add the products (all negative). The *difference* gives the *double area.* (4) Add the double areas of the two parts, divide by 2, and obtain the *end area* desired.

Left: $\frac{0}{0}\diagup\frac{0}{20}\diagup\frac{8}{36}\diagup\frac{7.5}{25}\diagup\frac{5.2}{0}\diagup\frac{0}{0}.$

$$A_1 = 0 + 160 + 270 + 130 + 0 - (0 + 0 + 200 + 0 + 0) = 360 \text{ sq ft.}$$

Right: $\frac{0}{0}\diagup\frac{0}{20}\diagup\frac{5}{30}\diagup\frac{5.2}{0}\diagup\frac{0}{0}.$

$$A_2 = 0 + 100 + 156 + 0 - (0 + 0 + 0 + 0) = 256 \text{ sq ft.}$$

Double area $= A_1 + A_2 = 616$ sq ft.
End area $= \frac{616}{2} = 308$ sq ft.

Note: This method is particularly adapted to *machine computation* for both highway and railroad sections.

Example 2. Same data as in Example 1. Required: to check the area by using Eq. (199).

Solution. Consider distances to the right of the center line as *positive* and distances to the left as *negative*, while all elevations are positive. Then proceeding clockwise around the *entire* area, Eq. (199) gives

$$\begin{aligned}\text{Area} &= \tfrac{1}{2}[8(-25 + 20) + 7.5(0 + 36) + 5.2(30 + 25) + 5.0(20 - 0)] \\ &= \tfrac{1}{2}(-40 + 270 + 286 + 100) = 308 \text{ sq ft.}\end{aligned}$$

Note: Since all the offsets (abscissas) in Example 1 were considered *positive*, that method appears simpler and so will be used in Example 3.

Example 3. *Embankment Section.* The base width (Fig. 126) is 20 ft, and the fractions giving the coordinates of the corners of the cross section are indicated. Required: area of the section.

Solution. Divide the area in two parts A_1 and A_2 as in Example 1 and apply the same method, noting that all offsets and elevations are considered positive.

Left: $\frac{0}{0}\diagup\frac{6.0}{0}\diagup\frac{7.2}{24.4}\diagup\frac{0}{10}\diagup\frac{0}{0}.$

$$A_1 = 0 - (146.4 + 72.0) = -218.4 \text{ sq ft.}$$

Right: $\frac{0}{0}\diagup\frac{6.0}{0}\diagup\frac{5.0}{20}\diagup\frac{0}{10}\diagup\frac{0}{0}.$

$$A_2 = 0 - (120 + 50) = -170.$$

Double area $= A_1 + A_2 = -388.4$ sq ft.

End area $= -\frac{388.4}{2} = -194.2$ sq ft.

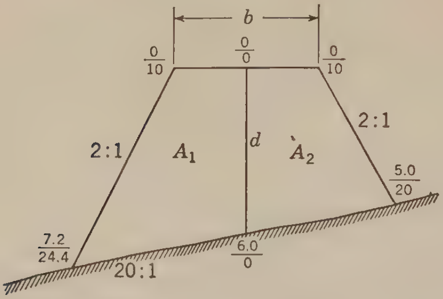


FIG. 126. Finding area of embankment section.

Example 4. *Sidehill Section.* The base width (Fig. 127) is 32 ft, and both *cut* and *fill* occur at the section. Required: areas in cut and fill treated separately.

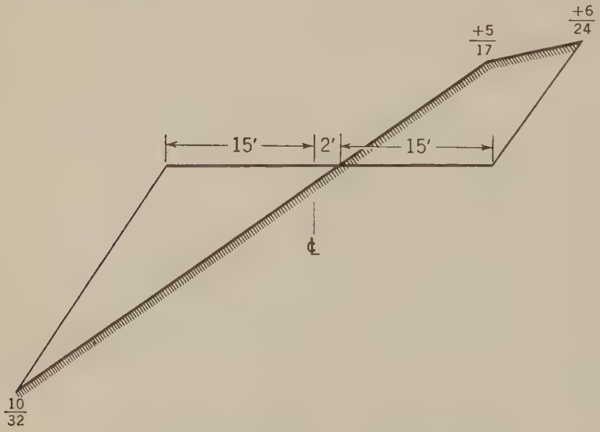


FIG. 127. Finding area of sidehill section.

Solution. For purposes of computation the vertical axis will be shifted 2 ft to the right of the center line, and the offsets (abscissas) will be adjusted accordingly. Otherwise the method is the same as in previous examples.

Fill:
$$\frac{0}{0} \diagup \frac{10}{34} \diagup \frac{0}{17} \diagup \frac{0}{0}.$$

Double area $= 0 - (170) = -170$ sq ft.

$$\text{Area (fill)} = -85 \text{ sq ft.}$$

Cut: $$\frac{0}{0}\Big/\frac{0}{15}\Big/\frac{6.0}{22}\Big/\frac{5.0}{15}\Big/\frac{0}{0}.$$

Double area $= 90 + 110 - (90) = 110$ sq ft.

$$\text{Area (cut)} = 55 \text{ sq ft.}$$

145. Formulas for Volume of Earthwork. The volume of earthwork may be found by means of either the *average end area* or the *prismoidal* formula. Although the former is less exact than the latter, it is generally accepted as the *standard earthwork formula,* on account of its simplicity.

146. Average End Area Formula. The volume of a right prism equals the *average area* multiplied by the *length.* Assuming the average area to be the same as the average *end* area, we have

$$\text{Volume} = V = \tfrac{1}{2}(A_1 + A_2)l \text{ (cu ft)} = \tfrac{1}{54}(A_1 + A_2)l \text{ (cu yd)}, \tag{200}$$

in which A_1 and A_2 = area of end sections in square feet and l = length of solid in feet.

This formula is applied to areas of any shape, but the results are slightly too large. The error is small if the sections do not change rapidly, as is the usual case in practice.

Table 37 gives volumes of 100-ft earthwork sections computed by the average end area formula. For any other length, the values are in proportion.

147. Prismoidal Formula. A prismoid is a solid whose ends are parallel and whose sides are plane or warped surfaces. Figure 128 represents a typical prismoid.

It may be shown by means of the calculus that the correct volume of a prismoid is

$$V = \frac{l}{6}(A_1 + 4A_m + A_2), \tag{201}$$

in which l is the distance between the two parallel bases A_1 and A_2 and A_m is a section midway between the two end bases and parallel to them. A_m is not an average of A_1 and A_2, but each of its linear dimensions is an average of the corresponding dimensions of A_1 and A_2.

148. Comparison of Average End Area and Prismoidal Formulas. Volumes computed by the average end area formula usually are slightly in excess of those by the prismoidal formula. The *difference* is called the *prismoidal correction.* If this correction is applied to volumes obtained by the more simple end area formula, then the results are as if the more exact prismoidal formula had been used.

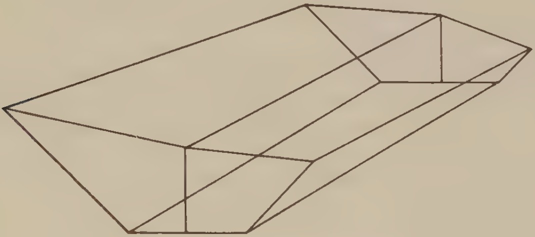


FIG. 128. Typical prismoid.

Let V_a = volume given by end area formula;
V_p = volume given by prismoidal formula;
$C_v = V_a - V_p$ = prismoidal correction;
d_1, d_2 = center cuts (or fills) at end sections A_1 and A_2;
X_1, X_2 = total distance between slope stakes at A_1 and A_2.

For three-level sections, it can be shown that

Prismoidal Correct

$$C_v \text{ (cu yd)} = \frac{l}{12 \times 27}(d_1 - d_2)(X_1 - X_2). \qquad (202)$$

For any form other than the typical three-level section, the foregoing expression is somewhat approximate.

Table 38 gives values of the prismoidal correction for lengths of 100 ft.

149. Borrow-pit Quantities. Excavations outside of the roadway are taken from so-called *borrow pits.* Data for the volume of material moved from borrow pits are determined by taking cross sections at definite intervals before and after the work is done. Two methods of procedure are as follows:

Method A

1. Establish a base line to one side of the proposed pit. Take cross sections at right angles thereto at intervals of 10, 20, or 25 ft extending well behind the probable limits of the pit.
2. After excavating, re-cross-section at the same points and intermediate ones if necessary.
3. Compute volumes excavated by the average end area formula.

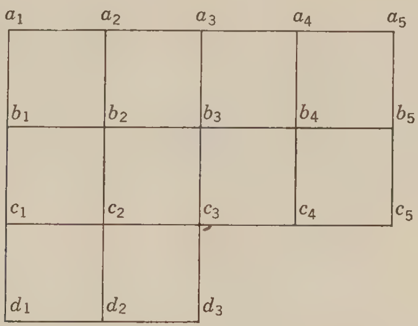


FIG. 129. Section of truncated prism.

Method B (Fig. 129)

1. Divide the original surface into squares, rectangles, or triangles, taking ground elevations at every corner.
2. Establish reference stakes that will not be disturbed during the construction.
3. After the borrow pit is abandoned, reproduce the same corners and take elevations on them again in the pit.
4. Compute volumes of the various vertical truncated prisms excavated, the altitudes being determined from the difference in levels at the corners, and bases from the dimensions of the squares or triangles (Art. 150).

Borrow pits may be avoided by widening the cut uniformly, but this will not be economical unless the material is easily obtainable and the haul is short.

150. Truncated Prisms. The volume of a rectangular prism truncated top and bottom is equal to $A\,\frac{h_1 + h_2 + h_3 + h_4}{4}$ cu ft, where A is the area of right section and h_1, h_2, h_3, h_1 are the corner heights, determined from the difference in levels.

The volume of a triangular prism truncated top and bottom is equal to $A\,\frac{h_1 + h_2 + h_3}{3}$ cu ft, where A is the area of right section and h_1, h_2, h_3 are the corner heights, determined from the difference in levels.

Instead of computing the volume of each prism separately in the system of equal squares shown in Fig. 129, it will save time to substitute in a combined formula.

Let Σh_1 = sum of corner heights used only once.
Σh_2 = sum of corner heights used twice.
Σh_3 = sum of corner heights used three times.
Σh_4 = sum of corner heights used four times.

Then the total volume is

$$V = \frac{A}{27}\,\frac{\Sigma h_1 + 2\Sigma h_2 + 3\Sigma h_3 + 4\Sigma h_4}{4} \text{ (cu yd).}$$

151. Correction for Road Curvature. The formulas for volumes of earthwork are based upon prismoids with parallel ends; when applied to curved roadways with end sections that are radial (not parallel), there is apt to be some error. The difference between the true volume and that determined by the prismoidal formula is called the *curvature correction.*

By the theorem of Pappus and Guldinus (see texts on mechanics):

If a plane area is made to revolve about an axis in the same plane, the volume of the solid generated by the revolution is equal to the product of the revolving area and the length of the path described by its center of gravity.

If the gravity curve is *outside* the center-line curve, the true volume is *more* than that given by the prismoidal formula; if *inside*, it is *less*.

Let R = radius of surveyed center line;

e = eccentricity of any cross section = distance from center line to a vertical through the center of gravity of the section (e_1 and e_2 applying to end areas A_1 and A_2, respectively);

C_e = volume correction for curvature.

Consider a curving mass of earthwork of center-line length = l ft. The true volume V equals the nominal volume as per straight alignment *plus* (or *minus*) the *correction for curvature*. That is,

$$V = \tfrac{1}{2}(A_1 + A_2)l \pm C_e. \tag{203}$$

If the end area A_1 and its eccentricity e_1 are *constant* throughout, the corresponding true volume V_1 is $A_1 l_1$, where l_1 equals the length of circular arc of radius = $(R + e_1)$.

From geometry we have the proportion $\frac{l_1}{l} = \frac{R + e_1}{R}$, whence

$$l_1 = \left(\frac{R + e_1}{R}\right) l.$$

Therefore,

$$V_1 = A_1 \left(\frac{R + e_1}{R}\right) l. \tag{204}$$

Similarly, we have

$$V_2 = A_2 \left(\frac{R + e_2}{R}\right) l, \tag{205}$$

as if the area A_2 at the other end of the mass, and the eccentricity e_2, were constant throughout.

The actual true volume is assumed to be the *average* of V_1 and V_2, that is,

$$\begin{aligned} V &= \tfrac{1}{2}(V_1 + V_2) \\ &= \frac{l}{2R}[A_1(R + e_1) + A_2(R + e_2)]. \end{aligned} \tag{206}$$

Equating Eqs. (203) and (206) and solving for C_e, we have

$$\begin{aligned} C_e &= \pm \frac{l}{2R}(A_1 e_1 + A_2 e_2) \text{ cu ft} \\ &= \pm \frac{l}{54R}(A_1 e_1 + A_2 e_2) \text{ cu yd}. \end{aligned} \tag{207}$$

Positive values of C_e occur when the excess area is on the *outside* of the center-line curve; *minus* values occur when on the *inside* of the center-line curve.

The centroid of a cross section may be found graphically. If the area is a triangle (Fig. 130), the centroid occurs at the intersection of the medians.

Correction for curvature is rarely considered except in special cases, such as sidehill construction or rock excavation.

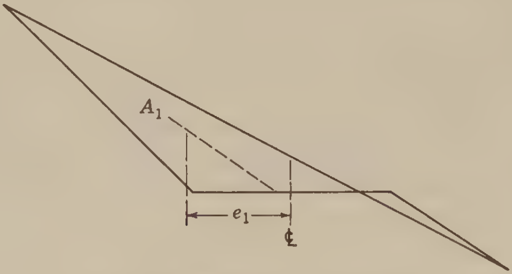


FIG. 130. Correction for road curvature.

Example. Given a curve with center-line radius = 573 ft (10° curve), $A_1 = 140$ sq ft, $e_1 = 8.2$ ft, $A_2 = 132$ sq ft, $e_2 = 7.1$ ft, and center-line length of earthwork solid = 50 ft. Assuming the gravity curve to be outside the center-line curve, it is required to find the correction for curvature.

Solution. Applying Eq. (207), we get

$$C_e = + \frac{50}{54(573)} [(140)(8.2) + (132)(7.1)] = +3.4 \text{ cu yd.}$$

152. Haul and Overhaul. In grading contracts for roads or railroads it is usually stipulated that the contractor shall be paid a certain price per cubic yard for excavating, hauling, and dumping the material, regardless of distance hauled, provided it does not exceed a specified limit called *free haul.* This free-haul distance may be 500 to 1,000 ft as per agreement. On large jobs with modern equipment, a free haul of 1,000 ft or more may be specified. Sometimes contracts make no allowance whatever for overhaul.

The word "haul" is here used with a double meaning; it may refer to the distance through which excavated material is moved, or, in a technical sense, it represents *yard stations*, that is, the volume of material mutiplied by the distance moved.

If there is an *overhaul* on some of the material, that is, if the distance from excavation to embankment is beyond the free-haul limit, then an extra charge may be allowed. The unit overhaul price is based on hauling 1 cu yd a distance of one station (100 ft). This price varies from about $1\frac{1}{2}$ to 5 cents. Thus if 30 cu yd is hauled an average distance of 1,500 ft when the free-haul limit is 500 ft, the overhaul would be $30 \times 10 = 300$ yd-sta.

A mass diagram (see Art. 154) is helpful in determining the amount of overhaul and the most economical distribution of the excavated materials. It may also indicate whether in some cases waste and borrow are economical.

153. Limit of Economical Haul. Where there are long hauls, it may be more economical to waste and borrow materials rather than pay for the cost of overhauling. Equating the cost of excavation plus overhaul to the cost of excavation from both the roadway and borrow pit, one can estimate the limit of economical haul for making the embankment. Thus, let

c = cost of roadway excavation per cubic yard;
b = cost of borrow per cubic yard;
h = cost of overhaul, on basis of 1 cu yd per station;
x = economical length of overhaul.

Cost to excavate and move 1 cu yd material from cut to fill

$$= c + hx. \qquad (a)$$

Cost to excavate from cut, waste, borrow, and place 1 cu yd material in fill

$$= b + c. \qquad (b)$$

Equating Eqs. (a) and (b) and solving for x, we have

$$x = \frac{b}{h}\,(\text{sta.}). \tag{208}$$

Adding the free-haul distance to Eq. (208), we get the *limit of economical haul.*

Example. The cost of roadway excavation [Eq. (208)] is 40 cents per cubic yard, the cost of borrow (b) is 45 cents, and the cost of overhaul

(h) is 4.5 cents per cubic yard per station. What is the economical length of overhaul?

Solution. Substituting in Eq. (208), we have

$$x = \frac{45}{4.5} = 10 \text{ sta.} = 1{,}000 \text{ ft.}$$

Hence, assuming a free-haul limit of 1,000 ft, the material may be hauled a distance of 2,000 ft for the same cost as to *waste* and *borrow* nearby.

154. Mass Diagram

1. *Definition.* A mass diagram is a continuous curve showing the accumulated algebraic sum of the yardage [cuts (+) and fills (−)] from some initial station to any succeeding station. It is drawn to the same horizontal scale as the profile.

Preliminary to drawing the mass curve, it is convenient to tabulate the cumulative yardage of cuts and fills at each station, in turn, as shown in Table A.

The points of the mass curve are plotted with reference to a horizontal scale of *distances* (same as the profile) and a vertical scale of *yards* (1 in. = 500 cu yd, for example).

TABLE A

Station	Cubic yards		Algebraic sum of cuts and fills
	Cut (+)	Fill plus shrinkage (−)	
12	. . .	. . .	0
	152		
+50	. . .	. . .	152
	132		
13	. . .	. . .	284
	210		
14	. . .	. . .	494
	. . .	89	
15	. . .	. . .	405
	. . .	133	
16	. . .	. . .	272

2. *Characteristic Properties.* A study of the mass diagram (or curve) shown in Fig. 131 will verify the following statements:

a. The ordinate at any point on the mass curve represents the cumulative yardage to that point on the profile.

b. Within the limits of a single cut, the curve rises from left to right; within the limits of a single fill, it falls from left to right.

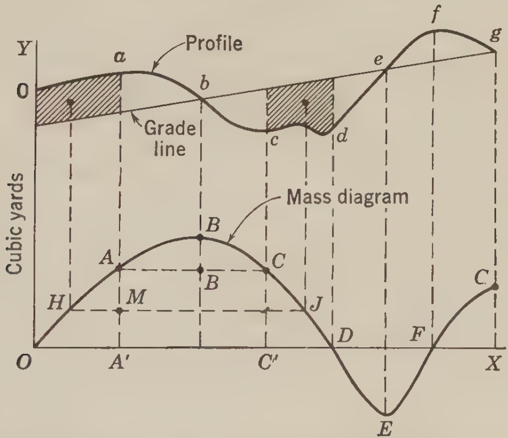


FIG. 131. Profile and mass diagram. AC = free-haul distance; $HJ - AC$ = overhaul distance; BB = free-haul yardage; $A'A = C'C$ = overhaul yardage; OD = length of balance.

c. Sections where the yardage changes from cut to fill correspond to a maximum; sections where the yardage changes from fill to cut correspond to a minimum. Evidently the maximum and minimum points on the mass diagram occur at, or near, grade points on the profile (see Fig. 131).

d. Any horizontal line, as AC, cutting off a loop of the mass curve, intersects the curve at two points between which the *cut is equal to the fill* (adjusted for shrinkage). Such a line is called a *balance line.*

e. The loops *convex* upward indicate that the haul from cut to fill is to be in one direction (to the right in this case); loops *concave* upward indicate a reverse direction of haul.

3. *Determination of Overhaul from the Mass Diagram.* One of the important uses of the mass diagram, aside from balancing cuts and fills and indicating the most advantageous distribution of the same, is to establish definitely the overhaul distance and the portion of the total yardage which is to be regarded as hauled beyond the specified free-haul limit.

Referring to Fig. 131, proceed as follows:

a. Assuming the free-haul distance to be 500 ft, find by trial a horizontal line intersecting the curve at points A and C, such that $AC = 500$ ft. Then the material above line AC will be hauled at no extra cost. The amount of this material is given by the ordinate from line AC to point B and is a measure of the volume in cut from a to b, which makes the fill from b to c.

b. Consider now the yardage above the balance line OD. A study of the mass curve and the corresponding profile shows that the cut from o to b will make the fill from b to d. But since part of this solidity, the part above the balance line AC, is included in the free-haul limit, the other part between lines OD and AC—which is measured by the ordinate $A'A$—is subject to *overhaul* unless wastage and borrow take place. That is, some or all of the yardage from o to a may be "overhauled" to make the fill from c to d.

The average length of haul of the solidity from o to a to make the fill from c to d is the distance between the centers of gravity of cut o to a and fill c to d. The gravity lines are found as follows: Bisect AA' at M and draw a horizontal line intersecting the mass curve at H and J. These points H and J are assumed to be vertically below the desired centers of gravity. Therefore the average haul is given by the length of line HJ, and the *overhaul* is this distance HJ *less* the *free-haul* distance AC. The overhaul distance (in stations) multiplied by the net yardage gives the *station-yards of overhaul.*

It should be noted that the foregoing graphical method of determining the center of gravity of the masses in cut and fill is inaccurate when there is abruptness in the mass curve. In such cases, a more accurate method is to divide the volume in parts and take moments about a vertical line of reference just as is done in finding the center of gravity of a system of forces.

4. *Other Advantages of the Mass Diagram.* The mass diagram may be used to indicate the most economical procedure for disposing of excavated material, what part of it should be moved forward or backward, and whether borrowing and wasting are advisable. Thus if the balance line OD is continued horizontally to point X, it will be seen that the cuts and fills from o to f are balanced, but the solidity represented by the ordinate at G is *excess cut* (from f to g) which may be carried forward, backward, or wasted. If the project ends at point g or if there are no fills immediately ahead, then this excavated material should be carried backward to help make the fill from b to c (it being downhill and within the free-haul limit), while an equivalent amount of yardage from the cut o to a would be wasted, thus reducing the station-yards of overhaul.

Regardless of whether an allowance for overhaul is included in the contract or whether the job is done on a lump-sum basis, it is usually advantageous to draw a mass diagram and indicate the balance points thereon. Such a diagram presents a graphical picture of a way to carry on the haulage of material that may be of service to a contractor or to anyone else concerned with the best and most economical procedure.

Sometimes situations arise where the ideal disposition of the material is not feasible or advisable, as at approaches to bridges or in towns where no embankments are allowed.

155. Earthwork Quantities by Electronic Computer. The earthwork program will handle both single- and dual-lane design through the use of a standard number of template points. The computer adjusts template points and cross-section data; it computes slopes and slope-stake data, areas of earthwork, volumes between adjacent cross sections including a shrinkage factor, and mass ordinates; it keeps accumulated totals of rock excavation and total excavation. All data are converted to absolute elevation by the computer for subsequent calculations.

The design engineer furnishes the typical section in X and Y coordinates, rate and direction of superelevation, any widening required, lump-sum earthwork volumes and the station at which they occur, and the station of the beginning and end of bridges on the project.

Input Data. The following data are required for each cross section: (1) project number; (2) station number; (3) equality station number (if any); (4) grade elevation (computed by separate program); (5) ground elevation (at survey center line); (6) rate and direction of superelevation; (7) widening, right and left; (8) lump-sum earthwork volumes; (9) cross-section points (in X and Y coordinates); (10) template points (in X and Y coordinates); and (11) rock-line points (in X and Y coordinates).

Type I data cards include items 1 through 8; Type II data cards include items 9 through 11 and the station of the cross section.

Data for each cross section are entered on two types of cards. The Type I card supplies station number, equality station (if any), grade elevation (which has been computed by a separate program), profile or ground elevation at the survey center line, rate and direction of superelevation, any widening required, and any lump-sum earthwork such as side roads and drives. All cross-section points, template points, and rock-line points are entered in the same X and Y coordinate system on Type II cards. A card code number indicates to the computer which type data are being entered. Only when the basic typical section changes from station to station is it necessary to enter new template points. The computer recognizes when all data for each station have been entered and proceeds with the calculations. The template points are modified for widening and/or superelevation and adjusted to grade elevation for this station; the cross-section and rock points are adjusted to ground elevation at the survey center line. All points are then adjusted to an assumed center line of 1,000 ft left of the survey center line so that all X distances will be positive.

Slopes and Slope Stakes. The difference in elevation between the left and right ditch points and the ground at these points is calculated and stored for use in calculating slopes and slope stakes. The computer recognizes a fill section and eliminates the ditch so that fill slopes are calculated from the shoulder point. Provision has been made so that slopes on either or both sides may conform to the practice of using the same slope through each separate cut or fill. If the slopes are not specified, the computer will use variable slopes as determined by the height of cut or fill at each cross section. From the ditch point or shoulder point, the computer applies appropriate cut or fill slopes and computes the intersection of that slope and the ground line. If the difference in elevation from the ditch or shoulder point to the intersection of slope and ground line exceeds the limit for the slope used, the computer will apply a steeper slope until the difference in elevation falls within the limits of the slope used. The elevation of this intersection is stored as the slope-stake elevation; its distance from the survey center line is stored as the slope-stake distance. Slopes used on both right and left are stored with the algebraic sign of the slope indicating a cut or a fill slope.

Areas. End areas of cut and fill are computed by the trapezoidal method and are accumulated to the nearest square foot. The computer starts at the left slope-stake position and breaks the area into trapezoids by means of the X distance between points and the difference in elevation between the ground line and the roadbed at these points. Cut and fill areas are separated by means of the algebraic sign. If the ground line and the roadbed cross between the slope stakes, the computer recognizes this condition and separates the cut area from the fill area, not explicitly finding the point of intersection, but by prorating as indicated by the difference in the distance from the ground line to the roadbed of the two points between which the intersection occurs. When the right slope stake is reached, cut and fill areas are stored and the computer tests for rock.

The top of the rock line, if any is present, has been entered into the computer in the same manner as cross-section points. The calculations of area proceed in the same manner as for earth cut and fill with the exception that areas where the roadbed is above the rock line are discarded. Areas where the roadbed is below the rock line are accumulated and stored.

Where rock cut is encountered, it may be the practice, as in North Carolina, to undercut the roadbed by 1 ft and to replace this undercut with suitable material. Accordingly, after the rock-cut area has been computed, the computer lowers the roadbed 1 ft from shoulder point to shoulder point and recomputes the rock-cut area. This area, which is the sum of the previous rock cut and the 1 ft undercut, is accumulated and stored.

Volumes. The volumes cut, fill, rock cut, and undercut are computed using the average end areas of adjacent cross sections and the distances between them. Answers are given to the nearest cubic yard. Where a grade point occurs between two adjacent cross sections, the areas are carried forward or back by a method of area proportion to obtain the volume up to or back to the grade point. Lump-sum earthwork volumes, such as drives and side roads, are added to the volumes of cut and fill for the stations where they occur. The computer recognizes the beginning and end of bridges and computes end bents required. These end-bent volumes are punched in cards separate from the cards for the cross section at which they occur for purposes of identification on the printed answer sheet.

Mass Ordinates. The mass ordinate is increased or decreased at each cross section by the amount of difference between cut and fill plus shrinkage factor. The algebraic sign of the mass ordinate indicates at each station "waste" or "borrow." At the beginning of each bridge on the project, the mass ordinate is reset to zero and restarted at the end of the bridge.

Rebalancing. Since the first earthwork run on any project will rarely balance the earthwork as desired, a provision for changing grades is required. Altered grades are computed by the grade profile program. At the cross section where the first change in grade elevation occurs, the previous areas, accumulated volumes, and mass ordinate are entered in the machine, the subsequent cross-section data cards are entered with a grade change card immediately following the Type I data card for each station which grade elevation changes, and the program proceeds with its computations as before.

Output. For each cross section, the computer punches out and another machine lists the following items: (1) project number; (2) station number; (3) equality station number (if any); (4) grade elevation; (5) total cut (CY) (includes earth and rock cut); (6) rock cut (CY) (includes undercut); (7) fill plus shrinkage (CY); (8) mass ordinate (CY); (9) accumulated total cut (CY); (10) accumulated rock cut (CY); (11) left slope; (12) left slope-stake elevation; (13) left slope-stake distance; (14) right slope; (15) right slope-stake elevation; (16) right slope-stake distance.

The following are punched for each station for rebalancing purposes but are not listed: (17) total cut area (sq ft); (18) fill area (sq ft); (19) rock-cut area (sq ft); (20) rock-cut and undercut area (sq ft).

Answers are punched in two types of cards. Type I includes items 1 through 10 in the preceding list; Type II includes items 11 through 20 and the station of the cross section.

Addendum; Earthwork by Applied Photogrammetry

The first requirement leading to the success of photogrammetric surveys is good, accurate photography. The photographs must be made when the tree foliage is at a minimum, making it possible to see the ground under the trees. This is best accomplished in February and March and when the sun is at its highest point. The aerial camera must be built to strict specifications and operated in a highly efficient manner.

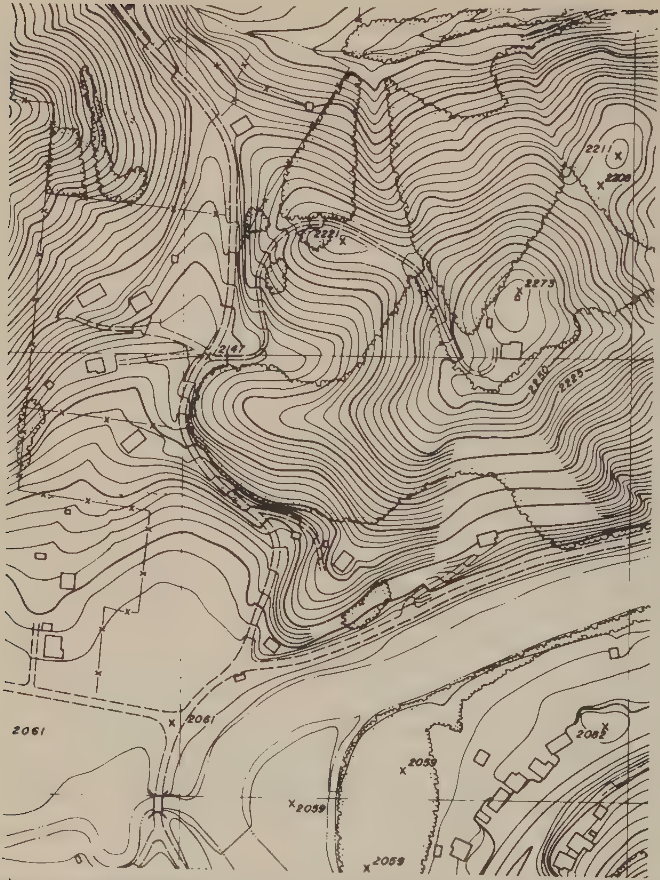
An aerial camera of foreign make, although enormously expensive, is considered the most satisfactory. It records all topographic information in minute detail (over approximately three square miles on each photograph), thus enabling the engineer to make his detailed plans for construction of highways, residential developments, reservoirs, or any other construction requiring a thorough knowledge of the topography.

The airplane serves as transportation for the camera and its operator. The accuracy with which the plane moves the camera from one exposure station to another is important. It is said that once the aerial photographic plane is flown and accepted for use, the final cost of the mapping operation is set. If it takes two or more flights to photograph an area that one flight on the predetermined line would have covered, the cost of mapping that segment automatically goes up proportionally to the number of photographs used. The fact that a man is a good aircraft pilot by no means qualifies him to fly an aerial photographic plane. The exact path and the altitude of the plane are determined before the plane leaves the ground. The pilot must be able to read maps, position the plane over that predetermined path within a few feet, and closely estimate a plumb line over a mile or more of height.

Next, the film must be developed and processed in a modern, well-equipped photographic laboratory. The lab technician must have a good knowledge of the lighting condition at the time the film was exposed. He must know the flight height of the plane and the relative contrast of the terrain photographed. Even the amount of relief can affect the exposure and development of the film. Strict control of the temperature of the water and chemicals in the lab is of utmost importance. All stresses or strains on the film must be avoided. Heat will render the film worthless. Contrary to popular thinking, measurements are not taken from paper prints of the aerial photograph; the lab technician must transfer the photographic image to precision plate glass to maintain stability before the photogrammetrist can start his operation.

Many of the highway departments require aerial photographs to be enlarged to 40 in. by 45 in. Other state agencies need photographs suitable for publication by lithography. These are but two of the many and varied types of reproduction the laboratory is called on to furnish.

Survey parties must be sent to the field to make horizontal and vertical surveys. These surveys serve to determine the scale of the photograph and to establish its vertical datum, enabling the pho-


2211
2208
2221
2273
2147
2250
2225
2061
2061
2082
2059
2059
2059

Section of topographic map used to measure distances and elevations of any points.

togrammetrist to orient the photograph and to take accurate measurements from it.

The survey parties are equipped with the latest and most accurate survey instruments consistent with the needs. Theodolites for measuring angles are used rather than the conventional engineer's transit. Horizontal distances are measured electronically by a tellurometer. This instrument works on the principle of radar and records the time it takes a signal to travel to a distant point and return to its starting point. There must be a clear line between the two points, but the distances, whether 100 ft or 50 miles, are measured with the same speed and accuracy. The time required for taking the measurement is about 15 min. All vertical surveys are carried out with the same high quality and exactness as the horizontal surveys.

All maps or topographic data must be checked in the field. The photograph is taken to the field to check and annotate every feature that will appear on the finished map. Each road, culvert, and bridge is classified by width and surface. Houses, churches, schools, and structures are classified by type of construction, number of stories, etc. Property lines are annotated, and all other information needed on the map is recorded. It should be noted that all the data are gathered before the map is drafted.

The making of the map by actually measuring and plotting the detail starts with the stereoplotter. This is an optical-mechanical instrument designed to duplicate the measurements to a very high degree of accuracy. As mentioned before, the aerial photograph is reproduced on a glass plate. The stereoplotter projectors are designed to the exact dimensions of the aerial camera. When the glass plates are inserted into the stereoplotter projectors, they take the position of the negative in the aerial camera. By the proper manipulation of the projectors, a miniature, third dimensional model can be formed; when it is properly oriented to the field surveys, it is correct in every detail. No measurements are attempted until these conditions have been satisfied.

The operation of the stereoplotter and the compilation of the map information are very tedious. With great restrictions on the vertical measurements for accuracy, contours are usually drawn at a vertical interval of one-fiftieth of an inch, and the contour must be accurate within one-half that amount. A man who has had several years experience on field surveys has a better chance of succeeding with a stereo operation than one with no engineering experience.

Aerial photography and the stereoplotter are by no means limited to drawing maps. Whenever two- or three-dimensional measurements are needed, photogrammetry is able to produce the desired results. Many states and counties are calling on photogrammetry for tax maps and in many cases finding enough land not appearing on tax records to pay for the cost of the mapping. The various highway departments are resorting more and more to photogrammetry for cross sections and detail plans to be used in contract negotiations.

There are many fields of highway engineering where photogrammetry could save considerable time and money.

Reference

Accepted Policy, Division of Engineering Design, North Carolina State Highway Commission.

Route 64-98 near Mt. Rushmore, S. D.

CHAPTER 11

ELECTRONIC EQUIPMENT AND APPLICATIONS

156. Foreword. The high-speed electronic digital computer has brought about a revolution in engineering analysis and design—a revolution that is just gathering momentum and one that will have far-reaching consequences in engineering education.

But no design which comes from machine calculations should be used without careful scrutiny by an experienced designer as to reasonableness. The engineer will never be replaced by the computer. Quite the reverse; the computer, by freeing him from routine labor, now challenges the engineer to grow into the scientific maturity our fast-moving technological society demands.

157. Descriptions. Digital computers work with digits of numbers.

Binary machines are simpler to build than decimal machines, and except for problems involving large amounts of input and output, are preferable because of their less complex structure. Certainly, in scientific and engineering calculations there is no good reason for requiring a machine with an internal decimal system. The unfamiliar binary system is no problem for the user because the machine can easily convert to and from its customer's decimal notation.

Different kinds of machines often embody different instruction codes. As a result, programs (the total list of instructions for a particular job) written in machine-level language cannot usually be run on other kinds of machines without being rewritten.

To overcome this handicap, algebraic language compilers have been written. The compilers accept a standardized mathematical notation and produce a machine-level code. The most common algebraic languages for scientific work are FORTRAN (from FORmula TRANslator) and ALGOL (from ALGOrithmic Language). For both of these languages, compilers are available that will produce code for most scientific computers.

COBOL (Common Business Oriented Language) is very much more like the English language and is used for many accounting jobs where reports are required with only a small amount of computations. The computer language contains terms like "add," "subtract," "multiply," "print," "read," "transfer," "branch," and many more.

But all these sophisticated compilers use more storage and take longer to run the problem data. For many scientific jobs the running time is not important, but in engineering or accounting work where a large volume of data has to be run every day, it is far more efficient to write the programs in the individual computer language by means of a symbolic compiler. This type of compiler translates an easily understood symbolic instruction into machine language instruction. It takes much longer to program, but is far more versatile and efficient in the hands of an expert programmer.

Most data have to be prepared manually in order to be read into the computer. The most common form of input medium is the punched card, which is keypunched and verified much like a typewriter. The cards are convenient because they can easily be corrected, changed, omitted, or inserted.

Data may be entered on paper tape, by an optical scanner, directly from some digital source via transmission lines, or from a console or remote-station typewriter.

Very often data may need to be stored for reference at a later date or for later references in the same program. Magnetic tape, disks, drums, and strips are all used for intermediate storage of large amounts of data, and each type has its advantages and disadvantages.

Although the components used in computers are becoming smaller and smaller, installations are becoming bigger because of the extra capacity and peripheral devices now demanded by the users.

Users of one particular type of computer often get together with others in the same field and form a users' group to swap ideas and programs.

All the computer manufacturers make available to the users subroutines for the solution of common problems, such as trigonometric functions, solutions of simultaneous equations, and many others. These can be incorporated into programs when required, thus reducing considerably the time spent in preparing new programs. In addition, the users of particular computers have formed exchange groups to share computer programs and reduce duplication of programming effort.

The digital computer, as the name implies, is a device for the making of calculations in digital form. It can do nothing more than an ordinary desk calculator in so far as computation is concerned. However, its computations are performed at a fantastic rate of speed. This speed, together with its capacity to store a multitude of intermediate results for use in later calculations and its ability to alter its course according to the intermediate results obtained, is what makes the digital computer so valuable for engineering calculations. The digital computer has, therefore, four important characteristics: (1) it can perform involved computations at a very high rate of speed; (2) it can be programmed to follow a very long and intricate series of computations; (3) it can store results of each separate computation for use in a later step; (4) it can make a comparison of intermediate results with predetermined quantities or other predetermined results and alter its course of action or program accordingly.

158. Universal Program Form. In the universal form it is of value to anyone who has a computer because he can code it for the computer he is using.

To be of value, a program must be such that it will solve the problem in any of its forms. It is not economical to write a program that covers only one set of circumstances and then to find, when you come to use it, that your data cover a different set of circumstances. The programs should also include all the problem decisions that are based on fact. In addition, any series of programs on the same problem should be set up in such a way that the output from one program can be used as input for the next program without the necessity of repunching cards or tape.

159. Survey Computations. In the field of survey computations, the first phase of highway construction, programs are available for computing unknown lengths and bearings, determining error of closure, adjusting traverses, computing coordinates, converting coordinates from one system to another, and computing areas of closed traverses. These operations are particularly important to the highway engineer just now, when he is so heavily involved in land acquisition and preliminary location work.

Integration of the electronic computer into the highway location and design process is most interesting. The first phase of this process is of course route reconnaissance, which in most cases can be accomplished by the inspection of existing maps or photography. If not, additional small-scale photography will usually supply the information needed.

It is in the second phase, the preliminary location phase, that electronic computation enters the picture. The highway engineer usually obtains a contour map at 200 or 250 ft to the inch, covering a width of a little over a mile on the selected route or routes. With this information, he visually selects what appears to be the most suitable line and grade and obtains cross-section data on that line. He then uses the computer to obtain the elevation of the highway center line at each point for which he has a cross section of the actual ground. The computer input for this operation consists of (1) the starting elevation, (2) the stations at grade changes, (3) the plus or minus grades, and (4) the vertical-curve data. The computer then determines the elevation of the center-line grade at each point where a ground cross section is available.

With the center-line elevation of the trial line established at each cross-section location, the design earthwork program is used to obtain earthwork quantities. The input for this calculation consists of the ground cross sections, the computed elevation of the center-line grade, the template of the road to be constructed, and the superelevation data. The output consists of the cut and/or fill at each cross section, the cut or fill adjusted for shrinkage or swell at each cross section, and the cumulative cut or fill balance at each station. With this information the engineer decides what changes are necessary in his trial line and grade, and the operation is repeated using the new grade data and the offsets of the new trial line from the line on which the cross sections were taken. See Art. 155 for full treatment of earthwork computation.

Methods and procedures have been developed by which cross-section data can be electronically punched onto cards or tape directly from photogrammetric instruments. This eliminates the necessity of reading the information out in visual English and then converting it into computer input by manually punching it on cards or tape.

Also, a program is being developed that will include not only the earthwork quantities and the cost of moving the earth but also such items as the cost of the base and subbase required, the cost of the structures involved, the cost of vehicle operation over the completed facility, and the like. With such information available at this stage, the engineer will be able to make a much better determination of the economics of the trial line locations and grades.

With this information the process used for the preliminary location is repeated but with the advantage of far greater accuracy. At this stage the real advantage of the electronic computer becomes evident. The engineer can program the computer to vary the width of the roadway depending on the height of fill encountered, to vary the slope of the cut or fill depending on its depth or height, and in fact to vary any dimension of the road design template that is dependent on exact measurements rather than on engineering judgment.

After the exact location has been established by several trials, the machine design is inspected by the engineer. Any parts of it that are not consistent with good practice, such as undue variations in cut or fill slopes or improper ditch grades, are corrected. With these changes the program is again run through. The output, consisting of the cut or fill at each section, the adjusted cut or fill at each section, the cumulative total adjusted cut or fill, and the slope-stake data, is printed out. These data are then photographically reproduced and become a part of the contract documents.

You will note that in the description of this process no mention was made of cross-section drawings. There was no need for them as far as the calculations were concerned. However some engineers and contractors feel that they need at least a few cross-section drawings for purposes of information. This problem has been solved quite successfully by the development of line plotters by at least two manufacturers. These devices will take the ground cross-section information used in the calculations and the final cross-section data developed in the computer and electronically translate them into complete cross-section drawings.

Another development under way in connection with the location and design process is the use of a digital model of the terrain rather than the center-line cross sections for earthwork determinations. The digital model is in essence a cross section except that it is based on a coordinate base line rather than on a proposed center line. The proposed center line can be at any angle to the coordinate base line. The computer program is written to compute earthwork quantities from coordinate cross sections that are not at right angles to the proposed center lines. This allows the engineer greater freedom in moving his proposed center line since it is unnecessary to prepare additional cross sections on the new center line.

Progress is being made in the development of an automatic or semiautomatic stereoplotter. It is understood that one of these is to scan the entire model and draw the contours and another is to adjust the instrument automatically as to elevation for the various locations chosen by the operator. Other instruments are available which will permit the direct printing or recording of the X, Y, Z coordinates on cards or tape.

160. Structural Design. In all fields of bridge geometrics, many programs have been prepared. Programs are available for determining all the geometrics of multispan skewed bridges on superelevated curved alignments and on vertical curves, with bents either parallel or not parallel. The computer produces chord distances between the intersections of the concentric areas with the bent-cap center lines, distances between the concentric arcs measured along the center line of each bent cap, center-line grade elevations, the slope of each beam, and the middle ordinates of the concentric arcs at the mid-point of each span.

Other programs have been worked out for computing influence lines, moments and shears for continuous beams; beam deflections; analysis of reinforced-concrete columns subject to axial load and either one-way or two-way bending; design of cantilever retaining walls; design of composite beams; determination of design constants for beams of variable moments of inertia; design of reinforced-concrete framed bents; analysis of a tied arch; design of reinforced-concrete continuous slab-and-girder bridges; and computation of the torsional resistance of suspension-bridge towers. In many cases these programs include the tabulation of material quantities.

161. Other Applications. For a number of years, punched-card processing and tabulating equipment has been used in the analysis of the large masses of data involved in highway planning studies. The electronic computer provides a means of further facilitating and greatly expediting the handling of these data and of increasing the quality of route planning through more thorough analysis than was feasible with conventional punched-card equipment. In a number of states programs are being developed to perform such studies on electronic computers.

The use of electronic computers in the solution of hydrology and hydraulic problems involved in highway engineering is being explored. One type of problem being investigated involves the multiple correlation of hydrologic data in estimating peak rates of runoff from a watershed for selected rainfall frequencies. A second type of problem adaptable to solution on electronic computers involves the determination of the economics of different sizes and shapes of bridge openings and clearances. Urban runoff and urban storm-sewer-design problems are particularly adaptable.

Numerous programs have been developed for extending, analyzing, and tabulating bid data at contract lettings. Several states have developed other programs for computing acreage of clearing, grubbing, and sodding and for computing borrow-pit excavation.

This new tool is so fast that it is now possible, in much engineering work, to use the original theory and formulas instead of the approximations now generally used to save time. Thus the electronic computer should improve the quality, as well as the speed, of highway engineering work. Use of the computer will:

1. Speed up and improve productivity.
2. Reduce the cost of engineering work.
3. Reduce the cost of construction by improving the quality of engineering work.

4. Reduce the cost of highway transportation by enabling the engineer to design a better and safer highway.

5. Determine the most efficient location of new roads by complex traffic analysis.

6. Raise the dignity of the highway engineering profession by relieving the highway engineer of much of the tedious and time-consuming computation work load.

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Covered bridge across the lower Ammonoosuc River, Bath, N.H.

CHAPTER 12

DRAINAGE SURVEYS AND FACILITIES

162. Scope. Drainage may be the controlling factor in the location of highways, railroads, or airports; it is a never-ending problem in the maintenance of these systems of transportation.

In all cases the purpose of drainage is the same: to remove surface water (from rain or snow) and to intercept and dispose of any underground water that might lower the stability of the subgrade. For highways and railroads, provision must be made for surface erosion and possible slides in the cuts or fills due to underground saturation; also numerous culverts or bridges of adequate size must be constructed to take care of cross drainage.

Generally speaking, the minor damage, or inconvenience, resulting from extreme flood conditions occurring once over a period of years is usually not sufficient to warrant the increased cost over a design based on ordinary floods. Exceptions to this practice are runways, highways, and railroads carrying heavy traffic flows in metropolitan areas and having high strategic importance in the transportation system.

As a means for diverting water falling on highways and runways, the surfaces are crowned so that there is a slope from the center to the outside shoulders. For highways and railroads, the water flows over the shoulders and spills uniformly over the embankment or into roadside ditches in cuts. Some states provide ditches on embankments and carry the water down the fill slopes in pipes or paved gutters for the protection of the embankments. Sometimes the side ditches must be paved to prevent excessive erosion; in other situations where the grade is too small for longitudinal drainage, the water in side ditches must be absorbed by underdrains. In built-up or urban districts requiring sidewalks, curb and gutter construction is often used.

To avoid private property damage, the utmost care should be exercised in diverting storm water from a highway or railroad along its natural channel.

Drainage of airports is more difficult than drainage of highways and railroads, largely because of the broad area of the airport with its wide runways, flat surfaces, and absence of side ditches. Where the airport is on porous soils, such as coarse sand or open gravel, it will drain naturally, except that storm drains will always be needed along runways and other paved areas. On other soils, extensive subdrainage and surface drainage may be necessary.

163. Structure Surveys. (Reference: *Manual of Practice, North Carolina State Highway Commission.*)

River and Stream Crossings. Consult the files for prior surveys of the same stream in the vicinity of the job. Note the date of the survey, the length and height of the existing structures, the drainage area, the elevation or height of high water above bed, and any other helpful information. USGS publication "Floods in North Carolina, Magnitude and Frequency" gives the location and flood data for all stream-gauging stations.

Obtain the best high-water information available from long-time local residents if possible. If flood marks are visible on an existing fill, take levels on the marks on both sides of the fill every 200 ft or so, starting at the structure and extending throughout the flood plain. If in the vicinity of a large stream which might cause backwater at the survey site, obtain the elevation of such backwater. Differentiate between high water from the stream that is being surveyed and backwater from another stream.

A center-line profile covering the entire flood plain should be obtained. When taking topographic data, locate all the natural features which may affect the layout. Make a stadia survey to show these conditions if considered advisable. On culvert surveys, show the channel width at intervals when running the bed profile.

Take levels from a local bench mark throughout the limits of the structure, and far enough on each side so that approach grades can be established. On culvert surveys, run a bed and natural-ground profile far enough to establish the stream-bed gradient. When drift is visible, take levels on it far enough upstream and downstream to establish the high-water gradient. Obtain elevations of existing structures in the vicinity. The normal water-surface elevation should be determined. On coastal areas, in waters influenced by wind tides only, the normal elevation can

14th St. Bridge, N.W., Washington, D.C.—an aerial view.

Open spandrell bridge on U.S. 52 west of Decorah, Iowa.

usually be determined from marks on structures, piles, etc. On waters influenced by ocean tides, a gauge should be established and observations made and recorded throughout the duration of the job. The elevations required are mean low water, mean high water, and maximum storm tide. If a tentative structure layout has been made, take rod soundings at each bent, including soundings left and right of the center line if necessary. An excellent check on a stream is frequently provided by existing structures in the vicinity when these structures and their approaches are above high water.

On surveys for structures over navigable waters, the mean or average low tide should be determined, and care should be taken to see that these levels check with the datum of the USGS (average tide), or the Coast and Geodetic Survey (mean low tide). The minimum allowable vertical clearances beneath bridges over navigable streams must be fixed by the Corps of Engineers, Department of the Army.

On surveys for widening existing structures, the following information is required:

Bridges. On bridges in which the superstructure can be widened, complete measurements are required except for the handrails. On other bridges only the substructure need be measured. If old plans are available, have them reproduced and check off the dimensions on them; otherwise make a sketch. The skew should be accurately determined. Measurements left and right of the survey center line should be obtained when it does not coincide with the center line of the bridge. If creosoted piles are involved, samples should be taken with the Swedish increment borer and the holes plugged with asphalt, if visual inspection indicates doubt as to soundness.

Levels should be taken at each end and bent of the bridge, showing center line, gutter, top of curb, top of wing, tip of wing, bridge seat, and top of footing. On skewed bridges, follow the skew of the bents. In addition, two or three sections should be taken at right angles to show the crown. The notes should state clearly which sections are skewed and which are normal. The thickness of any wearing surface should be measured. Cross sections should be taken at abutments, wing tips, end bents, and at other locations as required.

The condition of all structures which are to be retained or widened should be stated. Look for cracks, spall, undermined footings, unsound timber piles, or other structural defects, and note any found. Photos of these or any unusual conditions should be taken.

Widening Existing Culverts. The size, center-line station, and skew of all culverts are required. The center-line station is obtained by standing on the axis of the culvert prolonged upstream or downstream, and intersecting the survey center line by eye. The ends of the culvert are located in the usual manner. When plotted on the culvert survey sheet, the skew may be scaled with a protractor. As a further check, the skew should be checked by measuring the clear span normal to the axis of the culvert and dividing this quantity by the clear span along the skew. The quotient is the cosine of the skew angle. In some cases, a culvert is designed on an even skew and is then built in an existing stream channel which may have a different skew. If the fill over the culvert is not large, it will be seen that the headwall is not parallel to the roadway under these conditions.

The following measurements are required: overall length of barrel, excluding the headwall overhang; distance on each side from center line to end of barrel, normal to the center line; and thickness of top slab and interior walls when old plans are not available.

A profile along the axis of the culvert is also required, extending far enough upstream and downstream to cover any proposed channel changes or excavation. Use the transit level for this work when practicable. Elevations should be obtained on center line, each edge of pavement, shoulder, top of headwall, and floor slab, as well as stream bed and natural ground. At this time stream topography can be taken. Record stream channel widths at each level shot. Decide at this time, if possible, whether the culvert can be extended along existing alignment without excessive channel changes or must have a change of direction in order to lie more clearly within the existing channel.

Carefully inspect each culvert for structural and hydraulic conditions. Structural defects should be noted and photographed if possible. If evidence of insufficient capacity is noted, determine drainage area and highwater elevations upstream and downstream. Inspect other structures in the vicinity.

The *culvert survey sheet* (see Fig. 132) should be completed while on the site. Page 1 of this four-page folder is shown on the opposite page; page 2 is to show a profile of stream bed; page 3 is to show the location sketch; page 4 is to give elevations of points on the plan.

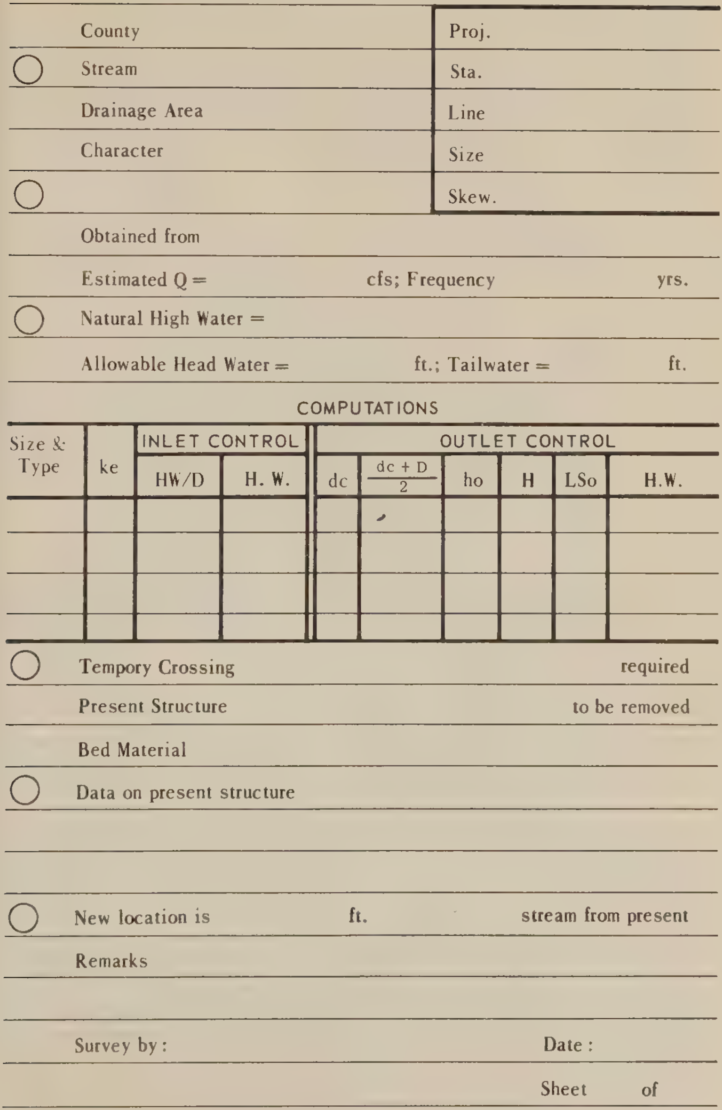
The symbols on page 1 of this culvert survey sheet (as defined by the North Carolina State Highway Commission) are as follows:

ke = entrance-loss coefficient;
HW = vertical distance, ft from culvert invert (flow line) at entrance to pool surface upstream;
TW = tailwater depth;
H = head loss, ft;
D = culvert height, ft;
dc = critical depth, ft, where water flows with least energy;
$ho = (dc + D)/2$ or TW, whichever is greater;
So = slope of barrel, ft/ft;
LSo = total fall in pipe from inlet to outlet.

Grade Separation Surveys

The reference point for all grade separation surveys is the intersection of the two located survey lines. In the case of a railroad grade separation, it is the intersection of the located survey line (the base line from which topography is taken) and the center line of the main track. In the case of a curved railroad, all track angles are measured at the reference point and all measured skew angles are doubled. Track topography is referenced to the base line. For parallel tracks, show center-to-center distances normal to the tracks and the center-line stations of the tracks. Where crossovers and spurs are involved, locate completely, showing points of switch and frog. Measure and record the distance to a milepost and the direction of progression. Obtain the width of railroad rights of way.

On curves, the stations of the offset lines are on radial lines from the center line of the survey. Take levels and rod soundings or earth-auger borings on the offset lines. A tentative structure layout computed on the site will be helpful. Existing pavement and track elevations are taken left and right from the reference point if no Y line is run in for the crossing. Cross sections in the area of the end bents should be taken from the center line of



	County	Proj.
○	Stream	Sta.
	Drainage Area	Line
	Character	Size
○		Skew.

Obtained from

Estimated Q = cfs; Frequency yrs.

○ Natural High Water =

Allowable Head Water = ft.; Tailwater = ft.

COMPUTATIONS

Size & Type	ke	INLET CONTROL		OUTLET CONTROL					
		HW/D	H. W.	dc	$\frac{dc + D}{2}$	ho	H	LSo	H.W.

○ Tempory Crossing required

Present Structure to be removed

Bed Material

○ Data on present structure

○ New location is ft. stream from present

Remarks

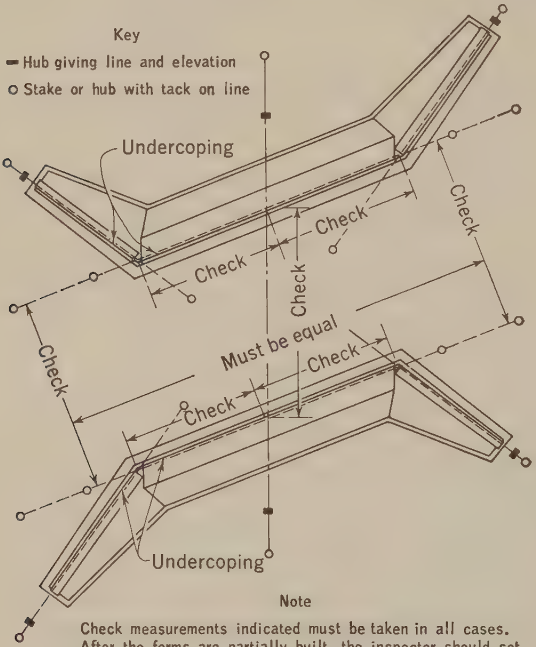
Survey by: Date:

Sheet of

FIG. 132. Culvert survey sheet.

survey as well as cross sections normal to the crossroad or railroad throughout the construction limits.

In the case of railroad overheads, it is important that no additional drainage be allowed to enter on existing railroad cut. Any drainage from the proposed road which will have to be discharged


Key
▬ Hub giving line and elevation
○ Stake or hub with tack on line
Undercoping
Check
Check
Check
Check
Check
Must be equal
Check
Check
Check
Undercoping

Note

Check measurements indicated must be taken in all cases. After the forms are partially built, the inspector should set elevations for the bridge seat. In case of no coping on bridge seat, the undercoping line becomes line at top of bridge seat.

FIG. 133. Method of staking a bridge.

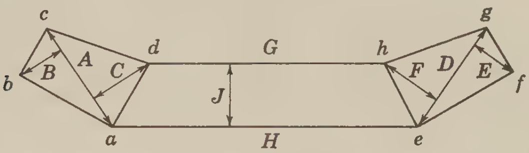
toward the railroad at an overhead crossing should be conducted to natural outlet outside the railroad section. For this reason, complete information on the existing railroad drainage in the vicinity should be obtained.

Hydrology

The science of hydrology deals with the behavior of water in the atmosphere, on the ground and underground. Hydrology draws on other natural sciences, most important of which are meteorology, climatology, hydraulics, soil science, agronomy, and forestry. Hydrologic principles and relationship provide the basis for determining certain factors essential to the planning and design of highway drainage structures and channels.

Hydraulics

After the discharge of the project-design frequency has been determined, it is necessary to determine by hydraulic principles



Horizontal measurements $= A \cdot B \cdot C \cdot D \cdot E \cdot F \cdot G \cdot H \cdot J$
Vertical measurements $= a \cdot b \cdot c \cdot d \cdot e \cdot f \cdot g \cdot h$

Formula:

$$\left[\left(\frac{A\times B}{2}+\frac{A\times C}{2}\right)\times\left(\frac{b+c+a+d}{4}\right)\right]+\left[\left(J\times\frac{G+H}{2}\right)\times\left(\frac{a+e+d+h}{4}\right)\right]+\left[\left(\frac{D\times E}{2}+\frac{D\times F}{2}\right)\times\left(\frac{f+g+e+h}{4}\right)\right]$$

FIG. 134. Formula for abutment excavation. This formula is approximate and holds only when the vertical measurements are about the same order of magnitude.

the structure required to adequately pass the discharge within controls established for the site. These controls will consist of allowable headwater, velocities, tailwater, and other hydraulic considerations, as well as structural requirements.

Structures normally requiring hydraulic design in highway work may be divided into three groups: (1) open channels, including roadside ditches and channels, median ditches and swales, gutters, chutes, berm ditches and channel changes; (2) culverts, including pipes, pipe arches, arches, concrete and corrugated metal, and reinforced concrete boxes, both single and multiple barrel; (3)

bridges, ranging from single spans over small streams to multiple structures across major flood plains.

The *Manning formula* is used for the design of open channels. This formula relates flow to three variable parameters: channel roughness, hydraulic radius, and slope, in the form

$$Q = \frac{1.486}{n} \sqrt[3]{R^2} \times \sqrt{S} \times A, \tag{209}$$

where Q is the discharge (cfs), n is the roughness coefficient, R is the hydraulic radius (wetted perimeter divided by the cross-sectional area), S is the average slope of the channel through the section under consideration in feet per foot, and A is the area of flow. These parameters are all subject to mathematical calculation except the roughness coefficient (n). The Bureau of Public Roads and other agencies have made many studies of the n values to be used as a guide.

Example. Given $n = 0.02$, $R = 2$, $S = 0.005$, $A = 6$ sq ft, to find Q.

Solution. $Q = \frac{1.486}{0.02} \sqrt[3]{4} \sqrt{0.005} \times 6 = 74.3(1.587)(0.07071) \times 6 = 50$ cfs.

Bridges

The method of hydraulic design for bridges is to provide for the waterway below the design high-water elevation to pass the design discharge with allowable velocities and backwater. To do this, the profile of the proposed center line, sufficient in length to show the entire flood plain, is plotted to suitable horizontal and vertical scale. The high-water elevation is plotted. This is usually obtained from the field survey giving elevations of marks or points of maximum high water pointed out by local residents or others familiar with the site. If reliable site information is lacking, an estimate is made by transferring high-water information from other sites, such as gauge or other structure sites, or by applying the Mannig formula.

Addendum: The most important dimension in a steel bridge substructure is the distance back-to-back of the curtain or back walls, as the superstructure has a definite and exact length which cannot be changed after the material is fabricated, to correct for errors in the substructure. Skew angles, where required, need to be set with extreme care, as it becomes almost impossible to erect steel on abutments improperly skewed.

Surveys for Long-span Bridges—An Adaptation of Virginia Practice (subject to possible changes with the advent of electronic devices). In the case of a long bridge crossing a wide channel, the base line should be extended by the triangulation method. Great accuracy is necessary in order that the results may be used when the construction survey or layout is made. It is desirable to have two base lines, one on each side of the stream, and each of a length about equal to the length of the proposed structure. This will produce sharp intersections with the base line of location, which are highly desirable, ensuring correct layout of the piers and avoidind expensive adjustments in the superstructure to fit the misplaced masonry.

The base line should be measured several times with a standard steel tape of known accuracy, which should be held level at all times by the use of plumb bobs at each end. It is desirable to use a spring balance to eliminate the effect of sag in the tape, and the readings should be corrected for variations in length caused by temperatures differing from the standard. Failure to observe these precautions may cause errors of several inches in a length of 1,000 ft. The details of making these measurements are given in almost all standard works on surveying.

The use of a 1-sec optical-reading theodolite is recommended. Two sets of directions should be observed, and the error of closure of any triangle should not exceed 3 sec.

The triangulation points should be carefully marked by heavy hubs driven 3 ft into the ground, or more if there is danger of heaving by frost. These points must be carefully referenced so that they can be easily located by the construction party.

Since 1955 two electronic measuring devices have been perfected which will measure long distances (from $\frac{1}{2}$ to 20 miles) precisely and rapidly. The geodimeter employs a visible light beam and the tellurometer employs a short-wave radio frequency for the measuring.

The tellurometer, which is still the only electronic instrument utilizing the microwave principle, has become standard equipment for major Federal mapping agencies.

164. Runoff Estimates. (*California Policy*)

1. *General.* The runoff estimate, or design discharge, depends upon many variables. Some of the more important are: duration and intensity of rainfall; storm frequency; the size, imperviousness, slope, and shape of the drainage area; and probable changes of land use. Judgment and care are essential in making runoff estimates. Such estimates may vary considerably because present methods of estimating are inexact and the watersheds encountered in highway practice usually are small basins less than 20 square miles in area, for which reliable data frequently are lacking.

2. *Chart Solutions.* Maps of the different states (California or North Carolina, for example) show the average rainfall intensity for the regions during a 60-min period once in 100 years. A larger scale map, available periodically, gives the latest hydrologic data with more accuracy.

This nomogram does not apply to urban areas; it applies only to natural conditions and is useful where reliable rain-intensity and runoff data are unavailable. In large watersheds, this nomogram should be used with caution. It is not a substitute for careful analysis based on the latest hydrologic data which must be continually accumulated by each district.

If the drainage basin is other than "pear-shaped," the time of concentration changes; then the correct runoff is obtained by entering the chart with a computed time of concentration.

3. *Time of Concentration.* For storm drains, the time of concentration consists of the inlet time and the time of flow in the drain; for culverts, the time of concentration and the inlet time are the same, but the design discharge nomogram for a small basin may consider only the inlet time and not the time of concentration.

4. *Field Inspections.* There is no substitute for a field review. Although aerial photographs and topographic maps are good media for the study of a drainage area, field inspections are necessary to verify such design assumptions as effects of ponding, the stream bed evaluations, size of existing culverts, and the size and volume of the debris. A field reconnaissance must be made for large culverts.

165. Criteria for Balanced Design. The culvert cross section shall be designed to discharge:

a. A 10-year flood without static head at entrance
b. A 100-year flood utilizing the available head at entrance

The available head may be limited by the fill height or the effects of ponding on upstream property.

For convenience, the ratio of the 10-year flood to the 100-year flood may be established for various geographic areas in the district and individual determinations omitted in the design of all but the important culverts in these areas.

166. Use of Available Head. It is not always economical or practical to utilize all of the available head. This particularly applies to situations where debris and detritus must pass through the culvert where a headwater pool cannot be tolerated, or where the natural gradient is steep and high inlet velocities are objectionable.

1. *Debris and Detritus.* There are two solutions to this problem:

a. Retain solids upstream from the entrance
b. Pass them through the culvert.

If drift and detritus are retained upstream, a riser or chimney may be added as a vertical extension to the culvert, thus increasing the head; however, this head should not be allowed to develop excessive velocities or cause pressures which might induce leakage in the culvert. If drift and detritus are passed through the culvert, the available head must be used to maintain or accelerate the velocity of the flow approaching the culvert instead of creating a pond at entrance, thereby inviting a blocked culvert.

2. *Steep Gradients.* Full use of the available head may develop excessive velocity resulting in abrasion of the culvert itself or in downstream scour. A larger culvert operating with less velocity frequently is more economical than an energy dissipator.

167. Chart Solutions. Detailed analysis for small culverts (less than 15 sq ft in cross section) seldom should be necessary; charts usually will suffice. Chart solutions for large culverts should be checked by other methods.

168. Entrance Design. The design of culvert entrances must consider hydraulics, debris and detritus, the effects on traffic and property, and economics.

1. *Hydraulic Considerations.* Entrance improvements lower the headwater pool, help maintain or increase the velocity of approach, and may reduce costs by permitting a smaller conduit to be used. Entrance improvements consist of rounded-lip entrances, expanded entrances, or highly developed transitions.

a. Rounded-lip Entrance. This type costs little, decreases contraction forced by a square-cornered entrance, and increases barrel capacity. Frequently it is an economical means of lowering the headwater pool without increasing the culvert size. Such entrances should be used for all box culverts and the sides of arch-culvert openings when joined to straight headwalls. The rounded-lip entrance may be omitted for concrete or corrugated metal pipe except for unusual combinations of bed load and large diameters of high velocity.

b. Expanded Entrance. This is a more highly developed entrance, such as a belled or throated entrance. The effect of an expanded entrance can frequently be economically achieved by using gunite, sacked concrete, slope paving, or other measures. Commercial flared-end sections provide an effective expanded entrance for small culverts and metal pipes; they usually cost less than headwalls for pipes under 36 in. in diameter.

c. Transitions. Elaborate transitions and throated openings for culverts are costly; such designs are warranted only in special cases.

d. Debris and Detritus. Where flows are likely to carry floating debris or detritus in sufficient quantity to block the entrance, it is imperative to maintain or increase the velocity of approach to transport such solids through the culvert. Under these conditions, conformity in shape with the approach channel and adequate transitioning assume greater importance.

169. Outlet Design. Improved culvert outlets are designed to restore natural flow conditions downstream. Outlets should be carefully scrutinized for conditions which produce scour. Where progressive erosion is to be expected, corrective measures such as bank protection or transitions should be considered. Since elaborate energy-dissipating structures are costly, they should be considered only in extreme cases.

Standard headwalls or wing walls should be used with care whenever supercritical velocities are involved and downstream property would be affected.

170. Choice of Type and Cross Section. The choice of type and cross section is based on the following factors:

1. *Physical and Structural Factors.* Some of the most important are: (*a*) durability; (*b*) headroom; (*c*) earth loads; (*d*) bedding conditions; (*e*) culvert rigidity; (*f*) impact.

2. *Hydraulic Factors.* Hydraulic considerations involve: (*a*) design discharge; (*b*) shape, slope, and cross-sectional area of channel; (*c*) velocity of approach; (*d*) total available head; (*e*) bed load; (*f*) inlet and outlet conditions; (*g*) slope of culvert; (*h*) smoothness of conduit; (*i*) length of culvert.

3. *Maintenance and Construction Factors.* These include: (*a*) local experience; (*b*) accessibility of site; (*c*) construction conditions.

4. *Economy.* Comparative costs should be weighed on a long-term basis. The overall costs of installation including inlet and outlet treatment are to be considered.

171. Diameter and Length for Pipe Culverts.

1. *Diameter.* The minimum diameter for cross pipes under the roadway shall be 18 in. For pipe runs exceeding 100 ft, an intermediate inlet or cleanout is necessary; otherwise a minimum 24-in. diameter is required. For 24-in. diameter pipe runs exceeding 300 ft, intermediate inlets or cleanouts are required. For other than cross pipes, the minimum diameter shall be 12 in. These standards do not apply to storm drains.

2. *Length.* The length of pipe culvert to be installed shall be determined as follows: (*a*) establish a theoretical length based on slope-stake requirements, making allowance for headwalls; (*b*) adjust the theoretical length for height of fill by applying these rules: for fills 12 ft high or less, no adjustment is required; for fills higher than 12 ft, add 1 ft of length at each end for each 10-ft increment of fill height or portion thereof, neglecting the first 12 ft; (*c*) use the nearest combination of commercial lengths which equals or exceeds the length in (*b*) above.

172. Median Drainage.

1. *Drainage across the Median.* Except for flush medians 10 ft wide or less, neither a concentrated flow nor a heavy-sheet flow shall cross the median into the oncoming traffic lanes.

2. *Grade.* The flattest desirable longitudinal slope or grade is 0.25 percent for earth medians and 0.12 percent for paved median waterways.

3. *Economy in Design.* In varying degrees, economy in median drainage may be achieved by: (*a*) providing temporary storage in the median in conjunction with inlet facilities; (*b*) locating inlets to utilize available nearby culverts or the collector system of a storm drain installation; (*c*) increasing inlet capacity by placing the intake opening in a local depression; (*d*) taking advantage of permeable soils of sufficient depth to ensure disposal by infiltration to the maximum extent without detrimental effects.

173. Ditches.

1. The flattest *grade* recommended for design is 0.25 percent for earth ditches and 0.12 percent for paved ditches.

2. *Slope ditches.* These ditches, sometimes called surface ditches, should be provided at the tops of cuts where it is necessary to intercept drainage from natural slopes inclined toward the highway.

Where the grade of the slope ditch is steep enough to invite erosion, the ditch should be paved. When the ditch grade exceeds a 4:1 slope, a downdrain is often advisable. Slope ditches may not be necessary where side slopes in favorable soils are flatter than 2:1 or where positive erosion-control measures are to be instituted during construction.

174. The Talbot Formula. An empirical formula that has long served as a guide in estimating the size of waterway opening is the *Talbot formula* (see Table 36):

$$a = C\sqrt[4]{A^3}, \tag{210}$$

where a = area of waterway opening, sq ft;
A = drainage, acres;
C = a constant depending on topographical and rainfall conditions pertaining to the watershed.

As a rule of thumb the Talbot formula is still an acceptable guide, particularly when the user has had many years of experience in a given locality and has had the opportunity to observe how culverts for highways and railroads have operated during flood runoff.

References

1. California Division of Highways, *Planning Manual*, 1963.
2. North Carolina State Highway Commission, *Bridge Location and Hydrographic Department*, 1965.
3. Virigina Department of Highways (Bridges), 1964.

CHAPTER 13

CONSTRUCTION SURVEYS AND PROCEDURES

175. The Contract System. The construction procedure will be presented here as if applied to highways; that for railroads, pipelines, and other facilities is similar but is often less complicated. When detailed plans and specifications for a project are completed, funds assigned for its construction, and necessary steps taken toward the acquirement of rights of way, notice is given to contractors in appropriate publications two weeks or more in advance of the date set for considering bids.

Prospective bidders are furnished plans, specifications, and bidding sheets prepared by the highway department or other organizations. These documents are specific, and a prospective bidder must exercise his best judgment in estimating what the cost of supplying materials and performing the construction items will be. The engineers are willing to supply all available information. The contractor knows that there can be no departure from the provisions prescribed in the contract document and that the primary advantages one bidder has over another are individual ingenuity and more effective management.

176. Construction Surveys and Stake Setting. As a guide to the contractor, engineers representing the highway department will need to do a considerable amount of surveying and stake setting just before and during construction. Among the surveying operations are the following: retracing the center line shown on the plans and referencing certain points on the curves; establishing sufficient parallel offset reference stakes; checking bench marks and running center-line levels over the retraced line, taking elevations at all stations, at all breaks in the ground, and at other points where it may be necessary to take cross sections for volume quantities; setting slope stakes and grade stakes; setting stakes for the complete layout of culverts and bridges; reporting and making advantageous changes, if any, in line or grade or minor adjustments of the drainage structures; progress reports; final estimates, etc. As the work progresses, the stakes that have been destroyed must be reset. All points of intersection should be carefully witnessed to permanent objects located outside the working area of the project.

The slope stakes (see Art. 138) are the contractor's guide in making his cuts and fills. These stakes are set for all heavy grading sections and on any section where the contractor would have difficulty in following grade stakes. On sections having both cut and fill, the "grade point" is staked. The attention of the contractor should be called to the importance of observing carefully the cuts and fills as shown on the slope stakes where superelevated curves are encountered, to avoid carrying the excavation too deep.

In case the grading is light and no slope stakes are required, grade stakes are set at 50-ft intervals on each side of the road on tangents and at 25-ft intervals on superelevated curves. When the grading is heavy, the grade stakes are set when the contractor is ready to begin his fine-grading operations. In pavement construction, the top of the stake should represent the grade for the forms.

177. Responsibilities of the Resident Engineer or Inspector. The following duties and responsibilities apply particularly to the resident engineer or the inspector.

a. Familiarity with the Work. All printed instructions relative to the project must be read, understood, and kept convenient for reference. Any doubtful points should be submitted to the district engineer for interpretation. Discrepancies, if any, should be reported at once.

Before construction starts, the engineer should go over the job in detail with the contractor and get acquainted with his method of procedure, the equipment he will use, his plans for handling and storing materials, and his sequence of operations.

b. Relations with the Contractor. In his dealings with the contractor, the inspector should follow a friendly, businesslike and ethical course. He should see that the contractor, as well as the state, is given a fair deal and that the work is performed in strict accordance with the specifications. He should not hesitate to offer, at the proper time and place, information and suggestions helpful to the solution of any problem at hand.

Orders should not be given directly to a laborer, but all matters should be taken up with the superintendent or foreman, and then the inspector should see that they are carried out. Always adhering to regulations and instructions, he must tactfully and with judgment strive to bring about, if possible, the desired results without losing the good will of the contractors, their employees, and the general public.

c. Relations with Public. Particular attention should be given to enforcement of that part of the specifications concerning public convenience and safety. Before construction starts, go over all detours, making certain that they are in a safe and passable condition, that they are properly posted, and that all barricades and lights are in place, so that there will be no reasonable complaint on the part of the public.

All dealings with the public should be courteous, agreeable, and cooperative without discussing too much the details of the work or the methods employed by the contractor.

d. Objectives. Competent inspection is one of the most important elements entering into the construction of any engineering project. The most careful and accurate plans and specifications may be prepared, and the structure may be accurately laid out in the field, but if faulty materials or poor workmanship is incorporated in the construction, the purpose of all the preceding effort is wasted.

The finished work should be pleasing to the eye. *What looks right may be wrong but what looks wrong cannot be right.* The conscientious inspector and the reputable contractor are generally working toward the same end: to carry out the contract and do a good job.

178. Miscellaneous Construction Survey Notes. *Setting Stakes for Parallel Curves*

a. Circular curves (see Chapter 3)
b. Simple spirals (see Chapter 6)
c. Combining spirals (see Chapter 7)

Locating the Center Line of Spirals. If feasible, the PI of the main tangents is established on the ground and the external angle Δ is measured directly; that is, with the transit set at the PI (vernier reading 0°00′), backsight to some point on the initial tangent (called POT), invert the telescope and turn to a distant point on the forward tangent, and then read the vernier for the angle Δ. The PI_s would next be located (Fig. 135) by measuring along the forward tangent the distance $T_s - U$, where T_s is found in Table 18, or it is given by the formula $T_s = (R_c + p)\tan\frac{1}{2}\Delta + k$ (p and k from Table 15), and in Table 15, U = LT and V = ST, where these distances are used to locate the PI_s (the intersection of the spiral tangents). Then, in the same direction, the ST might next be set, it being the distance U further along the tangent.

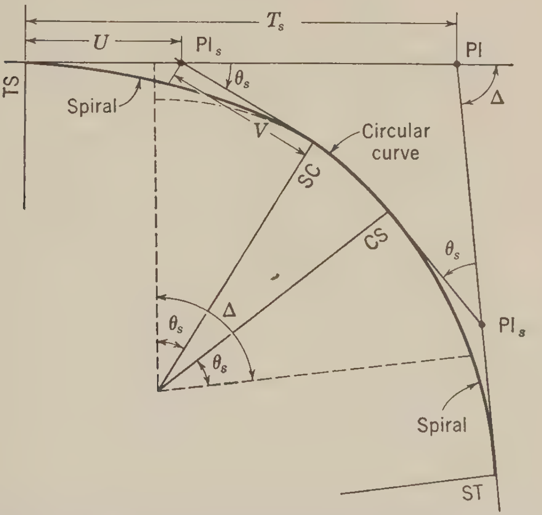


FIG. 135. Layout of spiraled curve.

Next, set up the transit at the PI_s, backsight on the ST, vernier reading 0°00′, invert the telescope, turn the angle θ_s, and measure the distance V to locate the CS. Then move the transit to the POT on the initial tangent, establish the line of sight on this tangent, then locate the PI_s and the TS, using the same values for

U and T_s as before. Next, set up the transit at the PI_s, vernier reading 0°00′, invert the telescope, turn the angle θ_s, and measure the distance V to locate the SC.

With tack points set at the SC and CS, they serve as important checks in the actual layout of the spiral, circle, and spiral, beginning at the TS and proceeding around to the ST, as follows:

Step 1. With the transit up at the TS, vernier reading 0°00′ on the tangent, even stations along the spiral are located by deflection angles that vary as the square of the distance along the curve. The total deflection to the SC equals $\frac{1}{2}\theta_s$ minus a correction (in Table 20) that is usually negligible.

Step 2. Set up transit at the SC, backsight on the PI_s, vernier reading 0°00′, invert telescope, and locate the circular curve.

Step 3. Set up transit at the ST, sight on the PI_s (preferably the PI for a longer sight), vernier reading 0°00′, and locate even stations in the usual way, the initial deflection angle to the CS being $\frac{1}{3}\theta_s$ minus a correction that is usually negligible.

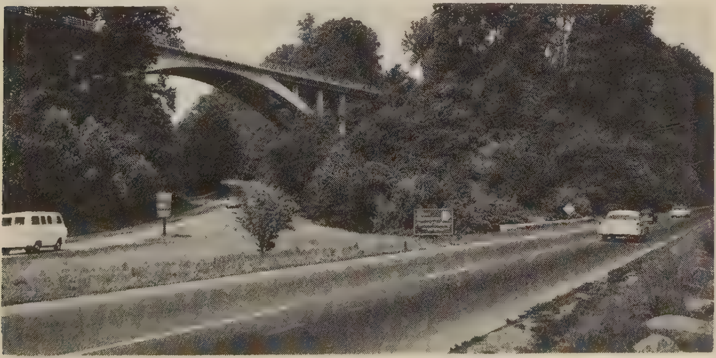
Superelevation (North Carolina Practice)

Practice varies depending on the type of facility being built—two-lane, two-way highways, or multilane divided highways separated by a median.

For two-lane or multilane undivided two-way facilities, the general practice is to accomplish superelevation by rotation about the center line, raising the outside edge and lowering the inside edge. In flat country, this could result in sags on the low side which may cause drainage problems. To prevent this, adjustments in grades or special drainage ditches may be found necessary in the design stage. On curves without spiral transitions, the length of runoff will normally be the same as if a transition were used; however, two-thirds of the runoff will be on the tangent and one-third on the curve. In eliminating crown on curves with spiral transitions, the outer edge is raised, beginning at some point on the tangent, so that it has the same elevation at the TS as the profile grade at that point. The point on the tangent where this begins is determined by the crown on the tangent section and the slope of the outside edge of pavement with relation to the profile grade. This relative slope should be no steeper than 1:400. Thus, for a 24-ft pavement crowned about the center line at a rate of 0.015

ft/ft, we have a tangent runout distance of 0.015 × 12 × 400 = 72 ft. This would normally be rounded to 75 ft. Beyond the TS the outside edge continues to rise at a uniform rate until full superelevation is reached at the SC. The slope of the outside edge with relation to the profile grade should not be steeper than 1:200 between the TS and SC and if necessary, the length of spiral should be increased to maintain this relationship. On the inside lane, no adjustment in crown is necessary until a point beyond the TS is reached where the two lanes are at the same slope. At this point, the entire pavement begins rotation about the center line, lowering the inside edge and raising the outside edge, until full superelevation across the full pavement is reached at the SC.

On multilane divided highways, the general practice is to maintain the profile grade on the median edge of pavement on both tangents and curves. On tangents the pavements slope downward from the median. Superelevation is obtained in the same manner as for two-lane, two-way pavement using the median edges of pavement as the center of rotation. However, as the pavements are at least twice as wide, the tangent runout and the transition lengths must be increased to maintain the 1:400 and 1:200 slope relation of pavement edges to profile grade. Due to the increased width, it is even more important on multilane pavements than on two-lane to be on the alert for drainage problems on the low side caused by superelevation.



George Washington Parkway, Va.

CHAPTER 14

STRING-LINING RAILROAD CURVES

179. Introduction. The purpose of string lining is to ascertain how much to adjust and "throw back" track that has slowly crept out of line laterally under the impact of traffic. This irregularity may become noticeable, particularly on curves where the lateral shock and sway in high-speed trains are unpleasant to passengers.

The trend toward higher speeds for both freight and passenger operation makes it more important than ever to maintain the track continuously in good alignment. On account of its simplicity as compared with use of a transit, flexibility, and satisfactory accuracy, string lining for smoother riding has become standard railroad practice. Aside from the services of an engineer, the equipment required is a strong fishline, a 50-ft steel tape, a 5-ft rule, marking crayon, pad, and notebook.

The method is based on the principle: *the mid-ordinate of a given length of chord varies directly with the degree of curvature;* in other words, it varies directly with the subtended angle. Hence the sum of the mid-ordinates is a measure of the angle between the tangents. Since it is not the purpose of string lining to change the angle between the tangents but merely to restore the curve to one having uniform properties, *the sum of the mid-ordinates before lining* must equal the sum of the mid-ordinates of the realigned curve.

180. Procedure. There are mainly four steps entering into the string-lining method:

1. Marking the outer rail at equal intervals (called stations) throughout the curve.

2. With a scale and strong fishline, of length equal to two stations, measure the middle ordinates at each intermediate station in turn.

3. Adjust the measured middle ordinates (called ordinates) to

determine the inward (or outward) "throw" of the track at each station for uniformity of ordinates, it being desirable that the ordinates along the central (circular) portion of the curve be *equal,* while those along the spiral approach and runoff curves should vary by equal increments from a full ordinate at the point of full curvature to a zero ordinate at the tangent.

4. Setting stakes for establishing the new positions of the track.

181. Chords, Ordinates, and Degree of Curvature. The ordinates are always measured at station points (at the middle of a chord); that is, the stations are spaced at intervals equal to one-half a chord length. Beginning at a considerable distance back on the tangents, the stations are marked and numbered in regular order throughout the curve and beyond. Various chord lengths have been used successfully.

Three men are needed to obtain the data. A strong light string is held taut against the gauge side of the outer rail by two men while the third man measures and records the ordinate at the intermediate station. In a similar manner the ordinate is found at the next station and so on for all the stations, except the first and last, which are supposedly on the tangents where the ordinate is zero.

The ordinates may be read in sixteenths of an inch or in hundredths of a foot. In either case for simplicity they should be recorded as whole numbers. Thus $2\frac{1}{2}$ in. would be recorded as 40, meaning $\frac{40}{16}$ in.

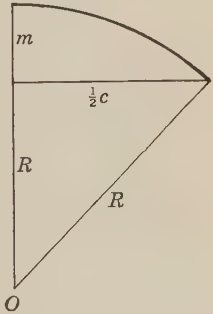


FIG. 136

From Fig. 136, we have

$$R^2 = (R - m)^2 + \tfrac{1}{4}c^2, \tag{a}$$

from which
$$R = \frac{m^2 + \frac{1}{4}c^2}{2m}. \tag{b}$$

Since m is very small in comparison with c, the term m^2 may be omitted without appreciable error. Thus,

$$R = \frac{c^2}{8m}. \tag{295}$$

Putting $R = 5{,}730/D$ in Eq. (212) and reducing, we get

$$D = \frac{45{,}840m}{c^2}, \tag{213}$$

where m and c are in feet.

With 31-ft stations (62-ft chords), Eq. (213) shows that D (degrees) $= m$ (inches), that is, the curvature is 1° per in. of ordinate. Thus, if $m = 24$ (meaning $\frac{24}{16}$ in.), $D = \frac{24°}{16} = 1°30'$.

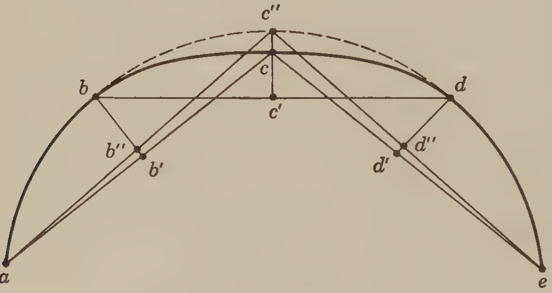


Fig. 137

A variation of one unit in the adjacent ordinates corresponds to a change of about 4 min in the degree of curve, which is not enough to be detected.

With 34-ft stations (68-ft chords), $D = 10m$, where m is expressed as hundredths of a foot. Thus, if $m = 15$ (0.15 ft), $D = 1.5° = 1°30'$.

With 39-ft stations (same as standard rail length) and with 78-ft chords, $D = 7.5m$, where m is in foot units, or $D = \frac{5}{8}m$, where m is in inch units.

Some engineers prefer the rail length as the station distance, where the stations occur just beyond each splice, the advantage claimed being that the joint is the weakest part of the track structure and kinks out of line faster than the main body of the rail.

182. Effect of Throw at One Station on Adjacent Stations. In Fig. 137, let a, b, c, d, e, etc., represent station points in consecutive order around the curve, while $b'b$, $c'c$, $d'd$ are the corresponding ordinates as originally measured.

Now if the track is shifted *outward* from c to c'', that is, if it is given a *plus* throw at point c, then the adjacent ordinates at b and d become $b''b$ $(= b'b - b'b'')$ and $d''d$ $(= d'd - d'd'')$; also the ordinate at c becomes $c'c''$ $(= c'c + cc'')$.

In actual practice the ordinates at b, c, and d are nearly parallel; hence Fig. 137 is greatly exaggerated, and, without appreciable error,

$$b'b'' = d'd'' = \tfrac{1}{2}cc''.$$

Letting t = the outward throw at c, it follows that the ordinate at station c is *increased* by t while the ordinates at the adjacent stations are each *decreased* by $\tfrac{1}{2}t$.

By a similar construction, if t is the *inward* (or negative) throw at c, then the ordinate at c is *decreased* by t while the ordinates at the adjacent stations are each *increased* by $\tfrac{1}{2}t$.

It is seen that the algebraic sum of the changes in ordinates equals zero for either outward or inward throws.

183. Derivation of Formula for "Throw." Let $m_1{}^1$, $m_2{}^1$, $m_3{}^1$, $m_4{}^1$, etc., represent the original (measured) ordinates at Sta. 1, 2, 3, 4, etc., while m_1, m_2, m_3, m_4, etc., are the corresponding "adjusted" ordinates at these stations.

Also, let d_1, d_2, d_3, d_4, etc., represent the "errors" at Sta. 1, 2, 3, 4, etc., that is,

$$d_1 = m_1{}^1 - m_1, \tag{a}$$
$$d_2 = m_2{}^1 - m_2, \tag{b}$$
$$d_3 = m_3{}^1 - m_3, \tag{c}$$
$$d_4 = m_4{}^1 - m_4, \text{ etc.} \tag{d}$$

Let t_1, t_2, t_3, t_4, etc., represent the "throws" at Sta. 1, 2, 3, 4, etc. Then combining the effects (see Art. 182) of a throw at a station with the throws at adjacent stations, we have

$$m_1 = m_1{}^1 + t_1 - \tfrac{1}{2}t_0 - \tfrac{1}{2}t_2, \tag{e}$$
$$m_2 = m_2{}^1 + t_2 - \tfrac{1}{2}t_1 - \tfrac{1}{2}t_3, \tag{f}$$
$$m_3 = m_3{}^1 + t_3 - \tfrac{1}{2}t_2 - \tfrac{1}{2}t_4, \tag{g}$$
$$m_4 = m_4{}^1 + t_4 - \tfrac{1}{2}t_3 - \tfrac{1}{2}t_5, \text{ etc.} \tag{h}$$

Substituting Eqs. (*a*), (*b*), (*c*), (*d*), etc., in (*e*), (*f*), (*g*), (*h*), etc., and reducing, we get

$$t_2 = 2(d_1 + t_1) - t_0, \tag{i}$$
$$t_3 = 2(d_2 + t_2) - t_1, \tag{j}$$
$$t_4 = 2(d_3 + t_3) - t_2, \text{ etc.} \tag{k}$$

In general,

$$t_n = 2(d_{n-1} + t_{n-1}) - t_{n-2}. \tag{214}$$

Hence the throw at any station equals twice the algebraic sum of the combined error and throw at the first preceding station minus the throw at the next preceding station.

Procedure. The data in Table A illustrate the author's method of determining the *throw*. A great number of solutions are possible, but the results given here are satisfactory for two reasons: (1) the final throw is zero and (2) the intermediate throws are moderate in amount.

TABLE A

Sta.	Ordinates		Error d	Throw [Eq. (197)] t	$2(d + t)$
	Measured m^1	Adjusted m			
(1)	(2)	(3)	(4)	(5)	(6)
1	0	0	0	0	0
2	5	4	1	0	2
3	7	8	−1	2	2
4	11	12	−1	2	2
5	16	16	0	0	0
6	17	16	1	−2	−2
7	16	16	0	−2	−4
8	13	12	1	−2	−2
9	7	8	−1	0	−2
10	4	4	0	0	0
11	0	0	0	0	
Sum......	96	96	0		

Column (2) gives the ordinates which were measured in the field at the corresponding stations.

Column (3) gives the *adjusted* ordinates, where the spiral approach and runoff curves vary by equal increments and the central (circular) portion has constant ordinates. It will be noted that the sum of the adjusted ordinates equals the sum of the measured ordinates, i.e., 96.

Column (4) gives the so-called "errors," d, at each station, obtained by subtracting the adjusted ordinate from the measured ordinate.

Column (5) gives t, the final throw [Eq. (214)], while Column (6) is used merely for the convenience of calculating the throw. Thus,

$$\begin{aligned} t_5 &= 2(d_4 + t_4) - t_3 = 2(-1 + 2) - 2 = 0; \\ t_6 &= 2(d_5 + t_5) - t_4 = 2(0 + 0) - 2 = -2; \\ t_7 &= 2(d_6 + t_6) - t_5 = 2(1 - 2) - 0 = -2, \text{ etc.} \end{aligned}$$

As a rule, the final throw is *not* zero in the initial sequence, but it can be reduced to zero with a slight adjustment in one or more pairs of ordinates, as will be shown in Art. 184.

184. Reducing the Final Throw to Zero. It is usually required that the resultant throws at the initial and final stations (those on the tangents) should be *zero*. Whatever may be the calculated throw (an accumulated quantity) at the last station, it can readily be reduced to zero by the algebraic addition of a quantity which is equal but of opposite sign. This result is produced by a change in one or more pairs of preceding ordinates.

Letting t_0 and $t_1 = 0$ and expressing the t's in terms of the d's alone, we get, from Eqs. (i), (j), (k), etc. (Art. 183),

$$t_2 = 2d_1, \tag{l}$$
$$t_3 = 4d_1 + 2d_2, \tag{m}$$
$$t_4 = 6d_1 + 4d_2 + 2d_3, \tag{n}$$
$$t_5 = 8d_1 + 6d_2 + 4d_3 + 2d_4, \text{ etc.} \tag{o}$$

In general,

$$t_n = 2(n - 1)d_1 + 2(n - 2)d_2 + \cdots + 2d_{n-1}. \tag{p}$$

Let Δd_k represent the *change* in the d at Sta. k alone; then the corresponding change in t at the nth station ahead is

$$\Delta t_n = 2(n - k)\ \Delta d_k. \tag{215}$$

Likewise if Δd_p represents the *change* in the d at Sta. p alone, then the corresponding change in t at the nth station ahead is

$$\Delta t_n = 2(n - p)\,\Delta d_p. \tag{216}$$

Combining the right-hand members of Eqs. (215) and (216), we have

$$\Delta t_n = 2(n - k)\,\Delta d_k + 2(n - p)\,\Delta d_p. \tag{217}$$

Ordinarily the changes in a pair of d's will be made numerically equal, but with opposite signs, in order that the sum of all the d's shall remain zero.

If $\Delta d_k = +1$ and $\Delta d_p = -1$, Eq. (200) becomes

$$\Delta t_n = 2(p - k); \tag{218}$$

also, if $\Delta d_k = +\frac{1}{2}$ and $\Delta d_p = -\frac{1}{2}$,

$$\Delta t_n = (p - k), \text{ etc.} \tag{219}$$

If $\Delta d_5 = +1$ (that is, the m at Sta. 5 is *decreased* by 1) and $\Delta d_{15} = -1$ (that is, the m at Sta. 15 is *increased* by 1), then

$$\Delta t_n = 2(15 - 5) = +20,$$

that is, the throw at Sta. 15 (or at all stations beyond Sta. 15) is *increased by* +20.

185. Determining the Revised Ordinates. It being essential that the sum of the adjusted ordinates shall equal the sum of the measured ordinates, the former may be estimated by first calculating the *average* of a group of intermediate ordinates which are approximately equal. This average ordinate at several consecutive stations represents the circular arc. The sum of the remaining ordinates, divided by 2, is the sum of an arithmetic series representing the approach and runoff spirals.

Thus in Column (3) of Table A, the ordinates 16, 16, 16 represent the circular arc, while the ordinates 4, 8, 12 (in arithmetic progression) represent the end spirals.

A trial set of revised (or adjusted) ordinates may also be determined graphically, as indicated in Fig. 138, where the measured ordinates are plotted to an enlarged scale and connected by dashed lines. The ordinates to the trapezoid, striking an average of the measured ordinates, are here considered to be the revised ordinates. Minor adjustments may later be necessary in order to make the final throw equal to zero. The horizontal line represents the intermediate circular arc while the sloping lines represent the end spirals.

Since the end ordinates are usually zero, the number of ordinates equals $(n - 2)$, where n is the number of stations. If b represents the number of *equal* ordinates (each $= y$, say), then the number of ordinates remaining for each spiral is $(n - 2 - b)/2$. Thus in Fig. 138, $n = 16$, $b = 6$, leaving four ordinates for each spiral.

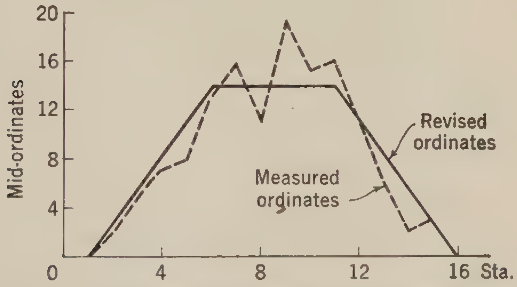


FIG. 138. Relation between measured and revised ordinates.

Sometimes the middle portion may consist of two or more horizontal lines representing compound curves.

Procedure. Column (2) of Table B gives the measured mid-ordinates at Sta. 1 to 16, the sum of the ordinates being 134. These ordinates are indicated by dashed lines (Fig. 138).

A trial set of revised ordinates, given in Column (3), are determined on the basis of equal ordinates at Sta. 6 to 11, inclusive. Letting y = the magnitude of these equal ordinates and noting that the sum of all the ordinates, including those to the sloping lines Fig. 138, equals 134, we have

$$2(\tfrac{1}{5}y + \tfrac{2}{5}y + \tfrac{3}{5}y + \tfrac{4}{5}y) + 6y = 134,$$

from which $y = 13.4$ and $\tfrac{1}{5}y = 2.6.$

The six ordinates that are equal (or nearly so) are 13, 14, 14, 14, 14, 13, making a total of 82. Subtracting 82 from 134 gives a total of 52, or 26 for each spiral. The ordinates 2, 5, 8, 11, giving a total of 26, are recorded in Column (3) of Table B.

TABLE B

Sta. (1)	Ordinates Old m^1 (2)	Ordinates New m (3)	Error d (4)	Throw [Eq. (197)] t (5)	$2(d + t)$ (6)	Δt (7)	Final throw $(t + \Delta t)$ (8)
1	0	0	0	0	0	...	0
2	2	2	0	0	0	...	0
3	5	5	0	0	0	...	0
4	7	8	−1	0	−2	...	0
5	8	11(10)	−3(−2)	−2	−10	...	−2
6	13	13	0	−10	−20	2	−8
7	16	14	2	−18	−32	4	−14
8	11	14	−3	−22	−50	6	−16
9	19	14	5	−32	−54	8	−24
10	15	14	1	−32	−62	10	−22
11	16	13(14)	3(2)	−30	−54	12	−18
12	11	11	0	−22	−44	12	−10
13	6	8	−2	−14	−32	12	−2
14	2	5	−3	−10	−26	12	2
15	3	2	1	−12	−22	12	0
16	0	0	0	−12	....	12	0
Sum	134	134	0				

The "errors" and "throws" are calculated as in Table A. It will be noted that the final throw (on the tangent) at Sta. 16 is not zero but −12. To reduce this value to 0, +12 must be added.

Let the adjusted ordinate at Sta. 5 be reduced from 11 to 10, causing the error at the station to be increased by 1; that is, $\Delta d_5 = +1$. Also let the adjusted ordinate at Sta. 11 be increased from 13 to 14; that is, $\Delta d_{11} = -1$. Then applying Eq. (218), we have

$$\Delta t_{16} = 2(11 - 5) = 12.$$

A new set of throws beyond Sta. 5 may be calculated independently in the usual way, as a check, or increments of 2, 4, 6, 8, etc., up to 12 may be added algebraically as shown in Column (7) of Table B.

186. Setting Points for Realigning Track. Substantial stakes, usually 2 by 2 by 18 in., are driven and tacks set so that the base of the rail will be a fixed distance, say 18 in., from the tack when the shift is completed.

Thus for a throw of $-\frac{1}{4}$ in., the tack would be set $18\frac{1}{4}$ in. from the base of the rail; for no throw, the tack would be set 18 in. from the base of the rail; for a throw of $+\frac{1}{2}$ in., the tack would be set $17\frac{1}{2}$ in. from the base of the rail; etc. *Plus* throws are *outward*, and *minus* throws are inward in all cases.

In case of multiple-track roadbeds, a point on the adjacent tie may be used for reference, thus dispensing with stakes. It is well to mark with crayon on top of the tie the amount of throw as well as the direction of throw at each station. This information is helpful to the track foreman, who prepares in advance of track shifting by digging out ballast, particularly stone, at the ends of the ties to allow for the required throw. This advance preparation saves time of the lining gang even though jacks are used to assist men with lining bars. However, sometimes by the use of jacks, a track is realigned with all ballast in place.

If the throws are large and traffic is heavy, the final adjustment may require going over the curve twice, the first shift being only approximate.

While the track is being realigned, the proper *superelevation* should be applied at the various stations along the curve. A level board placed on the outer rail will show its vertical height above the inner rail.

CHAPTER 15

RAILROAD TURNOUTS AND CROSSINGS

187. Trackwork Standards. Each railroad has its own trackwork plans which conform in general to those of the AREA. Switches, frogs, and other track structures needed in construction are determined from field layouts and are supplied from the railroads' frog and switch shops or by companies specializing in the manufacture of these items. Except in very special cases, standard track structures should be used in layouts to save time required to fabricate special structures.

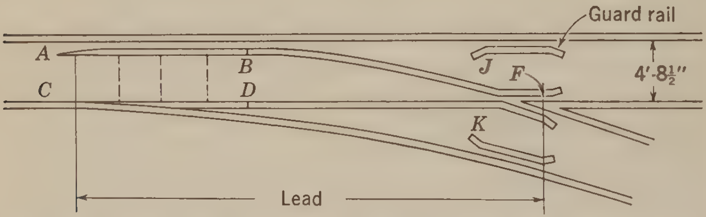


FIG. 139. Turnout from main track: AB, CD, switch rails (split switch); BJ, DK, closure rails; F, point of frog (½-in. point).

188. Definitions. 1. A *turnout* (Fig. 139) is an arrangement of a *switch* and a *frog* with closure rails, by means of which the wheels of trains may be diverted from one track to another. Standing at the switch point facing the frog, a turnout taking off to the right is a *right-hand turnout;* one springing to the left is a *left-hand turnout.*

2. A *switch* is a device for diverting the wheels of trains from the track on which they are running. A *split switch* (the form now in general use) consists essentially of two movable-point rails with the necessary fixtures; a *switch angle* is the angle at the *point of switch* between the gauge lines of the switch rails and the *stock rail,* the latter being the *running rail* against which the switch operates.

The *actual point of switch* is the end of the switch rail which is farther from the frog, the point where the spread between the gauge lines of the stock rail and the switch rail is $\frac{1}{4}$ in. The *theoretical point of switch* is the point where the gauge line of the switch point, if produced, would intersect the gauge line of the stock rail.

3. In order to keep car trucks on the track, the wheels must have *flanges* which impinge on the inside lines (gauge lines) of the rails. At points where the outer rail of the turnout crosses the rail of the main track, the flanges necessitate a special rail structure called a *frog*. This is a contrivance at the intersection of two rails which allows the flanges of wheels to pass from one rail to the other. The two channel ways, or narrow spaces through which the flanges of the wheels move, are called *flangeways*.

There are a number of types of frogs: spring rail frogs, plain rigid frogs, rail-bound manganese steel frogs, solid manganese steel frogs, and self-guarded frogs. The *spring rail frog*, generally used in main-track turnouts where speed is high, has a movable wing rail which is normally held against the frog point by a strong spring, thus making an unbroken running surface for wheels using the main track, whereas the flanges of wheels on the other track force the movable wing rail away from the point to provide a passageway. These frogs are made in left-hand and right-hand to fit the turnout being installed. All *rigid frogs* are interchangeable in either right- or left-hand turnouts. A *bolted rigid frog* is built essentially of rolled rails, with fillers between the rails, and held together with bolts. A *self-guarded frog* is a frog provided with guides or flanges above its running surface, which contact the tread rims for the purpose of safely guiding their flanges past the point of the frog, eliminating the necessity of a guard rail.

4. A *frog angle* is the angle formed by the intersecting gauge lines. Frogs are usually designed by *numbers* giving the ratio of the number of units of center-line length to the spread, or *one-half the cotangent* of *one-half the frog angle*.

The *toe end of a frog* is the end nearer the switch, while the *heel end of a frog* is the other end. The *throat* of a frog is the point at which converging wings of a frog are closest together.

5. The *theoretical point* is the intersection of the gauge lines of a frog. However the actual *point of frog*, from which measurements are usually made, is the $\frac{1}{2}$*-in. point*, because here it has a $\frac{1}{2}$-in. width. This point is located at a distance from the theoretical point toward the heel, which equals, in inches, one-half the frog number.

6. The *guard rails* are short rails, with flared ends, set securely opposite the frog to prevent the flange from bearing against the frog point and going to the wrong side.

7. The *lead* is the distance measured parallel to the main rail from the actual point of switch to the $\frac{1}{2}$-in. point of the frog. The *lead curve* is the curve in the turnout interposed between the heel of the switch rail and the toe of the frog, the switch and frog rails themselves being straight in most cases.

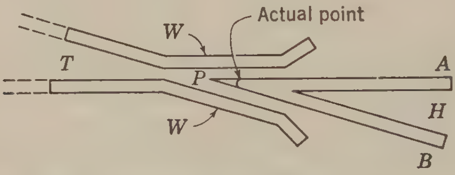


FIG. 140. Rigid frog: *P*, theoretical point; *H*, heel; *T*, toe; *APB*, frog angle (called *F*); *W*, wing rail.

8. A *crossing* is a track structure where one track crosses another at grade, and consists of four connected frogs.

9. The standard gauge of track is 4 ft $8\frac{1}{2}$ in. measured perpendicularly to the gauge lines.

189. Frog Number and Angle. Frogs are designated by their *number* (called n), which is the ratio of two dimensions, that is (see Fig. 125),

$$n = \frac{PH}{AB},$$

in which AB is twice the spread per unit length. Then if F is the frog angle,

$$2n = \frac{PH}{\frac{1}{2}AB} = \cot \tfrac{1}{2}F; \qquad n = \tfrac{1}{2} \cot \tfrac{1}{2}F. \tag{220}$$

190. Turnout and Crossover Data. It is the practice of railroad companies to base turnout and crossover data on standard *frog numbers*. Thus No. 16 to No. 20 turnouts are used where comparatively high-speed main-line movements are made through the turnout; No. 14 to No. 10 turnouts are used where the main-line speed is restricted; Nos. 7, 8, and 9 turnouts are used in yard work.

A No. 10 turnout, according to the AREA standard, calls for a frog angle of 5°43′29″, a 16 ft 6 in. switch rail with a switch angle of 1°46′22″, and an actual lead of 78 ft 9 in. with a lead curve of 7°21′24″.

191. Field Layouts. The field layouts include turnouts from both straight and curved track, but the latter should be avoided if possible. Ordinarily a few field measurements combined with the standard turnout plan are sufficient for fixing the layout. Thus if the position of the frog is established, then the point of switch, the lead, and other features are known from standards corresponding to the assumed frog number.

When a turnout is installed in an existing track, the heel of the frog is usually placed at an existing joint to avoid cutting the rail. In some instances the location of the frog is so fixed that it cannot be shifted to fit this joint, and the frog must be installed at an exact location and the rails cut to match it.

With the frog located, the position of the switch ties can be marked off from a standard turnout plan and the crossties in the track replaced with switch ties. The switch plates are then installed where the switch rails will be located. The rail on the turnout side of the existing track between the heel of the frog and the switch point is removed, while the opposite rail is undisturbed. The frog, guard rails, switch points with connections, and closure rails are then installed. The curved closure rail is lined by offsets from the gauge line of the rail in the existing track which was undisturbed.

Two of the simplest types of layout will be presented here briefly: (*a*) turnout from straight track (Fig. 141), (*b*) crossover to parallel track (Fig. 142).

a. Turnout from Straight Track. The engineer usually locates the frog point or the heel of frog and then the foreman constructs the turnout in accordance with standard turnout plans. Thus point *F* (Fig. 141) is located, from which the point of switch and other parts of the turnout are established. The lead is also known when any standard turnout is used. Figure 126 is drawn to a distorted scale; hence the lead *EF* as compared to the switch length is not in correct proportion.

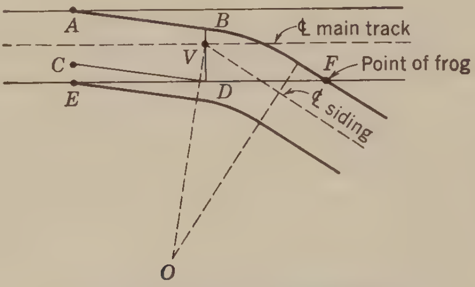


FIG. 141. Turnout from straight track.

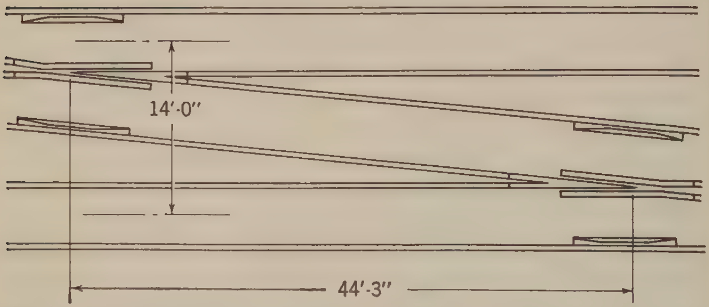


FIG. 142. Crossover between parallel tracks, No. 10 frog, 14-ft track centers.

To continue the turnout beyond the frog, it is helpful to run the center lines of the main track and the turnout to an intersection (at *V*) for point of reference.

b. Crossover to Parallel Track. Figure 142 shows a simple case of straight crossover between parallel tracks.

CHAPTER 16

MISCELLANEOUS

192. Surveying Instruments in Common Use. According to common usage, the surveying instruments are described as the *level* or *transit.* Formerly there were two types of levels, the Dumpy level and the Y level, but the latter is now obsolete and nearly extinct. Among the popular levels in use is the Zeiss self-leveling level which utilizes an inverted pendulum instead of a spirit level to give the direction of gravity; this innovation coupled with other automatic devices keeps the line of sight always horizontal by the pendulum.

A *theodolite* is the general term for a precise instrument designed to measure horizontal or vertical angles about a point.

A transit is a theodolite with a telescope that can be reversed in direction with respect to the rest of the alidade by rotating it about its horizontal axis without removing it from its bearings.

A tacheometer (or tachymeter) is the general term for an instrument that will measure distance by a single observation. An instrument equipped with stadia hairs is the most common example. Range finders or height finders are also tacheometers.

The transit is a general-purpose instrument designed to perform easily and quickly any surveying operation except accurate length measurement and extensive triangulation.

The term theodolite as used in the United States means a very precise direction theodolite designed especially for triangulation and often used for measuring angles in first- or second-order traverse. A popular type of instrument in use today for triangulation, and often for traverse, is a small, accurate type of theodolite called the Wild T-2 manufactured in Switzerland.

193. Care of Field Equipment. The field equipment and particularly the *transits* and *levels* are deserving of good care not only because they are delicate and expensive but because an accurate set of notes cannot be made with instruments in bad condition.

When a transit or level is being transported in automobiles, care should be exercised to see that it is placed in an instrument case and securely packed with shock-absorbing material. A cap should be screwed on the tripod head to protect the threads from injury.

When one is walking among trees or through doors of a building or other contracted spaces, the tripod should be carried under the arm with the instrument forward, and not over the shoulder. Before a transit is picked up, the compass needle should be raised from the pivot to prevent useless wear. Also, when the transit is carried, the spindle should not be clamped too rigidly, in order that it may yield slightly without too much injury, if accidentally struck, and the telescope should be lightly clamped in a vertical position. When set on the tripod, an instrument should never be left unattended or where it might be knocked over by vehicles, cattle, or pedestrians.

A hood should be placed over the transit when it is exposed to wet weather or to dust storms. Wet tapes should always be wiped dry before being put away or reeled up. Steel tapes should be rubbed with an oily cloth after cleaning to prevent rusting. All instruments should be kept clean and in good adjustment at all times. When in constant use, the adjustments should be checked once a week, or oftener if necessary.

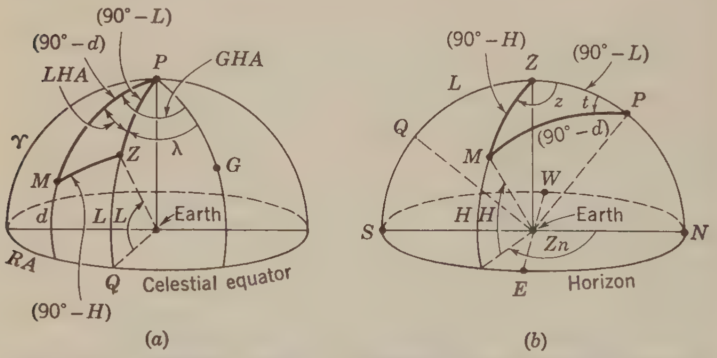


FIG. 143. (*a*) Equator system. Declination, pole distance, latitude, GHA, LHA, etc. (*b*) Horizon system. Zenith, zenith distance, altitude, azimuth, etc.

194. Determination of True Bearings. Among the methods of determining true north are (1) by observation of Polaris at elongation or at any time, (2) by the altitude of the sun, (3) by the altitude of a star which is preferably nearly east or west of the observer and 25 to 30° above the horizon. If stars are observed in pairs, one east and one west of the observer and at about the same altitude, the errors of estimating refraction are eliminated. *Time* does not have to be recorded because the declinations of stars remain practically constant during the period.

The procedure for observing Polaris will not be presented here. It is fully explained in *solar ephemerides* published annually by manufacturers of instruments and is available upon request.

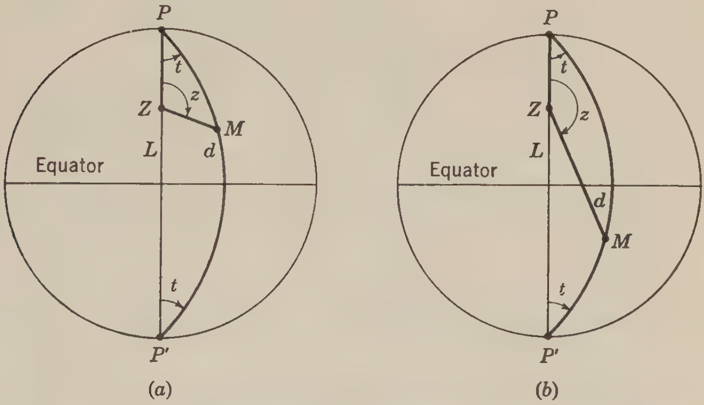


FIG. 144. (*a*) L and d same name (both positive). (*b*) L and d opposite name (L positive, d negative)

Figures 143 and 144, Eqs. (221) and (222), and the notation which follows apply to observations of either the *sun* or a *star*. For logarithmic computation (using Table 42),

$$\sec^2 \tfrac{1}{2}Z = \frac{\sec S \sec (S - p)}{\sec H \sec L}. \tag{221}$$

For machine computation (using Table 39),

$$\cos^2 \tfrac{1}{2}Z = \frac{\cos S \cos (S - p)}{\cos H \cos L}, \tag{222}$$

where Z = azimuth (or bearing) reckoned east or west from north depending upon whether observed body is in eastward or westward sky;

p = polar distance = $90° - d$;

d = declination obtained from solar ephemerides or a nautical almanac;

H = measured altitude corrected for refraction and parallax;

L = observer's latitude taken from a map.

$$S = \tfrac{1}{2}(p + H + L).$$

It should be noted that the cosine and secant functions of the negative quantities (if any) will always be *positive;* also log secants are *positive.* The fact that either Eq. (221) or (222) deals entirely with only one function is also an added element of simplicity as compared with other formulas. The work forms in the examples that follow expedite the calculations and help eliminate errors.

Equations (221) and (222) apply equally well to observations in the Southern Hemisphere. South latitudes must be used as *minus* quantities just as south declinations are negative. In all cases Z will be measured from the north, with values less than 180°, east or west of north.

Five-place tables give sufficient accuracy. Tables 39 and 42 will be used, the former rounded off to five places, if desired, to simplify interpolation.

Example 1. Assume the latitude (from map) = N30°09.0′.

Required: the sun's azimuth and the bearing of line AB.

Solution. With the transit set at point A of line AB, the field notes are:

Date: Sept. 11, 1950. Temperature, 80°F.

Watch on Eastern standard time.

All horizontal angles from line AB to the sun are measured clockwise.

The following are the averages of several readings:

POINTING TO SUN

Horizontal angle	Vertical angle	Time
146°30.2′	34°10.5′	3:33 P.M.

Standard time 75th meridian (referred to 0^h, midnight)	$15^h33.0^m$
Correction for time zone	+5
GCT	$20^h33.0^m$

d (for 0^h, Sept. 11, 1950)	4°52.0′
Change since 0^h; 20.55 × 0.95	− 19.5
d (for sun)	+ 4°32.5′

Measured altitude	34°10.5′
Refraction and parallax	− 1.2
True altitude, H	34°09.3′

$$\begin{array}{rlr}
d = & 4°32.5' & \textit{log sec} \\
 & & \\
p = & 85°27.5' & \\
H = & 34°09.5' \longrightarrow & 8{,}224 \\
L = & 30°09.0' \longrightarrow & 6{,}313 \\
 & 2)\overline{149°46.0'} \longrightarrow & \overline{14{,}537} \\
S = & \overline{74°53.0'} \longrightarrow & \overline{58{,}372} \\
S - p = & -\overline{10°34.5'} \longrightarrow & 744 \\
S - H = & 40°43.5' & \overline{59{,}116} \\
\text{Sum} = & \overline{30°09.0'} = L\ (\textit{check}) & 14{,}537 \\
 & & 2)\overline{44{,}579} \\
\tfrac{1}{2}Z = & 53°14.0' \longleftarrow & \overline{22{,}289}
\end{array}$$

$$\begin{array}{rl}
Z = & 106°28.0'\text{NW (see Fig. 145)} \\
 & \underline{146°30.2'} \\
 & \underline{252°58.2'} \\
 & \underline{180°} \\
 & \text{S}72°58.2'\text{E} = \text{true bearing line } AB \text{ (see Fig. 145)}
\end{array}$$

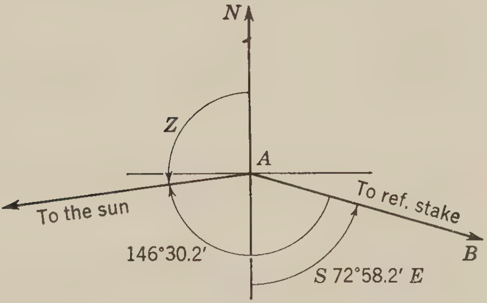


FIG. 145

Applying the foregoing data to Eq. (222), we have

$$\cos \tfrac{1}{2}Z = \sqrt{\frac{(0.26079)(0.98302)}{(0.82749)(0.86471)}} = 0.59856,$$

$$\tfrac{1}{2}Z = 53°14.0', \qquad Z = 106°28.0' \text{ NW.}$$

Example 2. Assume latitude (from map) = N35°10.5′. Required: the azimuth to the star Arcturus (in the west) and the bearing of line *AB*.

Solution. With the transit set at point A of line AB, the field notes are:

Date: July 3, 1950. Temperature, 75°F.

POINTING TO STAR ARCTURUS

Horizontal angle (clockwise from AB)	Vertical angle
48°10.5′	34°32.0′

d (for Arcturus)	+19°26.4′
Measured altitude, H	34°35.5′
Refraction correction	− 1.2′
True altitude, H	34°34.3′

$$\begin{array}{rlr}
d = & 19^\circ 26.4' & \textit{log sec} \\
 & & \\
p = & 70^\circ 33.6' & \\
H = & 34^\circ 34.3' \longrightarrow & 8{,}438 \\
L = & \underline{35^\circ 10.5'} \longrightarrow & \underline{8{,}757} \\
 & 2)\underline{140^\circ 18.4'} & 17{,}195 \rightarrow \\
S = & 70^\circ 09.2' \longrightarrow & 46{,}916 \\
S - p = & \overline{-0^\circ 24.4'} \longrightarrow & \underline{1} \\
S - H = & \underline{35^\circ 34.9'} & 46{,}917 \\
\text{Sum} = & 35^\circ 10.5' = L\ (\textit{check}) & \underline{17{,}195} \leftarrow \\
 & & 2)\underline{29{,}722} \\
\tfrac{1}{2}Z = & 44^\circ 44.9' \longleftarrow & 14{,}861 \\
Z = & 89^\circ 29.8'\text{NW} & \\
 & \underline{48^\circ 10.5'} & \\
 & 137^\circ 40.3' & \\
 & \text{S}42^\circ 19.7'\text{W} = \text{true bearing line } AB & \\
 & \overline{180^\circ 00.0'} &
\end{array}$$

Applying the foregoing data to Eq. (222), we have

$$\cos \tfrac{1}{2}Z = \sqrt{\frac{(0.33951)(0.99998)}{(0.82342)(0.81740)}} = 0.71022,$$

$$\tfrac{1}{2}Z = 44^\circ 44.85', \qquad Z = 89^\circ 29.7'\text{NW}.$$

This is practically the same result as that given by Eq. (221).

195. Funds Available and Relative Economy. As a supplement to the gasoline tax, transportation systems are usually financed either by the "pay-as-you-go" plan or on borrowed capital (bond issues). The former may be used successfully if the annual revenues are plentiful. It *should* be used in any case where the present bonded indebtedness is sufficiently heavy to affect adversely the public credit and the interest rate allowed the agency sponsoring the improvement, whether private or public.

The borrowing plan requires a definite program in advance. It permits relatively quick construction of projects large enough to secure the advantage of big contracts at lower prices; but in carrying out a mass-production plan, there is danger of cutting too short the time needed for preliminary planning and study. Cases may be cited where this error has resulted in faulty highway locations that soon became obsolete. With good engineering, the savings to the users—and the increased revenue to the owners of the facility—brought about by acceleration of the improvement program will usually more than compensate for the interest charges on the bond issues.

In recent years the *serial* method of bond retirement has become increasingly popular. Under this plan, a predetermined amount of the principal is scheduled to be paid each year, beginning at a certain time after the bonds are sold, usually not less than 1 year or more than 3. This plan of retirement does not require a sinking fund. The annual principal payments are usually made directly from current income, upon which they customarily have a prior claim. Bonds are retired in periods of 20 to 30 years, and many issues are sold at shorter terms.

When money is borrowed for a public or private project, it is customary to draw up a schedule of future payments on the debt, showing the interest and principal paid and the outstanding balance at the end of each year. Such a schedule is called an *amortization schedule.*

Example. A debt of $100,000 with interest at 4 percent is to be amortized by the payment of $12,000 at the end of each year. Required: a schedule of payments (see Table A) necessary to discharge the obligation.

Solution. Table A shows 10 equal annual payments plus a smaller payment at the end of the eleventh year. As a check on the work, it will be noted that the total of Column (5) equals the original principal; also the last entries of Columns (2) and (5) are equal.

TABLE A

Year (1)	Principal at beginning of year (2)	Annual payment (3)	Annual interest (4)	Paid on principal (5)
1	$100,000.00	$ 12,000.00	$ 4,000.00	$ 8,000.00
2	92,000.00	12,000.00	3,680.00	8,320.00
3	83,680.00	12,000.00	3,347.20	8,652.80
4	75,027.20	12,000.00	3,001.09	8,998.91
5	66,028.29	12,000.00	2,641.13	9,358.87
6	56,669.42	12,000.00	2,266.78	9,733.22
7	46,936.20	12,000.00	1,877.45	10,122.55
8	36,813.65	12,000.00	1,472.55	10,527.45
9	26,286.20	12,000.00	1,051.45	10,948.55
10	15,337.65	12,000.00	613.51	11,386.49
11	3,951.16	4,109.21	158.05	3,951.16
Totals. .	. . . .	$124,109.21	$24,109.21	$100,000.00

Relative Economy. For purposes of comparison or of economic evaluation, the cost of a transportation system, or any part of it, may be expressed in terms of capitalized cost or annual cost.

The *capitalized cost* is that sum which is sufficient to pay the initial cost of construction and provide for future maintenance and operation and renewals forever.

Let X = capitalized cost;
C = first cost = $A + B$, in which A is permanent part and B is temporary part;
M = annual maintenance and operating expense;
n = useful life of article;
i = annual rate of interest;
R = renewal or depreciation fund, being the amount which, when placed at compound interest during estimated useful life of parts which wear out or become obsolete, will yield a sum equal to replacement cost B, plus a perpetuating sum R for future renewals.

TABLE B. VALUES OF $\frac{i}{(1+i)^n - 1}$

n	i						
	2%	3%	$3\frac{1}{2}$%	4%	$4\frac{1}{2}$%	5%	6%
2	.495050	.492611	.491400	.490196	.488997	.487805	.485437
3	.326755	.323530	.321934	.320348	.318773	.317209	.314110
4	.242624	.239027	.237251	.235490	.233743	.232012	.228591
5	.192158	.188354	.186481	.184627	.182791	.180975	.177396
6	.158526	.154598	.152668	.150762	.148878	.147017	.143363
7	.134512	.130506	.128544	.126610	.124701	.122820	.119135
8	.116510	.112456	.110477	.108528	.106609	.104722	.101036
9	.102515	.098434	.096446	.094493	.092574	.090690	.087022
10	.091327	.087230	.085241	.083291	.081379	.079505	.075868
11	.082178	.078077	.076091	.074149	.072248	.070389	.066793
12	.074560	.070462	.068484	.066552	.064666	.062825	.059277
13	.068118	.064030	.062062	.060144	.058627	.056466	.052960
14	.062602	.058526	.056571	.054669	.052820	.051047	.047585
15	.057825	.053767	.051825	.049941	.049791	.046342	.042963
16	.053650	.049611	.047687	.045820	.044015	.042270	.038952
17	.049970	.045953	.044043	.042198	.040118	.038699	.035445
18	.046702	.042709	.040817	.038993	.037237	.035546	.032357
19	.043782	.039814	.037940	.036139	.034407	.032745	.029621
20	.041157	.037216	.035361	.033582	.031876	.030243	.027185
21	.038785	.034872	.033037	.031280	.029600	.027996	.025005
22	.036631	.032747	.030932	.029199	.027546	.025970	.023046
23	.034668	.030814	.029019	.027309	.025682	.024137	.021278
24	.032871	.029047	.027273	.025587	.023987	.022471	.019679
25	.031220	.027428	.025674	.024012	.022439	.020953	.018227
26	.029699	.025938	.024205	.022567	.021021	.019564	.016927
27	.028293	.024564	.022862	.021238	.019719	.018282	.015699
28	.026990	.023293	.021603	.020013	.018521	.017122	.014593
29	.025778	.022115	.020456	.018880	.017415	.016046	.013444
30	.024650	.021019	.019382	.017830	.016391	.015051	.012636

If the amount R is placed at compound interest as stated above, we have

$$R(1 + i)^n = B + R,$$

whence

$$R = \frac{B}{(1 + i)^n - 1}.$$

Therefore, the total capitalized cost is

$$X = C + \frac{M}{i} + \frac{B}{(1 + i)^n - 1}. \qquad (223)$$

The *annual cost* is the interest on the capitalized sum, that is,

$$\text{Annual cost} = Xi = Ci + M + \frac{Bi}{(1 + i)^n - 1}. \qquad (224)$$

Table B gives the quantity $\frac{i}{(1 + i)^n - 1}$ for various values of n and i.

APPENDIX A

ARC EXCESS

In Fig. 146, length of chord $AB = 2R \sin \frac{1}{2}\theta$; length of arc $AB = R\theta$ (θ in radians).

$$\text{Arc} - \text{chord} = \text{Arc excess} = R\theta - 2R \sin \tfrac{1}{2}\theta = R(\theta - 2 \sin \tfrac{1}{2}\theta). \quad (a)$$

Expressing $\sin \frac{1}{2}\theta$ as a series, (a) becomes

$$R\theta - R\left\{\theta - 2\left[\frac{\theta}{2} - \frac{(\theta/2)^3}{3!} + \frac{(\theta/2)^5}{5!} - \cdots\right]\right\} = R\left(\frac{1}{24}\theta^3 - \frac{1}{1{,}920}\theta^5 + \cdots\right). \quad (b)$$

By definition, $R = 100/D$ (D in radians); hence for 100-ft arc, $\theta = D$.

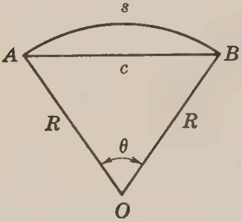


Fig. 146. Arc minus chord (arc excess).

Then $$\text{Arc excess (for 100-ft arc)} = \frac{100}{24}D^2 - \frac{100}{1{,}920}D^4 + \cdots. \quad (c)$$

Noting that D (radians) $= \frac{\pi}{180} D$ (degrees), we get

$$\text{Arc excess (100-ft arc)} = 0.1269\left(\frac{D}{10}\right)^2 - 0.48(10)^{-4}\left(\frac{D}{10}\right)^4 = 0.127\left(\frac{D}{10}\right)^2 \text{ approx.,} \quad (d)$$

where D is in degrees.

Equation (*d*) is true to the nearest hundredth of a foot for values of D less than 34°.

Equation (*b*) shows that for a given radius, the arc excess varies approximately as the cube of the arc. Hence it follows that the arc excess for any arc of length s ($s < 100$) is

$$\left(\frac{s}{100}\right)^3\left[0.127\left(\frac{D}{10}\right)^2\right]. \tag{e}$$

For 50-ft arcs, (*e*) becomes

$$\frac{1}{8}\left[0.127\left(\frac{D}{10}\right)^2\right] = 0.0159\left(\frac{D}{10}\right)^2. \tag{f}$$

If $D = 50°$ (arc def.), (*f*) gives $0.3975 = 0.40$ ft. and the corresponding length of chord $= 49.60$ ft. As a check,

$$\text{Chord length } = 2R \sin 12°30' = 2\left(\frac{5{,}729.58}{50}\right)(0.2164396) = 49.60 \text{ ft.}$$

Hence (*f*) is sufficiently exact for values of D as high as 50°.

If $D = 80°$ (*arc* definition), the arc excess for 30-ft arcs =

$$\left(\tfrac{30}{100}\right)^3\left[0.127\left(\tfrac{80}{10}\right)^2\right] = 0.22 \text{ ft.}$$

Then the corresponding length of chord is $30.00 - 0.22 = 29.78$ ft, as given in Table 7.

APPENDIX B

UNSYMMETRICAL VERTICAL CURVE

In Fig. 147, the diameter (or axis) VD of the parabola intersects the parallel chords AB and $A'B'$ at F and D, respectively.

The triangles VAB and $VA'B'$ are similar; hence

$$\frac{VB'}{VB} = \frac{A'B'}{AB} = \frac{1}{2}.$$

Draw a vertical through V intersecting $A'B'$ and AB at C and M, respectively. Then, from similar triangles,

$$\frac{VC}{VM} = \frac{VB'}{VB} = \frac{1}{2},$$

that is, $VC = CM = e$, say.

In Chap. 5, the unsymmetrical curve ACB consists of two parabolic arcs with a common tangent at C. The two arcs AC and CB of lengths l_1 and l_2, respectively, are therefore compounded at C.

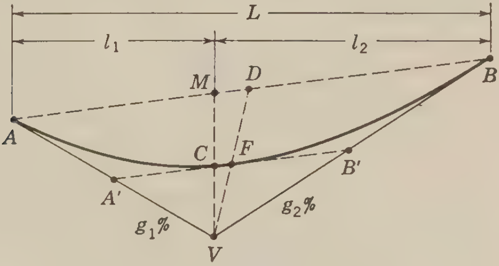


FIG. 147. Unsymmetrical parabola.

By the notation of Art. 74, the grade of the tangent to the curve at C (here called g_c) is parallel to the chord AB; hence

$$g_c = \frac{g_1 l_1 + g_2 l_2}{l_1 + l_2}. \tag{a}$$

Then

$$r_1 = \frac{g_c - g_1}{l_1} = \frac{l_2}{l_1} \frac{g_2 - g_1}{L}. \tag{b}$$

Likewise,

$$r_2 = \frac{g_2 - g_c}{l_2} = \frac{l_1}{l_2} \frac{g_2 - g_1}{L}. \tag{c}$$

APPENDIX C

TRANSITION SPIRAL

Derivation of the Coordinates y and x. Expanding the *sine* and *cosine* into series (see Figure 67), we have

$$dy = dl \sin\theta = dl\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \frac{\theta^9}{9!} - \cdots\right); \qquad (a)$$

$$dx = dl \cos\theta = dl\left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \frac{\theta^8}{8!} - \cdots\right). \qquad (b)$$

Substituting $\theta = (l/l_s)^2\theta_s$ in (a), we have

$$dy = dl\left[\theta_s\left(\frac{l}{l_s}\right)^2 - \frac{\theta_s^3}{3!}\left(\frac{l}{l_s}\right)^6 + \frac{\theta_s^5}{5!}\left(\frac{l}{l_s}\right)^{10} - \frac{\theta_s^7}{7!}\left(\frac{l}{l_s}\right)^{14} + \frac{\theta_s^9}{9!}\left(\frac{l}{l_s}\right)^{18}\right]. \qquad (c)$$

Integrating, reducing, and restoring θ, we get

$$y = l\left(\frac{\theta}{3} - \frac{\theta^3}{42} + \frac{\theta^5}{1{,}320} - \frac{\theta^7}{75{,}600} + \frac{\theta^9}{6{,}894{,}720}\right). \qquad (d)$$

Likewise, substituting in (b), integrating, and restoring θ, we get

$$x = l\left(1 - \frac{\theta^2}{10} + \frac{\theta^4}{216} - \frac{\theta^6}{9{,}360} + \frac{\theta^8}{685{,}440}\right). \qquad (e)$$

Converting radians into degrees and reducing, we get

$$y = \frac{l}{100}[0.5817764173\theta - 0.126585165\theta^3(10)^{-4} + 0.122691057\theta^5(10)^{-9} - 0.652559\theta^7(10)^{-15}], \qquad (f)$$

$$x = \frac{l}{100}[100 - 0.3046174198\theta^2(10)^{-2} + 0.429591539\theta^4(10)^{-7} - 0.301987076\theta^6(10)^{-12} + 0.135721\theta^8(10)^{-17}], \qquad (g)$$

where l is in feet and θ is measured in degrees.

Note: The last items in the series are practically negligible.

Derivation of the Formula for ϕ

$$\tan \phi = \frac{y}{x}, \tag{h}$$

where y and x are given by Eqs. (f) and (g).

Knowing l and θ, y and x are determined; then

$$\phi = \arctan \frac{y}{x} \tag{i}$$

Numerous calculations have shown that

$$\phi = \frac{\theta}{3} - C_s, \tag{j}$$

where C_s is a small correction that is practically negligible for values of θ under 16° (see Art. 84).

Derivation of p and k

$$p = y_c - R_c(1 - \cos \theta_s), \tag{k}$$

and

$$k = x_c - R_c \sin \theta_s. \tag{l}$$

Applying Eqs. (f) and (g), letting $R_c = \frac{1}{2}l_s/\theta_s$, and expanding $\sin \theta_s$ and $\cos \theta_s$ into series, we get

$$p = l_s\left(\frac{\theta_s}{12} - \frac{\theta_s^3}{336} + \frac{\theta_s^5}{15,840} - \frac{\theta_s^7}{1,209,600} + \cdots\right), \tag{m}$$

$$k = l_s\left(\frac{1}{2} - \frac{\theta_s^2}{60} + \frac{\theta_s^4}{2,160} - \frac{\theta_s^6}{131,040} + \cdots\right), \tag{n}$$

in which θ_s is measured in radians; or

$$p = l_s[0.001454440\theta_s - 1.5823150\theta_s^3\,(10)^{-8} + 1.0224260\theta_s^5\,(10)^{-13} - \cdots], \tag{o}$$

$$k = l_s[0.5 - 5.0769570\theta_s^2\,(10)^{-6} + 4.2959150\theta_s^4\,(10)^{-11} - \cdots], \tag{p}$$

in which θ_s is measured in degrees.

APPENDIX D

SPIRAL APPLIED TO UNSHIFTED CIRCULAR CURVE

The parts Δ, D, R, etc., of an existing circular curve are known. Let it be assumed that the PC and PT are established. By compounding this curve with a sharper curve, as in Fig. 75, to permit a spiral to be inserted between the original tangents and the new curves, the given curve is retained in its existing location along any central portion of its length.

It is required to derive formulas for the spiral functions l_s, a, D_c, x_c, y_c, LT, and ST, where

l_s = length of spiral;
a = distance from PC to TS, or from PT to ST;
D_c = degree of curve of sharpened circular curve;
x_c^0, y_c^0 = coordinates of spiral of length unity, given in Table 15 or by Eqs. (f) and (g) in Appendix C;
x_c, y_c = actual coordinates of spiral;
LT = long tangent;
ST = short tangent.

Since y_c is the same for both spiral and circle, we have (Fig. 75)

$$y_c = R \text{ vers } \theta_s; \qquad (a)$$

then

$$l_s = 100\,\frac{y_c}{y_c{}^0} = \frac{R \text{ vers } \theta_s}{y_c{}^0}. \qquad (b)$$

Also

$$D_c = \frac{200\theta_s}{l_s}, \qquad (c)$$

$$x_c = x_c{}^0\,\frac{l_s}{100}, \qquad (d)$$

$$a = x_c - R \sin \theta_s, \qquad (e)$$

$$\text{ST} = \frac{y_c}{\sin \theta_s} = \frac{R(1 - \cos \theta_s)}{\sin \theta_s} = R \tan \tfrac{1}{2}\theta_s, \qquad (f)$$

$$\text{LT} = a + \text{ST}. \qquad (g)$$

Table 26 gives these functions for applying spirals to existing 1° curves. The tabular quantities are to be divided by the actual degree of the existing curve, an exception being D_c, which is obtained by multiplying the tabular quantity by the degree of curve D.

APPENDIX E

COORDINATES X AND Y OF COMBINING SPIRAL

To find coordinates X and Y of a *combining spiral* with reference to point A as origin and AV as X axis (see Fig. 80, also Table 22).

Using the notation of Art. 96 and repeating Eq. (104), we have

$$\theta = D_1 \times \frac{l}{100} + \theta_s \times \left(\frac{l}{L_s}\right)^2. \qquad (a)$$

Also, $$dy = dl \times \sin\theta = dl\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots\right); \qquad (b)$$

and $$dx = dl \times \cos\theta = dl\left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \cdots\right). \qquad (c)$$

Substituting (a) in (b), we have

$$Y = \int_0^{L_s}\left(\frac{D_1}{100}\,l + \frac{\theta_s}{L_s^2}\,l^2\right)dl - \frac{1}{3!}\int_0^{L_s}\left(\frac{D_1}{100}\,l + \frac{\theta_s}{L_s^2}\,l^2\right)^3 dl + \frac{1}{5!}\int_0^{L_s}\left(\frac{D_1}{100}\,l + \frac{\theta_s}{L_s^2}\,l^2\right)^5 dl. \qquad (d)$$

Substituting (a) in (c), we have

$$X = L_s - \frac{1}{2!}\int_0^{L_s}\left(\frac{D_1}{100}\,l + \frac{\theta_s}{L_s^2}\,l^2\right)^2 dl + \frac{1}{4!}\int_0^{L_s}\left(\frac{D_1}{100}\,l + \frac{\theta_s}{L_s^2}\,l^2\right)^4 dl - \frac{1}{6!}\int_0^{L_s}\left(\frac{D_1}{100}\,l + \frac{\theta_s}{L_s^2}\,l^2\right)^6 dl. \qquad (e)$$

Integrating, changing from radians to degrees, and reducing, we get

$$Y = \frac{L_s}{100}\left[\theta_s \times A' - \left(\frac{\theta_s}{10}\right)^3 \times B' + \left(\frac{\theta_s}{10}\right)^5 \times C'\right], \qquad (f)$$

where $A' = \frac{1}{2}N + \frac{1}{3}$;

$$B' = \left[\tfrac{1}{4}N^3 + \tfrac{3}{5}N^2 + \tfrac{1}{2}N + \tfrac{1}{7}\right] \times \frac{1}{3!}$$

$$C' = \left[\tfrac{1}{6}N^5 + \tfrac{5}{7}N^4 + \tfrac{5}{4}N^3 + \tfrac{10}{9}N^2 + \tfrac{1}{2}N + \tfrac{1}{11}\right] \times \frac{1}{5!}$$

$$N = \frac{2D_1}{D_2 - D_1}.$$

Integrating, changing from radians to degrees, and reducing, we get

$$X = \frac{L_s}{100}\left[100 - \left(\frac{\theta_s}{10}\right)^2 \times D' + \left(\frac{\theta_s}{10}\right)^4 \times E' - \left(\frac{\theta_s}{10}\right)^6 \times F'\right];$$

where $D' = \left[\frac{1}{3}N^2 + \frac{1}{2}N + \frac{1}{5}\right] \times \frac{1}{2!},$

$$E' = \left[\frac{1}{5}N^4 + \frac{2}{3}N^3 + \frac{6}{7}N^2 + \frac{1}{2}N + \frac{1}{7}\right] \times \frac{1}{4!};$$

$$F' = \left[\frac{1}{7}N^6 + \frac{3}{4}N^5 + \frac{5}{3}N^4 + 2N^3 + \frac{15}{11}N^2 + \frac{1}{2}N + \frac{1}{13}\right] \times \frac{1}{6!};$$

$$A = A'\left(\frac{\pi}{1.8}\right), B = B'\left(\frac{\pi}{1.8}\right)^3\left(\frac{1}{10}\right); C = C'\left(\frac{\pi}{1.8}\right)^5\left(\frac{1}{10}\right)^3;$$

$$D = D'\left(\frac{\pi}{1.8}\right)^2; E = E'\left(\frac{\pi}{1.8}\right)^4\left(\frac{1}{10}\right)^2; F = F'\left(\frac{\pi}{1.8}\right)^6\left(\frac{1}{10}\right)^4;$$

where A, B, C, D, E, F are evaluated (correctly) in Table 22.

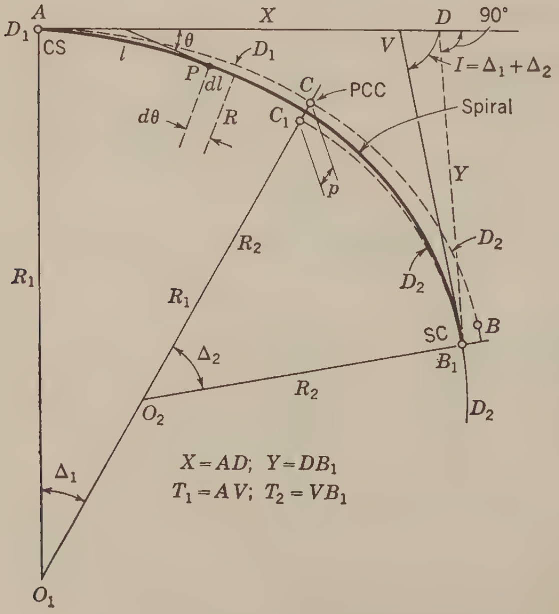

A
D_1
X
D
90°
CS
l
θ
D_1
V
$I = \Delta_1 + \Delta_2$
P
dl
C
PCC
Spiral
dθ
R
C_1
p
Y
R_2
D_2
R_1
R_1
D_2
SC
B
B_1
Δ_2
R_2
O_2
D_2
$X = AD$; $Y = DB_1$
$T_1 = AV$; $T_2 = VB_1$
Δ_1
O_1

FIG. 80. Combining spiral AB_1, (repeated).

Alternate Procedure

Let $J = \frac{1}{2}[(D_2 - D_1)/D_1]1/L_s$. Then Eq. (99) becomes

$$\theta = \frac{l}{100}\left[D_1 + \tfrac{1}{2}(D_2 - D_1)\frac{l}{L_s}\right] = \frac{D_1 l}{100}(1 + Jl)$$

$$= \frac{D_1 l}{100}(1 + K), \qquad (g)$$

where $K = Jl$.

Substituting Eq. (g) in Eq. (b), and expanding, integrating, and factoring,

$$\begin{aligned} y = {} & \left(\frac{D_1}{100}\right) l^2(\tfrac{1}{2} + \tfrac{1}{3}K) - \frac{1}{3!}\left(\frac{D_1}{100}\right)^3 l^4[\tfrac{1}{4} + \tfrac{3}{5}K + \tfrac{3}{6}K^2 + \tfrac{1}{7}K^3] \\ & + \frac{1}{5!}\left(\frac{D_1}{100}\right)^5 l^6[\tfrac{1}{6} + \tfrac{5}{7}K + \tfrac{10}{8}K^2 + \tfrac{10}{9}K^3 + \tfrac{5}{10}K^4 + \tfrac{1}{11}K^5] \\ & - \frac{1}{7!}\left(\frac{D_1}{100}\right)^7 l^8[\tfrac{1}{8} + \tfrac{7}{9}K + \tfrac{21}{10}K^2 + \tfrac{35}{11}K^3 + \tfrac{35}{12}K^4 + \tfrac{21}{13}K^5 \\ & + \tfrac{7}{14}K^6 + \tfrac{1}{15}K^7] + \cdots \qquad (h) \end{aligned}$$

Equation (h) gives the ordinate (y) in feet to any point on the spiral AB_1 (Fig. 80). D_1 is measured in radians; multiply D_1, $D_1{}^3$, $D_1{}^5$, $D_1{}^7$ by $\pi/180$, $(\pi/180)^3$, $(\pi/180)^5$, $(\pi/180)^7$, respectively, if D_1 is in degrees.

Similarly, from Eq. (c), we get

$$\begin{aligned} x = {} & l - \frac{1}{2}\left(\frac{D_1}{100}\right)^7 l^3[\tfrac{1}{3} + \tfrac{2}{4}K + \tfrac{1}{5}K^2] + \frac{1}{4!}\left(\frac{D_1}{100}\right)^4 l^5[\tfrac{1}{5} + \tfrac{4}{6}K \\ & + \tfrac{6}{7}K^2 + \tfrac{4}{8}K^3 + \tfrac{1}{9}K^4] - \frac{1}{6!}\left(\frac{D_1}{100}\right)^6 l^7[\tfrac{1}{7} + \tfrac{6}{8}K + \tfrac{15}{9}K^2 \\ & + \tfrac{20}{10}K^3 + \tfrac{15}{11}K^4 + \tfrac{6}{12}K^5 + \tfrac{1}{13}K^6] + \cdots \qquad (i) \end{aligned}$$

Equation (i) gives the value of x to any point on the spiral AB_1 (Fig. 80). D_1 is in radians; multiply each D_1 by $\pi/180$ if in degrees.

If $l = L_s$, then $x = X_s$, $y = Y_s$, etc.

Example. If $D_1 = 4°$, $D_2 = 10°$, and $L_s = 300$ ft, then $K = Jl = \frac{1}{2}(\frac{6}{4})\frac{1}{300} \times 300 = \frac{3}{4}$.

By Eqs. (h) and (i), $Y_s = 46.6475$ ft; $X_s = 294.6334$ ft. These results are identical with those obtained independently by Table 22.

APPENDIX F

MULTICENTERED COMPOUND CURVE

Figure 148 is drawn as a three-centered compound curve, but the expressions derived herewith may be easily extended to four or more centers.

Let R_1, R_2, R_3 equal the radii of arcs AP_1, P_1P_2, P_2P_3 with central angles of Δ_1, Δ_2, Δ_3 and long chords of $2R_1 \sin \frac{1}{2}\Delta_1$, $2R_2 \sin \frac{1}{2}\Delta_2$, and $2R_3 \sin \frac{1}{2}\Delta_3$.

At P_1 and P_2 draw lines P_1E and P_2F parallel to the initial tangent AX, also lines tangent to the curve at these points. Let AN_1, AN_2, and AN_3 represent the abscissas of points P_1, P_2, and P_3 with reference to point A as origin and axis AX, while N_1P_1, N_2P_2, and N_3P_3 are the corresponding ordinates. Then we have (Fig. 148)

$$x_1 = AN_1 = R_1 \sin \Delta_1, \qquad y_1 = N_1P_1 = R_1 \operatorname{vers} \Delta_1;$$
$$x_2 = N_1N_2 = (2R_2 \sin \tfrac{1}{2}\Delta_2) \cos (\Delta_1 + \tfrac{1}{2}\Delta_2),$$
$$y_2 = EP_2 = (2R_2 \sin \tfrac{1}{2}\Delta_2) \sin (\Delta_1 + \tfrac{1}{2}\Delta_2);$$
$$x_3 = N_2N_3 = (2R_3 \sin \tfrac{1}{2}\Delta_3) \cos (\Delta_1 + \Delta_2 + \tfrac{1}{2}\Delta_3),$$
$$y_3 = FP_3 = (2R_3 \sin \tfrac{1}{2}\Delta_3) \sin (\Delta_1 + \Delta_2 + \tfrac{1}{2}\Delta_3).$$

Hence,

$$X_1 = AN_3 = x_1 + x_2 + x_3; \qquad Y_1 = N_3P_3 = y_1 + y_2 + y_3.$$

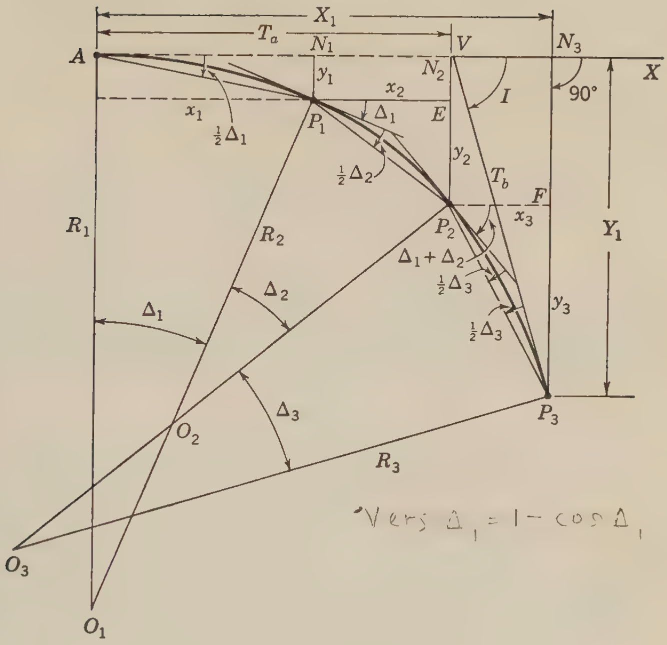
Note: These values of X_1 and Y_1 will check with Eqs. (46) and (47), determined by independent methods.

Extending the foregoing procedure to a four-centered curve, we get

$$x_4 = (2R_4 \sin \tfrac{1}{2}\Delta_4) \cos (\Delta_1 + \Delta_2 + \Delta_3 + \tfrac{1}{2}\Delta_4);$$
$$y_4 = (2R_4 \sin \tfrac{1}{2}\Delta_4) \sin (\Delta_1 + \Delta_2 + \Delta_3 + \tfrac{1}{2}\Delta_4); \text{ etc.}$$

P_1, P_2, P_3 and other points on the curve may be located by offsets or deflection angles. In the latter case, the transit would be set in turn at A, P_1, and P_2, while establishing a series of circular arcs as parts of a compound curve.

Example. Given $\Delta_1 = \Delta_3$; $R_1 = R_3 = 100$ ft; $R_2 = 20$ ft; and *offset* $(= y_1) = 2.5$ ft. Required: inner edge of pavement for 90° turn $(I = 90°)$.


X_1
T_a
N_1
V
N_3
A
X
y_1
N_2
I
90°
x_2
x_1
Δ_1
E
$\frac{1}{2}\Delta_1$
P_1
$\frac{1}{2}\Delta_2$
y_2
T_b
F
x_3
R_1
R_2
P_2
Y_1
$\Delta_1+\Delta_2$
$\frac{1}{2}\Delta_3$
Δ_2
y_3
Δ_1
$\frac{1}{2}\Delta_3$
Δ_3
O_2
P_3
R_3
O_3
O_1

Vers $\Delta_1 = 1 - \cos \Delta_1$

FIG. 148. Compound curve.

Solution. $y_1 = R_1 \text{ vers } \Delta_1 = 2.5$, from which vers $\Delta_1 = 0.025$ and $\Delta_1 = 12°50'20''$. Then $x_1 = R_1 \sin \Delta_1 = 22.221$ = distance N_1A (Fig. 148) locating the PC of the curve.

Next, since $I = 2\Delta_1 + \Delta_2$, $\Delta_2 = 90 - 2 \times \Delta_1 = 64°19'20''$.

Because of the symmetrical arrangement with $I = 90°$, it follows that $X_1 = Y_1 = T_a = T_b$. Also, since $X_1 = x_1 + x_2 + x_3$, we get

$$x_1 = 22.221$$
$$x_2 = (40 \times \sin 32°09'40'') \cos 45° = 15.056$$
$$x_3 = (200 \times \sin 6°25'10'') \cos 83°34'50'' = \underline{\ 2.500}$$
$$39.777 = X_1$$

Using the above data, the actual layout of the curve is simplified.

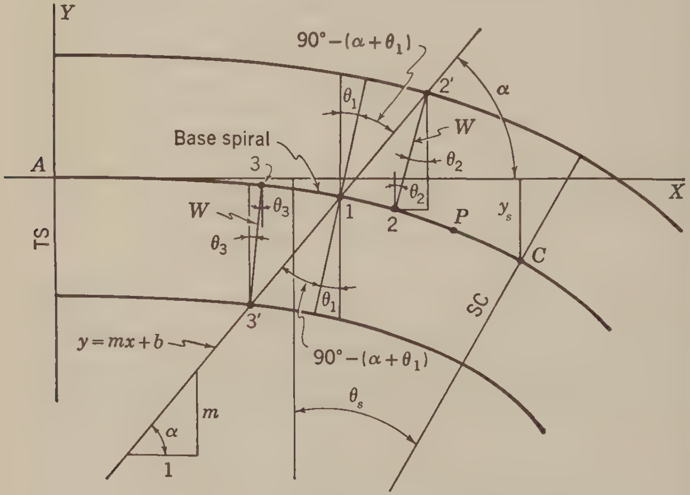
APPENDIX G

INTERSECTION OF A STRAIGHT LINE WITH A SPIRAL

Let x and y be the coordinates of any point on the center-line spiral (Fig. 149) referred to the TS as origin.

R = variable radius of the center-line spiral (the base spiral) referred to any point P distant l from the TS (at point A).

θ = angle subtended by spiral arc AP.



Y
$90° - (\alpha + \theta_1)$
θ_1
2'
α
W
Base spiral
A
3
θ_2
θ_2
X
W
θ_3
1
2
P
y_s
TS
θ_3
C
θ_1
SC
3'
$y = mx + b$
$90° - (\alpha + \theta_1)$
θ_s
m
α
1

Fig. 149

At point C, the SC, arc $AC = l_s$ and θ becomes θ_s, the so-called "spiral angle," and l_s is the total length of spiral connecting a tangent with a circular curve of degree D_c.

The following are well-known expressions: $\theta = (l/l_s)^2 \times \theta_s$; $l_s = 200(\theta_s/D_c)$.

Let W = radial distance from the base spiral to the outer (or inner) parallel spiral.

For an assigned θ the actual values of the coordinates of a point on the base spiral are lx and ly, where x and y (for the unit spiral) are as given in Table 15 or by the expanded series

$$y = \frac{\theta}{3} - \frac{\theta^3}{42} + \frac{\theta^5}{1{,}320}; \qquad x = 1 - \frac{\theta^2}{10} + \frac{\theta^4}{216} - \frac{\theta^6}{9{,}360}. \tag{a}$$

Let the equation of the line be designated by $y = mx + b$, or

$$y - y'' = m(x - x''), \tag{b}$$

where the origin of coordinates is at point A (the TS). Assume that the line crosses the X axis at a given point distant a from the origin, that is, at the point where $x'' = a$, $y'' = 0$. Then Eq. (b) becomes

$$y = m(x - a). \tag{c}$$

At point 1, the intersection of the line with the base spiral, let the corresponding values of l and θ equal l_1 and θ_1, while the coordinates are x_1 and y_1 respectively.

Since both the TS and the SC of the outer and inner spirals must be radially opposite the base spiral at these points (when normally they are not), then their true spiral characteristics are slightly modified, thus causing a slight (but inappreciable) compounding at the SC.

From points 2′ and 3′, where the line crosses the outer and inner spirals, let radial lines of length W extend to points 2 and 3 of the base spiral. Hence (see Fig. 149), $y_2' = W \cos \theta_2 - l_2 y_2$ and $x_2' = l_2 x_2 + W \sin \theta_2$, where l_2, θ_2, y_2, and x_2 apply to point 2 on the base spiral. Likewise, for the inner spiral, we have $y_3' = -(W \cos \theta_3 + l_3 y_3)$, $x_3' = l_3 x_3 - W \sin \theta_3$.

Given the following data: l_s, θ_s, W, a, m (slope of the given line), and α (where $\tan \alpha = m$), the coordinates of the intersection points 1, 2′, and 3′ may be derived from Eqs. (g), (h), and (i) below [leading to Eqs. (n), (o), and (p)], which are adapted to electronic computation.

Since the given line passes through points 1, 2′, and 3′, its equation is satisfied by the coordinates of these points; that is,

$$-l_1 y_1 - m l_1 x_1 + ma = 0; \tag{d}$$

$$y_2' - m x_2' + ma = 0,$$

or

$$W \cos \theta_2 - l_2 y_2 - m(l_2 x_2 + W \sin \theta_2) + ma = 0; \tag{e}$$

$$y_3' - m x_3' + ma = 0,$$

or

$$-(W \cos \theta_3 + l_3 y_3) - m(l_3 x_3 - W \sin \theta_3) + ma = 0. \tag{f}$$

Omitting subscripts for the moment and reducing, Eqs. (*d*), (*e*), and (*f*) become

$$l(y + mx) - ma = 0; \tag{g}$$

$$l(y + mx) - [W(\cos\theta - m\sin\theta) + ma] = 0; \tag{h}$$

$$l(y + mx) - [W(m\sin\theta - \cos\theta) + ma] = 0. \tag{i}$$

Substituting the expanded series for y and x in Eq. (*h*), we have

$$l\left[\left(\frac{\theta}{3} - \frac{\theta^3}{42} + \frac{\theta^5}{1{,}320}\right) + m\left(1 - \frac{\theta^2}{10} + \frac{\theta^4}{216} - \frac{\theta^6}{9{,}360}\right)\right] - [W(\cos\theta - m\sin\theta) + ma] = 0. \tag{j}$$

For an approximation, Taylor's theorem (given in treatises on the calculus) will be used.* Applying this theorem to Eq. (*j*), let $\theta_2 = \theta + K$, where θ is the solution, that is, $f(\theta) = 0$, and θ_2 is an approximation. Then

$$K \times f'(\theta) = f(\theta_2) = E, \tag{k}$$

where E is the error.

Noting that $l = (l_s/\sqrt{\theta_s})\sqrt{\theta}$, differentiating Eq. (*j*), neglecting terms θ^5 and higher, and reducing, we get

$$f'(\theta) = \frac{l_s}{\sqrt{\theta_s}}\left[\tfrac{1}{2}\theta^{\frac{1}{2}} - \tfrac{1}{12}\theta^{\frac{5}{2}} - m\left(\tfrac{1}{2}\theta^{-\frac{1}{2}} - \tfrac{1}{4}\theta^{\frac{3}{2}} + \tfrac{1}{48}\theta^{\frac{7}{2}}\right)\right] + W(\sin\theta + m\cos\theta) = E. \tag{l}$$

For this approximation put $\theta = \theta_2$ and divide both sides of Eq. (*k*) by $\sqrt{\theta}$ or $\sqrt{\theta_2}$. Then

$$K\left\{\frac{l_s}{\sqrt{\theta_s}}\left[\tfrac{1}{2} - \tfrac{1}{12}\theta_2{}^2 + \frac{m}{\theta_2}\left(\tfrac{1}{2} - \tfrac{1}{4}\theta_2{}^2 + \tfrac{1}{48}\theta_2{}^4\right)\right] + \frac{W}{\sqrt{\theta_2}}(\sin\theta_2 + m\cos\theta_2)\right\}$$
$$= \frac{l_s}{\sqrt{\theta_s}}\left[\tfrac{1}{3}\theta_2 - \tfrac{1}{42}\theta_2{}^3 + \frac{1}{1{,}320}\theta_2{}^5 + m\left(1 - \tfrac{1}{10}\theta_2{}^2 + \tfrac{1}{216}\theta_2{}^4 - \frac{1}{9{,}360}\theta_2{}^6\right)\right] - \frac{1}{\sqrt{\theta_2}}[W(\cos\theta_2 - m\sin\theta_2) + ma]. \tag{m}$$

---

* Thanks to L. A. Colbert for the suggestion.

Equation (m) can be solved directly, and the correction K found. The right-hand side of Eq. (m) has to be taken to more terms since greater accuracy is required, for this must eventually be computed to almost zero, whereas the coefficient of K in the left-hand side remains comparatively large, and as K approaches zero, such accuracy is not required. Then $\theta = \theta_2 - K$ = recurring value of θ_2.

For the sake of brevity, multiply by $\sqrt{\theta_s}$ and let

$$C_1 = l_s\left[\tfrac{1}{2} - \tfrac{1}{12}\theta_1^2 + \frac{m}{\theta_1}\left(\tfrac{1}{2} - \tfrac{1}{4}\theta_1^2 + \tfrac{1}{48}\theta_1^4\right)\right],$$

$$C_2 = l_s\left[\tfrac{1}{2} - \tfrac{1}{12}\theta_2^2 + \frac{m}{\theta_2}\left(\tfrac{1}{2} - \tfrac{1}{4}\theta_2^2 + \tfrac{1}{48}\theta_2^4\right)\right] + W\frac{\sqrt{\theta_s}}{\sqrt{\theta_2}}(\sin\theta_2 + m\cos\theta_2),$$

$$C_3 = l_s\left[\tfrac{1}{2} - \tfrac{1}{12}\theta_3^2 + \frac{m}{\theta_3}\left(\tfrac{1}{2} - \tfrac{1}{4}\theta_3^2 + \tfrac{1}{48}\theta_3^4\right)\right] + W\frac{\sqrt{\theta_s}}{\sqrt{\theta_3}}(\sin\theta_3 + m\cos\theta_3).$$

As values θ_1, θ_2, θ_3 in turn are assigned for θ, the corresponding y's and x's are y_1, x_1; y_2, x_2; y_3, x_3.

For a further simplification, let Eq. (a) represent the y's and x's in terms of special values of θ as indicated by the subscripts.

Solutions by iteration of Eqs. (n), (o), and (p) will give the desired points of intersection 1, 2′, and 3′:

$$KC_1 = l_s(y_1 + mx_1) - \frac{\sqrt{\theta_s}}{\sqrt{\theta_1}} \times ma, \qquad (n)$$

in which $\theta = \theta_1 - K$ = recurring value of θ_1;

$$KC_2 = l_s(y_2 + mx_2) - \frac{\sqrt{\theta_s}}{\sqrt{\theta_2}}[W(\cos\theta_2 - m\sin\theta_2) + ma], \qquad (o)$$

in which $\theta = \theta_2 - K$ = recurring value of θ_2;

$$KC_3 = l_s(y_3 + mx_3) - \frac{\sqrt{\theta_s}}{\sqrt{\theta_3}}[W(m\sin\theta_3 - \cos\theta_3) + ma], \qquad (p)$$

in which $\theta = \theta_3 - K$ = recurring value of θ_3.

Preliminary approximations for the θ's in applying Eqs. (n), (o), and (p) are as follows $\left[\text{with } y_s = l_s(\tfrac{1}{3}\theta_s)\left(\dfrac{\pi}{180}\right)\right]$:

$$\theta_1 = \left(\frac{l_1}{l_s}\right)^2 \times \theta_s,$$

where $l_1 = a - (a/l_s)^3 \times y_s \times \dfrac{1}{m} = a\left[1 - (a/l_s)^2\left(\dfrac{\theta_s}{3m}\right)\left(\dfrac{\pi}{180}\right)\right]$;

$$\theta_2 = \left(\frac{l_2}{l_s}\right)^2 \times \theta_s,$$

where $l_2 = l_1 + W \cot(\alpha + \theta_1)$;

$$\theta_3 = \left(\frac{l_3}{l_s}\right)^2 \times \theta_s,$$

where $l_3 = l_1 - W \cot(\alpha + \theta_1)$.

As each point of a bridge pier or whatnot is determined, its adjacent distances to the previously computed points may be compared with the results obtained by the use of Eqs. (b) and (c) (Art'l 93), which in itself may be considered an alternative procedure. This statement applies also to the use of Eq. (a) of the combining spiral (see Art'l 97).

Example. Given $l_s = 300$ ft, $\theta_s = 9°$, $W = 15$ ft, $m = 2$ ($\alpha = 63°26.3'$), $a = 160$ ft. Required: (1) x_1, y_1; (2) x'_2, y'_2; (3) x'_3, y'_3.

Solution. Because of limited space, the solution will not be carried beyond the second approximation for the θ's. A summary of the results is given below.

(1) $l_1 = 158.81$; $\theta_1 = 2°31.3'$; $C_1 = 6{,}958.32$; $K = -0.0000304$ radian.
$l_1 = 158.86$; $\theta_1 = 2°31.44'$; $x'_1 = 158.8291$; $y_1 = -2.3324$.
(2) $l_2 = 165.55$; $\theta_2 = 2°44.4'$; $C_2 = 6{,}473.76$; $K = +0.00003198$ radian.
$l_2 = 165.49$; $\theta_2 = 2°44.3'$; $x'_2 = 166.1688$; $y'_2 = 12.3469$.
(3) $l_3 = 152.12$; $\theta_3 = 2°18.8'$; $C_3 = 7{,}632.36$; $K = -0.0000004$ radian.
$l_3 = 152.12$; $\theta_3 = 2°18.84'$; $x'_3 = 151.4896$; $y'_3 = -17.0355$.

$$\textit{Check: } m = \frac{y'_2 - y'_3}{x'_2 - x'_3} = \frac{29.3824}{14.6792} = 2.0016 \text{ (exact value} = 2).$$

U.S. 52, from Winston-Salem to Pilot Mountain, N.C.

EXPLANATION OF THE TABLES

Table 1. Functions of Unit Circular Curves. Contains radii, tangents, externals, long chords, and middle ordinates for the unit circular curve of length 100 ft, also tangents and externals for the unit curve of radius 10,000 ft. For curves of any length L, multiply the tabular quantities for the 100-ft curve by $L/100$; for curves of any radius R, multiply the tabular quantities for the 10,000-ft curve by $R/10{,}000$. When interpreted properly, this table is applicable to both the chord and the arc definition.

To obtain quantities to the nearest minute without interpolation, the second part of this table should be used in conjunction with Tables 40 and 41, since for $R = 10{,}000$, $T = 10{,}000 \times \tan \frac{1}{2}\Delta$ and $E = 10{,}000 \times \text{exsec } \frac{1}{2}\Delta$.

When L is given, $D = (100/L) \times \Delta$; when R is given, L is found in Table 13.

Table 2. Tangents and Externals for a 1° Curve. T and E are given for every minute of the external angle Δ. For any other degree of curve D, divide the tabular quantities by D. This table is applicable to both the chord and the arc definition.

Table 3. Corrections for Tangent Distances in Table 2. Applying only to the chord definition of D. Based on $(R_c - R_a) \tan \frac{1}{2}\Delta$, where R_c = radius as per chord definition and R_a = radius as per arc definition.

Table 4. Corrections for External Distances in Table 2. Applying only to the chord definition of D. Based on $(R_c - R_a) \text{ exsec } \frac{1}{2}\Delta$, where R_c = radius as per chord definition and R_a = radius as per arc definition.

Table 5. Radius When Degree of Curve Is Given

$$R = \frac{5{,}729.58}{D} \text{ (arc definition)}; \; R = \frac{50}{\sin \frac{1}{2}D} \text{ (chord definition)}.$$

For the usual flat railway and highway curve, the difference in radii by the two definitions is negligible.

Table 6. Degree of Curve (D) When Radius (R) Is Given

$$D = \frac{5{,}729.58}{R} \text{ (arc definition)}; \; \sin \tfrac{1}{2}D = \frac{50}{R} \text{ (chord definition)}.$$

Table 7. Chord Lengths of Circular Arcs. Based on *arc excess over chord* $= R(\theta^3/24 - \theta^5/1{,}920 + \cdots)$; also Eq. (10).

Table 8. Chord Lengths for Short Radius Curves. Based on Eq. (10), *arc* definition.

Table 9. Arc Lengths for Corresponding Chords. Based on Eq. (11), *arc* definition.

Table 10. True Subchords (Chord Definition). Contains subchords for locating points on a curve less than 100 ft apart. Thus, if the PC is at Sta. 37 + 50, the subchord locating Sta. 38 + 00 is 50.19 for a 20° curve (chord definition).

Table 11. Actual Arcs (Chord Definition) for Various True Chords. Based on Eq. (12).

Table 12. Actual Length of Arc Subtended by a Chord of 100 Ft. Applies primarily to the chord definition. To find the actual arc for any number of full stations (chords of 100 ft), multiply the tabular value by the number of stations; to obtain the actual arcs for the given subchords, find the central angle and then multiply the radius by the value given in Table 10.

Table 13. Arc Lengths for Radius = 1. Based on the formula arc $= R\theta$, where θ is the angle in radians. To find the arc for a given angle and radius, write down the tabular numbers for the given degrees, minutes, and seconds. Add these and multiply by the radius. The result will be the length of the arc.

Example. Given radius = 2,000 ft, angle = 27°30′20″, to find the arc (feet).

Angle	*Arc (for R = 1)*
27°	0.4712389
30′	0.0087266
20″	0.0000970
	0.4800625

Actual radius = 2,000 × 0.4800625 = 960.125 ft.

Table 14. Deflections and Chords for Even-radius Curves

$$\text{Deflection (minutes)} = \frac{1{,}718.873}{R} \times \text{arc length.}$$

For any radius R, the deflections (minutes) per foot of arc is "that for a 100-ft radius" × $100/R$.

For any radius R, the degree of curve (degrees) is the "deflection (minutes) per foot of arc for that radius" × $\frac{10}{3}$.

The length of curve is found from Table 10.

Table 15. Functions of Unit Spiral of Length = 1. Gives various parts of the unit spiral, which, for any spiral angle θ_s are to be multiplied by the actual length of the spiral in feet.

Table 16. Deflection Angles to any Point on Spiral. Gives deflection angles ϕ to any point on the spiral corresponding to the Spiral Angle θ at that point. Thus if $D_c = 9°$ and $l_s = 300$ feet, then $\theta_s = (l_s/200) \times D_c = (300/200) \times 9° = 13°30'$, and ϕ_c (from Table) $= 4°19'53.2'' + 0.3329' \times 30 = 4°19'53.2'' + 9.99' = 4°29'52.6''$.

For an intermediate point on this spiral, say where $l = 200$ feet, $\theta = (l/l_s)^2 \times \theta_s = (200/300)^2 \times 13.5° = 6°00'$, ϕ (from Table) $= 1°59'59.3''$.

Table 17. Functions of the 10-chord Spiral. These coefficients for determining the coordinates p and k (chord definition) agree exactly with those given in the AREA manual. They are based on the formulas $p = y_c - R_c \text{ vers } \theta_s$ and $k = x_c - R_s \sin \theta_s$.

Table 18. T_s for 1° Circular Curve, Spiraled. Gives the total tangent distance T_s for the 1° osculating circle and various combinations of θ_s and Δ. Interpolation gives results to the nearest hundredth of a foot for values of Δ expressed in degrees and minutes.

To simplify interpolation, it is desirable that integral values of θ_s be selected; to expedite the layout of the spiral, fractional values of l_s are to be avoided. Accordingly, $D = 200\theta_s/l_s$.

Example. Given $\Delta = 41°12'$, $\theta_s = 6°$, $l_s = 400$ ft. Required: T_s for both arc and chord definitions.

Solution. $D = 200 \times 6/400 = 3°$. Interpolating for T_s, we have

$$\begin{array}{r l}
2{,}745.90 & \\
\underline{11.46} & \leftarrow 57.28 \times 12/60 = 11.46 \\
2{,}757.36 & = T_s \text{ for } 1° \text{ curve.}
\end{array}$$

$$\text{Then} \quad T_s \text{ (for } D = 3°) = \frac{2{,}757.36}{3} = 919.12 \text{ ft (arc definition).}$$

For the chord definition, a small correction is to be added to the foregoing value of T_s. Since $T_s = T + p \tan \frac{1}{2}\Delta + k$, this correction is based on the formula $C_T - C_p \tan \frac{1}{2}\Delta - C_k$, where C_T is found in Table 3, C_p and C_k in Table 17. In this example, the correction is $0.07 - 0 - 0.03 = 0.04$; hence

$$T_s \text{ (chord definition)} = 919.12 + 0.04 = 919.16 \text{ ft.}$$

Table 19. E_s for 1° Circular Curve, Spiraled. Similar to Table 18 (see explanation of Table 18).

Example. Given $\Delta = 41°12'$, $\theta_s = 6°$, $l_s = 400$ ft. Required: E_s for arc definition.

Solution. Interpolating for E_s, we have

$$\begin{array}{rl} 398.5 & \\ 4.1 & \leftarrow 20.3 \times 12/60 = 4.1 \\ \hline 402.6 & = E_s \text{ for } 1° \text{ curve.} \end{array}$$

Then $E_s \text{ (for } D = 3°) = \dfrac{402.6}{3} = 134.2 \text{ ft (arc definition).}$

Table 20. Values of the Correction C_s for Large Spiral Deflections.

Table 21. Values of the Correction C_a for Spiraled Compound Curves for Various Values of the Product $\Delta_2\Delta_1(\Delta_2 - \Delta_1)$.

Tables 20 and 21 are on the same page. These corrections (given in seconds) are negligible for flat curves. For the simple spiral connecting a tangent with a circular arc, only Table 20 is applicable; but for the spiral between two circular arcs of different radii, both Tables 20 and 21 are applicable.

Table 22. Values of A, B, C, D, E, F Giving Y and X for Spirals Combining Curves of Different Radii. See Appendix E for theoretical background.

Table 23. Deflection Angles to Chord Points of Spiral. Contains deflections to each tenth point of a 10-chord spiral. These deflections are sponsored by the AREA manual, but they apply also to the arc definition to a negligible fraction of a minute.

Table 24. Coefficients for Deflections from Intermediate Setup on Spiral. If a transit is set at any chord point of a 10-chord spiral, this table gives coefficients for determining deflections (forward or backward) to any other chord point. These deflections are to be multiplied by ϕ_1, the deflection from the TS

to point 1, given in Table 23, or found from the formula

$$\phi_1 = \frac{\theta_s}{300} \times 60 \text{ (minutes)}.$$

Example. Given spiral angle $\theta_s = 10°$. Required: deflections from point 6 on the spiral.

Solution. Table 23 gives $\phi_1 = 0°02.0'$. With transit at point 6, the backward deflection to the TS $= 0.02 \times 72 = 144' = 2°24'$.

Forward deflections to point $7 = 0.02 \times 19 = 0°38'$,
$8 = 0.02 \times 40 = 1°20'$,
$9 = 0.02 \times 63 = 2°06'$,
$10 = 0.02 \times 88 = 2°56'$.

Table 25. Tangents and Externals to the Unit Double-spiral Curve. The tabular quantities T_s and E_s to the unit double spiral, where each spiral equals 100 ft, are used in a manner similar to T and E to the unit circle (Table 1). For any other pair of spirals, each of length l_s, the tabular values of T_s and E_s would be multiplied by $l_s/100$.

Table 26. Functions of the Spiral Which Is Applied to an Existing 1° Circular Curve. Gives values of θ_s, a, D_c, x_c, y_c, LT, and ST for spiral lengths varying in increments of 10 ft from 900 to 2,800 ft. These quantities are calculated on the basis of 1° circular curve. For any specified value of D, all the tabular numbers except θ_s and D_c will be *divided* by D, while D_c is to be *multiplied* by D.

Integral values of l_s, preferably multiples of 10, 20, or 50, will by selected for simplicity of layout. This may require a slight adjustment of D. Corresponding values of θ_s and D_c may be found by interpolation, or with one of them known, the other may be found from $\theta_s = l_s D_c/200$, or $D_c = 200\theta_s/l_s$.

Example. Given a 3° circular curve whose central portion is fixed in position by a bridge or cliff. The end portions, however, may be shifted inward sufficiently to provide clearance for spirals. The maximum permissible spiral angle is approximately 8°.

Solution. By Table 26, $l_s = 1{,}200/3 = 400$ ft for $\theta_s = 8°00.11'$. Also $a = 400.06/3 = 133.35$ ft; $x_c = 1{,}197.66/3 = 399.22$ ft; $y_c = 55.79/3 = 18.60$ ft; LT $= 800.80/3 = 266.93$ ft; ST $= 400.73/3 = 133.58$ ft; and $D_c = 1.3336° \times 3 = 4.0008° = 4°00.05'$.

All the foregoing data may not actually be needed in the layout of the spiral. The distance a (= 133.35) is essential for locating the TS by measurement backward along the tangent from the PC. With the spiral angle known, which for practical purposes is 8°, the 400-ft spiral may be set out by deflections in the usual way.

Table 27. Minimum Length of Spirals for Railroads. Self-explanatory.

Table 28. Extra Area of the Two Spiraled Portions of Widened Pavement. Based on Art. 111. It is assumed that widening takes place gradually all the way from the TS to the SC. A slight adjustment of the forms by eye may be needed at the SC, but not at other places.

Table 29. Extra Area of the Intermediate Circular Portion of Widened Pavement (in Square Yards) **per Lineal Foot of Noninal ℄ Curve.** Based on Art. 111*b*. It is equally applicable whether the end portions are spiraled or unspiraled.

Table 30. Factors *A*, *B*, and *C* Giving the Extra Area of Widened Pavement, Where Surveyed Center Line Is a Circular Curve. Based on Eq. (163). The only argument is the angle θ_c, which is the central angle subtended by arc AB (Fig. 92). The given values of w, f, and D may be inserted in the formula.

Table 31. Coefficients *C* Giving Vertical Offset from Tangent to Vertical Curve. Applying to vertical curves of lengths in multiples of 50 ft. Merely multiply the coefficient by e, where $e = LA/8$, in which L is the length of curve in stations and $A = g_2 - g_1$.

Table 32. Minutes in Decimals of a Degree

Table 33. Inches in Decimals of a Foot

Table 34. Map Scales and Conversion Factors. Self-explanatory.

Table 35. Horizontal Distances and Elevations from Stadia Readings. Needed in connection with stadia traverses.

Table 36. Area of Waterway Calculated by Talbot's Formula. Self-explanatory. Serves as a guide in selecting the size of waterway openings for culverts or bridges.

Table 37. Volume of 100-ft Earthwork Solids (Cubic Yards). Based on the average end area formula [Eq. (200)].

Table 38. Prismoidal Correction (Cubic Yards) for 100-ft Earthwork Solids. Gives correction (cubic yards) which when

subtracted from the volumes computed by the average end area formula will give greater accuracy (see Art. 148). The use of this table will probably be limited to special situations such as heavy rock excavation.

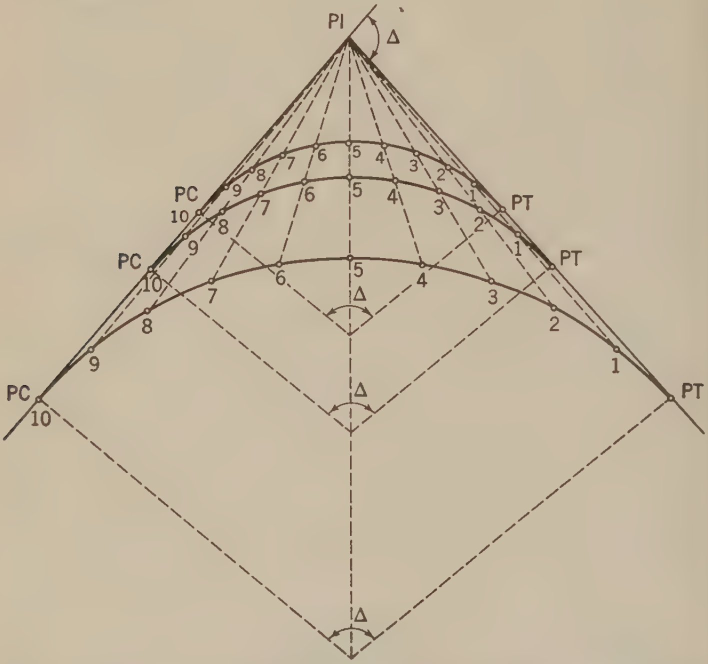


Fig. 150. Showing that deflection angles are independent of curve length. See explanation of Table 46.

Table 39. Natural Sines and Cosines

Table 40. Natural Tangents and Cotangents

Table 41. Natural Versines and External Secants. These quantities are especially useful in curve computation. It should be noted that vers $\theta = 1 - \cos\theta$; exsec $\theta = \sec\theta - 1$.

Table 42. Values of A and B, Where A = (Log Cosecant) × 10,000, B = (Log Secant) × 10,000. To be used [see Eq. (221)] in establishing true north by observations on sun or stars.

Table 43. Trigonometric Formulas

Table 44. Useful Numbers and Formulas

Table 45. Squares, Cubes, Square and Cube Roots, Circumferences, and Areas

Table 46. Deflections from the PI to the 10-point Circular Curve. Provides a quick way of laying out a circular curve when a transit is set at the PI. Each tenth point of the curve, regardless of its length, is located by continuous measurements along the curve as directed by tabular deflections from the PI. The midpoint of the curve must be located independently by the external distance E, thus giving another check in the curve layout. See Fig. 150.

Table 47. Direction Angle α and Distance r from the PI to the First (or Third) Quarter-points of a Circular Curve.

See Eqs. (22) and (23).

TABLE 1. FUNCTIONS OF UNIT CIRCULAR CURVES

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 ft						Tangents and externals for the unit circular curve whose radius is 10,000 ft		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
1°00′	5,729.58	50.001	0.218	99.999	0.218	87.27	0.38	1°00′
10	4,911.07	.002	.255	.998	.254	101.81	0.52	10
20	4,297.18	.002	.291	.998	.291	116.36	0.68	20
30	3,819.71	.003	.327	.997	.327	130.91	0.86	30
40	3,437.75	.004	.364	.996	.363	145.45	1.06	40
50	3,125.22	.004	.400	.996	.400	160.00	1.28	50
2°00	2,864.79	.005	.436	.995	.436	174.55	1.52	2°00
10	2,644.42	.006	.473	.994	.472	189.10	1.79	10
20	2,455.53	.007	.509	.993	.509	203.65	2.07	20
30	2,291.83	.008	.545	.992	.545	218.20	2.38	30
40	2,148.59	.009	.582	.991	.582	232.75	2.71	40
50	2,022.20	.010	.618	.990	.618	247.31	3.06	50
3°00	1,909.86	.011	.654	.989	.654	261.86	3.43	3°00
10	1,809.34	.013	.691	.987	.691	276.41	3.82	10
20	1,718.87	.014	.727	.986	.727	290.97	4.23	20
30	1,637.02	.016	.764	.984	.763	305.53	4.67	30
40	1,562.61	.017	.800	.983	.800	320.09	5.12	40
50	1,494.67	.019	.837	.981	.836	334.65	5.60	50
4°00	1,432.39	.020	.873	.980	.873	349.21	6.10	4°00
10	1,375.10	.022	.910	.978	.909	363.77	6.61	10
20	1,322.21	.024	.946	.976	.945	378.34	7.15	20
30	1,273.24	.026	.982	.974	.982	392.90	7.72	30
40	1,227.77	.028	1.019	.972	1.018	407.47	8.30	40
50	1,185.43	.030	.055	.970	.054	422.04	8.90	50
5°00	1,145.92	.032	.092	.968	.091	436.61	9.53	5°00
10	1,108.95	.034	.128	.966	.127	451.18	10.17	10
20	1,074.30	.036	.165	.964	.163	465.76	10.84	20
30	1,041.74	.038	.201	.961	.200	480.33	11.53	30
40	1,011.10	.041	.237	.959	.236	494.91	12.24	40
50	982.21	.043	.274	.957	.273	509.50	12.97	50
6°00	954.93	.046	.310	.954	.310	524.08	13.72	6°00
10	929.12	.048	.347	.952	.346	538.66	14.50	10
20	904.67	.051	.383	.949	.382	553.25	15.29	20
30	881.47	.054	.420	.946	.418	567.84	16.11	30
40	859.44	.056	.456	.943	.454	582.43	16.95	40
50	838.47	.059	.493	.941	.490	597.03	17.81	50

TABLE 1. FUNCTIONS OF UNIT CIRCULAR CURVES (*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 ft						Tangents and externals for the unit circular curve whose radius is 10,000 ft		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
7°00′	818.51	50.062	1.529	99.938	1.526	611.63	18.69	7°00′
10	799.48	.065	.566	.935	.563	626.23	19.59	10
20	781.31	.068	.602	.931	.599	640.83	20.51	20
30	763.94	.072	.639	.928	.635	655.44	21.46	30
40	747.34	.075	.676	.925	.672	670.04	22.42	40
50	731.44	.078	.712	.922	.708	684.65	23.41	50
8°00	716.20	.081	.749	.919	.745	699.27	24.42	8°00
10	701.58	.085	.785	.915	.781	713.89	25.45	10
20	687.55	.088	.822	.912	.817	728.51	26.50	20
30	674.07	.092	.859	.908	.854	743.13	27.57	30
40	661.11	.096	.895	.904	.890	757.76	28.67	40
50	648.63	.099	.932	.901	.926	772.38	29.79	50
9°00	636.62	.103	.969	.897	.962	787.02	30.92	9°00
10	625.04	.107	2.005	.893	.999	801.65	32.08	10
20	613.88	.111	.042	.889	2.035	816.29	33.26	20
30	603.11	.115	.079	.885	.071	830.94	34.46	30
40	592.72	.119	.115	.881	.108	845.58	35.69	40
50	582.67	.123	.152	.877	.144	860.23	36.93	50
10°00	572.96	.127	.189	.873	.180	874.89	38.20	10°00
10	563.56	.132	.225	.869	.217	889.54	39.49	10
20	554.48	.136	.262	.864	.253	904.21	40.80	20
30	545.67	.140	.299	.860	.289	918.87	42.13	30
40	537.15	.145	.335	.855	.325	933.54	43.48	40
50	528.88	.150	.372	.851	.362	948.21	44.86	50
11°00	520.87	.154	.409	.846	.398	962.89	46.25	11°00
10	513.10	.159	.446	.842	.434	977.57	47.67	10
20	505.55	.164	.483	.837	.471	992.26	49.11	20
30	498.22	.168	.519	.832	.507	1006.95	50.57	30
40	491.11	.174	.556	.827	.543	1021.64	52.05	40
50	484.19	.178	.593	.822	.579	1036.34	53.56	50
12°00	477.46	.184	.630	.817	.616	1051.04	55.08	12°00
10	470.92	.189	.667	.812	.652	1065.75	56.63	10
20	464.56	.194	.704	.807	.688	1080.46	58.20	20
30	458.37	.199	.740	.801	.724	1095.18	59.79	30
40	452.34	.205	.777	.796	.761	1109.90	61.41	40
50	446.46	.210	.814	.791	.797	1124.63	63.04	50

TABLE 1. FUNCTIONS OF UNIT CIRCULAR CURVES (*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 ft						Tangents and externals for the unit circular curve whose radius is 10,000 ft		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
13°00′	440.74	50.216	2.850	99.786	2.833	1139.36	64.70	13°00′
10	435.16	.221	.888	.780	.869	1154.09	66.38	10
20	429.72	.227	.925	.774	.906	1168.83	68.08	20
30	424.41	.232	.962	.768	.942	1183.58	69.80	30
40	419.24	.238	.999	.763	.978	1198.33	71.54	40
50	414.19	.244	3.036	.757	3.014	1213.09	73.31	50
14°00	409.26	.250	.074	.751	.050	1227.85	75.10	14°00
10	404.44	.256	.111	.745	.087	1242.61	76.91	10
20	399.74	.262	.148	.739	.123	1257.38	78.74	20
30	395.14	.269	.185	.733	.159	1272.16	80.60	30
40	390.65	.275	.222	.727	.195	1286.94	82.47	40
50	386.26	.281	.259	.721	.232	1301.73	84.37	50
15°00	381.97	.288	.296	.715	.268	1316.53	86.29	15°00
10	377.77	.294	.333	.708	.304	1331.32	88.23	10
20	373.67	.301	.370	.702	.340	1346.13	90.20	20
30	369.65	.307	.408	.695	.376	1360.94	92.18	30
40	365.72	.314	.445	.688	.413	1375.76	94.19	40
50	361.87	.321	.482	.682	.449	1390.58	96.22	50
16°00	358.10	.328	.519	.675	.485	1405.41	98.28	16°00
10	354.41	.334	.557	.668	.521	1420.24	100.35	10
20	350.79	.341	.594	.661	.557	1435.08	102.45	20
30	347.25	.348	.631	.654	.594	1449.93	104.57	30
40	343.77	.356	.668	.648	.630	1464.78	106.71	40
50	340.37	.363	.706	.641	.666	1479.64	108.88	50
17°00	337.03	.370	.743	.634	.702	1494.51	111.06	17°00
10	333.76	.377	.781	.626	.738	1509.38	113.27	10
20	330.55	.385	.818	.619	.774	1524.26	115.50	20
30	327.40	.392	.855	.611	.811	1539.15	117.76	30
40	324.32	.400	.893	.604	.847	1554.04	120.03	40
50	321.28	.408	.930	.597	.883	1568.94	122.33	50
18°00	318.31	.416	.968	.589	.919	1583.84	124.65	18°00
10	315.39	.423	4.005	.581	.955	1598.76	127.00	10
20	312.52	.431	.043	.574	.991	1613.68	129.36	20
30	309.71	.439	.080	.566	4.027	1628.60	131.75	30
40	306.94	.447	.118	.558	.063	1643.54	134.16	40
50	304.22	.455	.156	.550	.100	1658.48	136.60	50

TABLE 1. FUNCTIONS OF UNIT CIRCULAR CURVES (*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 ft						Tangents and externals for the unit circular curve whose radius is 10,000 ft		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
19°00′	301.56	50.463	4.193	99.542	4.136	1673.43	139.05	19°00′
10	298.93	.472	.231	.534	.172	1688.38	141.53	10
20	296.36	.480	.269	.526	.208	1703.34	144.03	20
30	293.82	.488	.306	.518	.244	1718.31	146.56	30
40	291.33	.497	.344	.509	.280	1733.29	149.10	40
50	288.89	.505	.382	.501	.316	1748.28	151.67	50
20°00	286.48	.514	.419	.493	.352	1763.27	154.27	20°00
10	284.11	.523	.457	.484	.388	1778.27	156.88	10
20	281.78	.531	.495	.476	.424	1793.28	159.52	20
30	279.49	.540	.533	.467	.460	1808.30	162.18	30
40	277.24	.549	.571	.458	.497	1823.32	164.87	40
50	275.02	.558	.609	.450	.533	1838.35	167.57	50
21°00	272.84	.567	.646	.441	.569	1853.39	170.30	21°00
10	270.69	.576	.684	.432	.605	1868.44	173.06	10
20	268.57	.586	.722	.423	.641	1883.50	175.83	20
30	266.49	.595	.760	.414	.677	1898.56	178.63	30
40	264.44	.604	.798	.405	.713	1913.63	181.45	40
50	262.42	.614	.836	.396	.749	1928.71	184.30	50
22°00	260.44	.624	.874	.387	.785	1943.80	187.17	22°00
10	258.48	.633	.913	.377	.821	1958.90	190.06	10
20	256.55	.643	.951	.368	.857	1974.01	192.97	20
30	254.65	.653	.989	.358	.893	1989.12	195.91	30
40	252.78	.662	5.027	.349	.929	2004.25	198.87	40
50	250.93	.672	.065	.339	.965	2019.38	201.86	50
23°00	249.11	.682	.103	.330	5.001	2034.52	204.87	23°00
10	247.32	.692	.142	.320	.037	2049.67	207.90	10
20	245.55	.703	.180	.310	.073	2064.83	210.95	20
30	243.81	.713	.218	.300	.109	2080.00	214.03	30
40	242.09	.723	.257	.290	.145	2095.18	217.13	40
50	240.40	.734	.295	.280	.181	2110.37	220.26	50
24°00	238.73	.744	.333	.271	.217	2125.57	223.41	24°00
10	237.09	.755	.372	.260	.253	2140.77	226.58	10
20	235.46	.765	.410	.250	.289	2155.99	229.77	20
30	233.86	.776	.449	.240	.325	2171.21	232.99	30
40	232.28	.787	.488	.229	.361	2186.45	236.24	40
50	230.72	.798	.526	.219	.397	2201.69	239.50	50

TABLE 1. FUNCTIONS OF UNIT CIRCULAR CURVES (*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 ft						Tangents and externals for the unit circular curve whose radius is 10,000 ft		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
25°00′	229.18	50.809	5.565	99.209	5.432	2216.95	242.80	25°00′
10	227.66	.820	.603	.198	.468	2232.21	246.11	10
20	226.17	.831	.642	.187	.504	2247.49	249.45	20
30	224.69	.842	.681	.176	.540	2262.77	252.81	30
40	223.23	.853	.719	.166	.576	2278.06	256.20	40
50	221.79	.865	.758	.155	.612	2293.37	259.61	50
26°00	220.37	.876	.797	.144	.648	2308.68	263.04	26°00
10	218.96	.888	.835	.133	.684	2324.01	266.50	10
20	217.58	.899	.874	.122	.720	2339.34	269.98	20
30	216.21	.911	.913	.111	.756	2354.69	273.49	30
40	214.86	.923	.952	.100	.792	2370.04	277.02	40
50	213.52	.934	.991	.088	.827	2385.41	280.57	50
27°00	212.21	.946	6.030	.077	.863	2400.79	284.15	27°00
10	210.90	.958	.069	.066	.899	2416.18	287.76	10
20	209.62	.970	.108	.054	.935	2431.58	291.38	20
30	208.35	.982	.147	.042	.971	2446.98	295.03	30
40	207.09	.995	.186	.031	6.007	2462.41	298.71	40
50	205.85	51.007	.225	.019	.042	2477.84	302.41	50
28°00	204.63	.020	.264	.008	.078	2493.28	306.14	28°00
10	203.42	.032	.304	98.996	.114	2508.73	309.89	10
20	202.22	.044	.343	.984	.150	2524.20	313.66	20
30	201.04	.057	.382	.972	.186	2539.68	317.46	30
40	199.87	.070	.422	.960	.222	2555.17	321.28	40
50	198.71	.083	.461	.948	.257	2570.66	325.13	50
29°00	197.57	.096	.500	.936	.293	2586.18	329.00	29°00
10	196.44	.108	.540	.924	.329	2601.70	332.90	10
20	195.33	.122	.579	.911	.365	2617.23	336.82	20
30	194.22	.135	.619	.899	.400	2632.78	340.77	30
40	193.13	.148	.658	.887	.436	2648.34	344.74	40
50	192.05	.161	.698	.874	.472	2663.91	348.74	50
30°00	190.98	.174	.737	.862	.508	2679.49	352.76	30°00
10	189.93	.188	.777	.849	.543	2695.09	356.81	10
20	188.89	.202	.817	.836	.579	2710.69	360.88	20
30	187.86	.215	.856	.823	.615	2726.31	364.98	30
40	186.83	.229	.896	.811	.651	2741.95	369.10	40
50	185.82	.243	.936	.798	.686	2757.59	373.25	50

TABLE 1. FUNCTIONS OF UNIT CIRCULAR CURVES (*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 ft						Tangents and externals for the unit circular curve whose radius is 10,000 ft		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
31°00′	184.82	51.256	6.976	98.785	6.722	2773.25	377.42	31°00′
10	183.84	.271	7.016	.772	.758	2788.92	381.62	10
20	182.86	.284	.055	.758	.793	2804.60	385.84	20
30	181.89	.299	.095	.745	.829	2820.29	390.09	30
40	180.93	.313	.135	.732	.865	2836.00	394.37	40
50	179.99	.327	.175	.719	.900	2851.72	398.67	50
32°00	179.05	.342	.215	.705	.936	2867.45	402.99	32°00
10	178.12	.356	.256	.692	.972	2883.20	407.35	10
20	177.20	.371	.296	.678	7.007	2898.96	411.72	20
30	176.29	.385	.336	.664	.043	2914.73	416.13	30
40	175.40	.400	.376	.651	.079	2930.52	420.55	40
50	174.50	.415	.417	.637	.114	2946.32	425.01	50
33°00	173.62	.430	.457	.624	.150	2962.14	429.49	33°00
10	172.75	.445	.497	.610	.185	2977.96	434.00	10
20	171.89	.460	.538	.595	.221	2993.80	438.53	20
30	171.03	.475	.578	.581	.257	3009.66	443.09	30
40	170.18	.490	.619	.567	.292	3025.53	447.67	40
50	169.35	.505	.659	.553	.328	3041.41	452.28	50
34°00	168.52	.521	.700	.539	.363	3057.31	456.92	34°00
10	167.70	.536	.740	.525	.399	3073.22	461.58	10
20	166.88	.552	.781	.511	.434	3089.14	466.27	20
30	166.07	.568	.822	.496	.470	3105.08	470.99	30
40	165.28	.583	.863	.482	.505	3121.04	475.73	40
50	164.48	.599	.904	.468	.541	3137.01	480.50	50
35°00	163.70	.615	.944	.454	.576	3152.99	485.29	35°00
10	162.93	.631	.985	.439	.612	3168.99	490.11	10
20	162.16	.647	8.026	.424	.647	3185.00	494.96	20
30	161.40	.664	.067	.409	.683	3201.03	499.84	30
40	160.64	.680	.108	.394	.718	3217.07	504.74	40
50	159.90	.696	.149	.378	.754	3233.13	509.67	50
36°00	159.15	.713	.190	.363	.790	3249.20	514.62	36°00
10	158.42	.729	.232	.348	.825	3265.28	519.61	10
20	157.69	.746	.274	.333	.861	3281.39	524.61	20
30	156.97	.762	.315	.317	.896	3297.51	529.65	30
40	156.26	.779	.356	.302	.931	3313.64	534.71	40
50	155.55	.796	.397	.287	.967	3329.79	539.81	50

TABLE 1. FUNCTIONS OF UNIT CIRCULAR CURVES (*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 ft						Tangents and externals for the unit circular curve whose radius is 10,000 ft		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
37°00′	154.85	51.813	8.438	98.271	8.002	3345.95	544.92	37°00′
10	154.16	.830	.480	.256	.038	3362.13	550.07	10
20	153.47	.848	.521	.240	.073	3378.33	555.24	20
30	152.79	.865	.563	.224	.108	3394.54	560.44	30
40	152.11	.882	.605	.209	.144	3410.77	565.67	40
50	151.44	.900	.646	.193	.179	3427.02	570.92	50
38°00	150.78	.917	.688	.177	.215	3443.28	576.21	38°00
10	150.12	.935	.730	.161	.250	3459.55	581.52	10
20	149.47	.952	.772	.145	.285	3475.85	586.86	20
30	148.82	.970	.814	.129	.321	3492.16	592.22	30
40	148.18	.988	.855	.113	.356	3508.48	597.62	40
50	147.54	52.006	.897	.097	.391	3524.83	603.04	50
39°00	146.91	.024	.939	.081	.427	3541.19	608.49	39°00
10	146.29	.042	.982	.064	.462	3557.56	613.97	10
20	145.67	.061	9.024	.048	.497	3573.96	619.47	20
30	145.05	.079	.066	.031	.533	3590.37	625.01	30
40	144.44	.098	.108	.015	.568	3606.80	630.57	40
50	143.84	.116	.151	97.998	.603	3623.24	636.16	50
40°00	143.24	.135	.193	.982	.638	3639.70	641.78	40°00
10	142.64	.154	.235	.965	.674	3656.18	647.43	10
20	142.06	.172	.278	.948	.709	3672.68	653.10	20
30	141.47	.191	.320	.931	.744	3689.20	658.81	30
40	140.89	.211	.363	.914	.779	3705.73	664.54	40
50	140.32	.230	.405	.897	.815	3722.28	670.30	50
41°00	139.74	.249	.448	.880	.850	3738.85	676.09	41°00
10	139.18	.268	.491	.863	.885	3755.43	681.91	10
20	138.62	.288	.534	.845	.920	3772.04	687.76	20
30	138.06	.307	.577	.828	.955	3788.66	693.64	30
40	137.51	.327	.620	.811	.991	3805.30	699.55	40
50	136.96	.346	.663	.793	9.026	3821.96	705.48	50
42°00	136.42	.366	.705	.776	.061	3838.64	711.45	42°00
10	135.88	.386	.749	.758	.096	3855.34	717.45	10
20	135.34	.406	.792	.741	.131	3872.05	723.47	20
30	134.81	.426	.835	.723	.166	3888.79	729.52	30
40	134.29	.466	.878	.705	.201	3905.54	735.61	40
50	133.76	.467	.922	.687	.236	3922.31	741.72	50

Table 1. Functions of Unit Circular Curves (*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 ft						Tangents and externals for the unit circular curve whose radius is 10,000 ft		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
43°00′	133.25	52.487	9.965	97.670	9.272	3939.1	747.86	43°00′
10	132.73	.508	10.009	.651	.307	3955.9	754.04	10
20	132.22	.528	.052	.633	.342	3972.7	760.24	20
30	131.71	.549	.096	.615	.377	3989.6	766.47	30
40	131.21	.570	.139	.597	.412	4006.5	772.73	40
50	130.71	.590	.183	.579	.447	4023.3	779.03	50
44°00	130.22	.611	.227	.561	.482	4040.2	785.35	44°00
10	129.73	.632	.271	.542	.517	4057.1	791.70	10
20	129.24	.654	.314	.523	.552	4074.1	798.08	20
30	128.75	.675	.358	.505	.587	4091.1	804.50	30
40	128.27	.696	.402	.486	.622	4108.1	810.94	40
50	127.80	.718	.446	.467	.657	4125.1	817.42	50
45°00	127.32	.739	.490	.448	.692	4142.1	823.92	45°00
10	126.85	.761	.534	.430	.727	4159.1	830.46	10
20	126.39	.783	.579	.411	.762	4176.2	837.03	20
30	125.92	.805	.623	.392	.797	4193.3	843.62	30
40	125.46	.827	.668	.373	.832	4210.4	850.25	40
50	125.01	.849	.712	.355	.867	4227.6	856.91	50
46°00	124.56	.871	.757	.336	.902	4244.7	863.60	46°00
10	124.11	.893	.801	.316	.936	4261.9	870.33	10
20	123.66	.916	.846	.297	.971	4279.1	877.08	20
30	123.22	.938	.891	.278	10.006	4296.3	883.87	30
40	122.78	.961	.936	.258	.041	4313.5	890.68	40
50	122.34	.983	.980	.239	.076	4330.8	897.53	50
47°00	121.91	53.006	11.025	.220	.111	4348.1	904.41	47°00
10	121.48	.029	.070	.200	.146	4365.4	911.32	10
20	121.05	.052	.115	.180	.180	4382.7	918.27	20
30	120.62	.075	.161	.160	.215	4400.1	925.24	30
40	120.20	.098	.206	.141	.250	4417.5	932.25	40
50	119.78	.122	.251	.121	.285	4434.9	939.29	50
48°00	119.37	.145	.296	.101	.320	4452.3	946.36	48°00
10	118.95	.169	.342	.081	.354	4469.7	953.47	10
20	118.54	.192	.387	.061	.389	4487.2	960.60	20
30	118.14	.216	.433	.041	.424	4504.7	967.77	30
40	117.73	.240	.479	.021	.459	4522.2	974.98	40
50	117.33	.264	.524	.000	.494	4539.7	982.21	50

TABLE 1. FUNCTIONS OF UNIT CIRCULAR CURVES (*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 ft						Tangents and externals for the unit circular curve whose radius is 10,000 ft		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
49°00′	116.93	53.288	11.570	96.980	10.528	4557.3	989.5	49°00′
10	116.53	.312	.616	.960	.563	4574.8	996.8	10
20	116.14	.337	.662	.939	.598	4592.4	1004.1	20
30	115.75	.361	.708	.919	.632	4610.0	1011.5	30
40	115.36	.386	.754	.898	.667	4627.7	1018.9	40
50	114.97	.410	.800	.878	.701	4645.4	1026.3	50
50°00	114.59	.435	.846	.857	.736	4663.1	1033.8	50°00
10	114.21	.460	.893	.836	.771	4680.8	1041.3	10
20	113.83	.485	.939	.815	.805	4698.5	1048.8	20
30	113.46	.510	.985	.794	.840	4716.3	1056.4	30
40	113.08	.535	12.032	.773	.875	4734.1	1064.0	40
50	112.71	.560	.079	.752	.909	4751.9	1071.6	50
51°00	112.34	.586	.125	.731	.944	4769.8	1079.3	51°00
10	111.98	.611	.172	.710	.979	4787.6	1087.0	10
20	111.62	.637	.219	.689	11.013	4805.5	1094.7	20
30	111.25	.662	.266	.667	.048	4823.4	1102.5	30
40	110.90	.688	.313	.646	.082	4841.4	1110.3	40
50	110.54	.714	.360	.625	.117	4859.3	1118.1	50
52°00	110.18	.741	.407	.603	.151	4877.3	1126.0	52°00
10	109.83	.767	.454	.581	.186	4895.3	1133.9	10
20	109.48	.793	.502	.560	.220	4913.4	1141.9	20
30	109.13	.819	.549	.538	.255	4931.4	1149.9	30
40	108.79	.846	.596	.516	.289	4949.5	1157.9	40
50	108.45	.872	.644	.494	.324	4967.7	1165.9	50
53°00	108.11	.899	.692	.473	.358	4985.8	1174.0	53°00
10	107.77	.926	.739	.450	.393	5004.0	1182.1	10
20	107.43	.953	.787	.428	.427	5022.2	1190.3	20
30	107.09	.980	.835	.406	.461	5040.4	1198.5	30
40	106.76	54.008	.883	.384	.496	5058.7	1206.7	40
50	106.43	.035	.931	.362	.530	5076.9	1215.0	50
54°00	106.10	.062	.979	.340	.565	5095.2	1223.3	54°00
10	105.78	.090	13.027	.317	.599	5113.6	1231.6	10
20	105.45	.118	.076	.295	.633	5131.9	1240.0	20
30	105.13	.146	.124	.272	.668	5150.3	1248.4	30
40	104.81	.173	.173	.250	.702	5168.8	1256.8	40
50	104.49	.202	.221	.227	.736	5187.2	1265.3	50

TABLE 1. FUNCTIONS OF UNIT CIRCULAR CURVES (*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 ft						Tangents and externals for the unit circular curve whose radius is 10,000 ft		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
55°00′	104.17	54.230	13.270	96.205	11.771	5205.7	1273.8	55°00′
10	103.86	.258	.319	.182	.805	5224.2	1282.4	10
20	103.55	.286	.368	.159	.839	5242.7	1291.0	20
30	103.24	.315	.416	.136	.873	5261.2	1299.6	30
40	102.93	.344	.466	.113	.908	5279.8	1308.3	40
50	102.62	.372	.515	.090	.942	5298.4	1317.0	50
56°00	102.31	.401	.564	.067	.976	5317.1	1325.7	56°00
10	102.01	.430	.613	.044	12.010	5335.8	1334.5	10
20	101.71	.460	.663	.020	.044	5354.5	1343.3	20
30	101.41	.489	.712	95.997	.079	5373.2	1352.1	30
40	101.11	.518	.762	.974	.113	5391.9	1361.0	40
50	100.81	.548	.811	.950	.147	5410.7	1370.0	50
57°00	100.52	.577	.861	.927	.181	5429.6	1378.9	57°00
10	100.22	.607	.911	.903	.215	5448.4	1387.9	10
20	99.93	.637	.961	.880	.249	5467.3	1397.0	20
30	99.64	.667	14.011	.856	.283	5486.2	1406.1	30
40	99.36	.697	.061	.832	.318	5505.1	1415.2	40
50	99.07	.728	.111	.808	.352	5524.1	1424.3	50
58°00	98.78	.758	.161	.785	.386	5543.1	1433.5	58°00
10	98.50	.788	.212	.761	.420	5562.1	1442.8	10
20	98.22	.819	.262	.736	.454	5581.2	1452.1	20
30	97.94	.850	.313	.712	.488	5600.3	1461.4	30
40	97.66	.881	.364	.688	.522	5619.4	1470.7	40
50	97.39	.912	.414	.664	.556	5638.5	1480.1	50
59°00	97.11	.943	.465	.640	.590	5657.7	1489.6	59°00
10	96.84	.974	.516	.615	.624	5676.9	1499.0	10
20	96.56	55.006	.567	.591	.658	5696.2	1508.5	20
30	96.30	.037	.619	.566	.692	5715.5	1518.1	30
40	96.03	.069	.670	.542	.726	5734.8	1527.7	40
50	95.76	.101	.721	.517	.760	5754.1	1537.3	50
60°00	95.49	.133	.773	.493	.794	5773.5	1547.0	60°00
10	95.23	.165	.824	.468	.828	5792.9	1556.7	10
20	94.96	.197	.876	.443	.861	5812.3	1566.5	20
30	94.70	.230	.928	.418	.895	5831.8	1576.3	30
40	94.44	.262	.980	.393	.929	5851.3	1586.1	40
50	94.18	.295	15.032	.368	.963	5870.9	1596.0	50

TABLE 1. FUNCTIONS OF UNIT CIRCULAR CURVES (*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 ft						Tangents and externals for the unit circular curve whose radius is 10,000 ft		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
61°00′	93.93	55.328	15.084	95.344	12.997	5890.5	1605.9	61°00′
10	93.67	.361	.136	.318	13.030	5910.1	1615.9	10
20	93.42	.394	.189	.293	.064	5929.7	1625.9	20
30	93.16	.427	.241	.268	.098	5949.4	1635.9	30
40	92.91	.460	.294	.243	.132	5969.1	1646.0	40
50	92.66	.493	.346	.217	.166	5988.8	1656.2	50
62°00	92.41	.527	.399	.192	.199	6008.6	1666.3	62°00
10	92.16	.561	.452	.166	.233	6028.4	1676.5	10
20	91.92	.595	.505	.141	.267	6048.3	1686.8	20
30	91.67	.629	.558	.115	.301	6068.2	1697.1	30
40	91.43	.663	.611	.089	.334	6088.1	1707.5	40
50	91.19	.697	.665	.064	.368	6108.0	1717.8	50
63°00	90.94	.732	.718	.038	.402	6128.0	1728.3	63°00
10	90.71	.766	.772	.012	.535	6148.0	1738.7	10
20	90.47	.801	.825	94.986	.469	6168.1	1749.3	20
30	90.23	.836	.879	.960	.503	6188.2	1759.8	30
40	89.99	.871	.833	.934	.536	6208.3	1770.4	40
50	89.76	.906	.987	.908	.570	6228.5	1782.0	50
64°00	89.52	.941	16.041	.882	.603	6248.7	1791.8	64°00
10	89.29	.977	.095	.855	.637	6268.9	1802.5	10
20	89.06	56.012	.150	.829	.671	6289.2	1813.3	20
30	88.83	.048	.204	.802	.704	6309.5	1824.1	30
40	88.60	.084	.259	.776	.738	6329.9	1835.0	40
50	88.37	.120	.313	.749	.771	6350.3	1845.9	50
65°00	88.15	.156	.368	.723	.805	6370.7	1856.9	65°00
10	87.92	.192	.423	.696	.838	6391.2	1867.9	10
20	87.70	.229	.478	.669	.871	6411.7	1878.9	20
30	87.47	.266	.533	.643	.905	6432.2	1890.1	30
40	87.25	.302	.589	.616	.938	6452.8	1901.2	40
50	87.03	.339	.644	.589	.972	6473.4	1912.4	50
66°00	86.81	.376	.699	.562	14.005	6494.1	1923.6	66°00
10	86.59	.413	.755	.535	.039	6514.8	1934.9	10
20	86.38	.451	.811	.508	.072	6535.5	1946.2	20
30	86.16	.488	.867	.481	.105	6556.3	1957.6	30
40	85.94	.526	.923	.453	.139	6577.1	1969.1	40
50	85.73	.564	.979	.426	.172	6598.0	1980.5	50

TABLE 1. FUNCTIONS OF UNIT CIRCULAR CURVES (*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 ft						Tangents and externals for the unit circular curve whose radius is 10,000 ft		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
67°00′	85.52	56.602	17.035	94.399	14.205	6618.9	1992.0	67°00′
10	85.30	.640	.092	.371	.239	6639.8	2003.6	10
20	85.09	.678	.148	.344	.272	6660.8	2015.2	20
30	84.88	.717	.205	.316	.305	6681.8	2026.9	30
40	84.67	.755	.262	.289	.338	6702.8	2038.6	40
50	84.46	.794	.319	.261	.372	6723.9	2050.4	50
68°00	84.26	.833	.376	.234	.405	6745.1	2062.2	68°00
10	84.05	.872	.433	.206	.438	6766.3	2074.0	10
20	83.85	.911	.490	.178	.471	6787.5	2085.9	20
30	83.64	.951	.548	.150	.505	6808.8	2097.9	30
40	83.44	.990	.605	.122	.538	6830.1	2109.9	40
50	83.24	57.030	.663	.095	.571	6851.4	2122.0	50
69°00	83.04	.070	.721	.067	.604	6872.8	2134.1	69°00
10	82.84	.110	.779	.038	.637	6894.3	2146.2	10
20	82.64	.150	.837	.010	.670	6915.7	2158.4	20
30	82.44	.191	.895	93.981	.703	6937.2	2170.7	30
40	82.24	.231	.954	.953	.736	6958.8	2183.0	40
50	82.05	.272	18.012	.924	.770	6980.4	2195.3	50
70°00	81.85	.313	.071	.896	.803	7002.1	2207.7	70°00
10	81.66	.354	.130	.867	.836	7023.8	2220.2	10
20	81.46	.395	.189	.838	.869	7045.5	2232.7	20
30	81.27	.436	.248	.810	.902	7067.3	2245.3	30
40	81.08	.478	.307	.781	.935	7089.1	2257.9	40
50	80.89	.520	.366	.752	.968	7111.0	2270.5	50
71°00	80.70	.561	.426	.723	15.001	7132.9	2283.3	71°00
10	80.51	.604	.485	.694	.034	7154.9	2296.0	10
20	80.32	.646	.545	.665	.066	7176.9	2308.9	20
30	80.13	.688	.605	.636	.099	7199.0	2321.7	30
40	79.95	.731	.665	.607	.132	7221.1	2334.7	40
50	79.76	.774	.725	.578	.165	7243.2	2347.6	50
72°00	79.58	.816	.786	.549	.198	7265.4	2360.7	72°00
10	79.39	.860	.846	.519	.231	7287.7	2373.8	10
20	79.21	.903	.907	.490	.264	7310.0	2386.9	20
30	79.03	.946	.968	.460	.296	7332.3	2400.1	30
40	78.85	.990	19.029	.430	.329	7354.7	2413.4	40
50	78.67	58.034	.090	.401	.362	7377.1	2426.7	50

TABLE 1. FUNCTIONS OF UNIT CIRCULAR CURVES (*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 ft						Tangents and externals for the unit circular curve whose radius is 10,000 ft		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
73°00′	78.49	58.078	19.151	93.371	15.395	7399.6	2440.0	73°00′
10	78.31	.122	.213	.342	.427	7422.1	2453.4	10
20	78.13	.166	.274	.312	.460	7444.7	2466.9	20
30	77.95	.211	.336	.282	.493	7467.4	2480.4	30
40	77.78	.255	.398	.253	.526	7490.0	2494.0	40
50	77.60	.300	.460	.223	.558	7512.8	2507.7	50
74°00	77.43	.345	.522	.193	.591	7535.5	2521.4	74°00
10	77.25	.390	.585	.163	.624	7558.4	2535.1	10
20	77.08	.436	.647	.132	.656	7581.2	2548.9	20
30	76.91	.482	.710	.102	.689	7604.2	2562.8	30
40	76.74	.527	.773	.072	.721	7627.2	2576.7	40
50	76.56	.573	.836	.041	.754	7650.2	2590.7	50
75°00	76.39	.620	.899	.011	.787	7673.3	2604.7	75°00
10	76.22	.666	.962	92.981	.819	7696.4	2618.8	10
20	76.06	.712	20.026	.950	.852	7719.6	2633.0	20
30	75.89	.759	.089	.920	.884	7742.8	2647.2	30
40	75.72	.806	.153	.889	.917	7766.1	2661.5	40
50	75.55	.853	.217	.859	.949	7789.5	2675.8	50
76°00	75.39	.901	.281	.828	.982	7812.9	2690.2	76°00
10	75.22	.948	.346	.797	16.014	7836.3	2704.6	10
20	75.06	.996	.410	.767	.046	7859.8	2719.1	20
30	74.90	59.044	.475	.736	.079	7883.4	2733.7	30
40	74.73	.092	.540	.705	.111	7907.0	2748.3	40
50	74.57	.140	.604	.674	.144	7930.6	2763.0	50
77°00	74.41	.188	.669	.643	.176	7954.3	2777.8	77°00
10	74.25	.237	.735	.611	.208	7978.1	2792.6	10
20	74.09	.286	.801	.580	.241	8002.0	2807.5	20
30	73.93	.335	.866	.549	.273	8025.9	2822.4	30
40	73.77	.384	.932	.517	.305	8049.8	2837.4	40
50	73.61	.434	.998	.486	.338	8073.8	2852.5	50
78°00	73.46	.484	21.064	.455	.370	8097.8	2867.6	78°00
10	73.30	.534	.131	.423	.402	8121.9	2882.8	10
20	73.14	.584	.197	.391	.434	8146.1	2898.0	20
30	72.99	.634	.264	.360	.467	8170.3	2913.3	30
40	72.83	.684	.331	.328	.499	8194.6	2928.7	40
50	72.68	.735	.398	.296	.531	8219.0	2944.2	50

TABLE 1. FUNCTIONS OF UNIT CIRCULAR CURVES (*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 ft						Tangents and externals for the unit circular curve whose radius is 10,000 ft		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
79°00′	72.53	59.786	21.465	92.265	16.563	8243.4	2959.7	79°00′
10	72.37	.837	.533	.233	.595	8267.8	2975.2	10
20	72.22	.889	.601	.201	.627	8292.3	2990.9	20
30	72.07	.940	.668	.169	.660	8316.9	3006.6	30
40	71.92	.992	.737	.137	.692	8341.5	3022.3	40
50	71.77	60.044	.805	.105	.724	8366.2	3038.2	50
80°00	71.62	.096	.873	.073	.756	8391.0	3054.1	80°00
10	71.47	.148	.942	.040	.788	8415.8	3070.0	10
20	71.32	.201	22.011	.008	.820	8440.7	3086.1	20
30	71.17	.254	.080	91.975	.852	8465.6	3102.2	30
40	71.03	.307	.149	.943	.884	8490.6	3118.3	40
50	70.88	.360	.218	.911	.916	8515.7	3134.6	50
81°00	70.74	.414	.288	.878	.948	8540.8	3150.9	81°00
10	70.59	.468	.358	.845	.980	8566.0	3167.2	10
20	70.44	.522	.428	.813	17.012	8591.2	3183.7	20
30	70.30	.576	.498	.780	.043	8616.6	3200.2	30
40	70.16	.630	.568	.747	.075	8641.9	3216.8	40
50	70.02	.685	.639	.714	.107	8667.4	3233.4	50
82°00	69.87	.740	.710	.682	.139	8692.9	3250.1	82°00
10	69.73	.795	.781	.648	.171	8718.4	3266.9	10
20	69.59	.850	.852	.615	.203	8744.1	3283.8	20
30	69.45	.906	.923	.582	.234	8769.8	3300.7	30
40	69.31	.961	.995	.549	.266	8795.5	3317.7	40
50	69.17	61.017	23.067	.516	.298	8821.4	3334.8	50
83°00	69.03	.074	.139	.483	.330	8847.3	3351.9	83°00
10	68.89	.130	.211	.449	.362	8873.2	3369.1	10
20	68.75	.187	.284	.416	.393	8899.2	3386.4	20
30	68.62	.244	.356	.382	.425	8925.3	3403.8	30
40	68.48	.301	.429	.349	.457	8951.5	3421.2	40
50	68.34	.358	.502	.315	.488	8977.7	3438.7	50
84°00	68.21	.416	.575	.282	.520	9004.0	3456.3	84°00
10	68.07	.474	.649	.248	.551	9030.4	3474.0	10
20	67.94	.532	.723	.214	.583	9056.9	3491.7	20
30	67.81	.590	.797	.180	.615	9083.4	3509.5	30
40	67.67	.649	.871	.146	.646	9109.9	3527.4	40
50	67.54	.708	.945	.113	.678	9136.6	3545.4	50

TABLE 1. FUNCTIONS OF UNIT CIRCULAR CURVES (*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 ft						Tangents and externals for the unit circular curve whose radius is 10,000 ft		
Δ	*R*	*T*	*E*	*L. C.*	*M Ord.*	*T*	*E*	Δ
85°00′	67.41	61.767	24.020	91.079	17.709	9163.3	3563.4	85°00′
10	67.27	.826	.095	.045	.741	9190.1	3581.5	10
20	67.14	.886	.170	.010	.772	9217.0	3599.7	20
30	67.01	.946	.245	90.976	.804	9243.9	3618.0	30
40	66.88	62.006	.321	.942	.835	9270.9	3636.3	40
50	66.75	.066	.397	.908	.867	9298.0	3654.8	50
86°00	66.62	.127	.472	.874	.898	9325.2	3673.3	86°00
10	66.49	.188	.549	.839	.929	9352.4	3691.9	10
20	66.36	.249	.625	.804	.961	9379.7	3710.5	20
30	66.24	.310	.702	.770	.992	9407.1	3729.3	30
40	66.11	.372	.779	.735	18.023	9434.5	3748.1	40
50	65.98	.434	.856	.701	.055	9462.0	3767.0	50
87°00	65.86	.496	.934	.666	.086	9489.6	3786.0	87°00
10	65.73	.559	25.011	.631	.117	9517.3	3805.0	10
20	65.61	.621	.089	.596	.149	9545.1	3824.2	20
30	65.48	.684	.167	.561	.180	9572.9	3843.4	30
40	65.36	.748	.246	.527	.211	9600.8	3862.8	40
50	65.23	.811	.324	.492	.242	9628.8	3882.2	50
88°00	65.11	.875	.403	.457	.273	9656.9	3901.6	88°00
10	64.98	.939	.482	.422	.305	9685.0	3921.2	10
20	64.86	63.003	.562	.386	.336	9713.3	3940.9	20
30	64.74	.068	.641	.351	.367	9741.6	3960.6	30
40	64.62	.133	.721	.316	.398	9770.0	3980.4	40
50	64.50	.198	.802	.280	.429	9798.4	4000.3	50
89°00	64.38	.263	.882	.245	.460	9827.0	4020.3	89°00
10	64.26	.329	.963	.210	.491	9855.6	4040.4	10
20	64.14	.395	26.044	.174	.522	9884.3	4060.6	20
30	64.02	.461	.124	.138	.553	9913.1	4080.8	30
40	63.90	.528	.206	.103	.584	9942	4101.2	40
50	63.78	.595	.288	.067	.615	9971	4121.6	50
90°00	63.66	.662	.370	.032	.646	10000	4142.1	90°00
10	63.54	.731	.452	89.996	.677	10029	4162.7	10
20	63.43	.797	.535	.960	.708	10058	4183.5	20
30	63.31	.865	.617	.924	.739	10088	4204.2	30
40	63.19	.934	.701	.888	.770	10117	4225.1	40
50	63.08	64.002	.784	.852	.801	10147	4246.1	50

TABLE 1. FUNCTIONS OF UNIT CIRCULAR CURVES (*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 ft						Tangents and externals for the unit circular curve whose radius is 10,000 ft		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
91°00′	62.96	64.071	26.867	89.816	18.831	10176	4267.2	91°00′
10	62.85	.140	.951	.780	.862	10206	4288.4	10
20	62.73	.210	27.036	.743	.893	10235	4309.6	20
30	62.62	.280	.120	.707	.924	10265	4330.9	30
40	62.50	.350	.205	.671	.954	10295	4352.4	40
50	62.39	.420	.290	.635	.985	10325	4373.9	50
92°00	62.28	.491	.375	.598	19.016	10355	4395.6	92°00
10	62.16	.562	.461	.561	.047	10385	4417.3	10
20	62.05	.633	.546	.525	.077	10416	4439.1	20
30	61.94	.705	.632	.488	.108	10446	4461.0	30
40	61.83	.777	.719	.452	.139	10477	4483.1	40
50	61.72	.849	.806	.415	.169	10507	4505.2	50
93°00	61.61	.922	.892	.378	.200	10538	4527.4	93°00
10	61.50	.995	.980	.341	.231	10569	4549.4	10
20	61.39	65.068	28.068	.304	.261	10599	4572.1	20
30	61.28	.141	.156	.267	.292	10630	4594.6	30
40	61.17	.215	.244	.230	.322	10661	4617.3	40
50	61.06	.290	.333	.193	.353	10692	4640.0	50
94°00	60.95	.364	.421	.156	.383	10724	4662.8	94°00
10	60.84	.439	.511	.119	.414	10755	4685.7	10
20	60.74	.514	.600	.082	.444	10786	4708.7	20
30	60.63	.590	.690	.044	.474	10818	4731.9	30
40	60.52	.666	.780	.007	.505	10850	4755.1	40
50	60.42	.742	.870	88.970	.535	10881	4778.4	50
95°00	60.31	.818	.961	.932	.565	10913	4801.9	95°00
10	60.21	.895	29.052	.895	.596	10945	4825.4	10
20	60.10	.972	.143	.857	.626	10977	4849.1	20
30	60.00	66.050	.235	.819	.656	11009	4872.8	30
40	59.89	.128	.327	.782	.687	11041	4896.7	40
50	59.79	.206	.420	.744	.717	11074	4920.7	50
96°00	59.68	.285	.512	.706	.747	11106	4944.8	96°00
10	59.58	.364	.605	.668	.777	11139	4969.0	10
20	59.48	.443	.699	.630	.808	11171	4993.3	20
30	59.37	.523	.792	.592	.838	11204	5017.7	30
40	59.27	.603	.886	.554	.868	11237	5042.2	40
50	59.17	.683	.981	.516	.898	11270	5066.8	50

TABLE 1. FUNCTIONS OF UNIT CIRCULAR CURVES (*Continued*)

Radii, tangents, externals, long chords and middle ordinates for the unit circular curve of length 100 ft						Tangents and externals for the unit circular curve whose radius is 10,000 ft		
Δ	*R*	*T*	*E*	*L. C.*	*M. Ord.*	*T*	*E*	Δ
97°00′	59.07	66.764	30.075	88.478	19.928	11303	5091.6	97°00′
10	58.97	.845	.170	.440	.958	11336	5116.5	10
20	58.86	.927	.266	.402	.988	11369	5141.5	20
30	58.76	67.009	.361	.363	20.018	11403	5166.5	30
40	58.66	.091	.458	.325	.048	11436	5191.8	40
50	58.56	.173	.554	.287	.079	11470	5217.1	50
98°00	58.46	.256	.651	.248	.109	11504	5242.5	98°00
10	58.36	.340	.748	.210	.138	11538	5268.1	10
20	58.27	.424	.846	.171	.168	11571	5293.8	20
30	58.17	.508	.944	.132	.198	11606	5319.6	30
40	58.07	.592	31.042	.094	.228	11640	5345.5	40
50	57.97	.677	.140	.055	.258	11674	5371.5	50
99°00	57.87	.762	.239	.016	.288	11708	5397.7	99°00
10	57.78	.848	.339	87.977	.318	11743	5424.0	10
20	57.68	.934	.438	.938	.348	11778	5450.4	20
30	57.58	68.021	.538	.899	.377	11812	5476.9	30
40	57.49	.108	.639	.860	.407	11847	5503.6	40
50	57.39	.195	.740	.821	.437	11882	5530.3	50
100°00	57.30	.282	.841	.782	.467	11918	5557.2	100°00
10	57.20	.371	.943	.743	.496	11953	5584.3	10
20	57.11	.459	32.045	.704	.526	11988	5611.4	20
30	57.01	.548	.147	.664	.556	12024	5638.7	30
40	56.92	.637	.250	.625	.585	12059	5666.1	40
50	56.82	.727	.353	.586	.615	12095	5693.7	50
101°00	56.73	.817	.456	.546	.645	12131	5721.3	101°00
10	56.64	.908	.561	.506	.674	12167	5749.1	10
20	56.54	.999	.665	.467	.704	12203	5777.1	20
30	56.45	69.090	.770	.427	.733	12239	5805.1	30
40	56.36	.182	.875	.388	.763	12276	5833.3	40
50	56.26	.274	.981	.348	.792	12312	5861.7	50
102°00	56.17	.367	33.086	.308	.822	12349	5890.2	102°00
10	56.08	.460	.193	.268	.851	12386	5918.8	10
20	55.99	.554	.300	.228	.881	12423	5947.5	20
30	55.90	.648	.407	.188	.910	12460	5976.4	30
40	55.81	.742	.515	.148	.940	12497	6005.4	40
50	55.72	.837	.623	.108	.969	12534	6034.6	50

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Table 2. Tangents and Externals for a 1° Curve

′	Δ = 0°		Δ = 1°		Δ = 2°		Δ = 3°		′
	T	E	T	E	T	E	T	E	
0	0.000	0.000	50.001	0.218	100.010	0.873	150.034	1.964	0
1	0.833	0.000	50.835	0.226	100.844	0.887	150.868	1.986	1
2	1.667	0.000	51.668	0.233	101.677	0.902	151.702	2.008	2
3	2.500	0.001	52.501	0.241	102.511	0.917	152.536	2.030	3
4	3.333	0.001	53.335	0.248	103.345	0.932	153.370	2.052	4
5	4.167	0.002	54.168	0.256	104.178	0.947	154.204	2.075	5
6	5.000	0.002	55.002	0.264	105.012	0.962	155.038	2.097	6
7	5.833	0.003	55.835	0.272	105.845	0.977	155.872	2.120	7
8	6.667	0.004	56.668	0.280	106.679	0.993	156.706	2.142	8
9	7.500	0.005	57.502	0.288	107.513	1.009	157.540	2.165	9
10	8.333	0.006	58.335	0.297	108.346	1.024	158.374	2.188	10
11	9.167	0.007	59.169	0.305	109.180	1.040	159.208	2.212	11
12	10.000	0.009	60.002	0.314	110.013	1.056	160.042	2.235	12
13	10.833	0.010	60.836	0.323	110.847	1.072	160.876	2.258	13
14	11.667	0.012	61.669	0.332	111.681	1.088	161.710	2.282	14
15	12.500	0.014	62.502	0.341	112.514	1.105	162.544	2.305	15
16	13.333	0.015	63.336	0.350	113.348	1.121	163.378	2.329	16
17	14.167	0.017	64.169	0.359	114.182	1.138	164.212	2.353	17
18	15.000	0.019	65.003	0.368	115.015	1.154	165.046	2.377	18
19	15.834	0.022	65.836	0.378	115.849	1.171	165.880	2.401	19
20	16.667	0.024	66.670	0.388	116.683	1.188	166.714	2.425	20
21	17.500	0.027	67.503	0.398	117.516	1.205	167.548	2.449	21
22	18.333	0.029	68.337	0.407	118.350	1.222	168.382	2.474	22
23	19.167	0.032	69.170	0.417	119.184	1.239	169.216	2.498	23
24	20.000	0.035	70.003	0.427	120.018	1.257	170.050	2.523	24
25	20.834	0.038	70.837	0.438	120.851	1.274	170.884	2.548	25
26	21.667	0.041	71.670	0.448	121.685	1.292	171.718	2.573	26
27	22.500	0.044	72.504	0.459	122.519	1.310	172.552	2.598	27
28	23.333	0.048	73.337	0.469	123.352	1.328	173.386	2.623	28
29	24.167	0.051	74.171	0.480	124.186	1.346	174.220	2.648	29
30	25.000	0.054	75.004	0.491	125.020	1.364	175.054	2.674	30
31	25.834	0.058	75.838	0.502	125.854	1.382	175.889	2.699	31
32	26.667	0.062	76.671	0.513	126.687	1.400	176.723	2.725	32
33	27.500	0.066	77.505	0.524	127.521	1.419	177.557	2.751	33
34	28.333	0.070	78.338	0.536	128.355	1.438	178.391	2.776	34
35	29.167	0.074	79.172	0.547	129.189	1.456	179.225	2.802	35
36	30.000	0.078	80.005	0.559	130.022	1.475	180.059	2.829	36
37	30.834	0.083	80.839	0.570	130.856	1.494	180.893	2.855	37
38	31.667	0.088	81.672	0.582	131.690	1.513	181.728	2.881	38
39	32.500	0.092	82.506	0.594	132.524	1.532	182.562	2.908	39
40	33.334	0.097	83.339	0.606	133.357	1.552	183.396	2.934	40
41	34.167	0.102	84.173	0.618	134.191	1.571	184.230	2.961	41
42	35.000	0.107	85.006	0.630	135.025	1.591	185.064	2.988	42
43	35.834	0.112	85.840	0.643	135.859	1.610	185.898	3.015	43
44	36.667	0.117	86.673	0.655	136.693	1.630	186.733	3.042	44
45	37.501	0.123	87.507	0.668	137.526	1.650	187.567	3.069	45
46	38.333	0.128	88.340	0.681	138.360	1.670	188.401	3.097	46
47	39.167	0.134	89.174	0.694	139.194	1.691	189.235	3.124	47
48	40.001	0.140	90.007	0.707	140.028	1.711	190.070	3.152	48
49	40.834	0.146	90.841	0.720	140.862	1.731	190.904	3.180	49
50	41.667	0.151	91.675	0.733	141.696	1.752	191.738	3.207	50
51	42.501	0.158	92.508	0.747	142.529	1.773	192.572	3.235	51
52	43.333	0.164	93.342	0.760	143.363	1.793	193.407	3.263	52
53	44.168	0.170	94.175	0.774	144.197	1.814	194.241	3.292	53
54	45.001	0.176	95.009	0.788	145.031	1.835	195.075	3.320	54
55	45.834	0.183	95.842	0.802	145.865	1.856	195.910	3.348	55
56	46.667	0.190	96.676	0.815	146.699	1.878	196.744	3.377	56
57	47.501	0.197	97.509	0.830	147.533	1.899	197.578	3.406	57
58	48.333	0.204	98.343	0.844	148.366	1.921	198.413	3.434	58
59	49.168	0.211	99.177	0.858	149.200	1.942	199.247	3.463	59

$D = \frac{T_{table}}{T_{field}}$ $D = \frac{E_{table}}{E_{field}}$

TABLE 2. TANGENTS AND EXTERNALS FOR A 1° CURVE (*Continued*)

′	Δ = 4°		Δ = 5°		Δ = 6°		Δ = 7°		′
	T	E	T	E	T	E	T	E	
0	200.081	3.492	250.159	5.458	300.274	7.863	350.436	10.707	0
1	200.916	3.522	250.994	5.495	301.110	7.907	351.272	10.758	1
2	201.750	3.551	251.829	5.532	301.946	7.951	352.109	10.809	2
3	202.584	3.580	252.664	5.568	302.781	7.995	352.945	10.861	3
4	203.419	3.610	253.499	5.605	303.617	8.039	353.782	10.912	4
5	204.253	3.640	254.333	5.642	304.453	8.083	354.618	10.964	5
6	205.088	3.669	255.168	5.679	305.288	8.128	355.455	11.015	6
7	205.922	3.699	256.003	5.716	306.124	8.172	356.291	11.067	7
8	206.756	3.729	256.838	5.754	306.960	8.217	357.128	11.119	8
9	207.591	3.759	257.673	5.791	307.796	8.261	357.965	11.171	9
10	208.425	3.790	258.509	5.829	308.631	8.306	358.801	11.224	10
11	209.260	3.820	259.344	5.866	309.467	8.351	359.638	11.276	11
12	210.094	3.851	260.179	5.904	310.303	8.397	360.474	11.328	12
13	210.929	3.881	261.014	5.942	311.139	8.442	361.311	11.381	13
14	211.763	3.912	261.849	5.980	311.974	8.487	362.148	11.434	14
15	212.597	3.943	262.684	6.018	312.810	8.533	362.984	11.486	15
16	213.432	3.974	263.519	6.057	313.646	8.578	363.821	11.539	16
17	214.266	4.005	264.354	6.095	314.482	8.624	364.658	11.593	17
18	215.101	4.036	265.189	6.134	315.318	8.670	365.495	11.646	18
19	215.935	4.068	266.024	6.172	316.154	8.716	366.331	11.699	19
20	216.770	4.099	266.859	6.211	316.989	8.762	367.168	11.753	20
21	217.604	4.131	267.694	6.250	317.825	8.808	368.005	11.806	21
22	218.439	4.162	268.530	6.289	318.661	8.855	368.842	11.860	22
23	219.274	4.194	269.365	6.328	319.497	8.901	369.678	11.914	23
24	220.108	4.226	270.200	6.368	320.333	8.948	370.515	11.968	24
25	220.943	4.258	271.035	6.407	321.169	8.994	371.352	12.022	25
26	221.777	4.291	271.870	6.447	322.005	9.041	372.189	12.076	26
27	222.612	4.323	272.706	6.486	322.841	9.088	373.026	12.130	27
28	223.446	4.356	273.541	6.526	323.677	9.135	373.863	12.185	28
29	224.281	4.388	274.376	6.566	324.513	9.183	374.699	12.239	29
30	225.116	4.421	275.211	6.606	325.349	9.230	375.536	12.294	30
31	225.950	4.454	276.047	6.646	326.185	9.277	376.373	12.349	31
32	226.785	4.486	276.882	6.686	327.021	9.325	377.210	12.404	32
33	227.620	4.519	277.717	6.727	327.857	9.373	378.047	12.459	33
34	228.454	4.553	278.552	6.767	328.693	9.420	378.884	12.514	34
35	229.289	4.586	279.388	6.808	329.529	9.468	379.721	12.569	35
36	230.124	4.620	280.223	6.849	330.365	9.516	380.558	12.624	36
37	230.958	4.653	281.058	6.889	331.201	9.565	381.395	12.680	37
38	231.793	4.687	281.894	6.930	332.038	9.613	382.232	12.736	38
39	232.628	4.721	282.729	6.971	332.874	9.661	383.069	12.791	39
40	233.462	4.754	283.565	7.013	333.710	9.710	383.906	12.847	40
41	234.297	4.788	284.400	7.054	334.546	9.759	384.743	12.903	41
42	235.132	4.823	285.235	7.096	335.382	9.807	385.580	12.959	42
43	235.967	4.857	286.071	7.137	336.218	9.856	386.418	13.016	43
44	236.801	4.891	286.906	7.179	337.055	9.905	387.255	13.072	44
45	237.636	4.926	287.742	7.221	337.891	9.955	388.092	13.129	45
46	238.471	4.961	288.577	7.263	338.727	10.004	388.929	13.185	46
47	239.306	4.995	289.412	7.305	339.563	10.053	389.766	13.242	47
48	240.140	5.030	290.248	7.347	340.400	10.103	390.603	13.299	48
49	240.975	5.065	291.083	7.389	341.236	10.152	391.441	13.356	49
50	241.810	5.100	291.919	7.432	342.072	10.202	392.278	13.413	50
51	242.645	5.136	292.754	7.474	342.909	10.252	393.115	13.470	51
52	243.480	5.171	293.590	7.517	343.745	10.302	393.952	13.528	52
53	244.315	5.207	294.425	7.560	344.581	10.352	394.790	13.585	53
54	245.149	5.242	295.261	7.603	345.418	10.403	395.627	13.643	54
55	245.984	5.278	296.096	7.646	346.254	10.453	396.464	13.700	55
56	246.819	5.314	296.932	7.689	347.090	10.503	397.302	13.758	56
57	247.654	5.350	297.768	7.732	347.927	10.554	398.139	13.816	57
58	248.489	5.386	298.603	7.776	348.763	10.605	398.976	13.874	58
59	249.324	5.422	299.439	7.819	349.600	10.656	399.814	13.933	59

$D = \frac{T_{table}}{T_{field}}$ $D = \frac{E_{table}}{E_{field}}$

TABLE 2. TANGENTS AND EXTERNALS FOR A 1° CURVE (*Continued*)

′	Δ = 8°		Δ = 9°		Δ = 10°		Δ = 11°		′
	T	E	T	E	T	E	T	E	
0	400.651	13.991	450.928	17.717	501.273	21.886	551.696	26.500	0
1	401.489	14.050	451.766	17.783	502.113	21.959	552.537	26.581	1
2	402.326	14.108	452.605	17.849	502.953	22.033	553.378	26.661	2
3	403.163	14.167	453.443	17.915	503.792	22.106	554.219	26.742	3
4	404.001	14.226	454.282	17.981	504.632	22.180	555.060	26.823	4
5	404.838	14.285	455.120	18.047	505.472	22.254	555.901	26.904	5
6	405.676	14.344	455.959	18.114	506.312	22.327	556.742	26.986	6
7	406.513	14.403	456.797	18.180	507.152	22.401	557.584	27.067	7
8	407.351	14.462	457.636	18.247	507.991	22.475	558.425	27.149	8
9	408.188	14.522	458.475	18.314	508.831	22.550	559.266	27.230	9
10	409.026	14.581	459.314	18.381	509.671	22.624	560.107	27.312	10
11	409.864	14.641	460.152	18.448	510.511	22.699	560.949	27.394	11
12	410.701	14.701	460.991	18.515	511.351	22.773	561.790	27.476	12
13	411.539	14.761	461.830	18.583	512.191	22.848	562.631	27.558	13
14	412.376	14.821	462.668	18.650	513.031	22.923	563.473	27.641	14
15	413.214	14.881	463.507	18.718	513.871	22.998	564.314	27.723	15
16	414.052	14.941	464.346	18.785	514.711	23.073	565.156	27.806	16
17	414.890	15.002	465.185	18.853	515.551	23.148	565.997	27.888	17
18	415.727	15.062	466.024	18.921	516.391	23.223	566.839	27.971	18
19	416.565	15.123	466.862	18.989	517.232	23.299	567.680	28.054	19
20	417.403	15.184	467.701	19.057	518.072	23.374	568.522	28.137	20
21	418.240	15.245	468.540	19.126	518.912	23.450	569.363	28.220	21
22	419.078	15.306	469.379	19.194	519.752	23.526	570.205	28.303	22
23	419.916	15.367	470.218	19.263	520.592	23.602	571.046	28.387	23
24	420.754	15.428	471.057	19.331	521.432	23.678	571.888	28.470	24
25	421.592	15.490	471.896	19.400	522.273	23.754	572.730	28.554	25
26	422.430	15.551	472.735	19.469	523.113	23.831	573.571	28.638	26
27	423.267	15.613	473.574	19.538	523.953	23.907	574.413	28.722	27
28	424.105	15.675	474.413	19.607	524.794	23.984	575.255	28.806	28
29	424.943	15.737	475.252	19.677	525.634	24.060	576.096	28.890	29
30	425.781	15.799	476.091	19.746	526.474	24.137	576.938	28.974	30
31	426.619	15.861	476.930	19.816	527.315	24.214	577.780	29.058	31
32	427.457	15.923	477.769	19.885	528.155	24.291	578.622	29.143	32
33	428.295	15.986	478.609	19.955	528.995	24.368	579.464	29.228	33
34	429.133	16.048	479.448	20.025	529.836	24.446	580.305	29.312	34
35	429.971	16.111	480.287	20.095	530.676	24.523	581.147	29.397	35
36	430.809	16.174	481.126	20.165	531.517	24.601	581.989	29.482	36
37	431.647	16.236	481.965	20.235	532.357	24.678	582.831	29.567	37
38	432.485	16.299	482.805	20.306	533.198	24.756	583.673	29.653	38
39	433.323	16.363	483.644	20.376	534.038	24.834	584.515	29.738	39
40	434.161	16.426	484.483	20.447	534.879	24.912	585.357	29.824	40
41	435.000	16.489	485.322	20.518	535.720	24.991	586.199	29.909	41
42	435.838	16.553	486.162	20.589	536.560	25.069	587.041	29.995	42
43	436.676	16.616	487.001	20.660	537.401	25.147	587.883	30.081	43
44	437.514	16.680	487.840	20.731	538.242	25.226	588.725	30.167	44
45	438.352	16.744	488.680	20.802	539.082	25.305	589.568	30.253	45
46	439.190	16.808	489.519	20.874	539.923	25.383	590.410	30.339	46
47	440.029	16.872	490.359	20.945	540.764	25.462	591.252	30.426	47
48	440.867	16.936	491.198	21.017	541.605	25.541	592.094	30.512	48
49	441.705	17.001	492.038	21.088	542.445	25.621	592.936	30.599	49
50	442.544	17.065	492.877	21.160	543.286	25.700	593.779	30.686	50
51	443.382	17.130	493.717	21.232	544.127	25.779	594.621	30.773	51
52	444.220	17.195	494.556	21.305	544.968	25.859	595.463	30.860	52
53	445.059	17.259	495.396	21.377	545.809	25.939	596.306	30.947	53
54	445.897	17.324	496.235	21.449	546.650	26.018	597.148	31.034	54
55	446.735	17.390	497.075	21.522	547.491	26.098	597.990	31.121	55
56	447.574	17.455	497.914	21.594	548.332	26.178	598.833	31.209	56
57	448.412	17.520	498.754	21.667	549.173	26.259	599.675	31.296	57
58	449.251	17.586	499.594	21.740	550.014	26.339	600.518	31.384	58
59	450.089	17.651	500.433	21.813	550.855	26.419	601.360	31.472	59

TABLE 2. TANGENTS AND EXTERNALS FOR A 1° CURVE (*Continued*)

′	Δ = 12°		Δ = 13°		Δ = 14°		Δ = 15°		′
	T	E	T	E	T	E	T	E	
0	602.203	31.560	652.803	37.069	703.504	43.028	754.313	49.440	0
1	603.045	31.648	653.647	37.164	704.350	43.131	755.161	49.551	1
2	603.888	31.736	654.491	37.260	705.195	43.235	756.009	49.662	2
3	604.731	31.825	655.335	37.356	706.041	43.338	756.857	49.773	3
4	605.573	31.913	656.180	37.452	706.887	43.442	757.705	49.884	4
5	606.416	32.002	657.024	37.548	707.733	43.545	758.552	49.995	5
6	607.258	32.091	657.868	37.644	708.580	43.649	759.400	50.107	6
7	608.101	32.180	658.713	37.741	709.426	43.753	760.248	50.218	7
8	608.944	32.269	659.557	37.837	710.272	43.857	761.096	50.330	8
9	609.787	32.358	660.401	37.934	711.118	43.961	761.944	50.441	9
10	610.630	32.447	661.246	38.031	711.964	44.065	762.793	50.553	10
11	611.472	32.536	662.090	38.128	712.810	44.170	763.641	50.665	11
12	612.315	32.626	662.935	38.225	713.657	44.274	764.489	50.777	12
13	613.158	32.716	663.779	38.322	714.503	44.379	765.337	50.889	13
14	614.001	32.805	664.624	38.419	715.349	44.484	766.185	51.002	14
15	614.844	32.895	665.468	38.516	716.195	44.589	767.033	51.114	15
16	615.687	32.985	666.313	38.614	717.042	44.694	767.882	51.227	16
17	616.530	33.075	667.157	38.711	717.888	44.799	768.730	51.340	17
18	617.373	33.166	668.002	38.809	718.735	44.904	769.578	51.453	18
19	618.216	33.256	668.847	38.907	719.581	45.010	770.427	51.566	19
20	619.059	33.346	669.692	39.005	720.428	45.115	771.275	51.679	20
21	619.902	33.437	670.536	39.103	721.274	45.221	772.124	51.792	21
22	620.745	33.528	671.381	39.201	722.121	45.327	772.972	51.905	22
23	621.588	33.619	672.226	39.300	722.967	45.432	773.821	52.019	23
24	622.431	33.710	673.071	39.398	723.814	45.539	774.669	52.133	24
25	623.274	33.801	673.915	39.497	724.661	45.645	775.518	52.246	25
26	624.118	33.892	674.760	39.596	725.507	45.751	776.366	52.360	26
27	624.961	33.983	675.605	39.695	726.354	45.857	777.215	52.474	27
28	625.804	34.075	676.450	39.794	727.201	45.964	778.064	52.588	28
29	626.647	34.167	677.295	39.893	728.047	46.071	778.913	52.703	29
30	627.491	34.258	678.140	39.992	728.894	46.177	779.761	52.817	30
31	628.334	34.350	678.985	40.091	729.741	46.284	780.610	52.931	31
32	629.177	34.442	679.830	40.191	730.588	46.391	781.459	53.046	32
33	630.021	34.534	680.675	40.290	731.435	46.499	782.308	53.161	33
34	630.864	34.627	681.520	40.390	732.282	46.606	783.157	53.276	34
35	631.708	34.719	682.366	40.490	733.129	46.713	784.005	53.391	35
36	632.551	34.811	683.211	40.590	733.976	46.821	784.854	53.506	36
37	633.395	34.904	684.056	40.690	734.823	46.929	785.703	53.621	37
38	634.238	34.997	684.901	40.791	735.670	47.036	786.552	53.737	38
39	635.082	35.090	685.746	40.891	736.517	47.144	787.402	53.852	39
40	635.925	35.183	686.592	40.991	737.364	47.252	788.251	53.968	40
41	636.769	35.276	687.437	41.092	738.211	47.361	789.100	54.084	41
42	637.613	35.369	688.282	41.193	739.058	47.469	789.949	54.200	42
43	638.456	35.462	689.128	41.294	739.906	47.577	790.798	54.316	43
44	639.300	35.556	689.973	41.395	740.753	47.686	791.647	54.432	44
45	640.114	35.649	690.819	41.496	741.600	47.795	792.496	54.548	45
46	640.987	35.743	691.664	41.597	742.447	47.903	793.346	54.665	46
47	641.831	35.837	692.509	41.699	743.295	48.012	794.195	54.781	47
48	642.675	35.931	693.355	41.800	744.142	48.122	795.045	54.898	48
49	643.519	36.025	694.201	41.902	744.990	48.231	795.894	55.015	49
50	644.363	36.119	695.046	42.004	745.837	48.340	796.743	55.132	50
51	645.207	36.214	695.892	42.105	746.684	48.449	797.593	55.249	51
52	646.051	36.308	696.737	42.207	747.532	48.559	798.442	55.366	52
53	646.895	36.403	697.583	42.310	748.379	48.669	799.292	55.483	53
54	647.738	36.498	698.429	42.412	749.227	48.779	800.141	55.601	54
55	648.582	36.593	699.274	42.514	750.075	48.889	800.991	55.718	55
56	649.426	36.688	700.120	42.617	750.922	48.999	801.841	55.836	56
57	650.270	36.783	700.966	42.719	751.770	49.109	802.690	55.954	57
58	651.115	36.878	701.812	42.822	752.618	49.219	803.540	56.072	58
59	651.959	36.973	702.658	42.925	753.465	49.330	804.390	56.190	59

Table 2. Tangents and Externals for a 1° Curve (*Continued*)

′	Δ = 16°		Δ = 17°		Δ = 18°		Δ = 19°		′
	T	E	T	E	T	E	T	E	
0	805.240	56.308	856.291	63.633	907.476	71.420	958.803	79.670	0
1	806.089	56.426	857.143	63.759	908.330	71.554	959.659	79.812	1
2	806.939	56.545	857.995	63.885	909.184	71.687	960.516	79.953	2
3	807.789	56.663	858.847	64.012	910.039	71.821	961.373	80.095	3
4	808.639	56.782	859.699	64.138	910.893	71.955	962.229	80.237	4
5	809.489	56.901	860.551	64.265	911.748	72.090	963.086	80.379	5
6	810.339	57.020	861.403	64.391	912.602	72.224	963.943	80.521	6
7	811.189	57.139	862.256	64.518	913.457	72.358	964.800	80.663	7
8	812.039	57.258	863.108	64.645	914.311	72.493	965.657	80.806	8
9	812.889	57.377	863.960	64.772	915.166	72.628	966.514	80.948	9
10	813.739	57.497	864.812	64.899	916.020	72.763	967.371	81.091	10
11	814.589	57.617	865.665	65.026	916.875	72.898	968.228	81.233	11
12	815.440	57.736	866.517	65.154	917.730	73.033	969.085	81.376	12
13	816.290	57.856	867.370	65.281	918.584	73.168	969.943	81.519	13
14	817.140	57.976	868.222	65.409	919.439	73.303	970.800	81.663	14
15	817.990	58.096	869.074	65.537	920.294	73.439	971.657	81.806	15
16	818.841	58.216	869.927	65.665	921.149	73.575	972.514	81.949	16
17	819.691	58.337	870.779	65.793	922.004	73.710	973.372	82.093	17
18	820.542	58.457	871.632	65.921	922.859	73.846	974.229	82.236	18
19	821.392	58.578	872.485	66.049	923.714	73.982	975.087	82.380	19
20	822.243	58.699	873.337	66.178	924.569	74.118	975.944	82.524	20
21	823.093	58.820	874.190	66.306	925.424	74.255	976.802	82.668	21
22	823.944	58.941	875.043	66.435	926.279	74.391	977.659	82.812	22
23	824.794	59.062	875.896	66.564	927.134	74.528	978.517	82.957	23
24	825.645	59.183	876.749	66.693	927.989	74.664	979.375	83.101	24
25	826.495	59.304	877.601	66.822	928.844	74.801	980.232	83.246	25
26	827.346	59.426	878.454	66.951	929.699	74.938	981.090	83.390	26
27	828.197	59.548	879.307	67.080	930.555	75.075	981.948	83.535	27
28	829.048	59.669	880.160	67.210	931.410	75.212	982.806	83.680	28
29	829.898	59.791	881.013	67.339	932.266	75.350	983.664	83.825	29
30	830.749	59.913	881.866	67.469	933.121	75.487	984.522	83.971	30
31	831.600	60.035	882.719	67.599	933.976	75.625	985.379	84.116	31
32	832.451	60.158	883.573	67.729	934.832	75.762	986.237	84.261	32
33	833.302	60.280	884.426	67.859	935.687	75.900	987.095	84.407	33
34	834.153	60.403	885.279	67.989	936.543	76.038	987.954	84.553	34
35	835.004	60.525	886.132	68.119	937.399	76.176	988.812	84.699	35
36	835.855	60.648	886.985	68.250	938.254	76.314	989.670	84.845	36
37	836.706	60.771	887.839	68.380	939.110	76.453	990.528	84.991	37
38	837.557	60.894	888.692	68.511	939.966	76.591	991.386	85.137	38
39	838.408	61.017	889.546	68.642	940.822	76.730	992.245	85.283	39
40	839.260	61.140	890.399	68.773	941.677	76.868	993.103	85.430	40
41	840.111	61.264	891.252	68.904	942.533	77.007	993.961	85.577	41
42	840.962	61.387	892.106	69.035	943.389	77.146	994.820	85.723	42
43	841.813	61.511	892.960	69.167	944.245	77.285	995.678	85.870	43
44	842.665	61.635	893.813	69.298	945.101	77.425	996.537	86.017	44
45	843.516	61.759	894.667	69.430	945.957	77.564	997.395	86.165	45
46	844.367	61.883	895.520	69.562	946.813	77.704	998.254	86.312	46
47	845.219	62.007	896.374	69.694	947.669	77.843	999.113	86.459	47
48	846.070	62.132	897.228	69.826	948.525	77.983	999.971	86.607	48
49	846.922	62.256	898.082	69.958	949.382	78.123	1000.830	86.755	49
50	847.774	62.381	898.936	70.090	950.238	78.263	1001.689	86.902	50
51	848.625	62.505	899.789	70.222	951.094	78.403	1002.548	87.050	51
52	849.477	62.630	900.643	70.355	951.950	78.543	1003.407	87.199	52
53	850.328	62.755	901.497	70.488	952.807	78.684	1004.265	87.347	53
54	851.180	62.880	902.351	70.620	953.663	78.824	1005.124	87.495	54
55	852.032	63.005	903.205	70.753	954.520	78.965	1005.983	87.644	55
56	852.884	63.131	904.059	70.886	955.376	79.106	1006.842	87.792	56
57	853.735	63.256	904.913	71.019	956.233	79.247	1007.702	87.941	57
58	854.587	63.382	905.767	71.153	957.089	79.388	1008.561	88.090	58
59	855.439	63.507	906.622	71.286	957.946	79.529	1009.420	88.239	59

TABLE 2. TANGENTS AND EXTERNALS FOR A 1° CURVE (*Continued*)

′	Δ = 20°		Δ = 21°		Δ = 22°		Δ = 23°		′
	T	E	T	E	T	E	T	E	
0	1010.28	88.388	1061.91	97.576	1113.72	107.239	1165.70	117.379	0
1	1011.14	88.537	1062.78	97.733	1114.58	107.404	1166.56	117.552	1
2	1012.00	88.687	1063.64	97.891	1115.45	107.569	1167.43	117.726	2
3	1012.86	88.836	1064.50	98.048	1116.31	107.734	1168.30	117.899	3
4	1013.72	88.986	1065.36	98.206	1117.18	107.900	1169.17	118.073	4
5	1014.58	89.136	1066.22	98.363	1118.04	108.065	1170.04	118.246	5
6	1015.44	89.285	1067.09	98.521	1118.91	108.231	1170.90	118.420	6
7	1016.29	89.436	1067.95	98.679	1119.77	108.397	1171.77	118.594	7
8	1017.15	89.586	1068.81	98.837	1120.64	108.563	1172.64	118.768	8
9	1018.01	89.736	1069.67	98.995	1121.50	108.729	1173.51	118.942	9
10	1018.87	89.887	1070.54	99.154	1122.37	108.896	1174.38	119.116	10
11	1019.73	90.037	1071.40	99.312	1123.23	109.062	1175.24	119.291	11
12	1020.59	90.188	1072.26	99.471	1124.10	109.229	1176.11	119.465	12
13	1021.45	90.339	1073.12	99.629	1124.96	109.395	1176.98	119.640	13
14	1022.31	90.490	1073.99	99.788	1125.83	109.562	1177.85	119.815	14
15	1023.17	90.641	1074.85	99.947	1126.69	109.729	1178.72	119.990	15
16	1024.03	90.792	1075.71	100.106	1127.56	109.896	1179.59	120.165	16
17	1024.89	90.943	1076.57	100.266	1128.43	110.063	1180.46	120.340	17
18	1025.75	91.095	1077.44	100.425	1129.29	110.231	1181.32	120.515	18
19	1026.61	91.246	1078.30	100.584	1130.16	110.398	1182.19	120.691	19
20	1027.47	91.398	1079.16	100.744	1131.02	110.566	1183.06	120.867	20
21	1028.33	91.550	1080.03	100.904	1131.89	110.733	1183.93	121.042	21
22	1029.19	91.702	1080.89	101.064	1132.75	110.901	1184.80	121.218	22
23	1030.05	91.854	1081.75	101.224	1133.62	111.069	1185.67	121.394	23
24	1030.91	92.007	1082.61	101.384	1134.49	111.237	1186.54	121.570	24
25	1031.77	92.159	1083.48	101.544	1135.35	111.406	1187.41	121.747	25
26	1032.63	92.312	1084.34	101.705	1136.22	111.574	1188.28	121.923	26
27	1033.49	92.464	1085.20	101.865	1137.09	111.743	1189.15	122.100	27
28	1034.36	92.617	1086.07	102.026	1137.95	111.911	1190.02	122.276	28
29	1035.22	92.770	1086.93	102.187	1138.82	112.080	1190.88	122.453	29
30	1036.08	92.923	1087.79	102.348	1139.68	112.249	1191.75	122.630	30
31	1036.94	93.076	1088.66	102.509	1140.55	112.418	1192.62	122.807	31
32	1037.80	93.230	1089.52	102.670	1141.42	112.587	1193.49	122.985	32
33	1038.66	93.383	1090.38	102.832	1142.28	112.757	1194.36	123.162	33
34	1039.52	93.537	1091.25	102.993	1143.15	112.926	1195.23	123.340	34
35	1040.38	93.690	1092.11	103.155	1144.02	113.096	1196.10	123.517	35
36	1041.24	93.844	1092.98	103.317	1144.88	113.266	1196.97	123.695	36
37	1042.10	93.998	1093.84	103.478	1145.75	113.435	1197.84	123.873	37
38	1042.96	94.152	1094.70	103.640	1146.62	113.605	1198.71	124.051	38
39	1043.82	94.307	1095.57	103.803	1147.48	113.775	1199.58	124.229	39
40	1044.68	94.461	1096.43	103.965	1148.35	113.946	1200.45	124.407	40
41	1045.55	94.616	1097.29	104.127	1149.22	114.116	1201.32	124.586	41
42	1046.41	94.770	1098.16	104.290	1150.08	114.287	1202.19	124.764	42
43	1047.27	94.925	1099.02	104.453	1150.95	114.457	1203.06	124.943	43
44	1048.13	95.080	1099.89	104.615	1151.82	114.628	1203.93	125.122	44
45	1048.99	95.235	1100.75	104.778	1152.68	114.799	1204.80	125.301	45
46	1049.85	95.340	1101.61	104.942	1153.55	114.970	1205.67	125.480	46
47	1050.71	95.545	1102.48	105.105	1154.42	115.141	1206.54	125.659	47
48	1051.57	95.701	1103.34	105.268	1155.29	115.313	1207.41	125.839	48
49	1052.44	95.856	1104.21	105.432	1156.15	115.484	1208.28	126.018	49
50	1053.30	96.012	1105.07	105.595	1157.02	115.656	1209.15	126.198	50
51	1054.16	96.168	1105.94	105.759	1157.89	115.828	1210.02	126.378	51
52	1055.02	96.324	1106.80	105.923	1158.75	116.000	1210.89	126.558	52
53	1055.88	96.481	1107.66	106.087	1159.62	116.172	1211.76	126.738	53
54	1056.74	96.636	1108.53	106.251	1160.49	116.344	1212.63	126.918	54
55	1057.61	96.792	1109.39	106.415	1161.36	116.516	1213.51	127.098	55
56	1058.47	96.949	1110.26	106.580	1162.22	116.688	1214.38	127.279	56
57	1059.33	97.106	1111.12	106.744	1163.09	116.861	1215.25	127.459	57
58	1060.19	97.262	1111.99	106.909	1163.96	117.034	1216.12	127.640	58
59	1061.05	97.419	1112.85	107.074	1164.83	117.206	1216.99	127.821	59

TABLE 2. TANGENTS AND EXTERNALS FOR A 1° CURVE (*Continued*)

′	Δ = 24°		Δ = 25°		Δ = 26°		Δ = 27°		′
	T	E	T	E	T	E	T	E	
0	1217.86	128.002	1270.22	139.111	1322.78	150.711	1375.55	162.807	0
1	1218.73	128.183	1271.09	139.301	1323.65	150.909	1376.43	163.013	1
2	1219.60	128.364	1271.97	139.490	1324.53	151.107	1377.31	163.219	2
3	1220.48	128.546	1272.84	139.680	1325.41	151.304	1378.19	163.425	3
4	1221.34	128.728	1273.71	139.869	1326.29	151.502	1379.08	163.631	4
5	1222.21	128.909	1274.59	140.059	1327.17	151.700	1379.96	163.838	5
6	1223.09	129.091	1275.46	140.249	1328.04	151.899	1380.84	164.044	6
7	1223.96	129.273	1276.34	140.439	1328.92	152.097	1381.72	164.251	7
8	1224.83	129.455	1277.21	140.630	1329.80	152.295	1382.60	164.458	8
9	1225.70	129.637	1278.09	140.820	1330.69	152.494	1383.48	164.664	9
10	1226.57	129.820	1278.96	141.011	1331.56	152.693	1384.37	164.872	10
11	1227.44	130.002	1279.84	141.201	1332.44	152.892	1385.25	165.079	11
12	1228.31	130.185	1280.71	141.392	1333.31	153.091	1386.13	165.286	12
13	1229.19	130.368	1281.59	141.583	1334.19	153.290	1387.01	165.494	13
14	1230.06	130.551	1282.46	141.774	1335.07	153.489	1387.90	165.701	14
15	1230.93	130.734	1283.34	141.965	1335.95	153.689	1388.78	165.909	15
16	1231.80	130.917	1284.21	142.157	1336.83	153.888	1389.66	166.117	16
17	1232.67	131.100	1285.09	142.348	1337.71	154.088	1390.54	166.325	17
18	1233.55	131.284	1285.96	142.540	1338.59	154.288	1391.42	166.533	18
19	1234.42	131.467	1286.84	142.731	1339.47	154.488	1392.31	166.741	19
20	1235.29	131.651	1287.71	142.923	1340.34	154.688	1393.19	166.950	20
21	1236.16	131.835	1288.59	143.115	1341.22	154.888	1394.07	167.158	21
22	1237.03	132.019	1289.46	143.307	1342.10	155.089	1394.95	167.367	22
23	1237.91	132.203	1290.34	143.500	1342.98	155.289	1395.84	167.576	23
24	1238.78	132.387	1291.22	143.692	1343.86	155.490	1396.72	167.785	24
25	1239.65	132.572	1292.09	143.885	1344.74	155.691	1397.60	167.994	25
26	1240.52	132.756	1292.97	144.077	1345.62	155.892	1398.49	168.203	26
27	1241.40	132.941	1293.84	144.270	1346.50	156.093	1399.37	168.413	27
28	1242.27	133.126	1294.72	144.463	1347.38	156.294	1400.25	168.623	28
29	1243.14	133.311	1295.60	144.656	1348.26	156.495	1401.14	168.832	29
30	1244.01	133.496	1296.47	144.850	1349.14	156.697	1402.02	169.042	30
31	1244.89	133.681	1297.35	145.043	1350.02	156.899	1402.90	169.252	31
32	1245.76	133.866	1298.22	145.237	1350.90	157.101	1403.79	169.462	32
33	1246.63	134.052	1299.10	145.430	1351.78	157.302	1404.67	169.673	33
34	1247.50	134.237	1299.98	145.624	1352.66	157.505	1405.55	169.883	34
35	1248.38	134.423	1300.85	145.818	1353.53	157.707	1406.44	170.094	35
36	1249.25	134.609	1301.73	146.012	1354.41	157.909	1407.32	170.304	36
37	1250.12	134.795	1302.60	146.206	1355.29	158.112	1408.20	170.515	37
38	1251.00	134.981	1303.48	146.401	1356.17	158.314	1409.09	170.726	38
39	1251.87	135.168	1304.36	146.595	1357.05	158.517	1409.97	170.937	39
40	1252.74	135.354	1305.23	146.790	1357.93	158.720	1410.85	171.149	40
41	1253.62	135.540	1306.11	146.985	1358.82	158.923	1411.74	171.360	41
42	1254.49	135.727	1306.99	147.180	1359.70	159.126	1412.62	171.571	42
43	1255.36	135.914	1307.86	147.375	1360.58	159.329	1413.51	171.783	43
44	1256.24	136.101	1308.74	147.570	1361.46	159.533	1414.39	171.995	44
45	1257.11	136.288	1309.62	147.765	1362.34	159.736	1415.27	172.207	45
46	1257.98	136.475	1310.49	147.961	1363.22	159.940	1416.16	172.419	46
47	1258.86	136.663	1311.37	148.156	1364.10	160.144	1417.04	172.631	47
48	1259.73	136.850	1312.25	148.352	1364.98	160.348	1417.93	172.844	48
49	1260.60	137.038	1313.13	148.548	1365.86	160.552	1418.81	173.056	49
50	1261.48	137.226	1314.00	148.744	1366.74	160.756	1419.70	173.269	50
51	1262.35	137.414	1314.88	148.940	1367.62	160.961	1420.58	173.482	51
52	1263.22	137.602	1315.76	149.136	1368.50	161.165	1421.47	173.695	52
53	1264.10	137.790	1316.63	149.332	1369.38	161.370	1422.35	173.908	53
54	1264.97	137.978	1317.51	149.529	1370.26	161.575	1423.23	174.121	54
55	1265.85	138.167	1318.39	149.726	1371.14	161.780	1424.12	174.334	55
56	1266.72	138.355	1319.27	149.923	1372.02	161.985	1425.00	174.548	56
57	1267.59	138.544	1320.14	150.120	1372.91	162.190	1425.89	174.761	57
58	1268.47	138.733	1321.02	150.317	1373.79	162.396	1426.77	174.975	58
59	1269.34	138.922	1321.90	150.514	1374.67	162.601	1427.66	175.189	59

TABLE 2. TANGENTS AND EXTERNALS FOR A 1° CURVE (*Continued*)

′	Δ = 28°		Δ = 29°		Δ = 30°		Δ = 31°		′
	T	E	T	E	T	E	T	E	
0	1428.54	175.403	1481.77	188.505	1535.24	202.118	1588.95	216.247	0
1	1429.43	175.617	1482.66	188.728	1536.13	202.349	1589.85	216 489	1
2	1430.31	175.832	1483.55	188.950	1537.02	202.580	1590.75	216.727	2
3	1431.20	176.046	1484.44	189.173	1537.92	202.812	1591.65	216.967	3
4	1432.09	176.261	1485.33	189.396	1538.81	203.043	1592.54	217.207	4
5	1432.97	176.476	1486.22	189.620	1539.70	203.275	1593.44	217.448	5
6	1433.86	176.690	1487.11	189.843	1540.60	203.507	1594.34	217.688	6
7	1434.74	176.906	1487.99	190.067	1541.49	203.739	1595.24	217.929	7
8	1435.63	177.121	1488.88	190.290	1542.38	203.972	1596.13	218.170	8
9	1436.51	177.336	1489.77	190.514	1543.28	204.204	1597.03	218.411	9
10	1437.40	177.552	1490.66	190.738	1544.17	204.437	1597.93	218.652	10
11	1438.28	177.767	1491.55	190.962	1545.06	204.669	1598.83	218.894	11
12	1439.17	177.983	1492.44	191.186	1545.96	204.902	1599.73	219.135	12
13	1440.06	178.199	1493.33	191.411	1546.85	205.135	1600.63	219.377	13
14	1440.94	178.415	1494.22	191.635	1547.75	205.368	1601.52	219.619	14
15	1441.83	178.631	1495.11	191.860	1548.64	205.601	1602.42	219.861	15
16	1442.71	178.847	1496.00	192.085	1549.54	205.835	1603.32	220.103	16
17	1443.60	179.064	1496.89	192.310	1550.43	206.068	1604.22	220 345	17
18	1444.49	179.280	1497.78	192.535	1551.32	206.302	1605.12	220.587	18
19	1445.37	179.497	1498.67	192.760	1552.22	206.536	1606.02	220.830	19
20	1446.26	179.714	1499.56	192.985	1553.11	206.770	1606.92	221.073	20
21	1447.15	179.931	1500.45	193.211	1554.01	207.004	1607 81	221.315	21
22	1448.03	180.148	1501.35	193.436	1554.90	207.238	1608.71	221.558	22
23	1448.92	180.365	1502.24	193.662	1555.80	207.473	1609.61	221.801	23
24	1449.81	180.583	1503.13	193.888	1556.69	207.707	1610.51	222.045	24
25	1450.69	180.800	1504.02	194.144	1557.59	207.942	1611.41	222.288	25
26	1451.58	181.018	1504.91	194.341	1558.48	208.177	1612.31	222.532	26
27	1452.47	181.236	1505.80	194.567	1559.38	208.412	1613.21	222.775	27
28	1453.35	181.454	1506.69	194.794	1560.27	208.647	1614.11	223.019	28
29	1454.24	181.672	1507.58	195.020	1561.17	208.882	1615.01	223.263	29
30	1455.13	181.890	1508.47	195.247	1562.06	209.117	1615.91	223.507	30
31	1456.01	182.109	1509.36	195.474	1562.96	209.353	1616.81	223.752	31
32	1456.90	182.327	1510.25	195.701	1563.85	209.589	1617.71	223.996	32
33	1457.79	182.546	1511.15	195.928	1564.75	209.825	1618.61	224.240	33
34	1458.68	182.765	1512.04	196.156	1565.64	210.061	1619.51	224 485	34
35	1459.56	182.984	1512.93	196.383	1566.54	210.297	1620.41	224.730	35
36	1460.45	183.203	1513.82	196.611	1567.44	210.533	1621.31	224.975	36
37	1461.34	183.422	1514.71	196.839	1568.33	210.769	1622.21	225.220	37
38	1462.23	183 642	1515.60	197.067	1569.23	211.006	1623.11	225.465	38
39	1463.11	183.861	1516.49	197.295	1570.12	211.243	1624.01	225.711	39
40	1464.00	184.081	1517.39	197.523	1571.02	211.479	1624.91	225.956	40
41	1464.89	184.301	1518.28	197.751	1571.91	211.716	1625.81	226.202	41
42	1465.78	184.521	1519.17	197.980	1572.81	211.954	1626.71	226.448	42
43	1466.66	184.741	1520.06	198.208	1573.71	212.191	1627.61	226.694	43
44	1467.55	184.961	1520.95	198.437	1574.60	212.428	1628.51	226.940	44
45	1468.44	185.182	1521.85	198.666	1575.50	212.666	1629.41	227.186	45
46	1469.33	185.402	1522 74	198.895	1576.40	212.904	1630.31	227.433	46
47	1470.22	185.623	1523.63	199.125	1577.29	213.141	1631 21	227.679	47
48	1471.11	185.844	1524.52	199.354	1578.19	213.380	1632.11	227.926	48
49	1471.99	186.065	1525.42	199.583	1579.09	213.618	1633.01	228.173	49
50	1472.88	186 286	1526.31	199.813	1579.98	213.856	1633.92	228.420	50
51	1473.77	186.507	1527.20	200.043	1580.88	214.094	1634.82	228.667	51
52	1474.66	186.729	1528.09	200.273	1581.78	214.333	1635 72	228.915	52
53	1475.55	186.950	1528.99	200.503	1582.67	214.572	1636.62	229.162	53
54	1476.44	187.172	1529.88	200.733	1583.57	214.811	1637.52	229.410	54
55	1477.33	187.394	1530.77	200.964	1584.47	215.050	1638.42	229.658	55
56	1478.21	187.616	1531.66	201.194	1585.36	215.289	1639.32	229.906	56
57	1479.10	187.838	1532.56	201.425	1586.26	215.528	1640.22	230.154	57
58	1479.99	188.060	1533.45	201.656	1587.16	215.768	1641.13	230.402	58
59	1480.88	188.282	1534.34	201.887	1588.06	216.007	1642.03	230.650	59

TABLE 2. TANGENTS AND EXTERNALS FOR A 1° CURVE (*Continued*)

′	Δ = 32°		Δ = 33°		Δ = 34°		Δ = 35°		′
	T	E	T	E	T	E	T	E	
0	1642.93	230.899	1697.18	246.079	1751.71	261.795	1806.53	278.051	0
1	1643.83	231.148	1698.08	246.337	1752.62	262.061	1807.45	278.327	1
2	1644.73	231.396	1698.99	246.594	1753.53	262.328	1808.36	278.603	2
3	1645.64	231.645	1699.90	246.852	1754.44	262.594	1809.28	278.879	3
4	1646.54	231.894	1700.80	247.110	1755.35	262.861	1810.19	279.155	4
5	1647.44	232.144	1701.71	247.368	1756.26	263.128	1811.11	279.431	5
6	1648.34	232.393	1702.62	247.626	1757.18	263.396	1812.03	279.707	6
7	1649.24	232.642	1703.53	247.885	1758.09	263.663	1812.94	279.983	7
8	1650.15	232.892	1704.43	248.143	1759.00	263.931	1813.86	280.260	8
9	1651.05	233.142	1705.34	248.402	1759.91	264.198	1814.78	280.537	9
10	1651.95	233.392	1706.25	248.661	1760.82	264.466	1815.70	280.814	10
11	1652.85	233.642	1707.15	248.920	1761.74	264.734	1816.61	281.091	11
12	1653.76	233.892	1708.06	249.179	1762.65	265.002	1817.53	281.368	12
13	1654.66	234.143	1708.97	249.439	1763.56	265.271	1818.45	281.646	13
14	1655.56	234.393	1709.88	249.698	1764.47	265.539	1819.36	281.923	14
15	1656.47	234.644	1710.78	249.958	1765.39	265.808	1820.28	282.201	15
16	1657.37	234.895	1711.69	250.217	1766.30	266.077	1821.20	282.479	16
17	1658.27	235.146	1712.60	250.477	1767.21	266.345	1822.12	282.757	17
18	1659.18	235.397	1713.51	250.737	1768.12	266.614	1823.03	283.035	18
19	1660.08	235.648	1714.41	250.997	1769.04	266.884	1823.95	283.314	19
20	1660.98	235.900	1715.32	251.258	1769.95	267.153	1824.87	283.592	20
21	1661.89	236.152	1716.23	251.518	1770.86	267.423	1825.79	283.871	21
22	1662.79	236.403	1717.14	251.779	1771.77	267.692	1826.71	284.149	22
23	1663.69	236.655	1718.05	252.040	1772.69	267.962	1827 62	284.428	23
24	1664.60	236.907	1718.96	252.301	1773.60	268.232	1828.54	284.707	24
25	1665.50	237.159	1719.86	252.562	1774.51	268.502	1829.46	284.987	25
26	1666.40	237.412	1720.77	252.823	1775.43	268.772	1830.38	285.266	26
27	1667.31	237.664	1721.68	253.084	1776.34	269.043	1831.30	285.546	27
28	1668.21	237.917	1722.59	253.346	1777.25	269.313	1832.21	285.825	28
29	1669.12	238.170	1723.50	253.608	1778.17	269.584	1833.13	286.105	29
30	1670.02	238.423	1724.41	253.869	1779.08	269.855	1834.05	286.385	30
31	1670.92	238.676	1725.32	254.131	1779.99	270.126	1834.97	286.665	31
32	1671.83	238.929	1726.22	254.394	1780.91	270.397	1835.89	286.946	32
33	1672.73	239.182	1727.13	254.656	1781.82	270.668	1836.81	287.226	33
34	1673.64	239.436	1728.04	254.918	1782.74	270.940	1837.73	287.507	34
35	1674.54	239.689	1728.95	255.181	1783.65	271.211	1838.65	287.787	35
36	1675.45	239.943	1729.86	255.444	1784.56	271.483	1839.57	288.068	36
37	1676.35	240.197	1730.77	255.707	1785.48	271.755	1840.49	288.350	37
38	1677.26	240.451	1731.68	255.970	1786.39	272.027	1841.40	288.631	38
39	1678.16	240.706	1732.59	256.233	1787.31	272.300	1842.32	288.912	39
40	1679.06	240.960	1733.50	256.496	1788.22	272.572	1843.24	289.194	40
41	1679.97	241.214	1734.41	256.760	1789.14	272.844	1844.16	289.475	41
42	1680.87	241.469	1735.32	257.023	1790.05	273.117	1845.08	289.757	42
43	1681.78	241.724	1736.23	257.287	1790.97	273.390	1846.00	290.039	43
44	1682.69	241.979	1737.14	257.551	1791.88	273.663	1846.92	290.321	44
45	1683.59	242.234	1738.05	257.815	1792.80	273.936	1847.84	290.604	45
46	1684.50	242.489	1738.96	258.079	1793.71	274.209	1848.76	290.886	46
47	1685.40	242.745	1739.87	258.344	1794.63	274.483	1849.68	291.169	47
48	1686.31	243.000	1740.78	258.608	1795.54	274.756	1850.60	291.452	48
49	1687.21	243.256	1741.69	258.873	1796.46	275.030	1851.52	291.735	49
50	1688.12	243.512	1742.60	259.138	1797.37	275.304	1852.44	292.018	50
51	1689.02	243.768	1743.51	259.403	1798.29	275.578	1853.36	292.301	51
52	1689.93	244.024	1744.42	259.668	1799.20	275.852	1854.28	292.584	52
53	1690.83	244.281	1745.33	259.933	1800.12	276.127	1855.21	292.868	53
54	1691.74	244.537	1746.24	260.199	1801.03	276.401	1856.13	293.151	54
55	1692.65	244.794	1747.15	260.464	1801.95	276.676	1857.05	293.435	55
56	1693.55	245.051	1748.06	260.730	1802.86	276.951	1857.97	293.719	56
57	1694.46	245.308	1748.97	260.996	1803.78	277.226	1858.89	294.004	57
58	1695.37	245.565	1749.89	261.262	1804.70	277.501	1859.81	294.288	58
59	1696.27	245.822	1750.80	261.528	1805.61	277.776	1860.73	294.572	59

TABLE 2. TANGENTS AND EXTERNALS FOR A 1° CURVE (*Continued*)

′	Δ = 36°		Δ = 37°		Δ = 38°		Δ = 39°		′
	T	E	T	E	T	E	T	E	
0	1861.65	294.857	1917.09	312.218	1972.85	330.142	2028.95	348.637	0
1	1862.57	295.142	1918.02	312.512	1973.78	330.446	2029.89	348.950	1
2	1863.50	295.427	1918.94	312.806	1974.72	330.749	2030.83	349.264	2
3	1864.42	295.712	1919.87	313.101	1975.65	331.053	2031.76	349.577	3
4	1865.34	295.997	1920.80	313.395	1976.58	331.357	2032.70	349.891	4
5	1866.26	296.282	1921.72	313.690	1977.51	331.662	2033.64	350.205	5
6	1867.18	296.568	1922.65	313.985	1978.45	331.966	2034.58	350.519	6
7	1868.10	296.853	1923.58	314.280	1979.38	332.270	2035.52	350.833	7
8	1869.03	297.139	1924.51	314.575	1980.31	332.575	2036.46	351.147	8
9	1869.95	297.426	1925.43	314.870	1981.24	332.880	2037.39	351.461	9
10	1870.87	297.712	1926.36	315.166	1982.18	333.185	2038.33	351.776	10
11	1871.79	297.998	1927.29	315.462	1983.11	333.490	2039.27	352.091	11
12	1872.71	298.284	1928.22	315.758	1984.04	333.795	2040.21	352.406	12
13	1873.64	298.571	1929.14	316.054	1984.98	334.101	2041.15	352.721	13
14	1874.56	298.858	1930.07	316.350	1985.91	334.406	2042.09	353.036	14
15	1875.48	299.145	1931.00	316.646	1986.84	334.712	2043.03	353.351	15
16	1876.40	299.432	1931.93	316.942	1987.78	335.018	2043.97	353.667	16
17	1877.33	299.719	1932.86	317.239	1988.71	335.324	2044.91	353.983	17
18	1878.25	300.006	1933.78	317.536	1989.65	335.630	2045.85	354.299	18
19	1879.17	300.294	1934.71	317.833	1990.58	335.937	2046.79	354.614	19
20	1880.10	300.582	1935.64	318.130	1991.51	336.243	2047.73	354.931	20
21	1881.02	300.870	1936.57	318.427	1992.45	336.550	2048.67	355.247	21
22	1881.94	301.158	1937.50	318.724	1993.38	336.857	2049.61	355.564	22
23	1882.87	301.446	1938.43	319.022	1994.32	337.164	2050.55	355.880	23
24	1883.79	301.734	1939.36	319.320	1995.25	337.471	2051.49	356.197	24
25	1884.71	302.023	1940.28	319.618	1996.18	337.779	2052.43	356.514	25
26	1885.64	302.311	1941.21	319.916	1997.12	338.086	2053.37	356.831	26
27	1886.56	302.600	1942.14	320.214	1998.05	338.394	2054.31	357.149	27
28	1887.48	302.889	1943.07	320.512	1998.99	338.702	2055.25	357.466	28
29	1888.41	303.178	1944.00	320.811	1999.92	339.010	2056.19	357.784	29
30	1889.33	303.467	1944.93	321.109	2000.86	339.318	2057.13	358.102	30
31	1890.26	303.757	1945.86	321.408	2001.79	339.626	2058.07	358.420	31
32	1891.18	304.047	1946.79	321.707	2002.73	339.934	2059.01	358.738	32
33	1892.10	304.336	1947.72	322.006	2003.66	340.243	2059.95	359.056	33
34	1893.03	304.626	1948.65	322.305	2004.60	340.552	2060.89	359.374	34
35	1893.95	304.916	1949.58	322.605	2005.53	340.861	2061.83	359.693	35
36	1894.88	305.206	1950.51	322.904	2006.47	341.170	2062.77	360.012	36
37	1895.80	305.497	1951.44	323.204	2007.41	341.480	2063.72	360.331	37
38	1896.73	305.787	1952.37	323.504	2008.34	341.789	2064.66	360.650	38
39	1897.65	306.078	1953.30	323.804	2009.28	342.099	2065.60	360.969	39
40	1898.58	306.369	1954.23	324.104	2010.21	342.408	2066.54	361.289	40
41	1899.50	306.660	1955.16	324.405	2011.15	342.718	2067.48	361.608	41
42	1900.43	306.951	1956.09	324.705	2012.08	343.029	2068.42	361.928	42
43	1901.35	307.242	1957.02	325.006	2013.02	343.339	2069.37	362.248	43
44	1902.28	307.534	1957.95	325.307	2013.96	343.649	2070.31	362.568	44
45	1903.20	307.825	1958.88	325.608	2014.89	343.960	2071.25	362.888	45
46	1904.13	308.117	1959.81	325.909	2015.83	344.270	2072.19	363.209	46
47	1905.05	308.409	1960.74	326.210	2016.77	344.581	2073.14	363.529	47
48	1905.98	308.701	1961.67	326.512	2017.70	344.892	2074.08	363.850	48
49	1906.90	308.993	1962.60	326.814	2018.64	345.203	2075.02	364.171	49
50	1907.83	309.286	1963.54	327.115	2019.58	345.515	2075.96	364.492	50
51	1908.75	309.578	1964.47	327.417	2020.51	345.826	2076.91	364.813	51
52	1909.68	309.871	1965.40	327.720	2021.45	346.138	2077.85	365.135	52
53	1910.61	310.164	1966.33	328.022	2022.39	346.450	2078.79	365.456	53
54	1911.53	310.457	1967.26	328.324	2023.32	346.762	2079.74	365.778	54
55	1912.46	310.750	1968.19	328.627	2024.26	347.074	2080.68	366.100	55
56	1913.38	311.043	1969.12	328.930	2025.20	347.387	2081.62	366.422	56
57	1914.31	311.336	1970.06	329.232	2026.14	347.699	2082.56	366.744	57
58	1915.24	311.630	1970.99	329.536	2027.07	348.012	2083.51	367.066	58
59	1916.16	311.924	1971.92	329.839	2028.01	348.324	2084.45	367.389	59

TABLE 2. TANGENTS AND EXTERNALS FOR A 1° CURVE (*Continued*)

′	Δ = 40°		Δ = 41°		Δ = 42°		Δ = 43°		′
	T	E	T	E	T	E	T	E	
0	2085.40	367.71	2142.20	387.37	2199.38	407.63	2256.94	428.49	0
1	2086.34	368.03	2143.15	387.71	2200.33	407.97	2257.90	428.85	1
2	2087.28	368.36	2144.10	388.04	2201.30	408.32	2258.86	429.20	2
3	2088.23	368.68	2145.05	388.37	2202.25	408.66	2259.83	429.55	3
4	2089.17	369.00	2146.00	388.70	2203.20	409.00	2260.79	429.91	4
5	2090.12	369.33	2146.95	389.04	2204.16	409.35	2261.76	430.26	5
6	2091.06	369.65	2147.90	389.37	2205.12	409.69	2262.72	430.61	6
7	2092.00	369.97	2148.85	389.70	2206.07	410.03	2263.68	430.97	7
8	2092.95	370.30	2149.80	390.04	2207.03	410.38	2264.65	431.32	8
9	2093.89	370.62	2150.75	390.37	2207.99	410.72	2265.61	431.68	9
10	2094.84	370.95	2151.70	390.71	2208.95	411.07	2266.57	432.03	10
11	2095.78	371.27	2152.66	391.04	2209.90	411.41	2267.54	432.38	11
12	2096.73	371.60	2153.61	391.38	2210.86	411.75	2268.50	432.74	12
13	2097.67	371.92	2154.56	391.71	2211.82	412.10	2269.46	433.09	13
14	2098.62	372.25	2155.51	392.05	2212.78	412.44	2270.43	433.45	14
15	2099.56	372.57	2156.46	392.38	2213.73	412.79	2271.39	433.80	15
16	2100.51	372.90	2157.41	392.72	2214.69	413.13	2272.36	434.16	16
17	2101.45	373.22	2158.36	393.05	2215.65	413.48	2273.32	434.52	17
18	2102.40	373.55	2159.31	393.39	2216.61	413.83	2274.29	434.87	18
19	2103.34	373.87	2160.27	393.72	2217.56	414.17	2275.25	435.23	19
20	2104.29	374.20	2161.22	394.06	2218.52	414.52	2276.22	435.58	20
21	2105.24	374.53	2162.17	394.40	2219.48	414.86	2277.18	435.94	21
22	2106.18	374.85	2163.12	394.73	2220.44	415.21	2278.15	436.30	22
23	2107.13	375.18	2164.07	395.07	2221.40	415.56	2279.11	436.65	23
24	2108.07	375.51	2165.03	395.40	2222.36	415.90	2280.08	437.01	24
25	2109.02	375.83	2165.98	395.74	2223.32	416.25	2281.04	437.37	25
26	2109.97	376.16	2166.93	396.08	2224.27	416.60	2282.01	437.72	26
27	2110.91	376.49	2167.88	396.42	2225.23	416.94	2282.97	438.08	27
28	2111.86	376.81	2168.84	396.75	2226.19	417.29	2283.94	438.44	28
29	2112.81	377.14	2169.79	397.09	2227.15	417.64	2284.90	438.80	29
30	2113.75	377.47	2170.74	397.43	2228.11	417.99	2285.87	439.15	30
31	2114.70	377.80	2171.70	397.76	2229.07	418.33	2286.84	439.51	31
32	2115.65	378.12	2172.65	398.10	2230.03	418.68	2287.80	439.87	32
33	2116.59	378.45	2173.60	398.44	2230.99	419.03	2288.77	440.23	33
34	2117.54	378.78	2174.56	398.78	2231.95	419.38	2289.74	440.59	34
35	2118.49	379.11	2175.51	399.12	2232.91	419.73	2290.70	440.95	35
36	2119.44	379.44	2176.46	399.46	2233.87	420.07	2291.67	441.31	36
37	2120.38	379.77	2177.42	399.79	2234.83	420.42	2292.63	441.66	37
38	2121.33	380.10	2178.37	400.13	2235.79	420.77	2293.60	442.02	38
39	2122.28	380.42	2179.32	400.47	2236.75	421.12	2294.57	442.38	39
40	2123.23	380.75	2180.28	400.81	2237.71	421.47	2295.54	442.74	40
41	2124.17	381.08	2181.23	401.15	2238.67	421.82	2296.50	443.10	41
42	2125.12	381.41	2182.19	401.49	2239.63	422.17	2297.47	443.46	42
43	2126.07	381.74	2183.14	401.83	2240.59	422.52	2298.44	443.82	43
44	2127.02	382.07	2184.09	402.17	2241.55	422.87	2299.40	444.18	44
45	2127.97	382.40	2185.05	402.51	2242.51	423.22	2300.37	444.54	45
46	2128.91	382.73	2186.00	402.85	2243.47	423.57	2301.34	444.90	46
47	2129.86	383.06	2186.96	403.19	2244.44	423.92	2302.31	445.26	47
48	2130.81	383.39	2187.91	403.53	2245.40	424.27	2303.28	445.63	48
49	2131.76	383.72	2188.87	403.87	2246.36	424.62	2304.24	445.99	49
50	2132.71	384.05	2189.82	404.21	2247.32	424.97	2305.21	446.35	50
51	2133.66	384.39	2190.78	404.55	2248.28	425.32	2306.18	446.71	51
52	2134.61	384.72	2191.73	404.89	2249.24	425.68	2307.15	447.07	52
53	2135.56	385.05	2192.69	405.24	2250.20	426.03	2308.12	447.43	53
54	2136.50	385.38	2193.64	405.58	2251.17	426.38	2309.09	447.79	54
55	2137.45	385.71	2194.60	405.92	2252.13	426.73	2310.05	448.16	55
56	2138.40	386.04	2195.56	406.26	2253.09	427.08	2311.02	448.52	56
57	2139.35	386.38	2196.51	406.60	2254.05	427.44	2311.99	448.88	57
58	2140.30	386.71	2197.47	406.95	2255.02	427.79	2312.96	449.24	58
59	2141.25	387.04	2198.42	407.29	2255.98	428.14	2313.93	449.61	59

Table 2. Tangents and Externals for a 1° Curve (*Continued*)

′	Δ = 44°		Δ = 45°		Δ = 46°		Δ = 47°		′
	T	E	T	E	T	E	T	E	
0	2314.90	449.97	2373.27	472.07	2432.06	494.81	2491.29	518.19	0
1	2315.87	450.33	2374.25	472.45	2433.05	495.19	2492.28	518.59	1
2	2316.84	450.70	2375.22	472.82	2434.03	495.58	2493.27	518.98	2
3	2317.81	451.06	2376.20	473.19	2435.01	495.96	2494.26	519.38	3
4	2318.78	451.42	2377.17	473.57	2436.00	496.35	2495.26	519.77	4
5	2319.75	451.79	2378.15	473.94	2436.98	496.73	2496.25	520.17	5
6	2320.72	452.15	2379.13	474.32	2437.96	497.12	2497.24	520.56	6
7	2321.69	452.51	2380.11	474.69	2438.95	497.50	2498.23	520.96	7
8	2322.66	452.88	2381.08	475.07	2439.93	497.89	2499.22	521.36	8
9	2323.63	453.25	2382.06	475.44	2440.92	498.27	2500.21	521.75	9
10	2324.60	453.61	2383.04	475.82	2441.90	498.66	2501.21	522.15	10
11	2325.57	453.98	2384.02	476.19	2442.89	499.05	2502.20	522.55	11
12	2326.54	454.34	2384.99	476.57	2443.87	499.43	2503.19	522.94	12
13	2327.51	454.71	2385.97	476.94	2444.86	499.82	2504.18	523.34	13
14	2328.48	455.07	2386.95	477.32	2445.84	500.21	2505.18	523.74	14
15	2329.45	455.44	2387.93	477.70	2446.83	500.59	2506.17	524.14	15
16	2330.42	455.80	2388.90	478.07	2447.81	500.98	2507.16	524.53	16
17	2331.40	456.17	2389.88	478.45	2448.80	501.37	2508.15	524.93	17
18	2332.37	456.54	2390.86	478.83	2449.78	501.76	2509.15	525.33	18
19	2333.34	456.90	2391.84	479.20	2450.77	502.14	2510.14	525.73	19
20	2334.31	457.27	2392.82	479.58	2451.76	502.53	2511.13	526.13	20
21	2335.28	457.63	2393.80	479.96	2452.74	502.92	2512.13	526.53	21
22	2336.25	458.00	2394.78	480.34	2453.73	503.31	2513.12	526.93	22
23	2337.23	458.37	2395.76	480.71	2454.71	503.69	2514.11	527.33	23
24	2338.20	458.74	2396.73	481.09	2455.70	504.08	2515.11	527.72	24
25	2339.17	459.10	2397.71	481.47	2456.69	504.47	2516.10	528.12	25
26	2340.14	459.47	2398.69	481.85	2457.67	504.86	2517.10	528.52	26
27	2341.11	459 84	2399.67	482.23	2458.66	505.25	2518.09	528.92	27
28	2342.09	460.21	2400.65	482.60	2459.65	505.64	2519.09	529.32	28
29	2343.06	460.57	2401.63	482.98	2460.63	506.03	2520.08	529.72	29
30	2344.03	460.94	2402.61	483.36	2461.62	506.42	2521.07	530.13	30
31	2345.01	461.31	2403.59	483.74	2462.61	506.81	2522.07	530.53	31
32	2345.98	461.68	2404.57	484.12	2463.60	507.20	2523.06	530.93	32
33	2346.95	462.05	2405.55	484.50	2464.58	507.59	2524.06	531.33	33
34	2347.92	462.42	2406.53	484.88	2465.57	507.98	2525.05	531.73	34
35	2348.90	462.79	2407.51	485.26	2466.56	508.37	2526.05	532.13	35
36	2349.87	463.16	2408.49	485.64	2467.55	508.76	2527.04	532.53	36
37	2350.84	463.53	2409.47	486.02	2468.53	509.15	2528.04	532.93	37
38	2351.82	463.90	2410.45	486.40	2469.52	509.54	2529.04	533.34	38
39	2352.79	464.27	2411.44	486.78	2470.51	509.93	2530.03	533.74	39
40	2353.77	464.64	2412.42	487.16	2471.50	510.32	2531.03	534.14	40
41	2354.74	465.01	2413.40	487.54	2472.49	510.71	2532.02	534.54	41
42	2355.71	465.38	2414.38	487.92	2473.48	511.11	2533.02	534.94	42
43	2356.69	465.75	2415.36	488.30	2474.46	511.50	2534.02	535.35	43
44	2357.66	466.12	2416.34	488.68	2475.45	511.89	2535.01	535.75	44
45	2358.64	466.49	2417.32	489.06	2476.44	512.28	2536.01	536.15	45
46	2359.61	466.86	2418.30	489.45	2477.43	512.68	2537.01	536.56	46
47	2360.59	467.23	2419.29	489.83	2478.42	513.07	2538.00	536.96	47
48	2361.56	467.60	2420.27	490.21	2479.41	513.46	2539 00	537.37	48
49	2362.54	467.97	2421.25	490.59	2480.40	513.85	2540.00	537.77	49
50	2363.51	468.35	2422.23	490.97	2481.39	514.25	2540.99	538.17	50
51	2364.49	468.72	2423.22	491.36	2482.38	514.64	2541.99	538.58	51
52	2365.46	469.09	2424.20	491.74	2483.37	515.03	2542.99	538.98	52
53	2366.44	469.46	2425.18	492.12	2484.36	515.43	2543.99	539.39	53
54	2367.41	469.83	2426.16	492.51	2485.35	515.82	2544.98	539.79	54
55	2368.39	470.21	2427.15	492.89	2486.34	516.22	2545.98	540.20	55
56	2369.36	470.58	2428.13	493.27	2487.33	516.61	2546.98	540.60	56
57	2370.34	470.95	2429.11	493.66	2488.32	517.00	2547.98	541.01	57
58	2371.32	471.33	2430.09	494.04	2489.31	517.40	2548.98	541.41	58
59	2372.29	471.70	2431.08	494.42	2490.30	517.79	2549.97	541.82	59

TABLE 2. TANGENTS AND EXTERNALS FOR A 1° CURVE (*Continued*)

′	Δ = 48°		Δ = 49°		Δ = 50°		Δ = 51°		′
	T	E	T	E	T	E	T	E	
0	2550.97	542.23	2611.12	566.93	2671.75	592.31	2732.87	618.32	0
1	2551.97	542.63	2612.13	567.35	2672.76	592.74	2733.89	618.82	1
2	2552.97	543.04	2613.13	567.77	2673.78	593.17	2734.91	619.27	2
3	2553.97	543.45	2614.14	568.18	2674.79	593.60	2735.94	619.71	3
4	2554.97	543.85	2615.15	568.60	2675.81	594.03	2736.96	620.15	4
5	2555.97	544.26	2616.15	569.02	2676.82	594.46	2737.98	620.59	5
6	2556.97	544.67	2617.16	569.44	2677.84	594.90	2739.01	621.03	6
7	2557.97	545.07	2618.17	569.86	2678.85	595.32	2740.03	621.47	7
8	2558.96	545.48	2619.17	570.27	2679.87	595.75	2741.06	621.91	8
9	2559.96	545.89	2620.18	570.69	2680.88	596.18	2742.08	622.36	9
10	2560.96	546.30	2621.19	571.11	2681.90	596.61	2743.10	622.80	10
11	2561.96	546.71	2622.20	571.53	2682.91	597.04	2744.13	623.24	11
12	2562.96	547.11	2623.21	571.95	2683.93	597.47	2745.15	623.68	12
13	2563.96	547.52	2624.21	572.37	2684.95	597.90	2746.18	624.13	13
14	2564.96	547.93	2625.22	572.79	2685.96	598.33	2747.20	624.57	14
15	2565.96	548.34	2626.23	573.21	2686.98	598.76	2748.23	625.01	15
16	2566.97	548.75	2627.24	573.63	2688.00	599.20	2749.25	625.46	16
17	2567.97	549.16	2628.25	574.05	2689.01	599.63	2750.28	625.90	17
18	2568.97	549.57	2629.26	574.47	2690.03	600.06	2751.30	626.34	18
19	2569.97	549.98	2630.26	574.89	2691.05	600.50	2752.33	626.79	19
20	2570.97	550.39	2631.27	575.31	2692.06	600.93	2753.36	627.23	20
21	2571.97	550.80	2632.28	575.74	2693.08	601.36	2754.38	627.68	21
22	2572.97	551.21	2633.29	576.16	2694.10	601.79	2755.41	628.12	22
23	2573.97	551.62	2634.30	576.58	2695.12	602.22	2756.43	628.56	23
24	2574.97	552.03	2635.31	577.00	2696.13	602.66	2757.46	629.01	24
25	2575.98	552.44	2636.32	577.42	2697.15	603.09	2758.49	629.46	25
26	2576.98	552.85	2637.33	577.84	2698.17	603.52	2759.51	629.90	26
27	2577.98	553.26	2638.34	578.27	2699.19	603.96	2760.54	630.35	27
28	2578.98	553.67	2639.35	578.69	2700.21	604.39	2761.57	630.79	28
29	2579.98	554.08	2640.36	579.11	2701.23	604.83	2762.59	631.24	29
30	2580.99	554.49	2641.37	579.54	2702.24	605.26	2763.62	631.68	30
31	2581.99	554.91	2642.38	579.96	2703.26	605.70	2764.65	632.13	31
32	2582.99	555.32	2643.39	580.38	2704.28	606.13	2765.67	632.58	32
33	2583.99	555.73	2644.40	580.80	2705.30	606.57	2766.70	633.02	33
34	2585.00	556.14	2645.41	581.23	2706.32	607.00	2767.73	633.47	34
35	2586.00	556.55	2646.42	581.65	2707.34	607.44	2768.76	633.92	35
36	2587.00	556.97	2647.44	582.08	2708.36	607.87	2769.79	634.37	36
37	2588.01	557.38	2648.45	582.50	2709.38	608.31	2770.81	634.81	37
38	2589.01	557.79	2649.46	582.93	2710.40	608.74	2771.84	635.26	38
39	2590.01	558.21	2650.47	583.35	2711.42	609.18	2772.87	635.71	39
40	2591.02	558.62	2651.48	583.77	2712.44	609.62	2773.90	636.16	40
41	2592.02	559.03	2652.49	584.20	2713.46	610.05	2774.93	636.60	41
42	2593.03	559.45	2653.51	584.62	2714.48	610.49	2775.96	637.05	42
43	2594.03	559.86	2654.52	585.05	2715.50	610.93	2776.99	637.50	43
44	2595.03	560.28	2655.53	585.48	2716.52	611.36	2778.02	637.95	44
45	2596.04	560.69	2656.54	585.90	2717.54	611.80	2779.05	638.40	45
46	2597.04	561.10	2657.56	586.33	2718.56	612.24	2780.07	638.85	46
47	2598.05	561.52	2658.57	586.75	2719.58	612.68	2781.10	639.30	47
48	2599.05	561.93	2659.58	587.18	2720.60	613.12	2782.13	639.75	48
49	2600.06	562.35	2660.59	587.61	2721.62	613.55	2783.16	640.20	49
50	2601.06	562.77	2661.61	588.03	2722.65	613.99	2784.19	640.65	50
51	2602.07	563.18	2662.62	588.46	2723.67	614.43	2785.22	641.10	51
52	2603.07	563.60	2663.63	588.89	2724.69	614.87	2786.25	641.55	52
53	2604.08	564.01	2664.65	589.31	2725.71	615.31	2787.28	642.00	53
54	2605.08	564.43	2665.66	589.74	2726.73	615.75	2788.32	642.45	54
55	2606.09	564.85	2666.68	590.17	2727.76	616.19	2789.35	642.90	55
56	2607.09	565.26	2667.69	590.60	2728.78	616.63	2790.38	643.35	56
57	2608.10	565.68	2668.70	591.03	2729.80	617.07	2791.41	643.81	57
58	2609.11	566.09	2669.72	591.45	2730.82	617.51	2792.44	644.26	58
59	2610.11	566.51	2670.73	591.88	2731.85	617.95	2793.47	644.71	59

TABLE 2. TANGENTS AND EXTERNALS FOR A 1° CURVE (*Continued*)

′	Δ = 52°		Δ = 53°		Δ = 54°		Δ = 55°		′
	T	E	T	E	T	E	T	E	
0	2794.50	645.16	2856.66	672.65	2919.37	700.88	2982.63	729.84	0
1	2795.53	645.61	2857.70	673.12	2920.42	701.36	2983.69	730.33	1
2	2796.57	646.07	2858.74	673.58	2921.47	701.83	2984.75	730.82	2
3	2797.60	646.52	2859.78	674.05	2922.52	702.31	2985.81	731.31	3
4	2798.63	646.97	2860.83	674.51	2923.57	702.79	2986.87	731.80	4
5	2799.66	647.42	2861.87	674.98	2924.62	703.26	2987.93	732.29	5
6	2800.69	647.88	2862.91	675.44	2925.67	703.74	2988.99	732.78	6
7	2801.73	648.33	2863.95	675.91	2926.72	704.22	2990.05	733.27	7
8	2802.76	648.79	2864.99	676.37	2927.77	704.70	2991.11	733.76	8
9	2803.79	649.24	2866.03	676.84	2928.82	705.18	2992.17	734.26	9
10	2804.82	649.69	2867.07	677.31	2929.87	705.65	2993.23	734.75	10
11	2805.86	650.15	2868.12	677.77	2930.92	706.13	2994.29	735.24	11
12	2806.89	650.60	2869 16	678.24	2931.97	706.61	2995 35	735.73	12
13	2807.92	651.06	2870.20	678.71	2933.02	707.09	2996.41	736.22	13
14	2808.96	651.51	2871.24	679.17	2934.08	707.57	2997.47	736.71	14
15	2809.99	651.97	2872.29	679.64	2935.13	708.05	2998.53	737.20	15
16	2811.03	652.42	2873.33	680.11	2936.18	708.53	2999.60	737.70	16
17	2812.06	652.88	2874.37	680.58	2937.23	709.01	3000.66	738.19	17
18	2813.09	653.33	2875.42	681.04	2938.29	709.49	3001.72	738.68	18
19	2814.13	653.79	2876 46	681.51	2939.34	709.97	3002.78	739.17	19
20	2815.16	654.25	2877.50	681.98	2940.39	710.45	3003.84	739.67	20
21	2816.20	654.70	2878.55	682.45	2941.44	710.93	3004.91	740.16	21
22	2817.23	655.16	2879.59	682.92	2942.50	711.41	3005.97	740.65	22
23	2818.27	655.62	2880.63	683.39	2943.55	711.89	3007.03	741.15	23
24	2819.30	656.07	2881.68	683.86	2944.60	712.37	3008.10	741.64	24
25	2820.34	656.53	2882.72	684.33	2945.66	712.86	3009.16	742.14	25
26	2821.37	656.99	2883.77	684.79	2946.71	713.34	3010.22	742.63	26
27	2822.41	657.44	2884.81	685.26	2947.76	713.82	3011.29	743.13	27
28	2823.44	657.90	2885.86	685.73	2948.82	714.30	3012.35	743.62	28
29	2824.48	658.36	2886.90	686.20	2949.87	714.78	3013.41	744.12	29
30	2825.52	658.82	2887.95	686.67	2950.93	715.27	3014.48	744.61	30
31	2826.55	659.28	2888.99	687.14	2951.98	715.75	3015.54	745.11	31
32	2827.59	659.74	2890.04	687.61	2953.04	716.23	3016.61	745.60	32
33	2828.62	660.19	2891.08	688.09	2954.09	716 72	3017.67	746.10	33
34	2829.66	660.65	2892.13	688.56	2955.15	717.20	3018.73	746.59	34
35	2830.70	661.11	2893.17	689.03	2956.20	717.68	3019.80	747.09	35
36	2831.73	661 57	2894.22	689.50	2957.26	718.17	3020.86	747.59	36
37	2832.77	662.03	2895.26	689.97	2958.31	718.65	3021.93	748.08	37
38	2833.81	662.49	2896.31	690.44	2959.37	719.14	3022.99	748.58	38
39	2834.84	662.95	2897.36	690.92	2960.42	719.62	3024.06	749.08	39
40	2835.88	663.41	2898.40	691.39	2961.48	720.11	3025.13	749.58	40
41	2836.92	663 87	2899.45	691.86	2962.53	720.59	3026.19	750.07	41
42	2837.96	664.33	2900.50	692.33	2963.59	721.08	3027.26	750.57	42
43	2839.00	664.79	2901.54	692.81	2964.65	721.56	3028.32	751.07	43
44	2840.03	665.25	2902.59	693.28	2965 70	722.05	3029.39	751.57	44
45	2841.07	665.71	2903.64	693.75	2966.76	722.53	3030.46	752.07	45
46	2842.11	666.17	2904.69	694.23	2967.82	723.02	3031.52	752.57	46
47	2843.15	666.64	2905 73	694.70	2968.87	723.51	3032.59	753.07	47
48	2844.19	667.10	2906.78	695.17	2969.93	723.99	3033.66	753.56	48
49	2845.23	667.56	2907.83	695.65	2970.99	724.48	3034.72	754.06	49
50	2846.26	668.02	2908.88	696.12	2972.05	724.96	3035.79	754.56	50
51	2847.30	668.48	2909.92	696.60	2973.10	725.45	3036.86	755.06	51
52	2848.34	668.95	2910.97	697.07	2974.16	725.94	3037.92	755.56	52
53	2849.38	669.41	2912.02	697.55	2975.22	726.43	3038.99	756.06	53
54	2850.42	669.87	2913.07	698.02	2976.28	726.91	3040.06	756.56	54
55	2851.46	670.34	2914.12	698.50	2977.34	727.40	3041.13	757.06	55
56	2852.50	670.80	2915.17	698.97	2978.39	727.89	3042.20	757.56	56
57	2853.54	671.26	2916.22	699.45	2979.45	728.38	3043.26	758.07	57
58	2854.58	671.73	2917.27	699.93	2980.51	728.87	3044.33	758.57	58
59	2855.62	672.19	2918.32	700.40	2981.57	729.36	3045.40	759.07	59

TABLE 2. TANGENTS AND EXTERNALS FOR A 1° CURVE (*Continued*)

′	Δ = 56°		Δ = 57°		Δ = 58°		Δ = 59°		′
	T	E	T	E	T	E	T	E	
0	3046.47	759.57	3110.91	790.07	3175.96	821.36	3241.64	853.45	0
1	3047.54	760.07	3111.99	790.58	3177.05	821.89	3242.74	853.99	1
2	3048.61	760.57	3113.07	791.10	3178.14	822.41	3243.84	854.54	2
3	3049.68	761.08	3114.14	791.61	3179.23	822.94	3244.94	855.08	3
4	3050.75	761.58	3115.22	792.13	3180.32	823.47	3246.04	855.62	4
5	3051.82	762.08	3116.30	792.65	3181.41	824.00	3247.14	856.16	5
6	3052.89	762.58	3117.38	793.16	3182.50	824.53	3248.24	856.71	6
7	3053.96	763.09	3118.46	793.68	3183.59	825.06	3249.34	857.25	7
8	3055.03	763.59	3119.54	794.20	3184.68	825.59	3250.45	857.79	8
9	3056.10	764.10	3120.62	794.71	3185.77	826.12	3251.55	858.34	9
10	3057.17	764.60	3121.71	795.23	3186.86	826.65	3252.65	858.88	10
11	3058.24	765.10	3122.79	795.75	3187.95	827.18	3253.75	859.42	11
12	3059.31	765.61	3123.87	796.26	3189.04	827.71	3254.85	859.97	12
13	3060.38	766.11	3124.95	796.78	3190.13	828.24	3255.96	860.51	13
14	3061.45	766.62	3126.03	797.30	3191.23	828.77	3257.06	861.06	14
15	3062.52	767.12	3127.11	797.82	3192.32	829.31	3258.16	861.60	15
16	3063.59	767.63	3128.19	798.33	3193.41	829.84	3259.26	862.15	16
17	3064.67	768.13	3129.27	798.85	3194.50	830.37	3260.37	862.70	17
18	3065.74	768.64	3130.36	799.37	3195.59	830.90	3261.47	863.24	18
19	3066.81	769.14	3131.44	799.89	3196.69	831.43	3262.57	863.79	19
20	3067.88	769.65	3132.52	800.41	3197.78	831.97	3263.68	864.33	20
21	3068.95	770.16	3133.60	800.93	3198.87	832.50	3264.78	864.88	21
22	3070.03	770.66	3134.69	801.45	3199.97	833.03	3265.88	865.42	22
23	3071.10	771.17	3135.77	801.97	3201.06	833.56	3266.99	865.97	23
24	3072.17	771.68	3136.85	802.49	3202.15	834.10	3268.09	866.52	24
25	3073.25	772.18	3137.94	803.01	3203.25	834.63	3269.20	867.07	25
26	3074.32	772.69	3139.02	803.53	3204.34	835.17	3270.30	867.61	26
27	3075.39	773.20	3140.10	804.05	3205.43	835.70	3271.41	868.16	27
28	3076.47	773.71	3141.19	804.57	3206.53	836.23	3272.51	868.71	28
29	3077.54	774.21	3142.27	805.09	3207.62	836.77	3273.62	869.26	29
30	3078.61	774.72	3143.35	805.61	3208.72	837.30	3274.72	869.81	30
31	3079.69	775.23	3144.44	806.14	3209.81	837.84	3275.83	870.36	31
32	3080.76	775.74	3145.52	806.66	3210.91	838.37	3276.93	870.90	32
33	3081.84	776.25	3146.61	807.18	3212.00	838.91	3278.04	871.45	33
34	3082.91	776.76	3147.69	807.70	3213.10	839.45	3279.15	872.00	34
35	3083.98	777.27	3148.78	808.22	3214.19	839.98	3280.25	872.55	35
36	3085.06	777.78	3149.86	808.75	3215.29	840.52	3281.36	873.10	36
37	3086.13	778.29	3150.95	809.27	3216.38	841.05	3282.47	873.65	37
38	3087.21	778.80	3152.03	809.79	3217.48	841.59	3283.57	874.20	38
39	3088.29	779.31	3153.12	810.32	3218.58	842.13	3284.68	874.75	39
40	3089.36	779.82	3154.20	810.84	3219.67	842.66	3285.79	875.30	40
41	3090.44	780.33	3155.29	811.36	3220.77	843.20	3286.90	875.85	41
42	3091.51	780.84	3156.38	811.89	3221.87	843.74	3288.00	876.41	42
43	3092.59	781.35	3157.46	812.41	3222.96	844.28	3289.11	876.96	43
44	3093.66	781.86	3158.55	812.94	3224.06	844.81	3290.22	877.51	44
45	3094.74	782.37	3159.64	813.46	3225.16	845.35	3291.33	878.06	45
46	3095.82	782.88	3160.72	813.99	3226.26	845.89	3292.44	878.61	46
47	3096.89	783.39	3161.81	814.51	3227.35	846.43	3293.54	879.17	47
48	3097.97	783.91	3162.90	815.04	3228.45	846.97	3294.65	879.72	48
49	3099.05	784.42	3163.98	815.56	3229.55	847.51	3295.76	880.27	49
50	3100.13	784.93	3165.07	816.09	3230.65	848.05	3296.87	880.82	50
51	3101.20	785.44	3166.16	816.61	3231.75	848.59	3297.98	881.38	51
52	3102.28	785.96	3167.25	817.14	3232.84	849.13	3299.09	881.93	52
53	3103.36	786.47	3168.34	817.67	3233.94	849.67	3300.20	882.49	53
54	3104.44	786.98	3169.42	818.19	3235.04	850.21	3301.31	883.04	54
55	3105.51	787.50	3170.51	818.72	3236.14	850.75	3302.42	883.59	55
56	3106.59	788.01	3171.60	819.25	3237.24	851.29	3303.53	884.15	56
57	3107.67	788.53	3172.69	819.77	3238.34	851.83	3304.64	884.70	57
58	3108.75	789.04	3173.78	820.30	3239.44	852.37	3305.75	885.26	58
59	3109.83	789.55	3174.87	820.83	3240.54	852.91	3306.86	885.81	59

TABLE 2. TANGENTS AND EXTERNALS FOR A 1° CURVE (*Continued*)

′	Δ = 60°		Δ = 61°		Δ = 62°		Δ = 63°		′
	T	E	T	E	T	E	T	E	
0	3308.0	886.4	3375.0	920.1	3442.7	954.7	3511.1	990.2	0
1	3309.1	886.9	3376.1	920.7	3443.8	955.3	3512.2	990.8	1
2	3310.2	887.5	3377.3	921.3	3445.0	955.9	3513.4	991.4	2
3	3311.3	888.0	3378.4	921.8	3446.1	956.5	3514.5	992.0	3
4	3312.5	888.6	3379.5	922.4	3447.2	957.1	3515.7	992.6	4
5	3313.6	889.2	3380.6	923.0	3448.4	957.7	3516.8	993.2	5
6	3314.7	889.7	3381.7	923.6	3449.5	958.3	3518.0	993.8	6
7	3315.8	890.3	3382.9	924.1	3450.6	958.8	3519.1	994.4	7
8	3316.9	890.8	3384.0	924.7	3451.8	959.4	3520.3	995.0	8
9	3318.0	891.4	3385.1	925.3	3452.9	960.0	3521.4	995.6	9
10	3319.1	891.9	3386.2	925.8	3454.0	960.6	3522.6	996.2	10
11	3320.2	892.5	3387.4	926.4	3455.2	961.2	3523.7	996.8	11
12	3321.4	893.1	3388.5	927.0	3456.3	961.8	3524.9	997.4	12
13	3322.5	893.6	3389.6	927.6	3457.5	962.4	3526.0	998.0	13
14	3323.6	894.2	3390.7	928.1	3458.6	963.0	3527.2	998.6	14
15	3324.7	894.7	3391.9	928.7	3459.7	963.5	3528.3	999.3	15
16	3325.8	895.3	3393.0	929.3	3460.9	964.1	3529.5	999.9	16
17	3326.9	895.9	3394.1	929.9	3462.0	964.7	3530.6	1000.5	17
18	3328.0	896.4	3395.2	930.4	3463.1	965.3	3531.8	1001.1	18
19	3329.1	897.0	3396.4	931.0	3464.3	965.9	3532.9	1001.7	19
20	3330.3	897.5	3397.5	931.6	3465.4	966.5	3534.1	1002.3	20
21	3331.4	898.1	3398.6	932.2	3466.6	967.1	3535.2	1002.9	21
22	3332.5	898.7	3399.7	932.7	3467.7	967.7	3536.4	1003.5	22
23	3333.6	899.2	3400.9	933.3	3468.8	968.2	3537.5	1004.1	23
24	3334.7	899.8	3402.0	933.9	3470.0	968.8	3538.7	1004.7	24
25	3335.8	900.3	3403.1	934.5	3471.1	969.4	3539.8	1005.3	25
26	3337.0	900.9	3404.2	935.0	3472.3	970.0	3541.0	1005.9	26
27	3338.1	901.5	3405.4	935.6	3473.4	970.6	3542.1	1006.5	27
28	3339.2	902.0	3406.5	936.2	3474.5	971.2	3543.3	1007.1	28
29	3340.3	902.6	3407.6	936.8	3475.7	971.8	3544.4	1007.7	29
30	3341.4	903.2	3408.8	937.3	3476.8	972.4	3545.6	1008.3	30
31	3342.5	903.7	3409.9	937.9	3478.0	973.0	3546.7	1008.9	31
32	3343.7	904.3	3411.0	938.5	3479.1	973.6	3547.9	1009.5	32
33	3344.8	904.8	3412.1	939.1	3480.2	974.2	3549.0	1010.1	33
34	3345.9	905.4	3413.3	939.6	3481.4	974.8	3550.2	1010.7	34
35	3347.0	906.0	3414.4	940.2	3482.5	975.3	3551.4	1011.3	35
36	3348.1	906.5	3415.5	940.8	3483.7	975.9	3552.5	1012.0	36
37	3349.2	907.1	3416.7	941.4	3484.8	976.5	3553.7	1012.6	37
38	3350.4	907.7	3417.8	942.0	3485.9	977.1	3554.8	1013.2	38
39	3351.5	908.2	3418.9	942.5	3487.1	977.7	3556.0	1013.8	39
40	3352.6	908.8	3420.0	943.1	3488.2	978.3	3557.1	1014.4	40
41	3353.7	909.4	3421.2	943.7	3489.4	978.9	3558.3	1015.0	41
42	3354.8	909.9	3422.3	944.3	3490.5	979.5	3559.4	1015.6	42
43	3356.0	910.5	3423.4	944.9	3491.7	980.1	3560.6	1016.2	43
44	3357.1	911.1	3424.6	945.4	3492.8	980.7	3561.7	1016.8	44
45	3358.2	911.6	3425.7	946.0	3493.9	981.3	3562.9	1017.4	45
46	3359.3	912.2	3426.8	946.6	3495.1	981.9	3564.1	1018.1	46
47	3360.4	912.7	3428.0	947.2	3496.2	982.5	3565.2	1018.7	47
48	3361.6	913.3	3429.1	947.8	3497.4	983.1	3566.4	1019.3	48
49	3362.7	913.9	3430.2	948.3	3498.5	983.7	3567.5	1019.9	49
50	3363.8	914.5	3431.4	948.9	3499.7	984.3	3568.7	1020.5	50
51	3364.9	915.0	3432.5	949.5	3500.8	984.9	3569.8	1021.1	51
52	3366.0	915.6	3433.6	950.1	3501.9	985.5	3571.0	1021.7	52
53	3367.2	916.2	3434.8	950.7	3503.1	986.1	3572.1	1022.3	53
54	3368.3	916.7	3435.9	951.2	3504.2	986.7	3573.3	1022.9	54
55	3369.4	917.3	3437.0	951.8	3505.4	987.2	3574.5	1023.5	55
56	3370.5	917.9	3438.2	952.4	3506.5	987.8	3575.6	1024.2	56
57	3371.6	918.4	3439.3	953.0	3507.7	988.4	3576.8	1024.8	57
58	3372.8	919.0	3440.4	953.6	3508.8	989.0	3577.9	1025.4	58
59	3373.9	919.6	3441.6	954.2	3509.9	989.6	3579.1	1026.0	59

TABLE 2. TANGENTS AND EXTERNALS FOR A 1° CURVE (*Continued*)

′	Δ = 64°		Δ = 65°		Δ = 66°		Δ = 67°		′
	T	E	T	E	T	E	T	E	
0	3580.2	1026.6	3650.2	1063.9	3720.9	1102.2	3792.3	1141.4	0
1	3581.4	1027.2	3651.3	1064.5	3722.0	1102.8	3793.5	1142.0	1
2	3582.6	1027.8	3652.5	1065.2	3723.2	1103.5	3794.7	1142.7	2
3	3583.7	1028.5	3653.7	1065.8	3724.4	1104.1	3795.9	1143.4	3
4	3584.9	1029.1	3654.9	1066.4	3725.6	1104.8	3797.1	1144.0	4
5	3586.1	1029.7	3656.0	1067.1	3726.8	1105.4	3798.3	1144.7	5
6	3587.2	1030.3	3657.2	1067.7	3728.0	1106.0	3799.5	1145.3	6
7	3588.4	1030.9	3658.4	1068.3	3729.1	1106.7	3800.7	1146.0	7
8	3589.5	1031.6	3659.5	1069.0	3730.3	1107.3	3801.9	1146.7	8
9	3590.7	1032.2	3660.7	1069.6	3731.5	1108.0	3803.1	1147.3	9
10	3591.9	1032.8	3661.9	1070.2	3732.7	1108.6	3804.3	1148.0	10
11	3593.0	1033.4	3663.1	1070.9	3733.9	1109.3	3805.5	1148.7	11
12	3594.2	1034.0	3664.2	1071.5	3735.1	1109.9	3806.7	1149.3	12
13	3595.3	1034.6	3665.4	1072.1	3736.3	1110.6	3807.9	1150.0	13
14	3596.5	1035.2	3666.6	1072.8	3737.5	1111.2	3809.1	1150.7	14
15	3597.7	1035.9	3667.8	1073.4	3738.7	1111.9	3810.3	1151.3	15
16	3598.8	1036.5	3668.9	1074.0	3739.8	1112.5	3811.5	1152.0	16
17	3600.0	1037.1	3670.1	1074.7	3741.0	1113.2	3812.7	1152.7	17
18	3601.1	1037.7	3671.3	1075.3	3742.2	1113.8	3813.9	1153.3	18
19	3602.3	1038.3	3672.5	1075.9	3743.4	1114.5	3815.1	1154.0	19
20	3603.5	1039.0	3673.6	1076.6	3744.6	1115.1	3816.4	1154.7	20
21	3604.6	1039.6	3674.8	1077.2	3745.8	1115.8	3817.6	1155.3	21
22	3605.8	1040.2	3676.0	1077.8	3747.0	1116.4	3818.8	1156.0	22
23	3607.0	1040.8	3677.2	1078.5	3748.2	1117.1	3820.0	1156.7	23
24	3608.1	1041.4	3678.3	1079.1	3749.4	1117.7	3821.2	1157.3	24
25	3609.3	1042.1	3679.5	1079.7	3750.5	1118.4	3822.4	1158.0	25
26	3610.5	1042.7	3680.7	1080.4	3751.7	1119.0	3823.6	1158.7	26
27	3611.6	1043.3	3681.9	1081.0	3752.9	1119.7	3824.8	1159.3	27
28	3612.8	1043.9	3683.1	1081.6	3754.1	1120.3	3826.0	1160.0	28
29	3613.9	1044.5	3684.2	1082.3	3755.3	1121.0	3827.2	1160.7	29
30	3615.1	1045.2	3685.4	1082.9	3756.5	1121.6	3828.4	1161.3	30
31	3616.3	1045.8	3686.6	1083.5	3757.7	1122.3	3829.6	1162.0	31
32	3617.4	1046.4	3687.8	1084.2	3758.9	1123.0	3830.8	1162.7	32
33	3618.6	1047.0	3688.9	1084.8	3760.1	1123.6	3832.0	1163.3	33
34	3619.8	1047.6	3690.1	1085.5	3761.2	1124.3	3833.2	1164.0	34
35	3620.9	1048.3	3691.3	1086.1	3762.6	1124.9	3834.4	1164.7	35
36	3622.1	1048.9	3692.5	1086.7	3763.8	1125.6	3835.6	1165.4	36
37	3623.3	1049.5	3693.7	1087.4	3764.9	1126.2	3836.8	1166.0	37
38	3624.4	1050.1	3694.8	1088.0	3766.0	1126.9	3838.0	1166.7	38
39	3625.6	1050.8	3696.0	1088.7	3767.2	1127.5	3839.2	1167.4	39
40	3626.8	1051.4	3697.2	1089.3	3768.4	1128.2	3840.5	1168.0	40
41	3627.9	1052.0	3698.4	1089.9	3769.6	1128.9	3841.7	1168.7	41
42	3629.1	1052.6	3699.6	1090.6	3770.8	1129.5	3842.9	1169.4	42
43	3630.3	1053.3	3700.7	1091.2	3772.0	1130.2	3844.1	1170.1	43
44	3631.4	1053.9	3701.9	1091.9	3773.2	1130.8	3845.3	1170.7	44
45	3632.6	1054.5	3703.1	1092.5	3774.4	1131.5	3846.5	1171.4	45
46	3633.8	1055.1	3704.3	1093.1	3775.6	1132.1	3847.7	1172.1	46
47	3635.0	1055.8	3705.5	1093.8	3776.8	1132.8	3848.9	1172.8	47
48	3636.1	1056.4	3706.7	1094.4	3778.0	1133.5	3850.1	1173.4	48
49	3637.3	1057.0	3707.8	1095.1	3779.2	1134.1	3851.3	1174.1	49
50	3638.5	1057.6	3709.0	1095.7	3780.4	1134.8	3852.5	1174.8	50
51	3639.6	1058.3	3710.2	1096.4	3781.6	1135.4	3853.7	1175.5	51
52	3640.8	1058.9	3711.4	1097.0	3782.8	1136.1	3855.0	1176.1	52
53	3642.0	1059.5	3712.6	1097.6	3784.0	1136.7	3856.2	1176.8	53
54	3643.1	1060.2	3713.7	1098.3	3785.2	1137.4	3857.4	1177.5	54
55	3644.3	1060.8	3714.9	1098.9	3786.4	1138.1	3858.6	1178.2	55
56	3645.5	1061.4	3716.1	1099.6	3787.6	1138.7	3859.8	1178.8	56
57	3646.7	1062.0	3717.3	1100.2	3788.8	1139.4	3861.0	1179.5	57
58	3647.8	1062.7	3718.5	1100.9	3789.9	1140.0	3862.2	1180.2	58
59	3649.0	1063.3	3719.7	1101.5	3791.1	1140.7	3863.4	1180.9	59

TABLE 2. TANGENTS AND EXTERNALS FOR A 1° CURVE (*Continued*)

′	Δ = 68°		Δ = 69°		Δ = 70°		Δ = 71°		′
	T	E	T	E	T	E	T	E	
0	3864.7	1181.6	3937.9	1222.7	4011.9	1265.0	4086.9	1308.2	0
1	3865.9	1182.2	3939.1	1223.4	4013.2	1265.7	4088.1	1308.9	1
2	3867.1	1182.9	3940.3	1224.1	4014.4	1266.4	4089.4	1309.7	2
3	3868.3	1183.6	3941.5	1224.8	4015.6	1267.1	4090.7	1310.4	3
4	3869.5	1184.3	3942.7	1225.5	4016.9	1267.8	4091.9	1311.2	4
5	3870.7	1184.9	3944.0	1226.2	4018.1	1268.5	4093.2	1311.9	5
6	3871.9	1185.6	3945.2	1226.9	4019.4	1269.2	4094.4	1312.6	6
7	3873.2	1186.3	3946.4	1227.6	4020.6	1269.9	4095.7	1313.3	7
8	3874.4	1187.0	3947.7	1228.3	4021.9	1270.7	4097.0	1314.1	8
9	3875.6	1187.7	3948.9	1229.0	4023.1	1271.4	4098.2	1314.8	9
10	3876.8	1188.3	3950.1	1229.7	4024.3	1272.1	4099.5	1315.5	10
11	3878.0	1189.0	3951.4	1230.4	4025.6	1272.8	4100.7	1316.3	11
12	3879.2	1189.7	3952.6	1231.1	4026.8	1273.5	4102.0	1317.0	12
13	3880.5	1190.4	3953.8	1231.8	4028.1	1274.2	4103.3	1317.7	13
14	3881.7	1191.1	3955.0	1232.5	4029.3	1275.0	4104.5	1318.5	14
15	3882.9	1191.8	3956.3	1233.2	4030.6	1275.7	4105.8	1319.2	15
16	3884.1	1192.4	3957.5	1233.9	4031.8	1276.4	4107.0	1319.9	16
17	3885.3	1193.1	3958.7	1234.6	4033.1	1277.1	4108.3	1320.7	17
18	3886.5	1193.8	3960.0	1235.3	4034.3	1277.8	4109.6	1321.4	18
19	3887.7	1194.5	3961.2	1236.0	4035.6	1278.5	4110.8	1322.2	19
20	3889.0	1195.2	3962.4	1236.7	4036.8	1279.3	4112.1	1322.9	20
21	3890.2	1195.9	3963.7	1237.4	4038.1	1280.0	4113.4	1323.6	21
22	3891.4	1196.5	3964.9	1238.1	4039.3	1280.7	4114.6	1324.4	22
23	3892.6	1197.2	3966.1	1238.8	4040.5	1281.4	4115.9	1325.1	23
24	3893.8	1197.9	3967.4	1239.5	4041.8	1282.1	4117.1	1325.8	24
25	3895.0	1198.6	3968.6	1240.2	4043.0	1282.9	4118.4	1326.6	25
26	3896.3	1199.3	3969.8	1240.9	4044.3	1283.6	4119.7	1327.3	26
27	3897.5	1200.0	3971.1	1241.6	4045.5	1284.3	4120.9	1328.0	27
28	3898.7	1200.6	3972.3	1242.3	4046.8	1285.0	4122.2	1328.8	28
29	3899.9	1201.3	3973.5	1243.0	4048.0	1285.7	4123.5	1329.5	29
30	3901.1	1202.0	3974.8	1243.7	4049.3	1286.5	4124.7	1330.3	30
31	3902.4	1202.7	3976.0	1244.4	4050.5	1287.2	4126.0	1331.0	31
32	3903.6	1203.4	3977.2	1245.1	4051.8	1287.9	4127.2	1331.7	32
33	3904.8	1204.1	3978.5	1245.8	4053.0	1288.6	4128.5	1332.5	33
34	3906.0	1204.8	3979.7	1246.5	4054.3	1289.3	4129.8	1333.2	34
35	3907.2	1205.4	3980.9	1247.2	4055.5	1290.0	4131.0	1334.0	35
36	3908.5	1206.1	3982.2	1247.9	4056.8	1290.8	4132.3	1334.7	36
37	3909.7	1206.8	3983.4	1248.6	4058.0	1291.5	4133.6	1335.4	37
38	3910.9	1207.5	3984.7	1249.4	4059.3	1292.2	4134.8	1336.2	38
39	3912.1	1208.2	3985.9	1250.1	4060.5	1292.9	4136.1	1336.9	39
40	3913.4	1208.9	3987.1	1250.8	4061.8	1293.7	4137.4	1337.7	40
41	3914.6	1209.6	3988.4	1251.5	4063.0	1294.4	4138.6	1338.4	41
42	3915.8	1210.3	3989.6	1252.2	4064.3	1295.1	4139.9	1339.2	42
43	3917.0	1211.0	3990.8	1252.9	4065.6	1295.8	4141.2	1339.9	43
44	3918.2	1211.7	3992.1	1253.6	4066.8	1296.6	4142.4	1340.6	44
45	3919.5	1212.3	3993.3	1254.3	4068.1	1297.3	4143.7	1341.4	45
46	3920.7	1213.0	3994.6	1255.0	4069.3	1298.0	4145.0	1342.1	46
47	3921.9	1213.7	3995.8	1255.7	4070.6	1298.7	4146.3	1342.9	47
48	3923.1	1214.4	3997.0	1256.4	4071.9	1299.5	4147.5	1343.6	48
49	3924.4	1215.1	3998.3	1257.1	4073.1	1300.2	4148.8	1344.4	49
50	3925.6	1215.8	3999.5	1257.8	4074.3	1300.9	4150.1	1345.1	50
51	3926.8	1216.5	4000.7	1258.6	4075.6	1301.6	4151.3	1345.8	51
52	3928.0	1217.2	4002.0	1259.3	4076.8	1302.4	4152.6	1346.6	52
53	3929.3	1217.9	4003.2	1260.0	4078.1	1303.1	4153.9	1347.3	53
54	3930.5	1218.6	4004.5	1260.7	4079.3	1303.8	4155.1	1348.1	54
55	3931.7	1219.2	4005.7	1261.4	4080.6	1304.6	4156.4	1348.8	55
56	3932.9	1219.9	4006.9	1262.1	4081.9	1305.3	4157.7	1349.6	56
57	3934.2	1220.6	4008.2	1262.8	4083.1	1306.0	4159.0	1350.3	57
58	3935.4	1221.3	4009.4	1263.5	4084.4	1306.7	4160.2	1351.1	58
59	3936.6	1222.0	4010.7	1264.2	4085.6	1307.5	4161.5	1351.8	59

TABLE 2. TANGENTS AND EXTERNALS FOR A 1° CURVE (*Continued*)

′	Δ = 72°		Δ = 73°		Δ = 74°		Δ = 75°		′
	T	E	T	E	T	E	T	E	
0	4162.8	1352.6	4239.7	1398.0	4317.6	1444.6	4396.5	1492.4	0
1	4164.1	1353.3	4241.0	1398.8	4318.9	1445.4	4397.8	1493.2	1
2	4165.3	1354.1	4242.2	1399.6	4320.2	1446.2	4399.1	1494.0	2
3	4166.6	1354.8	4243.5	1400.3	4321.5	1447.0	4400.4	1494.8	3
4	4167.9	1355.6	4244.8	1401.1	4322.8	1447.8	4401.8	1495.6	4
5	4169.2	1356.3	4246.1	1401.9	4324.1	1448.6	4403.1	1496.4	5
6	4170.4	1357.1	4247.4	1402.6	4325.4	1449.3	4404.4	1497.2	6
7	4171.7	1357.8	4248.7	1403.4	4326.7	1450.1	4405.7	1498.1	7
8	4173.0	1358.6	4250.0	1404.2	4328.0	1450.9	4407.1	1498.9	8
9	4174.3	1359.3	4251.3	1404.9	4329.3	1451.7	4408.4	1499.7	9
10	4175.5	1360.1	4252.6	1405.7	4330.6	1452.5	4409.7	1500.5	10
11	4176.8	1360.8	4253.9	1406.5	4331.9	1453.3	4411.0	1501.3	11
12	4178.1	1361.6	4255.2	1407.3	4333.2	1454.1	4412.4	1502.1	12
13	4179.4	1362.3	4256.5	1408.0	4334.6	1454.9	4413.7	1502.9	13
14	4180.6	1363.1	4257.8	1408.8	4335.9	1455.7	4415.0	1503.7	14
15	4181.9	1363.8	4259.0	1409.6	4337.2	1456.5	4416.4	1504.5	15
16	4183.2	1364.6	4260.3	1410.4	4338.5	1457.2	4417.7	1505.3	16
17	4184.5	1365.3	4261.6	1411.1	4339.8	1458.0	4419.0	1506.2	17
18	4185.7	1366.1	4262.9	1411.9	4341.1	1458.8	4420.3	1507.0	18
19	4187.0	1366.8	4264.2	1412.7	4342.4	1459.6	4421.7	1507.8	19
20	4188.3	1367.6	4265.5	1413.4	4343.7	1460.4	4423.0	1508.6	20
21	4189.6	1368.4	4266.8	1414.2	4345.0	1461.2	4424.3	1509.4	21
22	4190.9	1369.1	4268.1	1415.0	4346.4	1462.0	4425.7	1510.2	22
23	4192.1	1369.9	4269.4	1415.8	4347.7	1462.8	4427.0	1511.0	23
24	4193.4	1370.6	4270.7	1416.5	4349.0	1463.6	4428.3	1511.8	24
25	4194.7	1371.4	4272.0	1417.3	4350.3	1464.4	4429.7	1512.6	25
26	4196.0	1372.1	4273.3	1418.1	4351.6	1465.2	4431.0	1513.5	26
27	4197.3	1372.9	4274.6	1418.9	4352.9	1466.0	4432.3	1514.3	27
28	4198.5	1373.7	4275.9	1419.6	4354.2	1466.8	4433.7	1515.1	28
29	4199.8	1374.4	4277.2	1420.4	4355.6	1467.6	4435.0	1515.9	29
30	4201.1	1375.2	4278.5	1421.2	4356.9	1468.4	4436.3	1516.7	30
31	4202.4	1375.9	4279.8	1422.0	4358.2	1469.2	4437.6	1517.5	31
32	4203.7	1376.7	4281.1	1422.8	4359.5	1470.0	4439.0	1518.4	32
33	4204.9	1377.4	4282.4	1423.5	4360.8	1470.8	4440.3	1519.2	33
34	4206.2	1378.2	4283.7	1424.3	4362.1	1471.6	4441.7	1520.0	34
35	4207.5	1379.0	4285.0	1425.1	4363.5	1472.4	4443.0	1520.8	35
36	4208.8	1379.7	4286.3	1425.9	4364.8	1473.2	4444.3	1521.6	36
37	4210.1	1380.5	4287.6	1426.6	4366.1	1474.0	4445.7	1522.4	37
38	4211.4	1381.2	4288.9	1427.4	4367.4	1474.8	4447.0	1523.3	38
39	4212.6	1382.0	4290.2	1428.2	4368.7	1475.6	4448.3	1524.1	39
40	4213.9	1382.8	4291.5	1429.0	4370.0	1476.4	4449.7	1524.9	40
41	4215.2	1383.5	4292.8	1429.8	4371.4	1477.2	4451.0	1525.7	41
42	4216.5	1384.3	4294.1	1430.5	4372.7	1478.0	4452.3	1526.5	42
43	4217.8	1385.0	4295.4	1431.3	4374.0	1478.8	4453.7	1527.4	43
44	4219.1	1385.8	4296.7	1432.1	4375.3	1479.6	4455.0	1528.2	44
45	4220.4	1386.6	4298.0	1432.9	4376.6	1480.4	4456.3	1529.0	45
46	4221.6	1387.3	4299.3	1433.7	4378.0	1481.2	4457.7	1529.8	46
47	4222.9	1388.1	4300.6	1434.4	4379.3	1482.0	4459.0	1530.6	47
48	4224.2	1388.9	4301.9	1435.2	4380.6	1482.8	4460.4	1531.5	48
49	4225.5	1389.6	4303.2	1436.0	4381.9	1483.6	4461.7	1532.3	49
50	4226.8	1390.4	4304.5	1436.8	4383.2	1484.4	4463.0	1533.1	50
51	4228.1	1391.2	4305.8	1437.6	4384.6	1485.2	4464.4	1533.9	51
52	4229.4	1391.9	4307.1	1438.4	4385.9	1486.0	4465.7	1534.8	52
53	4230.6	1392.7	4308.4	1439.1	4387.2	1486.8	4467.1	1535.6	53
54	4231.9	1393.4	4309.7	1439.9	4388.5	1487.6	4468.4	1536.4	54
55	4233.2	1394.2	4311.0	1440.7	4389.8	1488.4	4469.7	1537.2	55
56	4234.5	1395.0	4312.3	1441.5	4391.2	1489.2	4471.1	1538.1	56
57	4235.8	1395.7	4313.6	1442.3	4392.5	1490.0	4472.4	1538.9	57
58	4237.1	1396.5	4314.9	1443.0	4393.8	1490.8	4473.8	1539.7	58
59	4238.4	1397.3	4316.2	1443.8	4395.1	1491.6	4475.1	1540.5	59

TABLE 2. TANGENTS AND EXTERNALS FOR A 1° CURVE (*Continued*)

′	Δ = 76°		Δ = 77°		Δ = 78°		Δ = 79°		′
	T	E	T	E	T	E	T	E	
0	4476.4	1541.4	4557.5	1591.6	4639.7	1643.0	4723.1	1695.8	0
1	4477.8	1542.2	4558.9	1592.4	4641.1	1643.9	4724.5	1696.7	1
2	4479.1	1543.0	4560.2	1593.3	4642.5	1644.7	4725.9	1697.6	2
3	4480.5	1543.9	4561.6	1594.1	4643.9	1645.6	4727.3	1698.4	3
4	4481.8	1544.7	4563.0	1595.0	4645.2	1646.5	4728.7	1699.3	4
5	4483.2	1545.5	4564.3	1595.8	4646.6	1647.3	4730.1	1700.2	5
6	4484.5	1546.3	4565.7	1596.7	4648.0	1648.2	4731.5	1701.1	6
7	4485.8	1547.2	4567.0	1597.5	4649.4	1649.1	4732.9	1702.0	7
8	4487.2	1548.0	4568.4	1598.4	4650.8	1650.0	4734.3	1702.9	8
9	4488.5	1548.9	4569.8	1599.2	4652.2	1650.8	4735.7	1703.8	9
10	4489.9	1549.6	4571.1	1600.1	4653.5	1651.7	4737.1	1704.7	10
11	4491.2	1550.5	4572.5	1600.9	4654.9	1652.6	4738.5	1705.6	11
12	4492.6	1551.3	4573.9	1601.8	4656.3	1653.5	4739.9	1706.5	12
13	4493.9	1552.1	4575.2	1602.6	4657.7	1654.3	4741.3	1707.4	13
14	4495.3	1553.0	4576.6	1603.5	4659.1	1655.2	4742.7	1708.3	14
15	4496.6	1553.6	4578.0	1604.3	4660.5	1656.1	4744.1	1709.2	15
16	4498.0	1554.6	4579.3	1605.2	4661.8	1657.0	4745.5	1710.1	16
17	4499.3	1555.5	4580.7	1606.0	4663.2	1657.8	4746.9	1711.0	17
18	4500.6	1556.3	4582.1	1606.9	4664.6	1658.7	4748.4	1711.9	18
19	4502.0	1557.2	4583.4	1607.7	4666.0	1659.6	4749.8	1712.8	19
20	4503.3	1558.0	4584.8	1608.6	4667.4	1660.5	4751.2	1713.7	20
21	4504.7	1558.8	4586.2	1609.4	4668.8	1661.3	4752.6	1714.6	21
22	4506.0	1559.6	4587.5	1610.3	4670.2	1662 2	4754.0	1715.5	22
23	4507.4	1560.5	4588.9	1611.1	4671.5	1663.1	4755.4	1716.3	23
24	4508.7	1561.3	4590.3	1612.0	4672.9	1664.0	4756.8	1717.2	24
25	4510.1	1562.1	4591.6	1612.8	4674.3	1664.8	4758.2	1718.2	25
26	4511.4	1563.0	4593.0	1613.7	4675.7	1665.7	4759.6	1719.1	26
27	4512.8	1563.8	4594.4	1614.5	4677.1	1666.6	4761.0	1719.9	27
28	4514.1	1564.6	4595.7	1615.4	4678.5	1667.5	4762.4	1720.9	28
29	4515.5	1565.5	4597.1	1616.3	4679.9	1668.3	4763.8	1721.8	29
30	4516.8	1566.3	4598.5	1617.1	4681.3	1669.2	4765.2	1722.7	30
31	4518.2	1567.1	4599.8	1618.0	4682.7	1670.1	4766.7	1723.6	31
32	4519.5	1568.0	4601.2	1618.8	4684.0	1671.0	4768.1	1724.5	32
33	4520.9	1568.8	4602.6	1619.7	4685.4	1671.9	4769.5	1725.4	33
34	4522.2	1569.7	4604.0	1620.5	4686.8	1672.8	4770.9	1726.3	34
35	4523.6	1570.5	4605.3	1621.4	4688.2	1673.6	4772.3	1727.2	35
36	4525.0	1571.3	4606.7	1622.3	4689.6	1674.5	4773.7	1728.1	36
37	4526.3	1572.2	4608.1	1623.1	4691.0	1675.4	4775.1	1729.0	37
38	4527.7	1573.0	4609.4	1624.0	4692.4	1676.5	4776.5	1729.9	38
39	4529.0	1573.9	4610.8	1624.8	4693.8	1677.2	4777.9	1730.8	39
40	4530.4	1574.7	4612.2	1625.7	4695.2	1678.1	4779.4	1731.7	40
41	4531.7	1575.5	4613.6	1626.6	4696.6	1678.9	4780.8	1732.6	41
42	4533.1	1576 4	4614.9	1627.4	4698.0	1679.8	4782.2	1733.5	42
43	4534.4	1577.2	4616.3	1628.3	4699.4	1680.7	4783.6	1734.4	43
44	4535.8	1578.1	4617.7	1629.2	4700.8	1681.6	4785.0	1735.3	44
45	4537.1	1578.9	4619.1	1630.0	4702.1	1682.5	4786.4	1736.2	45
46	4538.5	1579.7	4620.4	1630.9	4703.5	1683.3	4787.8	1737.1	46
47	4539.9	1580.6	4621.8	1631.8	4704.9	1684.3	4789.3	1738.1	47
48	4541.2	1581.4	4623.2	1632.6	4706.3	1685.1	4790.7	1738.9	48
49	4542.6	1582.3	4624.6	1633.5	4707.7	1686.0	4792.1	1739.8	48
50	4543.9	1583.1	4625.9	1634.4	4709.1	1686.9	4793.5	1740.8	50
51	4545.3	1583.9	4627.3	1635.2	4710.5	1687.8	4794.9	1741.7	51
52	4546.6	1584.8	4628.7	1636.1	4711.9	1688.7	4796.3	1742.6	52
53	4548.0	1585.6	4630.1	1637.0	4713.3	1689.6	4797.8	1743.5	53
54	4549.4	1586.5	4631.5	1637.8	4714.7	1690.4	4799.2	1744.4	54
55	4550.7	1587.3	4632.8	1638.7	4716.1	1691.3	4800.6	1745.3	55
56	4552.1	1588.2	4634.2	1639.6	4717.5	1692.2	4802.0	1746.2	56
57	4553.4	1589.0	4635.6	1640.4	4718.9	1693.1	4803.4	1747.1	57
58	4554.8	1589.9	4637.0	1641.3	4720.3	1694.0	4804.9	1748.0	58
59	4556.2	1590.7	4638.3	1642.1	4721.7	1694.9	4806.3	1749.0	59

Table 2. Tangents and Externals for a 1° Curve (*Continued*)

′	Δ = 80°		Δ = 81°		Δ = 82°		Δ = 83°		′
	T	E	T	E	T	E	T	E	
0	4807.7	1749.9	4893.5	1805.3	4980.6	1862.2	5069.1	1920.5	0
1	4809.1	1750.8	4895.0	1806.2	4982.1	1863.2	5070.6	1921.5	1
2	4810.5	1751.7	4896.4	1807.2	4983.6	1864.1	5072.1	1922.5	2
3	4812.0	1752.6	4897.9	1808.1	4985.0	1865.1	5073.6	1923.5	3
4	4813.4	1753.5	4899.3	1809.1	4986.5	1866.0	5075.1	1924.5	4
5	4814.8	1754.4	4900.7	1810.0	4988.0	1867.0	5076.5	1925.4	5
6	4816.2	1755.3	4902.2	1810.9	4989.4	1868.0	5078.0	1926.4	6
7	4817.6	1756.3	4903.6	1811.9	4990.9	1868.9	5079.5	1927.4	7
8	4819.1	1757.2	4905.1	1812.8	4992.4	1869.9	5081.0	1928.4	8
9	4820.5	1758.1	4906.5	1813.8	4993.8	1870.8	5082.5	1929.4	9
10	4821.9	1759.0	4908.0	1814.7	4995.3	1871.8	5084.0	1930.4	10
11	4823.3	1759.9	4909.4	1815.6	4996.8	1872.8	5085.5	1931.4	11
12	4824.8	1760.8	4910.8	1816.6	4998.2	1873.7	5087.0	1932.4	12
13	4826.2	1761.8	4912.3	1817.5	4999.7	1874.7	5088.5	1933.4	13
14	4827.6	1762.7	4913.7	1818.5	5001.2	1875.7	5089.9	1934.3	14
15	4829.0	1763.6	4915.2	1819.4	5002.6	1876.6	5091.4	1935.3	15
16	4830.5	1764.5	4916.6	1820.3	5004.1	1877.6	5092.9	1936.3	16
17	4831.9	1765.4	4918.1	1821.3	5005.6	1878.6	5094.4	1937.3	17
18	4833.3	1766.4	4919.5	1822.2	5007.0	1879.6	5095.9	1938.3	18
19	4834.7	1767.3	4921.0	1823.2	5008.5	1880.5	5097.4	1939.3	19
20	4836.2	1768.2	4922.4	1824.1	5010.0	1881.5	5098.9	1940.3	20
21	4837.6	1769.1	4923.9	1825.0	5011.5	1882.4	5100.4	1941.3	21
22	4839.0	1770.0	4925.3	1826.0	5012.9	1883.4	5101.9	1942.3	22
23	4840.4	1771.0	4926.8	1826.9	5014.4	1884.4	5103.4	1943.3	23
24	4841.9	1771.9	4928.2	1827.9	5015.9	1885.4	5104.9	1944.3	24
25	4843.3	1772.8	4929.7	1828.8	5017.3	1886.3	5106.4	1945.3	25
26	4844.7	1773.7	4931.1	1829.8	5018.8	1887.3	5107.9	1946.3	26
27	4846.2	1774.6	4932.6	1830.7	5020.3	1888.3	5109.4	1947.3	27
28	4847.6	1775.6	4934.0	1831.7	5021.8	1889.3	5110.9	1948.3	28
29	4849.0	1776.5	4935.5	1832.6	5023.2	1890.2	5112.3	1949.3	29
30	4850.4	1777.4	4936.9	1833.6	5024.7	1891.2	5113.8	1950.3	30
31	4851.9	1778.3	4938.4	1834.5	5026.2	1892.1	5115.3	1951.3	31
32	4853.3	1779.3	4939.8	1835.5	5027.7	1893.1	5116.8	1952.3	32
33	4854.7	1780.2	4941.3	1836.4	5029.1	1894.1	5118.3	1953.3	33
34	4856.2	1781.1	4942.7	1837.4	5030.6	1895.1	5119.8	1954.3	34
35	4857.6	1782.0	4944.2	1838.3	5032.1	1896.0	5121.3	1955.2	35
36	4859.0	1783.0	4945.6	1839.3	5033.6	1897.0	5122.8	1956.2	36
37	4860.5	1783.9	4947.1	1840.2	5035.0	1898.0	5124.3	1957.2	37
38	4861.9	1784.8	4948.6	1841.2	5036.5	1899.0	5125.8	1958.2	38
39	4863.3	1785.8	4950.0	1842.1	5038.0	1899.9	5127.3	1959.2	39
40	4864.8	1786.7	4951.5	1843.1	5039.5	1900.9	5128.8	1960.2	40
41	4866.2	1787.6	4952.9	1844.0	5040.9	1901.9	5130.3	1961.2	41
42	4867.6	1788.5	4954.4	1845.0	5042.4	1902.9	5131.8	1962.2	42
43	4869.1	1789.5	4955.8	1845.9	5043.9	1903.8	5133.3	1963.2	43
44	4870.5	1790.4	4957.3	1846.9	5045.4	1904.8	5134.8	1964.2	44
45	4871.9	1791.3	4958.7	1847.8	5046.9	1905.8	5136.4	1965.2	45
46	4873.4	1792.3	4960.2	1848.8	5048.3	1906.8	5137.9	1966.3	46
47	4874.8	1793.2	4961.7	1849.7	5049.8	1907.8	5139.4	1967.3	47
48	4876.3	1794.1	4963.1	1850.7	5051.3	1908.7	5140.9	1968.3	48
49	4877.7	1795.1	4964.6	1851.6	5052.8	1909.7	5142.4	1969.3	49
50	4879.1	1796.0	4966.0	1852.6	5054.3	1910.7	5143.9	1970.3	50
51	4880.6	1796.9	4967.5	1853.6	5055.7	1911.7	5145.4	1971.3	51
52	4882.0	1797.9	4969.0	1854.5	5057.2	1912.7	5146.9	1972.3	52
53	4883.4	1798.8	4970.4	1855.5	5058.7	1913.6	5148.4	1973.3	53
54	4884.9	1799.7	4971.9	1856.4	5060.2	1914.6	5149.9	1974.3	54
55	4886.3	1800.6	4973.3	1857.4	5061.7	1915.6	5151.4	1975.3	55
56	4887.8	1801.6	4974.8	1858.4	5063.2	1916.6	5152.9	1976.4	56
57	4889.2	1802.5	4976.3	1859.3	5064.7	1917.6	5154.4	1977.4	57
58	4890.6	1803.4	4977.7	1860.3	5066.1	1918.5	5155.9	1978.4	58
59	4892.1	1804.4	4979.2	1861.2	5067.6	1919.5	5157.4	1979.4	59

TABLE 2. TANGENTS AND EXTERNALS FOR A 1° CURVE (*Continued*)

′	Δ = 84°		Δ = 85°		Δ = 86°		Δ = 87°		′
	T	E	T	E	T	E	T	E	
0	5158.9	1980.3	5250.2	2041.7	5342.9	2104.6	5437.2	2169.2	0
1	5160.4	1981.3	5251.7	2042.7	5344.5	2105.7	5438.8	2170.3	1
2	5162.0	1982.4	5253.3	2043.8	5346.0	2106.8	5440.3	2171.4	2
3	5163.5	1983.4	5254.8	2044.8	5347.6	2107.8	5441.9	2172.5	3
4	5165.0	1984.4	5256.3	2045.8	5349.2	2108.9	5443.5	2173.6	4
5	5166.5	1985.4	5257.9	2046.9	5350.7	2110.0	5445.1	2174.7	5
6	5168.0	1986.4	5259.4	2047.9	5352.3	2111.0	5446.7	2175.8	6
7	5169.5	1987.5	5260.9	2049.0	5353.8	2112.1	5448.3	2176.9	7
8	5171.0	1988.5	5262.5	2050.0	5355.4	2113.2	5449.9	2178.0	8
9	5172.5	1989.5	5264.0	2051.0	5357.0	2114.2	5451.4	2179.1	9
10	5174.0	1990.5	5265.5	2052.1	5358.5	2115.3	5453.0	2180.2	10
11	5175.6	1991.5	5267.1	2053.1	5360.1	2116.4	5454.6	2181.2	11
12	5177.1	1992.5	5268.6	2054.2	5361.6	2117.4	5456.2	2182.3	12
13	5178.6	1993.5	5270.2	2055.2	5363.2	2118.5	5457.8	2183.4	13
14	5180.1	1994.5	5271.7	2056.2	5364.8	2119.6	5459.4	2184.5	14
15	5181.6	1995.5	5273.2	2057.3	5366.3	2120.6	5461.0	2185.6	15
16	5183.1	1996.5	5274.8	2058.3	5367.9	2121.7	5462.6	2186.7	16
17	5184.6	1997.6	5276.3	2059.4	5369.5	2122.8	5464.2	2187.8	17
18	5186.2	1998.6	5277.9	2060.4	5371.0	2123.8	5465.7	2188.9	18
19	5187.7	1999.6	5279.4	2061.5	5372.6	2124.9	5467.3	2190.0	19
20	5189.2	2000.6	5280.9	2062.5	5374.2	2126.0	5468.9	2191.1	20
21	5190.7	2001.6	5282.5	2063.5	5375.7	2127.1	5470.5	2192.2	21
22	5192.2	2002.6	5284.0	2064.6	5377.3	2128.1	5472.1	2193.3	22
23	5193.7	2003.7	5285.6	2065.6	5378.9	2129.2	5473.7	2194.4	23
24	5195.3	2004.7	5287.1	2066.7	5380.4	2130.3	5475.3	2195.5	24
25	5196.8	2005.7	5288.7	2067.7	5382.0	2131.3	5476.9	2196.6	25
26	5198.3	2006.7	5290.2	2068.8	5383.6	2132.4	5478.5	2197.7	26
27	5199.8	2007.7	5291.7	2069.8	5385.1	2133.5	5480.1	2198.8	27
28	5201.3	2008.8	5293.3	2070.9	5386.7	2134.6	5481.7	2199.9	28
29	5202.9	2009.8	5294.8	2071.9	5388.3	2135.6	5483.3	2201.0	29
30	5204.4	2010.8	5296.4	2073.0	5389.8	2136.7	5484.9	2202.1	30
31	5205.9	2011.8	5297.9	2074.0	5391.4	2137.8	5486.5	2203.2	31
32	5207.4	2012.9	5299.5	2075.1	5393.0	2138.9	5488.1	2204.3	32
33	5209.0	2013.9	5301.0	2076.1	5394.6	2139.9	5489.7	2205.5	33
34	5210.5	2014.9	5302.6	2077.2	5396.1	2141.0	5491.3	2206.6	34
35	5212.0	2015.9	5304.1	2078.2	5397.7	2142.1	5492.9	2207.7	35
36	5213.5	2017.0	5305.7	2079.3	5399.3	2143.2	5494.5	2208.8	36
37	5215.0	2018.0	5307.2	2080.3	5400.9	2144.3	5496.1	2209.9	37
38	5216.6	2019.0	5308.7	2081.4	5402.4	2145.3	5497.7	2211.0	38
39	5218.1	2020.0	5310.3	2082.4	5404.0	2146.4	5499.3	2212.1	39
40	5219.6	2021.1	5311.8	2083.5	5405.6	2147.5	5500.9	2213.2	40
41	5221.1	2022.1	5313.4	2084.5	5407.2	2148.6	5502.5	2214.3	41
42	5222.7	2023.2	5314.9	2085.6	5408.7	2149.7	5504.1	2215.4	42
43	5224.2	2024.2	5316.5	2086.6	5410.3	2150.8	5505.7	2216.5	43
44	5225.7	2025.2	5318.0	2087.7	5411.9	2151.8	5507.3	2217.6	44
45	5227.2	2026.2	5319.6	2088.8	5413.5	2152.9	5508.9	2218.7	45
46	5228.8	2027.2	5321.2	2089.9	5415.0	2154.0	5510.5	2219.9	46
47	5230.3	2028.3	5322.7	2090.9	5416.6	2155.1	5512.1	2221.0	47
48	5231.8	2029.3	5324.3	2091.9	5418.2	2156.2	5513.7	2222.1	48
49	5233.4	2030.3	5325.8	2093.0	5419.8	2157.3	5515.3	2223.2	49
50	5234.9	2031.4	5327.4	2094.0	5421.4	2158.3	5516.9	2224.3	50
51	5236.4	2032.4	5328.9	2095.1	5422.9	2159.4	5518.5	2225.4	51
52	5237.9	2033.4	5330.5	2096.2	5424.5	2160.5	5520.1	2226.5	52
53	5239.5	2034.5	5332.0	2097.2	5426.1	2161.6	5521.7	2227.7	53
54	5241.0	2035.5	5333.6	2098.3	5427.7	2162.7	5523.3	2228.8	54
55	5242.5	2036.5	5335.1	2099.3	5429.3	2163.8	5524.9	2229.9	55
56	5244.1	2037.6	5336.7	2100.4	5430.8	2164.9	5526.6	2231.0	56
57	5245.6	2038.6	5338.3	2101.5	5432.4	2166.0	5528.2	2232.1	57
58	5247.1	2039.6	5339.8	2102.5	5434.0	2167.0	5529.8	2232.2	58
59	5248.7	2040.7	5341.4	2103.6	5435.6	2168.1	5531.4	2234.4	59

TABLE 2. TANGENTS AND EXTERNALS FOR A 1° CURVE (*Continued*)

′	Δ = 88°		Δ = 89°		Δ = 90°		Δ = 91°		′
	T	E	T	E	T	E	T	E	
0	5533.0	2235.5	5630.4	2303.5	5729.6	2373.3	5830.5	2444.9	0
1	5534.6	2236.6	5632.1	2304.6	5731.2	2374.5	5832.2	2446.1	1
2	5536.2	2237.7	5633.7	2305.8	5732.9	2375.6	5833.9	2447.3	2
3	5537.8	2238.8	5635.4	2306.9	5734.6	2376.8	5835.6	2448.6	3
4	5539.4	2240.0	5637.0	2308.1	5736.3	2378.0	5837.3	2449.8	4
5	5541.1	2241.1	5638.6	2309.2	5737.9	2379.2	5839.0	2451.0	5
6	5542.7	2242.2	5640.3	2310.4	5739.6	2380.4	5840.7	2452.2	6
7	5544.3	2243.3	5641.9	2311.5	5741.3	2381.5	5842.4	2453.4	7
8	5545.9	2244.4	5643.6	2312.7	5742.9	2382.7	5844.1	2454.6	8
9	5547.5	2245.6	5645.2	2313.8	5744.6	2383.9	5845.8	2455.8	9
10	5549.1	2246.7	5646.8	2315.0	5746.3	2385.1	5847.4	2457.0	10
11	5550.7	2247.8	5648.5	2316.1	5747.9	2386.3	5849.2	2458.3	11
12	5552.4	2248.9	5650.1	2317.3	5749.6	2387.5	5850.9	2459.5	12
13	5554.0	2250.1	5651.8	2318.5	5751.3	2388.6	5852.6	2460.8	13
14	5555.6	2251.2	5653.4	2319.6	5753.0	2389.8	5854.3	2461.9	14
15	5557.2	2252.3	5655.1	2320.8	5754.6	2391.0	5856.0	2463.1	15
16	5558.8	2253.4	5656.7	2321.9	5756.3	2392 2	5857.7	2464.3	16
17	5560.4	2254.6	5658.4	2323.1	5758.0	2393.4	5859.4	2465.6	17
18	5562.1	2255.7	5660.0	2324.2	5759.7	2394.6	5861.1	2466.9	18
19	5563.7	2256.8	5661.7	2325.4	5761.3	2395.8	5862.8	2468.0	19
20	5565.3	2258.0	5663.3	2326.5	5763.0	2397.0	5864.5	2469.2	20
21	5566.9	2259.1	5664.9	2327.7	5764.7	2398.1	5866.2	2470.4	21
22	5568.5	2260.2	5666.6	2328.9	5766.4	2399.3	5867.9	2471.7	22
23	5570.2	2261.3	5668.2	2330.0	5768.0	2400.5	5869.6	2472.9	23
24	5571.8	2262.5	5669.9	2331.2	5769.7	2401.7	5871.3	2474.1	24
25	5573.4	2263.6	5671.5	2332.3	5771.4	2402.9	5873.0	2475.3	25
26	5575.0	2264.7	5673.2	2333.5	5773.1	2404.1	5874.7	2476.6	26
27	5576.6	2265.9	5674.8	2334.7	5774.8	2405.3	5876.4	2477.8	27
28	5578.3	2267.0	5676.5	2335.8	5776.4	2406.5	5878.2	2479.0	28
29	5579.9	2268.1	5678.1	2337.0	5778.1	2407.7	5879.9	2480.2	29
30	5581.5	2269.3	5679.8	2338.2	5779.8	2408.9	5881.6	2481.5	30
31	5583.1	2270.4	5681.4	2339.3	5781.5	2410.1	5883.3	2482.7	31
32	5584.8	2271.5	5683.1	2340.5	5783.2	2411.3	5885.0	2483.9	32
33	5586.4	2272.7	5684.8	2341.6	5784.8	2412.5	5886.7	2485.1	33
34	5588.0	2273.8	5686.4	2342.8	5786.5	2413.6	5888.4	2486.4	34
35	5589.6	2274.9	5688.1	2344.0	5788.2	2414.8	5890.1	2487.6	35
36	5591.3	2276.1	5689.7	2345.1	5789.9	2416.0	5891.9	2488.8	36
37	5592.9	2277.2	5691.4	2346.3	5791.6	2417.2	5893.6	2490.1	37
38	5594.5	2278.3	5693.0	2347.5	5793.3	2418.4	5895.3	2491.3	38
39	5596.1	2279.5	5694.7	2348.6	5795.0	2419.6	5897.0	2492.5	39
40	5597.8	2280.6	5696.3	2349.8	5796.6	2420.8	5898.7	2493.7	40
41	5599.4	2281.7	5698.0	2351.0	5798.3	2422.0	5900.4	2495.0	41
42	5601 0	2282.9	5699.7	2352.1	5800.0	2423.2	5902.2	2496.2	42
43	5602.7	2284.0	5701.3	2353.3	5801.7	2424.4	5903.9	2497.4	43
44	5604.3	2285.2	5703.0	2354.5	5803.4	2425.6	5905.6	2498.7	44
45	5605.9	2286.3	5704.6	2355.6	5805.1	2426.8	5907.3	2499.9	45
46	5607.6	2287.4	5706.3	2356.8	5806.8	2428.0	5909.0	2501.1	46
47	5609.2	2288.6	5708.0	2358.0	5808.5	2429.2	5910.8	2502.4	47
48	5610.8	2289.7	5709.6	2359.2	5810.1	2430.4	5912.5	2503.6	48
49	5612.5	2290.9	5711.3	2360.3	5811.8	2431.6	5914.2	2504.9	49
50	5614.1	2292.0	5712.9	2361.5	5813.5	2432.9	5915.9	2506.1	50
51	5615.7	2293.2	5714.6	2362.7	5815.2	2434.1	5917.6	2507.3	51
52	5617.4	2294.3	5716.3	2363.9	5816.9	2435.3	5919.4	2508.6	52
53	5619.0	2295.4	5717.9	2365.0	5818.6	2436.5	5921.1	2509.8	53
54	5620.6	2296.6	5719.6	2366.2	5820.3	2437.7	5922.8	2511.0	54
55	5622.3	2297.7	5721.3	2367.4	5822.0	2438.9	5924.5	2512.3	55
56	5623.9	2298.9	5722.9	2368.6	5823.7	2440.1	5926 3	2513.5	56
57	5625.5	2300.0	5724.6	2369.7	5825.4	2441.3	5928.0	2514.8	57
58	5627.2	2301.2	5726.2	2370.9	5827.1	2442.5	5929.7	2516.0	58
59	5628.8	2302.3	5727.9	2372.1	5828.8	2443.7	5931.4	2517.2	59

TABLE 2. TANGENTS AND EXTERNALS FOR A 1° CURVE (*Continued*)

′	Δ = 92°		Δ = 93°		Δ = 94°		Δ = 95°		′
	T	E	T	E	T	E	T	E	
0	5933.1	2518.5	6037.7	2594.0	6144.2	2671.6	6252.7	2751.3	0
1	5934.9	2519.7	6039.5	2595.3	6146.0	2672.9	6254.6	2752.6	1
2	5936.6	2521.0	6041.2	2596.6	6147.8	2674.2	6256.4	2754.0	2
3	5938.3	2522.2	6043.0	2597.8	6149.6	2675.5	6258.2	2755.3	3
4	5940.1	2523.5	6044.8	2599.1	6151.4	2676.8	6260.1	2756.7	4
5	5941.8	2524.7	6046.5	2600.4	6153.2	2678.1	6261.9	2758.0	5
6	5943.5	2525.9	6048.3	2601.7	6155.0	2679.5	6263.7	2759.4	6
7	5945.3	2527.2	6050.0	2603.0	6156.8	2680.8	6265.5	2760.7	7
8	5947.0	2528.4	6051.8	2604.2	6158.6	2682.1	6267.4	2762.1	8
9	5948.7	2529.7	6053.6	2605.5	6160.4	2683.4	6269.2	2763.4	9
10	5950.4	2530.9	6055.3	2606.8	6162.2	2684.7	6271.0	2764.8	10
11	5952.2	2532.2	6057.1	2608.1	6164.0	2686.0	6272.9	2766.1	11
12	5953.9	2533.4	6058.9	2609.4	6165.8	2687.4	6274.7	2767.5	12
13	5955.7	2534.7	6060.6	2610.6	6167.6	2688.7	6276.5	2768.8	13
14	5957.4	2535.9	6062.4	2611.9	6169.4	2690.0	6278.4	2770.2	14
15	5959.1	2537.2	6064.2	2613.2	6171.2	2691.3	6280.2	2771.5	15
16	5960.9	2538.4	6065.9	2614.5	6173.0	2692.6	6282.0	2772.9	16
17	5962.6	2539.7	6067.7	2615.8	6174.8	2693.9	6283.9	2774.2	17
18	5964.3	2540.9	6069.5	2617.1	6176.6	2695.3	6285.7	2775.6	18
19	5966.1	2542.2	6071.2	2618.4	6178.4	2696.6	6287.5	2777.0	19
20	5967.8	2543.4	6073.0	2619.6	6180.2	2697.9	6289.4	2778.3	20
21	5969.5	2544.7	6074.8	2620.9	6182.0	2699.2	6291.2	2779.7	21
22	5971.3	2545.9	6076.5	2622.2	6183.8	2700.6	6293.0	2781.0	22
23	5973.0	2547.2	6078.3	2623.5	6185.6	2701.9	6294.9	2782.4	23
24	5974.8	2548.5	6080.1	2624.8	6187.4	2703.2	6296.7	2783.8	24
25	5976.5	2549.7	6081.9	2626.1	6189.2	2704.5	6298.6	2785.1	25
26	5978.2	2551.0	6083.6	2627.4	6191.0	2705.9	6300.4	2786.5	26
27	5980.0	2552.2	6085.4	2628.7	6192.8	2707.2	6302.3	2787.8	27
28	5981.7	2553.5	6087.2	2630.0	6194.6	2708.5	6304.1	2789.2	28
29	5983.5	2554.7	6088.9	2631.2	6196.4	2709.8	6305.9	2790.6	29
30	5985.2	2556.0	6090.7	2632.5	6198.2	2711.2	6307.8	2791.9	30
31	5986.9	2557.3	6092.5	2633.8	6200.0	2712.5	6309.6	2793.3	31
32	5988.7	2558.5	6094.3	2635.1	6201.8	2713.8	6311.5	2794.7	32
33	5990.4	2559.8	6096.1	2636.4	6203.7	2715.2	6313.3	2796.0	33
34	5992.2	2561.0	6097.8	2637.7	6205.5	2716.5	6315.2	2797.4	34
35	5993.9	2562.3	6099.6	2639.0	6207.3	2717.8	6317.0	2798.8	35
36	5995.7	2563.6	6101.4	2640.3	6209.1	2719.1	6318.9	2800.1	36
37	5997.4	2564.8	6103.2	2641.6	6210.9	2720.5	6320.7	2801.5	37
38	5999.2	2566.1	6104.9	2642.9	6212.7	2721.8	6322.5	2802.9	38
39	6000.9	2567.3	6106.7	2644.2	6214.5	2723.1	6324.4	2804.2	39
40	6002.6	2568.6	6108.5	2645.5	6216.3	2724.5	6326.2	2805.6	40
41	6004.4	2569.9	6110.3	2646.8	6218.2	2725.8	6328.1	2807.0	41
42	6006.2	2571.1	6112.1	2648.1	6220.0	2727.1	6329.9	2808.4	42
43	6007.9	2572.4	6113.8	2649.4	6221.8	2728.5	6331.8	2809.7	43
44	6009.7	2573.7	6115.6	2650.7	6223.6	2729.8	6333.6	2811.1	44
45	6011.4	2574.9	6117.4	2652.0	6225.4	2731.2	6335.5	2812.5	45
46	6013.2	2576.2	6119.2	2653.3	6227.2	2732.5	6337.3	2813.8	46
47	6014.9	2577.5	6121.0	2654.6	6229.1	2733.8	6339.2	2815.2	47
48	6016.7	2578.7	6122.8	2655.9	6230.9	2735.2	6341.1	2816.6	48
49	6018.4	2580.0	6124.5	2657.2	6232.7	2736.5	6342.9	2818.0	49
50	6020.2	2581.3	6126.3	2658.5	6234.5	2737.8	6344.8	2819.3	50
51	6021.9	2582.6	6128.1	2659.8	6236.3	2739.2	6346.6	2820.7	51
52	6023.7	2583.8	6129.9	2661.1	6238.2	2740.5	6348.5	2822.1	52
53	6025.4	2585.1	6131.7	2662.4	6240.0	2741.9	6350.3	2823.5	53
54	6027.2	2586.4	6133.5	2663.7	6241.8	2743.2	6352.2	2824.9	54
55	6028.9	2587.6	6135.3	2665.0	6243.6	2744.6	6354.0	2826.2	55
56	6030.7	2588.9	6137.1	2666.4	6245.4	2745.9	6355.9	2827.6	56
57	6032.4	2590.2	6138.9	2667.7	6247.3	2747.2	6357.8	2829.0	57
58	6034.2	2591.5	6140.6	2669.0	6249.1	2748.6	6359.6	2830.4	58
59	6036.0	2592.7	6142.4	2670.3	6250.9	2749.9	6361.5	2831.8	59

TABLE 2. TANGENTS AND EXTERNALS FOR A 1° CURVE (*Continued*)

'	Δ = 96°		Δ = 97°		Δ = 98°		Δ = 99°		'
	T	E	T	E	T	E	T	E	
0	6363.3	2833.1	6476.1	2917.3	6591.1	3003.7	6708.5	3092.6	0
1	6365.2	2834.5	6478.0	2918.7	6593.1	3005.2	6710.5	3094.2	1
2	6367.1	2835.9	6479.9	2920.1	6595.0	3006.7	6712.4	3095.7	2
3	6368.9	2837.3	6481.8	2921.5	6596.9	3008.1	6714.4	3097.2	3
4	6370.8	2838.7	6483.7	2923.0	6598.9	3009.6	6716.4	3098.7	4
5	6372.7	2840.1	6485.6	2924.4	6600.8	3011.1	6718.4	3100.2	5
6	6374.5	2841.5	6487.5	2925.8	6602.8	3012.5	6720.3	3101.7	6
7	6376.4	2842.8	6489.4	2927.2	6604.7	3014.0	6722.3	3103.2	7
8	6378.3	2844.2	6491.3	2928.7	6606.6	3015.5	6724.3	3104.7	8
9	6380.1	2845.6	6493.2	2930.1	6608.6	3016.9	6726.3	3106.2	9
10	6382.0	2847.0	6495.1	2931.5	6610.5	3018.4	6728.3	3107.7	10
11	6383.9	2848.4	6497.0	2933.0	6612.5	3019.9	6730.3	3109.2	11
12	6385.7	2849.8	6498.9	2934.4	6614.4	3021.3	6732.2	3110.7	12
13	6387.6	2851.2	6500.8	2935.8	6616.4	3022.8	6734.2	3112.2	13
14	6389.5	2852.6	6502.8	2937.2	6618.3	3024.3	6736.2	3113.8	14
15	6391.3	2854.0	6504.7	2938.7	6620.2	3025.8	6738.2	3115.3	15
16	6393.2	2855.4	6506.6	2940.1	6622.2	3027.2	6740.2	3116.8	16
17	6395.1	2856.8	6508.5	2941.5	6624.1	3028.7	6742.2	3118.3	17
18	6396.9	2858.1	6510.4	2943.0	6626.1	3030.2	6744.2	3119.8	18
19	6398.8	2859.5	6512.3	2944.4	6628.0	3031.6	6746.1	3121.3	19
20	6400.7	2860.9	6514.2	2945.8	6630.0	3033.1	6748.1	3122.8	20
21	6402.6	2862.3	6516.1	2947.3	6631.9	3034.6	6750.1	3124.4	21
22	6404.4	2863.7	6518.0	2948.7	6633.9	3036.1	6752.1	3125.9	22
23	6406.3	2865.1	6519.9	2950.1	6635.8	3037.5	6754.1	3127.4	23
24	6408.2	2866.5	6521.8	2951.6	6637.8	3039.0	6756.1	3128.9	24
25	6410.1	2867.9	6523.8	2953.0	6639.7	3040.5	6758.1	3130.4	25
26	6411.9	2869.3	6525.7	2954.5	6641.7	3042.0	6760.1	3132.0	26
27	6413.8	2870.7	6527.6	2955.9	6643.6	3043.5	6762.1	3133.5	27
28	6415.7	2872.1	6529.5	2957.3	6645.6	3044.9	6764.1	3135.0	28
29	6417.6	2873.5	6531.4	2958.8	6647.5	3046.4	6766.1	3136.5	29
30	6419.5	2874.9	6533.3	2960.2	6649.5	3047.9	6768.1	3138.0	30
31	6421.3	2876.3	6535.3	2961.7	6651.5	3049.4	6770.1	3139.6	31
32	6423.2	2877.7	6537.2	2963.1	6653.4	3050.9	6772.0	3141.1	32
33	6425.1	2879.1	6539.1	2964.5	6655.4	3052.3	6774.0	3142.6	33
34	6427.0	2880.5	6541.0	2966.0	6657.3	3053.8	6776.0	3144.1	34
35	6428.9	2881.9	6542.9	2967.4	6659.3	3055.3	6778.0	3145.7	35
36	6430.7	2883.4	6544.9	2968.9	6661.3	3056.8	6780.0	3147.2	36
37	6432.6	2884.8	6546.8	2970.3	6663.2	3058.3	6782.0	3148.7	37
38	6434.5	2886.2	6548.7	2971.8	6665.2	3059.8	6784.1	3150.3	38
39	6436.4	2887.6	6550.6	2973.2	6667.1	3061.3	6786.0	3151.8	39
40	6438.3	2889.0	6552.5	2974.7	6669.1	3062.7	6788.0	3153.3	40
41	6440.2	2890.4	6554.5	2976.1	6671.1	3064.2	6790.1	3154.8	41
42	6442.1	2891.8	6556.4	2977.6	6673.0	3065.7	6792.1	3156.4	42
43	6443.9	2893.2	6558.3	2979.0	6675.0	3067.2	6794.1	3157.9	43
44	6445.8	2894.6	6560.2	2980.5	6677.0	3068.7	6796.1	3159.4	44
45	6447.7	2896.0	6562.2	2981.9	6678.9	3070.2	6798.1	3161.0	45
46	6449.6	2897.4	6564.1	2983.4	6680.9	3071.7	6800.1	3162.5	46
47	6451.5	2898.9	6566.0	2984.8	6682.9	3073.2	6802.1	3164.0	47
48	6453.4	2900.3	6567.9	2986.3	6684.8	3074.7	6804.1	3165.6	48
49	6455.3	2901.7	6569.9	2987.7	6686.8	3076.2	6806.1	3167.1	49
50	6457.2	2903.1	6571.8	2989.2	6688.8	3077.7	6808.1	3168.6	50
51	6459.1	2904.5	6573.7	2990.6	6690.7	3079.2	6810.1	3170.2	51
52	6461.0	2905.9	6575.7	2992.1	6692.7	3080.7	6812.1	3171.7	52
53	6462.8	2907.3	6577.6	2993.5	6694.7	3082.2	6814.2	3173.3	53
54	6464.7	2908.8	6579.5	2995.0	6696.6	3083.7	6816.2	3174.8	54
55	6466.6	2910.2	6581.5	2996.5	6698.6	3085.2	6818.2	3176.4	55
56	6468.5	2911.6	6583.4	2997.9	6700.6	3086.7	6820.2	3177.9	56
57	6470.4	2913.0	6585.3	2999.4	6702.6	3088.2	6822.2	3179.4	57
58	6472.3	2914.4	6587.3	3000.8	6704.5	3089.7	6824.2	3181.0	58
59	6474.2	2915.9	6589.2	3002.3	6706.5	3091.2	6826.2	3182.5	59

TABLE 2. TANGENTS AND EXTERNALS FOR A 1° CURVE (*Continued*)

′	Δ = 100°		Δ = 101°		Δ = 102°		Δ = 103°		′
	T	E	T	E	T	E	T	E	
0	6828.2	3184.1	6950.5	3278.1	7075.4	3374.8	7203.1	3474.3	0
10	6848.4	3199.6	6971.2	3294.0	7096.5	3391.2	7224.6	3491.2	10
20	6868.7	3215.1	6991.9	3310.0	7117.7	3407.7	7246.2	3508.2	20
30	6889.1	3230.7	7012.7	3326.1	7138.9	3424.2	7267.9	3525.2	30
40	6909.5	3246.4	7033.5	3342.3	7160.2	3440.8	7289.7	3542.3	40
50	6930.0	3262.2	7054.4	3358.5	7181.6	3457.6	7311.6	3559.5	50
60	6950.5	3278.1	7075.4	3374.8	7203.1	3474.3	7333.5	3576.8	60

′	Δ = 104°		Δ = 105°		Δ = 106°		Δ = 107°		′
	T	E	T	E	T	E	T	E	
0	7333.5	3576.8	7466.9	3682.3	7603.4	3790.9	7743.1	3902.8	0
10	7355.6	3594.2	7489.5	3700.2	7626.5	3809.3	7766.7	3921.8	10
20	7377.7	3611.6	7512.1	3718.1	7649.6	3827.9	7790.4	3940.9	20
30	7399.9	3629.2	7534.8	3736.2	7672.8	3846.5	7814.2	3960.1	30
40	7422.1	3646.8	7557.6	3754.3	7696.2	3865.2	7838.0	3979.3	40
50	7444.5	3664.5	7580.4	3772.6	7719.6	3883.9	7862.0	3998.7	50
60	7466.9	3682.3	7603.4	3790.9	7743.1	3902.8	7886.1	4018.2	60

′	Δ = 108°		Δ = 109°		Δ = 110°		Δ = 111°		′
	T	E	T	E	T	E	T	E	
0	7886.1	4018.2	8032.6	4137.0	8182.7	4259.6	8336.6	4386.1	0
10	7910.3	4037.7	8057.3	4157.2	8208.1	4280.4	8362.6	4407.6	10
20	7934.5	4057.4	8082.2	4177.5	8233.6	4301.4	8388.8	4429.1	20
30	7958.9	4077.2	8107.2	4197.9	8259.2	4322.4	8415.0	4450.8	30
40	7983.3	4097.0	8132.2	4218.4	8284.9	4343.5	8441.4	4472.6	40
50	8007.9	4117.0	8157.4	4238.9	8310.7	4364.7	8467.9	4494.5	50
60	8032.6	4137.0	8182.7	4259.6	8336.6	4386.1	8494.4	4516.6	60

′	Δ = 112°		Δ = 113°		Δ = 114°		Δ = 115°		′
	T	E	T	E	T	E	T	E	
0	8494.4	4516.6	8656.4	4651.3	8822.8	4790.4	8993.6	4934.1	0
10	8521.2	4538.7	8683.9	4674.1	8850.9	4814.0	9022.6	4958.5	10
20	8548.0	4561.0	8711.4	4697.1	8879.2	4837.8	9051.6	4983.0	20
30	8574.9	4583.4	8739.1	4720.3	8907.6	4861.6	9080.8	5007.7	30
40	8602.0	4605.9	8766.8	4743.5	8936.2	4885.7	9110.2	5032.5	40
50	8629.2	4628.5	8794.7	4766.9	8964.8	4909.8	9139.6	5057.5	50
60	8656.4	4651.3	8822.8	4790.4	8993.6	4934.1	9169.2	5082.6	60

′	Δ = 116°		Δ = 117°		Δ = 118°		Δ = 119°		′
	T	E	T	E	T	E	T	E	
0	9169.2	5082.6	9349.8	5236.2	9535.6	5395.0	9726.9	5559.4	0
10	9199.0	5107.8	9380.4	5262.3	9567.1	5422.0	9759.3	5587.3	10
20	9228.9	5133.2	9411.2	5288.5	9598.8	5449.2	9791.9	5615.5	20
30	9258.9	5158.7	9442.1	5314.9	9630.6	5476.5	9824.7	5643.7	30
40	9289.1	5184.4	9473.1	5341.4	9662.5	5504.0	9857.6	5672.2	40
50	9319.4	5210.2	9504.3	5368.1	9694.6	5531.6	9890.7	5700.8	50
60	9349.8	5236.2	9535.6	5395.0	9726.9	5559.4	9923.9	5729.6	60

TABLE 3. CORRECTIONS FOR TANGENT DISTANCES IN TABLE 2
Applying only to the "chord" definition
Add after dividing (T for 1° curve) by D

Δ	D =												Δ
	2°	3°	4°	5°	6°	7°	8°	9°	10°	15°	20°	25°	
10°	.01	.02	.02	.03	.04	.04	.05	.06	.06	.09	.13	.16	10°
12	.01	.02	.03	.04	.04	.05	.06	.07	.08	.11	.15	.19	12
14	.01	.02	.03	.04	.05	.06	.07	.08	.09	.13	.17	.22	14
16	.02	.03	.04	.05	.06	.07	.08	.09	.10	.15	.20	.25	16
18	.02	.03	.04	.06	.07	.08	.09	.10	.11	.17	.23	.28	18
20	.02	.03	.05	.06	.07	.09	.10	.11	.13	.19	.26	.32	20
22	.02	.04	.05	.07	.08	.10	.11	.13	.14	.21	.29	.35	22
24	.02	.04	.06	.07	.09	.11	.12	.14	.15	.23	.32	.38	24
26	.03	.04	.06	.08	.10	.12	.13	.15	.17	.25	.35	.42	26
28	.03	.05	.07	.09	.11	.12	.14	.16	.18	.27	.37	.45	28
30	.03	.05	.07	.09	.11	.13	.15	.17	.19	.29	.40	.49	30
32	.03	.06	.08	.10	.12	.14	.16	.19	.21	.31	.43	.53	32
34	.03	.06	.08	.11	.13	.15	.18	.20	.22	.33	.46	.56	34
36	.04	.06	.09	.11	.14	.16	.19	.21	.23	.35	.48	.59	36
38	.04	.07	.09	.12	.15	.17	.20	.22	.25	.38	.51	.63	38
40	.04	.07	.10	.13	.15	.18	.21	.24	.26	.40	.53	.67	40
42	.04	.07	.10	.13	.16	.19	.22	.25	.28	.42	.56	.70	42
44	.04	.08	.11	.14	.17	.20	.23	.26	.29	.44	.59	.74	44
46	.05	.08	.12	.15	.18	.21	.24	.27	.31	.46	.62	.77	46
48	.05	.09	.12	.16	.19	.22	.26	.29	.32	.49	.65	.81	48
50	.05	.09	.13	.16	.20	.23	.27	.30	.34	.51	.68	.85	50
52	.05	.09	.13	.17	.21	.24	.28	.32	.35	.53	.71	.89	52
54	.06	.10	.14	.18	.22	.25	.29	.33	.37	.56	.74	.93	54
56	.06	.10	.15	.19	.23	.27	.30	.34	.38	.58	.77	.97	56
58	.06	.11	.15	.19	.24	.28	.32	.36	.40	.61	.80	1.01	58
60	.06	.11	.16	.20	.24	.29	.33	.37	.42	.63	.84	1.05	60
62	.07	.12	.16	.21	.25	.30	.34	.39	.43	.65	.88	1.10	62
64	.07	.12	.17	.22	.27	.31	.36	.40	.45	.68	.91	1.14	64
66	.07	.13	.18	.23	.28	.32	.37	.42	.47	.71	.95	1.19	66
68	.07	.13	.18	.24	.29	.34	.39	.44	.49	.74	.98	1.23	68
70	.08	.14	.19	.24	.30	.35	.40	.45	.50	.76	1.02	1.28	70
72	.08	.14	.20	.25	.31	.36	.42	.47	.52	.79	1.06	1.33	72
74	.08	.15	.21	.26	.32	.38	.43	.49	.54	.82	1.10	1.38	74
76	.09	.15	.21	.27	.33	.39	.45	.51	.56	.85	1.14	1.43	76
78	.09	.16	.22	.28	.34	.40	.46	.52	.58	.88	1.18	1.48	78
80	.09	.16	.23	.29	.36	.42	.48	.54	.60	.91	1.22	1.53	80
82	.09	.17	.24	.30	.37	.43	.50	.56	.63	.95	1.26	1.59	82
84	.10	.17	.25	.31	.38	.45	.52	.58	.65	.98	1.31	1.65	84
86	.10	.18	.25	.33	.40	.47	.53	.60	.67	1.02	1.36	1.71	86
88	.11	.19	.26	.34	.41	.48	.55	.62	.70	1.05	1.40	1.77	88
90	.11	.19	.27	.35	.42	.50	.57	.65	.72	1.09	1.45	1.83	90
92	.11	.20	.28	.36	.44	.52	.59	.67	.75	1.13	1.49	1.90	92
94	.12	.21	.29	.37	.46	.53	.61	.69	.77	1.17	1.53	1.96	94
96	.12	.22	.30	.39	.47	.55	.64	.72	.80	1.21	1.61	2.03	96
98	.13	.22	.31	.40	.49	.57	.66	.74	.83	1.26	1.68	2.11	98
100	13	.23	.33	.42	.51	.59	.68	.77	.86	1.30	1.74	2.18	100

TABLE 4. CORRECTIONS FOR EXTERNAL DISTANCES IN TABLE 2
Applying only to the "chord" definition
Add after dividing (E for 1° curve) by D

Δ	D =												Δ
	2°	3°	4°	5°	6°	7°	8°	9°	10°	15°	20°	25°	
10°	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.01	.01	10°
12	.00	.00	.00	.00	.00	.00	.00	.00	.00	.01	.01	.01	12
14	.00	.00	.00	.00	.00	.00	.00	.00	.01	.01	.01	.01	14
16	.00	.00	.00	.00	.00	.00	.00	.01	.01	.01	.01	.02	16
18	.00	.00	.00	.00	.00	.00	.01	.01	.01	.01	.02	.02	18
20	.00	.00	.00	.01	.00	.01	.01	.01	.01	.02	.02	.03	20
22	.00	.00	.01	.01	.01	.01	.01	.01	.01	.02	.02	.03	22
24	.00	.00	.01	.01	.01	.01	.01	.01	.02	.02	.02	.03	24
26	.00	.01	.01	.01	.01	.01	.02	.02	.02	.03	.04	.05	26
28	.00	.01	.01	.01	.01	.02	.02	.02	.02	.03	.04	.06	28
30	.00	.01	.01	.01	.02	.02	.02	.02	.03	.04	.05	.06	30
32	.01	.01	.01	.01	.02	.02	.02	.03	.03	.04	.06	.07	32
34	.01	.01	.01	.02	.02	.02	.03	.03	.03	.05	.07	.08	34
36	.01	.01	.02	.02	.02	.03	.03	.03	.04	.06	.08	.09	36
38	.01	.01	.02	.02	.03	.03	.03	.04	.04	.06	.08	.11	38
40	.01	.01	.02	.02	.03	.03	.04	.04	.05	.07	.09	.12	40
42	.01	.02	.02	.03	.03	.04	.04	.05	.05	.08	.10	.13	42
44	.01	.02	.02	.03	.03	.04	.05	.05	.06	.09	.11	.14	44
46	.01	.02	.03	.03	.04	.04	.05	.06	.06	.09	.13	.16	46
48	.01	.02	.03	.03	.04	.05	.06	.06	.07	.10	.14	.17	48
50	.01	.02	.03	.04	.05	.05	.06	.07	.08	.11	.15	.19	50
52	.02	.02	.03	.04	.05	.06	.07	.07	.08	.12	.16	.21	52
54	.02	.03	.04	.04	.05	.06	.07	.08	.09	.13	.18	.22	54
56	.02	.03	.04	.05	.06	.07	.08	.09	.10	.14	.19	.24	56
58	.02	.03	.04	.05	.06	.07	.08	.09	.10	.16	.21	.26	58
60	.02	.03	.05	.06	.07	.08	.09	.10	.11	.17	.23	.28	60
62	.02	.04	.05	.06	.07	.08	.10	.11	.12	.18	.24	.30	62
64	.02	.04	.05	.06	.08	.09	.10	.12	.13	.20	.26	.33	64
66	.03	.04	.06	.07	.08	.10	.11	.13	.14	.21	.28	.35	66
68	.03	.05	.06	.07	.09	.11	.12	.14	.15	.23	.30	.38	68
70	.03	.05	.07	.08	.10	.11	.13	.14	.16	.24	.32	.40	70
72	.03	.05	.07	.08	.10	.12	.14	.16	.17	.26	.34	.43	72
74	.04	.06	.08	.09	.11	.13	.15	.17	.18	.28	.37	.46	74
76	.04	.06	.08	.10	.12	.14	.16	.18	.20	.29	.39	.49	76
78	.04	.06	.09	.10	.13	.15	.17	.19	.21	.31	.42	.52	78
80	.04	.07	.09	.11	.13	.16	.18	.20	.22	.33	.45	.56	80
82	.05	.07	.10	.12	.14	.17	.19	.21	.24	.36	.47	.59	82
84	.05	.08	.10	.12	.15	.18	.20	.23	.25	.38	.50	.63	84
86	.05	.08	.11	.13	.16	.19	.21	.24	.27	.40	.54	.67	86
88	.05	.09	.12	.14	.17	.20	.23	.26	.28	.43	.57	.71	88
90	.06	.09	.12	.15	.18	.21	.24	.27	.30	.45	.60	.76	90
92	.06	.10	.13	.16	.19	.22	.26	.29	.32	.48	.64	.80	92
94	.07	.10	.14	.17	.20	.24	.27	.31	.34	.51	.68	.85	94
96	.07	.11	.15	.18	.22	.25	.29	.32	.36	.54	.72	.90	96
98	.07	.12	.16	.19	.23	.27	.31	.34	.38	.57	.77	.96	98
100	.08	.12	.17	.20	.24	.28	.32	.36	.40	.61	.81	1.0	100

TABLE 5. RADIUS WHEN DEGREE OF CURVE IS GIVEN

Deg. D	Radius R (ft.)		Deg. D	Radius R (ft.)		Deg. D	Radius R (ft.)	
	Arc definition	Chord definition		Arc definition	Chord definition		Arc definition	Chord definition
0° 0′	∞	∞	1° 0′	**5,729.58**	5,729.65	2° 0′	**2,864.79**	2,864.93
1	**343,774.7**	343,774.7	1	**5,635.65**	5,635.72	1	**2,841.11**	2,841.26
2	**171,887.3**	171,887.3	2	**5,544.75**	5,544.83	2	**2,817.83**	2,817.97
3	**114,591.6**	114,591.6	3	**5,456.74**	5,456.82	3	**2,794.92**	2,795.06
4	**85,943.7**	85,943.7	4	**5,371.48**	5,371.56	4	**2,772.38**	2,772.53
5	**68,754.9**	68,754.9	5	**5,288.84**	5,288.92	5	**2,750.20**	2,750.35
6	**57,295.8**	57,295.8	6	**5,208.71**	5,208.79	6	**2,728.37**	2,728.52
7	**49,110.7**	49,110.7	7	**5,130.97**	5,131.05	7	**2,706.89**	2,707.04
8	**42,971.8**	42,971.8	8	**5,055.51**	5,055.59	8	**2,685.74**	2,685.89
9	**38,197.2**	38,197.2	9	**4,982.24**	4,982.33	9	**2,664.92**	2,655.08
10	**34,377.5**	34,377.5	10	**4,911.07**	4,911.15	10	**2,644.42**	2,644.58
11	**31,252.2**	31,252.3	11	**4,841.90**	4,841.98	11	**2,624.23**	2,624.39
12	**28,647.8**	28,647.9	12	**4,774.65**	4,774.74	12	**2,604.35**	2,604.51
13	**26,444.2**	26,444.2	13	**4,709.24**	4,709.33	13	**2,584.77**	2,584.93
14	**24,555.3**	24,555.4	14	**4,645.60**	4,645.69	14	**2,565.48**	2,565.65
15	**22,918.3**	22,918.3	15	**4,583.66**	4,583.75	15	**2,546.48**	2,546.64
16	**21,485.9**	21,485.9	16	**4,523.35**	4,523.44	16	**2,527.76**	2,527.92
17	**20,222.0**	20.222.1	17	**4,464.61**	4,464.70	17	**2,509.30**	2,509.47
18	**19,098.6**	19,098.6	18	**4,407.37**	4,407.46	18	**2,491.12**	2,491.29
19	**18,093.4**	18,093.4	19	**4,351.58**	4,351.67	19	**2,473.20**	2,473.37
20	**17,188.8**	17,188.8	20	**4,297.18**	4,297.28	20	**2,455.53**	2,455.70
21	**16,370.2**	16,370.3	21	**4,244,13**	4,244.23	21	**2,438.12**	2,438.29
22	**15,626.1**	15,626.2	22	**4,192.37**	4,192.47	22	**2,420.95**	2,421.12
23	**14,946.7**	14,946.8	23	**4,141.86**	4,141.96	23	**2,404.02**	2,404.19
24	**14,324.0**	14,324.0	24	**4,092.56**	4,092.66	24	**2,387.32**	2,387.50
25	**13,751.0**	13,751.0	25	**4,044.41**	4,044.51	25	**2,370.86**	2,371.04
26	**13,222.1**	13,222.1	26	**3,997.38**	3,997.48	26	**2,354.62**	2,354.80
27	**12,732.4**	12,732.4	27	**3,951.43**	3,951.54	27	**2,338.60**	2,338.78
28	**12,277.7**	12,277.7	28	**3,906.53**	3,906.64	28	**2,322.80**	2,322.98
29	**11,854.3**	11,854.3	29	**3,862.64**	3,862.74	29	**2,307.21**	2,307.39
30	**11,459.2**	11,459.2	30	**3,819.71**	3,819.83	30	**2,291.83**	2,292.01
31	**11,089.5**	11,089.5	31	**3,777.74**	3,777.85	31	**2,276.65**	2,276.84
32	**10,743.0**	10,743.0	32	**3,736.68**	3,736.79	32	**2,261.68**	2,261.86
33	**10,417.4**	10,417.5	33	**3,696.50**	3,696.61	33	**2,246.89**	2,247.08
34	**10,111.0**	10,111.1	34	**3,657.18**	3,657.29	34	**2,232.30**	2,232.49
35	**9,822.13**	9,822.18	35	**3,618.68**	3,618.80	35	**2,217.90**	2,218.09
36	**9,549.30**	9,549.34	36	**3,580.99**	3,581.10	36	**2,203.68**	2,203.87
37	**9,291.21**	9,291.25	37	**3,544.07**	3,544.19	37	**2,189.65**	2,189.84
38	**9,046.70**	9,046.75	38	**3,507.91**	3,508.02	38	**2,175.79**	2,175.98
39	**8,814.74**	8,814.78	39	**3,472.47**	3,472.59	39	**2,162.10**	2,162.30
40	**8,594.37**	8,594.42	40	**3,437.75**	3,437.87	40	**2,148.59**	2,148.79
41	**8,384.75**	8,384.80	41	**3,403.71**	3,403.83	41	**2,135.25**	2,135.44
42	**8,185.11**	8,185.16	42	**3,370.34**	3,370.46	42	**2,122.07**	2,122.26
43	**7,994.76**	7,994.81	43	**3,337.62**	3,337.74	43	**2,109.05**	2,109.24
44	**7,813.06**	7,813.11	44	**3,305.53**	3,305.65	44	**2,096.19**	2,096.39
45	**7,639.44**	7,639.49	45	**3,274,05**	3,274.17	45	**2,083.48**	2,083.68
46	**7,473.36**	7,473.42	46	**3,243.16**	3,243.29	46	**2,070.93**	2,071.13
47	**7,314.35**	7,314.41	47	**3,212.85**	3,212.98	47	**2,058.53**	2,058.73
48	**7,161.97**	7,162.03	48	**3,183.10**	3,183.23	48	**2,046.28**	2,046.48
49	**7,015.81**	7,015.87	49	**3,153.90**	3,154.03	49	**2,034.17**	2,034.37
50	**6,875.50**	6,875.55	50	**3,125.22**	3,125.36	50	**2,022.20**	2,022.41
51	**6,740.68**	6,740.74	51	**3,097.07**	3,097.20	51	**2,010.38**	2,010.59
52	**6,611.05**	6,611.12	52	**3,069.42**	3,069.55	52	**1,998.69**	1,998.90
53	**6,486.31**	6,486.38	53	**3,042.25**	3,042.39	53	**1,987.14**	1,987.35
54	**6,366.20**	6,366.26	54	**3,015.57**	3,015.71	54	**1,975.71**	1,975.93
55	**6,250.45**	6,250.51	55	**2,989.35**	2,989.48	55	**1,964.43**	1,964.64
56	**6,138.83**	6,138.90	56	**2,963.58**	2,963.72	56	**1,953.27**	1,953.48
57	**6,031.13**	6,031.20	57	**2,938.25**	2,938.39	57	**1,942.23**	1,942.44
58	**5,927.15**	5,927.22	58	**2,913.34**	2,913.49	58	**1,931.32**	1,931.53
59	**5,826.69**	5,826.76	59	**2,888.86**	2,889.01	59	**1,920.53**	1,920.75
60	**5,729.58**	5,729.65	60	**2,864.79**	2,864.93	60	**1,909.86**	1,910.08

TABLE 5. RADIUS WHEN DEGREE OF CURVE IS GIVEN (*Continued*)

Deg. *D*	Radius *R* (ft.)		Deg. *D*	Radius *R* (ft.)		Deg. *D*	Radius *R* (ft.)	
	Arc definition	Chord definition		Arc definition	Chord definition		Arc definition	Chord definition
3° 0′	1,909.86	1,910.08	4° 0′	1,432.39	1,432.69	5° 0′	1,145.92	1,146.28
1	1,899.31	1,899.53	1	1,426.45	1,426.74	1	1,142.11	1,142.47
2	1,888.86	1,889.09	2	1,420.56	1,420.85	2	1,138.33	1,138.69
3	1,878.55	1,878.77	3	1,414.71	1,415.01	3	1,134.57	1,134.94
4	1,868.33	1,868.56	4	1,408.91	1,409.21	4	1,130.84	1,131.21
5	1,858.24	1,858.47	5	1,403.16	1,403.46	5	1,127.13	1,127.50
6	1,848.25	1,848.48	6	1,397.46	1,397.76	6	1,123.45	1,123.82
7	1,838.37	1,838.59	7	1,391.80	1,392.10	7	1,119.79	1,120.16
8	1,828.59	1,828.82	8	1,386.19	1,386.49	8	1,116.15	1,116.52
9	1,818.91	1,819.14	9	1,380.62	1,380.92	9	1,112.54	1,112.91
10	1,809.34	1,809.57	10	1,375.10	1,375.40	10	1,108.95	1,109.33
11	1,799.87	1,800.10	11	1,369.62	1,369.92	11	1,105.39	1,105.76
12	1,790.49	1,790.73	12	1,364.19	1,364.49	12	1,101.84	1,102.22
13	1,781.22	1,781.45	13	1,358.79	1,359.10	13	1,098.32	1,098.70
14	1,772.03	1,772.27	14	1,353.44	1,353.75	14	1,094.82	1,095.20
15	1,762.95	1,763.18	15	1,348.14	1,348.45	15	1,091.35	1,091.73
16	1,753.95	1,754.19	16	1,342.87	1,343.18	16	1,087.89	1,088.28
17	1,745.05	1,745.29	17	1,337.65	1,337.96	17	1,084.46	1,084.85
18	1,736.24	1,736.48	18	1,332.46	1,332.77	18	1,081.05	1,081.44
19	1,727.51	1,727.75	19	1,327.32	1,327.63	19	1,077.66	1,078.05
20	1,718.87	1,719.12	20	1,322.21	1,322.53	20	1,074.30	1,074.68
21	1,710.32	1,710.57	21	1,317.14	1,317.46	21	1,070.95	1,071.34
22	1,701.86	1,702.10	22	1,312.12	1,312.43	22	1,067.62	1,068.01
23	1,693.47	1,693.72	23	1,307.13	1,307.45	23	1,064.32	1,064.71
24	1,685.17	1,685.42	24	1,302.18	1,302.50	24	1,061.03	1,061.43
25	1,676.95	1,677.20	25	1,297.26	1,297.58	25	1,057.77	1,058.16
26	1,668.81	1,669.06	26	1,292.39	1,292.71	26	1,054.52	1,054.92
27	1,660.75	1,661.00	27	1,287.55	1,287.87	27	1,051.30	1,051.70
28	1,652.76	1,653.02	28	1,282.74	1,283.07	28	1,048.09	1,048.49
29	1,644.85	1,645.11	29	1,277.97	1,278.30	29	1,044.91	1,045.31
30	1,637.02	1,637.28	30	1,273.24	1,273.57	30	1,041.74	1,042.14
31	1,629.26	1,629.52	31	1,268.54	1,268.87	31	1,038.59	1,039.00
32	1,621.58	1,621.84	32	1,263.88	1,264.21	32	1,035.47	1,035.87
33	1,613.97	1,614.22	33	1,259.25	1,259.58	33	1,032.36	1,032.76
34	1,606.42	1,606.68	34	1,254.65	1,254.98	34	1,029.27	1,029.67
35	1,598.95	1,599.21	35	1,250.09	1,250.42	35	1,026.19	1,026.60
36	1,591.55	1,591.81	36	1,245.56	1,245.89	36	1,023.14	1,023.55
37	1,584.22	1,584.48	37	1,241.06	1,241.40	37	1,020.10	1,020.51
38	1,576.95	1,577.21	38	1,236.60	1,236.94	38	1,017.09	1,017.49
39	1,569.75	1,570.01	39	1,232.17	1,232.51	39	1,014.08	1,014.50
40	1,562.61	1,562.88	40	1,227.77	1,228.11	40	1,011.10	1,011.51
41	1,555.54	1,555.81	41	1,223.40	1,223.74	41	1,008.14	1,008.55
42	1,548.53	1,548.80	42	1,219.06	1,219.40	42	1,005.19	1,005.60
43	1,541.59	1,541.86	43	1,214.75	1,215.09	43	1,002.26	1,002.67
44	1,534.71	1,534.98	44	1,210.47	1,210.82	44	999.35	999.76
45	1,527.89	1,528.16	45	1,206.23	1,206.57	45	996.45	996.87
46	1,521.13	1,521.40	46	1,202.01	1,202.36	46	993.57	993.99
47	1,514.43	1,514.70	47	1,197.82	1,198.17	47	990.71	991.13
48	1,507.78	1,508.06	48	1,193.66	1,194.01	48	987.86	988.28
49	1,501.20	1,501.48	49	1,189.53	1,189.88	49	985.03	985.45
50	1,494.67	1,494.95	50	1,185.43	1,185.78	50	982.21	982.64
51	1,488.20	1,488.48	51	1,181.36	1,181.71	51	979.42	979.84
52	1,481.79	1,482.07	52	1,177.31	1,177.66	52	976.63	977.06
53	1,475.43	1,475.71	53	1,173.29	1,173.65	53	973.87	974.29
54	1,469.12	1,469.41	54	1,169.30	1,169.66	54	971.12	971.54
55	1,462.87	1,463.16	55	1,165.34	1,165.70	55	968.38	968.81
56	1,456.67	1,456.96	56	1,161.40	1,161.76	56	965.66	966.09
57	1,450.53	1,450.81	57	1,157.49	1,157.85	57	962.95	963.39
58	1,444.43	1,444.72	58	1,153.61	1,153.97	58	960.26	960.70
59	1,438.39	1,438.68	59	1,149.75	1,150.11	59	957.59	958.02
60	1,432.39	1,432.69	60	1,145.92	1,146.28	60	954.93	955.37

TABLE 5. RADIUS WHEN DEGREE OF CURVE IS GIVEN (*Continued*)

Deg. D	Radius R (ft.)		Deg. D	Radius R (ft.)		Deg. D	Radius R (ft.)	
	Arc definition	Chord definition		Arc definition	Chord definition		Arc definition	Chord definition
6° 0′	954.93	955.37	7° 0′	818.51	819.02	8° 0′	716.20	716.78
1	952.28	952.72	1	816.57	817.08	1	714.71	715.29
2	949.65	950.09	2	814.63	815.14	2	713.23	713.81
3	947.04	947.48	3	812.70	813.22	3	711.75	712.33
4	944.44	944.88	4	810.79	811.30	4	710.28	710.87
5	941.85	942.29	5	808.88	809.40	5	708.81	709.40
6	939.28	939.72	6	806.98	807.50	6	707.35	707.94
7	936.72	937.16	7	805.09	805.61	7	705.90	706.49
8	934.17	934.62	8	803.21	803.73	8	704.46	705.05
9	931.64	932.09	9	801.34	801.86	9	703.02	703.61
10	929.12	929.57	10	799.48	800.00	10	701.58	702.18
11	926.62	927.07	11	797.62	798.14	11	700.16	700.75
12	924.13	924.58	12	795.78	796.30	12	698.73	699.33
13	921.65	922.10	13	793.94	794.46	13	697.31	697.91
14	919.18	919.64	14	792.11	792.63	14	695.90	696.50
15	916.73	917.19	15	790.29	790.81	15	694.49	695.09
16	914.29	914.75	16	788.47	789.00	16	693.09	693.70
17	911.87	912.33	17	786.67	787.20	17	691.70	692.30
18	909.46	909.92	18	784.87	785.40	18	690.31	690.91
19	907.06	907.52	19	783.09	783.62	19	688.92	689.53
20	904.67	905.13	20	781.31	781.84	20	687.55	688.16
21	902.30	902.76	21	779.53	780.07	21	686.18	686.78
22	899.93	900.40	22	777.77	778.31	22	684.81	685.42
23	897.58	898.05	23	776.02	776.55	23	683.45	684.06
24	895.25	895.71	24	774.27	774.81	24	682.09	682.70
25	892.92	893.39	25	772.53	773.07	25	680.74	681.35
26	890.61	891.08	26	770.80	771.34	26	679.40	680.01
27	888.31	888.78	27	769.07	769.61	27	678.06	678.67
28	886.02	886.49	28	767.35	767.90	28	676.72	677.34
29	883.74	884.21	29	765.65	766.19	29	675.39	676.01
30	881.47	881.95	30	763.94	764.49	30	674.07	674.69
31	879.22	879.69	31	762.25	762.80	31	672.75	673.37
32	876.98	877.45	32	760.56	761.11	32	671.43	672.06
33	874.75	875.22	33	758.88	759.43	33	670.13	670.75
34	872.52	873.00	34	757.21	757.76	34	668.82	669.45
35	870.32	870.79	35	755.55	756.10	35	667.52	668.15
36	868.12	868.60	36	753.89	754.44	36	666.23	666.86
37	865.93	866.41	37	752.24	752.80	37	664.94	665.57
38	863.76	864.24	38	750.60	751.16	38	663.66	664.29
39	861.59	862.07	39	748.96	749.52	39	662.38	663.01
40	859.44	859.92	40	747.34	747.89	40	661.11	661.74
41	857.29	857.78	41	745.72	746.27	41	659.84	660.47
42	855.16	855.65	42	744.10	744.66	42	658.57	659.21
43	853.04	853.53	43	742.49	743.06	43	657.31	657.95
44	850.93	851.42	44	740.89	741.46	44	656.06	656.69
45	848.83	849.32	45	739.30	739.87	45	654.81	655.45
46	846.74	847.23	46	737.71	738.28	46	653.56	654.20
47	844.66	845.15	47	736.13	736.70	47	652.32	652.96
48	842.59	843.08	48	734.56	735.13	48	651.09	651.73
49	840.52	841.02	49	733.00	733.56	49	649.86	650.50
50	838.47	838.97	50	731.44	732.01	50	648.63	649.27
51	836.43	836.93	51	729.88	730.45	51	647.41	648.05
52	834.40	834.90	52	728.34	728.91	52	646.19	646.84
53	832.38	832.88	53	726.80	727.37	53	644.98	645.63
54	830.37	830.88	54	725.26	725.84	54	643.77	644.42
55	828.37	828.88	55	723.74	724.31	55	642.57	643.22
56	826.38	826.89	56	722.23	722.79	56	641.37	642.02
57	824.40	824.91	57	720.70	721.28	57	640.18	640.83
58	822.43	822.93	58	719.19	719.77	58	638.99	639.64
59	820.46	820.97	59	717.69	718.27	59	637.80	638.45
60	818.51	819.02	60	716.20	716.78	60	636.62	637.27

TABLE 5. RADIUS WHEN DEGREE OF CURVE IS GIVEN (*Continued*)

Deg. D	Radius R (ft.)		Deg. D	Radius R (ft.)		Deg. D	Radius R (ft.)	
	Arc definition	Chord definition		Arc definition	Chord definition		Arc definition	Chord definition
9° 0′	636.62	637.27	10° 0′	572.96	573.69	11° 0′	520.87	521.67
1	635.44	636.10	1	572.00	572.73	1	520.01	520.88
2	634.27	634.93	2	571.05	571.78	2	519.30	520.10
3	633.10	633.76	3	570.11	570.84	3	518.52	519.32
4	631.94	632.60	4	569.16	569.90	4	517.73	518.54
5	630.78	631.44	5	568.22	568.96	5	516.95	517.76
6	629.62	630.29	6	567.29	568.02	6	516.18	516.99
7	628.47	629.14	7	566.35	567.09	7	515.40	516.21
8	627.33	627.99	8	565.42	566.16	8	514.63	515.44
9	626.18	626.85	9	564.49	565.23	9	513.86	514.68
10	625.04	625.71	10	563.56	564.31	10	513.10	513.91
11	623.91	624.58	11	562.64	563.38	11	512.33	513.15
12	622.78	623.45	12	561.72	562.47	12	511.57	512.38
13	621.65	622.32	13	560.81	561.55	13	510.81	511.63
14	620.53	621.20	14	559.89	560.64	14	510.05	510.87
15	619.41	620.09	15	558.98	559.73	15	509.30	510.11
16	618.30	618.97	16	558.08	558.82	16	508.54	509.36
17	617.19	617.87	17	557.17	557.92	17	507.79	508.61
18	616.08	616.76	18	556.27	557.02	18	507.04	507.86
19	614.98	615.66	19	555.37	556.12	19	506.30	507.12
20	613.88	614.56	20	554.48	555.23	20	505.55	506.38
21	612.79	613.47	21	553.58	554.34	21	504.81	505.64
22	611.70	612.38	22	552.69	553.45	22	504.07	504.90
23	610.61	611.30	23	551.81	552.56	23	503.33	504.16
24	609.53	610.21	24	550.92	551.68	24	502.59	503.42
25	608.45	609.14	25	550.04	550.80	25	501.86	502.69
26	607.38	608.06	26	549.16	549.92	26	501.13	501.96
27	606.30	606.99	27	548.28	549.05	27	500.40	501.23
28	605.24	605.93	28	547.41	548.17	28	499.67	500.51
29	604.17	604.86	29	546.54	547.30	29	498.95	499.78
30	603.11	603.80	30	545.67	546.44	30	498.22	499.06
31	602.06	602.75	31	544.81	545.57	31	497.50	498.34
32	601.00	601.70	32	543.95	544.71	32	496.78	497.62
33	599.96	600.65	33	543.09	543.86	33	496.07	496.91
34	598.91	599.61	34	542.23	543.00	34	495.35	496.19
35	597.87	598.57	35	541.38	542.15	35	494.64	495.48
36	596.83	597.53	36	540.53	541.30	36	493.93	494.77
37	595.80	596.50	37	539.68	540.45	37	493.22	494.07
38	594.77	595.47	38	538.83	539.61	38	492.51	493.36
39	593.74	594.44	39	537.99	538.76	39	491.81	492.66
40	592.72	593.42	40	537.15	537.92	40	491.11	491.96
41	591.69	592.40	41	536.31	537.09	41	490.41	491.26
42	590.68	591.38	42	535.47	536.25	42	489.71	490.56
43	589.68	590.37	43	534.64	535.42	43	489.01	489.86
44	588.67	589.36	44	533.81	534.59	44	488.32	489.17
45	587.66	588.36	45	532.99	533.77	45	487.62	488.48
46	586.65	587.36	46	532.16	532.94	46	486.93	487.79
47	585.65	586.36	47	531.34	532.12	47	486.24	487.10
48	584.66	585.36	48	530.52	531.30	48	485.56	486.42
49	583.66	584.37	49	529.70	530.49	49	484.87	485.73
50	582.67	583.38	50	528.88	529.67	50	484.19	485.05
51	581.68	582.40	51	528.07	528.86	51	483.51	484.37
52	580.70	581.42	52	527.26	528.05	52	482.83	483.69
53	579.72	580.44	53	526.45	527.25	53	482.15	483.02
54	578.75	579.46	54	525.65	526.44	54	481.48	482.34
55	577.77	578.49	55	524.85	525.64	55	480.80	481.67
56	576.80	577.53	56	524.05	524.84	56	480.13	481.00
57	575.84	576.56	57	523.25	524.05	57	479.46	480.33
58	574.87	575.60	58	522.45	523.25	58	478.79	479.67
59	573.91	574.64	59	521.66	522.46	59	478.13	479.00
60	572.96	573.69	60	520.87	521.67	60	477.46	478.34

Table 5. Radius When Degree of Curve Is Given (*Continued*)

Deg. *D*	Radius *R* (ft.)		Deg. *D*	Radius *R* (ft.)		Deg. *D*	Radius *R* (ft.)	
	Arc definition	Chord definition		Arc definition	Chord definition		Arc definition	Chord definition
12° 0′	477.46	478.34	14° 0′	409.26	410.28	16° 0′	358.10	359.26
2	476.14	477.02	2	408.28	409.31	2	357.35	358.52
4	474.83	475.71	4	407.32	408.34	4	356.61	357.78
6	473.52	474.40	6	406.35	407.38	6	355.87	357.05
8	472.22	473.10	8	405.39	406.42	8	355.14	356.32
10	470.92	471.81	10	404.44	405.47	10	354.41	355.59
12	469.64	470.53	12	403.49	404.53	12	353.68	354.86
14	468.36	469.25	14	402.55	403.58	14	352.95	354.13
16	467.09	467.98	16	401.61	402.65	16	352.23	353.41
18	465.82	466.72	18	400.66	401.71	18	351.51	352.70
20	464.56	465.46	20	399.74	400.78	20	350.79	351.98
22	463.31	464.21	22	398.81	399.86	22	350.08	351.02
24	462.06	462.96	24	397.89	398.94	24	349.36	350.56
26	460.82	461.73	26	396.97	398.02	26	348.66	349.85
28	459.59	460.50	28	396.05	397.11	28	347.95	349.15
30	458.37	459.28	30	395.14	396.20	30	347.25	348.45
32	457.15	458.06	32	394.24	395.30	32	346.55	347.75
34	455.93	456.85	34	393.33	394.40	34	345.85	347.06
36	454.73	455.65	36	392.44	393.50	36	345.16	346.37
38	453.53	454.45	38	391.54	392.61	38	344.46	345.68
40	452.34	453.26	40	390.65	391.72	40	343.77	344.99
42	451.15	452.07	42	389.77	390.84	42	343.09	344.31
44	449.97	450.89	44	388.89	389.96	44	342.41	343.62
46	448.79	449.72	46	388.01	389.08	46	341.72	342.95
48	447.62	448.56	48	387.18	388.21	48	341.05	342.27
50	446.46	447.40	50	386.26	387.34	50	340.37	341.60
52	445.30	446.24	52	385.40	386.48	52	339.70	340.93
54	444.15	445.09	54	384.54	385.62	54	339.03	340.26
56	443.01	443.95	56	383.68	384.77	56	338.36	339.60
58	441.87	442.81	58	382.82	383.91	58	337.70	338.93
13° 0′	440.74	441.68	15° 0′	381.97	383.06	17° 0′	337.03	338.27
2	439.61	440.56	2	381.12	382.22	2	336.38	337.62
4	438.49	439.44	4	380.28	381.38	4	335.72	336.96
6	437.37	438.33	6	379.44	380.54	6	335.06	336.31
8	436.23	437.22	8	378.61	379.71	8	334.41	335.66
10	435.16	436.12	10	377.77	378.88	10	333.76	335.01
12	434.06	435.02	12	376.95	378.05	12	333.12	334.37
14	432.97	433.93	14	376.12	377.23	14	332.47	333.73
16	431.88	432.84	16	375.30	376.41	16	331.83	333.09
18	430.79	431.76	18	374.48	375.60	18	331.19	332.45
20	429.72	430.69	20	373.67	374.79	20	330.55	331.82
22	428.65	429.62	22	372.86	373.98	22	329.92	331.18
24	427.58	428.56	24	372.05	373.17	24	329.29	330.55
26	426.52	427.50	26	371.25	372.37	26	328.66	329.93
28	425.46	426.44	28	370.45	371.57	28	328.03	329.30
30	424.41	425.40	30	369.65	370.78	30	327.40	328.68
32	423.37	424.35	32	368.86	369.99	32	326.78	328.06
34	422.33	423.32	34	368.07	369.20	34	326.16	327.44
36	421.29	422.28	36	367.28	368.42	36	325.54	326.83
38	420.26	421.26	38	366.50	367.64	38	324.93	326.22
40	419.24	420.23	40	365.72	366.86	40	324.32	325.60
42	418.22	419.22	42	364.94	366.09	42	323.70	325.00
44	417.20	418.20	44	364.17	365.31	44	323.09	324.39
46	416.19	417.19	46	363.40	364.55	46	322.49	323.79
48	415.19	416.19	48	362.63	363.78	48	321.89	323.18
50	414.19	415.19	50	361.87	363.02	50	321.28	322.59
52	413.20	414.20	52	361.11	362.26	52	320.69	321.99
54	412.20	413.21	54	360.35	361.51	54	320.09	321.39
56	411.21	412.23	56	359.59	360.76	56	319.49	320.81
58	410.23	411.25	58	358.85	360.01	58	318.90	320.21
60	409.26	410.28	60	358.10	359.26	60	318.31	319.62

TABLE 5. RADIUS WHEN DEGREE OF CURVE IS GIVEN (*Continued*)

Deg. *D*	Radius *R* (ft.)	
	Arc definition	Chord definition
18° 0′	**318.31**	319.62
2	**317.72**	319.03
4	**317.14**	318.45
6	**316.55**	317.87
8	**315.97**	317.29
10	**315.39**	316.71
12	**314.81**	316.14
14	**314.24**	315.56
16	**313.66**	314.99
18	**313.09**	314.42
20	**312.52**	313.86
22	**311.96**	313.30
24	**311.39**	312.73
26	**310.83**	312.17
28	**310.27**	311.61
30	**309.71**	311.06
32	**309.15**	310.51
34	**308.59**	309.95
36	**308.04**	309.39
38	**307.49**	308.85
40	**306.94**	308.30
42	**306.39**	307.76
44	**305.85**	307.22
46	**305.31**	306.68
48	**304.76**	306.14
50	**304.22**	305.60
52	**303.69**	305.06
54	**303.15**	304.53
56	**302.62**	304.00
58	**302.09**	303.47
19° 0′	**301.56**	302.94
2	**301.03**	302.42
4	**300.50**	301.89
6	**299.98**	301.37
8	**299.38**	300.85
10	**298.93**	300.33
12	**298.42**	299.82
14	**297.90**	299.30
16	**297.38**	298.78
18	**296.87**	298.27
20	**296.36**	297.77
22	**295.85**	297.26
24	**295.34**	296.75
26	**294.83**	296.25
28	**294.33**	295.75
30	**293.82**	295.25
32	**293.32**	294.75
34	**292.82**	294.25
36	**292.33**	293.75
38	**291.83**	293.27
40	**291.33**	292.77
42	**290.84**	292.28
44	**290.35**	291.79
46	**289.86**	291.31
48	**289.37**	290.82
50	**288.89**	290.33
52	**288.40**	289.85
54	**287.92**	289.37
56	**287.43**	288.89
58	**286.96**	288.41
60	**286.48**	287.94

Deg. *D*	Radius *R* (ft.)	
	Arc definition	Chord definition
20° 0′	**286.48**	287.94
10	**284.11**	285.58
20	**281.78**	283.27
30	**279.49**	280.99
40	**277.24**	278.75
50	**275.02**	276.54
21° 0′	**272.84**	274.37
10	**270.69**	272.23
20	**268.57**	270.13
30	**266.49**	268.06
40	**264.44**	266.02
50	**262.42**	264.02
22° 0′	**260.44**	262.04
10	**258.48**	260.10
20	**256.55**	258.18
30	**254.65**	256.29
40	**252.78**	254.43
50	**250.93**	252.60
23° 0′	**249.11**	250.79
10	**247.32**	249.01
20	**245.55**	247.26
30	**243.81**	245.53
40	**242.09**	243.82
50	**240.40**	242.14
24° 0′	**238.73**	240.49
10	**237.09**	238.85
20	**235.46**	237.24
30	**233.86**	235.65
40	**232.28**	234.08
50	**230.72**	232.54
25° 0′	**229.18**	231.01
10	**227.66**	229.51
20	**226.17**	228.02
30	**224.69**	226.55
40	**223.23**	225.11
50	**221.79**	223.68
26° 00′	**220.37**	222.27
10	**218.96**	220.88
20	**217.58**	219.51
30	**216.21**	218.15
40	**214.86**	216.81
50	**213.52**	215.49
27° 0′	**212.21**	214.18
10	**210.90**	212.89
20	**209.62**	211.62
30	**208.35**	210.36
40	**207.09**	209.12
50	**205.85**	207.89
28° 0′	**204.63**	206.68
10	**203.42**	205.48
20	**202.22**	204.30
30	**201.04**	203.13
40	**199.87**	201.97
50	**198.71**	200.83
29° 0′	**197.57**	199.70
10	**196.44**	198.58
20	**195.33**	197.48
30	**194.22**	196.38
40	**193.13**	195.31
50	**192.05**	194.24

Deg. *D*	Radius *R* (ft.)	
	Arc definition	Chord definition
30° 0′	**190.98**	193.19
10	**189.93**	192.14
20	**188.89**	191.11
30	**187.86**	190.09
40	**186.83**	189.08
50	**185.82**	188.09
31° 0′	**184.82**	187.10
10	**183.84**	186.12
20	**182.86**	185.16
30	**181.89**	184.20
40	**180.93**	183.26
50	**179.99**	182.32
32° 0′	**179.05**	181.40
10	**178.12**	180.48
20	**177.20**	179.58
30	**176.29**	178.68
40	**175.40**	177.79
50	**174.50**	176.92
33° 0′	**173.62**	176.05
10	**172.75**	175.19
20	**171.89**	174.34
30	**171.03**	173.49
40	**170.18**	172.66
50	**169.35**	171.83
34° 0′	**168.52**	171.02
10	**167.70**	170.21
20	**166.88**	169.40
30	**166.07**	168.61
40	**165.28**	167.82
50	**164.48**	167.05
35° 0′	**163.70**	166.28
10	**162.93**	165.51
20	**162.16**	164.76
30	**161.40**	164.01
40	**160.64**	163.27
50	**159.90**	162.53
36° 0′	**159.15**	161.80
10	**158.42**	161.08
20	**157.69**	160.37
30	**156.97**	159.66
40	**156.26**	158.96
50	**155.55**	158.27
37° 0′	**154.85**	157.58
10	**154.16**	156.90
20	**153.47**	156.22
30	**152.79**	155.55
40	**152.11**	154.89
50	**151.44**	154.23
38° 0′	**150.78**	153.58
10	**150.12**	152.93
20	**149.47**	152.29
30	**148.82**	151.66
40	**148.18**	151.03
50	**147.54**	150.41
39° 0′	**146.91**	149.79
10	**146.29**	149.17
20	**145.67**	148.57
30	**145.05**	147.97
40	**144.44**	147.37
50	**143.84**	146.78

TABLE 5. RADIUS WHEN DEGREE OF CURVE IS GIVEN (*Continued*)

Deg. *D*	Radius *R* (ft.)		Deg. *D*	Radius *R* (ft.)		Deg. *D*	Radius *R* (ft.)	
	Arc definition	Chord definition		Arc definition	Chord definition		Arc definition	Chord definition
40° 0′	**143.24**	146.19	50° 0′	**114.59**	118.31	60° 0′	**95.49**	100.00
10	**142.64**	145.61	10	**114.21**	117.94	10	**95.23**	99.75
20	**142.06**	145.03	20	**113.83**	117.58	20	**94.96**	99.50
30	**141.47**	144.46	30	**113.46**	117.21	30	**94.70**	99.25
40	**140.89**	143.89	40	**113.08**	116.85	40	**94.44**	99.00
50	**140.32**	143.33	50	**112.71**	116.50	50	**94.18**	98.76
41° 0′	**139.74**	142.77	51° 0′	**112.34**	116.14	61° 0′	**93.93**	98.52
10	**139.18**	142.22	10	**111.98**	115.79	10	**93.67**	98.27
20	**138.62**	141.67	20	**111.62**	115.44	20	**93.42**	98.03
30	**138.06**	141.13	30	**111.25**	115.09	30	**93.16**	97.79
40	**137.51**	140.59	40	**110.90**	114.74	40	**92.91**	97.55
50	**136.96**	140.05	50	**110.54**	114.40	50	**92.66**	97.32
42° 0′	**136.42**	139.52	52° 0′	**110.18**	114.06	62° 0′	**92.41**	97.08
10	**135.88**	138.99	10	**109.83**	113.72	10	**92.16**	96.85
20	**135.34**	138.47	20	**109.48**	113.38	20	**91.92**	96.61
30	**134.81**	137.95	30	**109.13**	113.05	30	**91.67**	96.38
40	**134.29**	137.44	40	**108.79**	112.72	40	**91.43**	96.15
50	**133.76**	136.93	50	**108.45**	112.39	50	**91.19**	95.92
43° 0′	**133.25**	136.43	53° 0′	**108.11**	112.06	63° 0′	**90.94**	95.69
10	**132.73**	135.92	10	**107.77**	111.73	10	**90.71**	95.47
20	**132.22**	135.43	20	**107.43**	111.41	20	**90.47**	95.24
30	**131.71**	134.93	30	**107.09**	111.09	30	**90.23**	95.02
40	**131.21**	134.44	40	**106.76**	110.77	40	**89.99**	94.80
50	**130.71**	133.96	50	**106.43**	110.45	50	**89.76**	94.57
44° 0′	**130.22**	133.47	54° 0′	**106.10**	110.13	64° 0′	**89.52**	94.35
10	**129.73**	132.99	10	**105.78**	109.82	10	**89.29**	94.13
20	**129.24**	132.52	20	**105.45**	109.51	20	**89.06**	93.92
30	**128.75**	132.05	30	**105.13**	109.20	30	**88.83**	93.70
40	**128.27**	131.58	40	**104.81**	108.89	40	**88.60**	93.49
50	**127.80**	131.12	50	**104.49**	108.59	50	**88.37**	93.27
45° 0′	**127.32**	130.66	55° 0′	**104.17**	108.28	65° 0′	**88.15**	93.06
10	**126.85**	130.20	10	**103.86**	107.98	10	**87.92**	92.85
20	**126.39**	129.75	20	**103.55**	107.68	20	**87.70**	92.64
30	**125.92**	129.30	30	**103.24**	107.38	30	**87.47**	92.43
40	**125.46**	128.85	40	**102.93**	107.09	40	**87.25**	92.22
50	**125.01**	128.41	50	**102.62**	106.79	50	**87.03**	92.01
46° 0′	**124.56**	127.97	56° 0′	**102.31**	106.50	66° 0′	**86.81**	91.81
10	**124.11**	127.53	10	**102.01**	106.21	10	**86.59**	91.60
20	**123.66**	127.09	20	**101.71**	105.92	20	**86.38**	91.39
30	**123.22**	126.66	30	**101.41**	105.63	30	**86.16**	91.19
40	**122.78**	126.24	40	**101.11**	105.35	40	**85.94**	90.99
50	**122.34**	125.81	50	**100.81**	105.07	50	**85.73**	90.79
47° 0′	**121.91**	125.39	57° 0′	**100.52**	104.79	67° 0′	**85.52**	90.59
10	**121.48**	124.97	10	**100.22**	104.51	10	**85.30**	90.39
20	**121.05**	124.56	20	**99.93**	104.23	20	**85.09**	90.19
30	**120.62**	124.15	30	**99.64**	103.95	30	**84.88**	90.00
40	**120.20**	123.74	40	**99.36**	103.68	40	**84.67**	89.80
50	**119.78**	123.33	50	**99.07**	103.40	50	**84.46**	89.61
48° 0′	**119.37**	122.93	58° 0′	**98.78**	103.13	68° 0′	**84.26**	89.42
10	**118.95**	122.53	10	**98.50**	102.86	10	**84.05**	89.22
20	**118.54**	122.13	20	**98.22**	102.60	20	**83.85**	89.03
30	**118.14**	121.74	30	**97.94**	102.33	30	**83.64**	88.84
40	**117.73**	121.35	40	**97.66**	102.06	40	**83.44**	88.65
50	**117.33**	120.96	50	**97.39**	101.80	50	**83.24**	88.46
49° 0′	**116.93**	120.57	59° 0′	**97.11**	101.54	69° 0′	**83.04**	88.27
10	**116.53**	120.19	10	**96.84**	101.28	10	**82.84**	88.09
20	**116.14**	119.81	20	**96.56**	101.02	20	**82.64**	87.90
30	**115.75**	119.43	30	**96.30**	100.76	30	**82.44**	87.72
40	**115.36**	119.05	40	**96.03**	100.51	40	**82.24**	87.54
50	**114.97**	118.68	50	**95.76**	100.25	50	**82.05**	87.35

TABLE 5. RADIUS WHEN DEGREE OF CURVE IS GIVEN (*Continued*)

Deg. D	Radius R (ft.)		Deg. D	Radius R (ft.)		Deg. D	Radius R (ft.)	
	Arc definition	Chord definition		Arc definition	Chord definition		Arc definition	Chord definition
70° 0′	**81.85**	87.17	79° 0′	**72.53**	78.61	88° 0′	**65.11**	71.98
10	**81.66**	86.99	10	**72.37**	78.48	10	**64.98**	71.87
20	**81.46**	86.81	20	**72.22**	78.33	20	**64.86**	71.76
30	**81.27**	86.63	30	**72.07**	78.19	30	**64.74**	71.65
40	**81.08**	86.46	40	**71.92**	78.06	40	**64.62**	71.55
50	**80.89**	86.28	50	**71.77**	77.92	50	**64.50**	71.44
71° 0′	**80.70**	86.10	80° 0′	**71.62**	77.78	89° 0′	**64.38**	71.33
10	**80.51**	85.93	10	**71.47**	77.65	10	**64.26**	71.23
20	**80.32**	85.75	20	**71.32**	77.52	20	**64.14**	71.13
30	**80.13**	85.58	30	**71.17**	77.38	30	**64.02**	71.02
40	**79.95**	85.41	40	**71.03**	77.25	40	**63.90**	70.92
50	**79.76**	85.24	50	**70.88**	77.12	50	**63.78**	70.81
72° 0′	**79.58**	85.07	81° 0′	**70.74**	76.99	90° 0′	**63.66**	70.71
10	**79.39**	84.89	10	**70.59**	76.86	10	**63.54**	70.61
20	**79.21**	84.73	20	**70.44**	76.73	20	**63.43**	70.51
30	**79.03**	84.56	30	**70.30**	76.60	30	**63.31**	70.40
40	**78.85**	84.39	40	**70.16**	76.47	40	**63.19**	70.30
50	**78.67**	84.25	50	**70.02**	76.34	50	**63.08**	70.20
73° 0′	**78.49**	84.07	82° 0′	**69.87**	76.21	91° 0′	**62.96**	70.10
10	**78.31**	83.89	10	**69.73**	76.09	10	**62.85**	70.00
20	**78.13**	83.73	20	**69.59**	75.96	20	**62.73**	69.90
30	**77.95**	83.57	30	**69.45**	75.83	30	**62.62**	69.80
40	**77.78**	83.40	40	**69.31**	75.71	40	**62.50**	69.70
50	**77.60**	83.24	50	**69.17**	75.58	50	**62.39**	69.61
74° 0′	**77.43**	83.08	83° 0′	**69.03**	75.45	92° 0′	**62.28**	69.51
10	**77.25**	82.92	10	**68.89**	75.33	10	**62.16**	69.41
20	**77.08**	82.76	20	**68.75**	75.21	20	**62.05**	69.31
30	**76.91**	82.60	30	**68.62**	75.09	30	**61.94**	69.22
40	**76.74**	82.45	40	**68.48**	74.97	40	**61.83**	69.12
50	**76.56**	82.29	50	**68.34**	74.84	50	**61.72**	69.03
75° 0′	**76.39**	82.13	84° 0′	**68.21**	74.72	93° 0′	**61.61**	68.93
10	**76.22**	81.98	10	**68.07**	74.60	10	**61.50**	68.83
20	**76.06**	81.82	20	**67.94**	74.48	20	**61.39**	68.74
30	**75.89**	81.67	30	**67.81**	74.36	30	**61.28**	68.65
40	**75.72**	81.52	40	**67.67**	74.25	40	**61.17**	68.55
50	**75.55**	81.37	50	**67.54**	74.11	50	**61.06**	68.45
76° 0′	**75.39**	81.22	85° 0′	**67.41**	74.00	94° 0′	**60.95**	68.36
10	**75.22**	81.06	10	**67.27**	73.89	10	**60.84**	68.27
20	**75.06**	80.91	20	**67.14**	73.78	20	**60.74**	68.18
30	**74.90**	80.77	30	**67.01**	73.66	30	**60.63**	68.09
40	**74.73**	80.61	40	**66.88**	73.54	40	**60.52**	68.00
50	**74.57**	80.47	50	**66.75**	73.43	50	**60.42**	67.91
77° 0′	**74.41**	80.32	86° 0′	**66.62**	73.32	95° 0′	**60.31**	67.81
10	**74.25**	80.17	10	**66.49**	73.20	10	**60.21**	67.72
20	**74.09**	80.03	20	**66.36**	73.09	20	**60.10**	67.64
30	**73.93**	79.88	30	**66.24**	72.97	30	**60.00**	67.56
40	**73.77**	79.74	40	**66.11**	72.86	40	**59.89**	67.46
50	**73.61**	79.59	50	**65.98**	72.75	50	**59.79**	67.37
78° 0′	**73.46**	79.45	87° 0′	**65.86**	72.64	96° 0′	**59.68**	67.28
10	**73.30**	79.31	10	**65.73**	72.53	10	**59.58**	67.19
20	**73.14**	79.17	20	**65.61**	72.42	20	**59.48**	67.11
30	**72.99**	79.03	30	**65.48**	72.31	30	**59.37**	67.02
40	**72.83**	78.89	40	**65.36**	72.20	40	**59.27**	66.93
50	**72.68**	78.75	50	**65.23**	72.09	50	**59.17**	66.85

TABLE 6. DEGREE OF CURVE (D) WHEN RADIUS (R) IS GIVEN

R(ft)	D	R(ft)	D	R(ft)	D	R(ft)	D
100	57.29578°	650	8.81474°	1200	4.77465°	5900	0.97111°
110	52.08707°	660	8.68118°	1250	4.58366°	6000	0.95493°
120	47.74648°	670	8.55161°	1300	4.40737°	6100	0.93928°
130	44.07368°	680	8.42585°	1350	4.24413°	6200	0.92413°
140	40.92556°	690	8.30374°	1400	4.09256°	6300	0.90946°
150	38.19719°	700	8.18511°	1450	3.95143°	6400	0.89525°
160	35.80986°	710	8.06983°	1500	3.81972°	6500	0.88147°
170	33.70340°	720	7.95775°	1550	3.69650°	6600	0.86812°
180	31.83099°	730	7.84874°	1600	3.58099°	6700	0.85516°
190	30.15567°	740	7.74267°	1650	3.47247°	6800	0.84258°
200	28.64789°	750	7.63944°	1700	3.37034°	6900	0.83037°
210	27.28370°	760	7.53892°	1750	3.27404°	7000	0.81851°
220	26.04354°	770	7.44101°	1800	3.18310°	7100	0.80698°
230	24.91121°	780	7.34561°	1850	3.09707°	7200	0.79577°
240	23.87324°	790	7.25263°	1900	3.01557°	7300	0.78487°
250	22.91831°	800	7.16197°	1950	2.93825°	7400	0.77427°
260	22.03684°	810	7.07355°	2000	2.86479°	7500	0.76394°
270	21.22066°	820	6.98729°	2100	2.72837°	7600	0.75389°
280	20.46278°	830	6.90311°	2200	2.60435°	7700	0.74410°
290	19.75717°	840	6.82093°	2300	2.49112°	7800	0.73456°
300	19.09859°	850	6.74068°	2400	2.38732°	7900	0.72526°
310	18.48251°	860	6.66230°	2500	2.29183°	8000	0.71620°
320	17.90493°	870	6.58572°	2600	2.20368°	8100	0.70736°
330	17.36236°	880	6.51088°	2700	2.12207°	8200	0.69873°
340	16.85170°	890	6.43773°	2800	2.04628°	8300	0.69031°
350	16.37022°	900	6.36620°	2900	1.97572°	8400	0.68209°
360	15.91549°	910	6.29624°	3000	1.90986°	8500	0.67407°
370	15.48535°	920	6.22780°	3100	1.84825°	8600	0.66623°
380	15.07784°	930	6.16084°	3200	1.79049°	8700	0.65857°
390	14.69123°	940	6.09530°	3300	1.73624°	8800	0.65109°
400	14.32394°	950	6.03113°	3400	1.68517°	8900	0.64377°
410	13.97458°	960	5.96831°	3500	1.63702°	9000	0.63662°
420	13.64185°	970	5.90678°	3600	1.59155°	9100	0.62962°
430	13.32460°	980	5.84651°	3700	1.54853°	9200	0.62278°
440	13.02177°	990	5.78745°	3800	1.50778°	9300	0.61608°
450	12.73240°	1000	5.72958°	3900	1.46912°	9400	0.60953°
460	12.45560°	1010	5.67285°	4000	1.43239°	9500	0.60311°
470	12.19059°	1020	5.61723°	4100	1.39746°	9600	0.59683°
480	11.93662°	1030	5.56270°	4200	1.36419°	9700	0.59068°
490	11.69302°	1040	5.50921°	4300	1.33246°	9800	0.58465°
500	11.45916°	1050	5.45674°	4400	1.30216°	9900	0.57875°
510	11.23447°	1060	5.40526°	4500	1.27324°	10000	0.57296°
520	11.01842°	1070	5.35475°	4600	1.24556°	11000	0.52087°
530	10.81052°	1080	5.30516°	4700	1.21906°	12000	0.47746°
540	10.61033°	1090	5.25649°	4800	1.19366°	13000	0.44074°
550	10.41741°	1100	5.20871°	4900	1.16930°	14000	0.40926°
560	10.23139°	1110	5.16178°	5000	1.14592°	15000	0.38197°
570	10.05189°	1120	5.11569°	5100	1.12345°	16000	0.35810°
580	9.87858°	1130	5.07042°	5200	1.10184°	17000	0.33703°
590	9.71115°	1140	5.02595°	5300	1.08105°	18000	0.31831°
600	9.54930°	1150	4.98224°	5400	1.06103°	19000	0.30156°
610	9.39275°	1160	4.93929°	5500	1.04174°	20000	0.28648°
620	9.24125°	1170	4.89708°	5600	1.02314°	21000	0.27284°
630	9.09457°	1180	4.85557°	5700	1.00519°	22000	0.26044°
640	8.95247°	1190	4.81477°	5800	0.98786°	23000	0.24911°

TABLE 7. CHORD LENGTHS OF CIRCULAR ARCS
(Arc definition)

D	For arcs of							
	50 ft	60 ft	70 ft	75 ft	80 ft	85 ft	90 ft	100 ft
1°	50	60	70	75	80	85	90	99.999
2	50	60	70	75	80	85	89.99	99.995
3	50	60	69.99	74.99	79.99	84.99	89.99	99.989
4	50	59.99	69.99	74.99	79.99	84.99	89.98	99.980
5	50	59.99	69.99	74.99	79.98	84.98	89.98	99.968
6	49.99	59.99	69.98	74.98	79.97	84.97	89.96	99.954
7	49.99	59.99	69.98	74.97	79.96	84.96	89.95	99.938
8	49.99	59.98	69.97	74.96	79.95	84.95	89.94	99.919
9	49.99	59.98	69.96	74.95	79.94	84.93	89.92	99.897
10	49.98	59.97	69.96	74.95	79.94	84.92	89.91	99.873
	25 ft	30 ft	40 ft	45 ft	50 ft	60 ft	75 ft	100 ft
11	25	30	39.99	44.99	49.98	59.97	74.94	99.846
12	25	30	39.99	44.98	49.98	59.96	74.92	99.817
13	24.99	29.99	39.99	44.98	49.97	59.95	74.91	99.786
14	24.99	29.99	39.98	44.98	49.97	59.94	74.89	99.751
15	24.99	29.99	39.98	44.97	49.96	59.94	74.88	99.715
16	24.99	29.99	39.98	44.97	49.96	59.93	74.86	99.675
17	24.99	29.99	39.98	44.97	49.95	59.92	74.84	99.634
18	24.99	29.99	39.97	44.96	49.95	59.91	74.83	99.589
19	24.99	29.99	39.97	44.96	49.94	59.90	74.81	99.542
20	24.99	29.99	39.97	44.95	49.94	59.89	74.79	99.493
21	24.99	29.98	39.96	44.95	49.93	59.88	74.76	99.441
22	24.99	29.98	39.96	44.94	49.92	59.87	74.74	99.387
23	24.99	29.98	39.96	44.94	49.92	59.85	74.71	99.330
24	24.99	29.98	39.95	44.93	49.91	59.84	74.69	99.271
25	24.99	29.98	39.95	44.93	49.90	59.83	74.67	99.209
26	24.99	29.98	39.94	44.92	49.89	59.81	74.64	99.144
27	24.98	29.97	39.94	44.91	49.88	59.80	74.61	99.077
28	24.98	29.97	39.94	44.91	49.88	59.79	74.58	99.008
29	24.98	29.97	39.93	44.90	49.87	59.77	74.55	98.936
30	24.98	29.97	39.93	44.90	49.86	59.75	74.52	98.862
31	24.98	29.97	39.92	44.89	49.85	59.74	74.49	98.785
32	24.98	29.96	39.92	44.88	49.84	59.72	74.45	98,705
33	24.98	29.96	39.91	44.87	49.83	59.70	74.42	98.623
34	24.98	29.96	39.91	44.87	49.82	59.69	74.38	98.539
35	24.98	29.96	39.90	44.86	49.81	59.67	74.35	98.452
36	24.97	29.96	39.89	44.85	49.79	59.65	74.31	98.363
37	24.97	29.95	39.89	44.84	49.78	59.63	74.27	98.271
38	24.97	29.95	39.88	44.83	49.77	59.61	74.23	98.177
39	24.97	29.95	39.88	44.82	49.76	59.58	74.19	98.081
40	24.97	29.95	39.87	44.82	49.75	59.56	74.15	97.982
41	24.97	29.94	39.86	44.81	49.73	59.54	74.10	97.880
42	24.97	29.94	39.86	44.80	49.72	59.52	74.06	97.776
43	24.96	29.94	39.85	44.79	49.71	59.49	74.01	97.670
44	24.96	29.93	39.84	44.78	49.69	59.47	73.96	97.561
45	24.96	29.93	39.84	44.77	49.68	59.45	73.92	97.450
46	24.96	29.93	39.83	44.75	49.66	59.42	73.87	97.336
47	24.96	29.92	39.82	44.74	49.65	59.39	73.82	97.220
48	24.95	29.92	39.81	44.73	49.64	59.37	73.77	97.101
49	24.95	29.92	39.81	44.72	49.62	59.34	73.72	96.980
50	24.95	29.91	39.80	44.71	49.60	59.32	73.67	96.857

TABLE 7. CHORD LENGTHS OF CIRCULAR ARCS (*Continued*)
(Arc definition)

D	For arcs of							
	10 ft	20 ft	25 ft	30 ft	40 ft	50 ft	60 ft	100 ft
51°	10	19.97	24.95	29.91	39.79	49.59	59.29	96.731
52	10	19.97	24.95	29.91	39.78	49.57	59.26	96.603
53	10	19.97	24.94	29.90	39.77	49.55	59.23	96.473
54	10	19.97	24.94	29.90	39.76	49.54	59.20	96.340
55	10	19.97	24.94	29.90	39.75	49.52	59.17	96.205
56	10	19.97	24.94	29.89	39.75	49.50	59.14	96.067
57	10	19.97	24.94	29.89	39.74	49.49	59.11	95.927
58	10	19.97	24.93	29.88	39.73	49.47	59.08	95.785
59	10	19.96	24.93	29.88	39.72	49.45	59.05	95.640
60	10	19.96	24.93	29.88	39.71	49.43	59.02	95.493
61	10	19.96	24.93	29.87	39.70	49.41	58.98	95.344
62	10	19.96	24.92	29.87	39.69	49.39	58.95	95.192
63	9.99	19.96	24.92	29.86	39.68	49.37	58.92	95.038
64	9.99	19.96	24.92	29.86	39.67	49.35	58.88	94.882
65	9.99	19.96	24.92	29.86	39.66	49.33	58.85	94.723
66	9.99	19.96	24.91	29.85	39.65	49.31	58.81	94.562
67	9.99	19.95	24.91	29.85	39.64	49.29	58.78	94.399
68	9.99	19.95	24.91	29.84	39.63	49.27	58.74	94.234
69	9.99	19.95	24.91	29.84	39.61	49.25	58.70	94.066
70	9.99	19.95	24.90	29.83	39.60	49.23	58.67	93.896
71	9.99	19.95	24.90	29.83	39.59	49.20	58.63	93.723
72	9.99	19.95	24.90	29.82	39.58	49.18	58.60	93.549
73	9.99	19.95	24.89	29.82	39.57	49.16	58.55	93.372
74	9.99	19.94	24.89	29.81	39.56	49.13	58.51	93.193
75	9.99	19.94	24.89	29.81	39.54	49.11	58.47	93.012
76	9.99	19.94	24.89	29.80	39.53	49.09	58.43	92.828
77	9.99	19.94	24.88	29.80	39.52	49.06	58.39	92.643
78	9.99	19.94	24.88	29.79	39.51	49.04	58.35	92.455
79	9.99	19.94	24.88	29.79	39.50	49.01	58.30	92.265
80	9.99	19.94	24.87	29.78	39.48	48.99	58.26	92.073
81	9.99	19.93	24.87	29.78	39.47	48.97	58.22	91.878
82	9.99	19.93	24.87	29.77	39.46	48.94	58.17	91.681
83	9.99	19.93	24.86	29.76	39.44	48.91	58.13	91.483
84	9.99	19.93	24.86	29.76	39.43	48.89	58.08	91.282
85	9.99	19.93	24.86	29.75	39.42	49.86	58.04	91.079
86	9.99	19.93	24.85	29.75	39.40	48.83	57.99	90.874
87	9.99	19.92	24.85	29.74	39.39	48.81	57.94	90.666
88	9.99	19.92	24.85	29.74	39.37	48.78	57.90	90.457
89	9.99	19.92	24.84	29.73	39.36	48.75	57.85	90.245
90	9.99	19.92	24.84	29.72	39.35	48.72	57.80	90.032
91	9.99	19.92	24.84	29.72	39.33	48.70	57.75	98.816
92	9.99	19.91	24.83	29.71	39.32	48.67	57.71	89.598
93	9.99	19.91	24.83	29.70	39.30	48.64	57.66	89.378
94	9.99	19.91	24.82	29.70	39.29	48.61	57.60	89.156
95	9.99	19.91	24.82	29.69	39.27	48.58	57.55	88.932
96	9.99	19.91	24.82	29.68	39.25	48.55	57.50	88.706
97	9.99	10.90	24.81	29.68	39.24	48.52	57.45	88.478
98	9.99	19.90	23.81	29.67	39.22	48.49	57.40	88.248
99	9.99	19.90	24.81	29.67	39.21	48.46	57.35	88.016
100	9.99	19.90	24.80	29.66	39.19	48.43	57.30	87.782

TABLE 8. CHORD LENGTHS FOR SHORT RADIUS CURVES
(Arc definition)

For arcs of 10 ft						For arcs of 5 ft					
D	*R* (ft)	Chords	*D*	*R* (ft)	Chords	*D*	*R* (ft)	Chords	*D*	*R* (ft)	Chords
100°	57.30	9.99	152°	37.69	9.97	253°	22.65	4.99	404°	14.18	4.97
101	56.73	9.99	154	37.21	9.97	256	22.38	4.99	408	14.04	4.97
102	56.17	9.99	156	36.73	9.97	259	22.12	4.99	412	13.91	4.97
103	55.63	9.99	158	36.26	9.97	262	21.87	4.99	416	13.77	4.97
104	55.09	9.99	160	35.81	9.97	265	21.62	4.99	420	13.64	4.97
105	54.57	9.99	162	35.37	9.97	268	21.38	4.99	424	13.51	4.97
106	54.05	9.99	164	34.94	9.97	271	21.14	4.99	428	13.39	4.97
107	53.55	9.99	166	34.52	9.97	274	20.91	4.99	432	13.26	4.97
108	53.05	9.99	168	34.10	9.96	277	20.68	4.99	436	13.14	4.97
109	52.56	9.98	170	33.70	9.96	280	20.46	4.99	440	13.02	4.97
110	52.09	9.98	172	33.31	9.96	283	20.25	4.99	444	12.90	4.97
111	51.62	9.98	174	32.93	9.96	286	20.03	4.99	448	12.79	4.97
112	51.16	9.98	176	32.55	9.96	289	19.83	4.99	452	12.68	4.97
113	50.70	9.98	178	32.19	9.96	292	19.62	4.99	456	12.56	4.97
114	50.26	9.98	180	31.83	9.96	295	19.42	4.99	460	12.46	4.97
115	49.82	9.98	182	31.48	9.96	298	19.23	4.99	464	12.35	4.97
116	49.39	9.98	184	31.14	9.96	301	19.04	4.99	468	12.24	4.97
117	48.97	9.98	186	30.80	9.96	304	18.85	4.99	472	12.14	4.96
118	48.56	9.98	188	30.48	9.96	307	18.66	4.99	476	12.04	4.96
119	48.15	9.98	190	30.16	9.96	310	18.48	4.98	480	11.94	4.96
120	47.75	9.98	192	29.84	9.95	313	18.31	4.98	484	11.84	4.96
121	47.35	9.98	194	29.53	9.95	316	18.13	4.98	488	11.74	4.96
122	46.96	9.98	196	29.23	9.95	319	17.96	4.98	492	11.65	4.96
123	46.58	9.98	198	28.94	9.95	322	17.79	4.98	496	11.55	4.96
124	46.21	9.98	200	28.65	9.95	325	17.63	4.98	500	11.46	4.96
125	45.84	9.98	202	28.36	9.95	328	17.47	4.98	504	11.37	4.96
126	45.47	9.98	204	28.09	9.95	331	17.31	4.98	508	11.28	4.96
127	45.11	9.98	206	27.81	9.95	334	17.15	4.98	512	11.19	4.96
128	44.76	9.98	208	27.55	9.95	337	17.00	4.98	516	11.10	4.96
129	44.42	9.98	210	27.28	9.94	340	16.85	4.98	520	11.02	4.96
130	44.07	9.98	212	27.03	9.94	343	16.70	4.98	524	10.93	4.96
131	43.74	9.98	214	26.77	9.94	346	16.56	4.98	528	10.85	4.96
132	43.41	9.98	216	26.53	9.94	349	16.42	4.98	532	10.77	4.96
133	43.08	9.98	218	26.28	9.94	352	16.28	4.98	536	10.69	4.95
134	42.76	9.98	220	26.04	9.94	355	16.14	4.98	540	10.61	4.95
135	42.44	9.98	222	25.81	9.94	358	16.00	4.98	544	10.53	4.95
136	42.13	9.98	224	25.58	9.94	361	15.87	4.98	548	10.46	4.95
137	41.82	9.98	226	25.35	9.94	364	15.74	4.98	552	10.38	4.95
138	41.52	9.98	228	25.13	9.93	367	15.61	4.98	556	10.30	4.95
139	41.22	9.98	230	24.91	9.93	370	15.49	4.98	560	10.23	4.95
140	40.93	9.98	232	24.70	9.93	373	15.36	4.98	564	10.16	4.95
141	40.64	9.97	234	24.49	9.93	376	15.24	4.98	568	10.09	4.95
142	40.35	9.97	236	24.29	9.93	379	15.12	4.98	572	10.02	4.95
143	40.07	9.97	238	24.07	9.93	382	15.00	4.98	576	9.95	4.95
144	39.79	9.97	240	23.87	9.93	385	14.88	4.98	580	9.88	4.95
145	39.51	9.97	242	23.68	9.93	388	14.77	4.98	584	9.81	4.95
146	39.24	9.97	244	23.48	9.92	391	14.65	4.98	588	9.74	4.95
147	38.98	9.97	246	23.29	9.92	394	14.54	4.98	592	9.68	4.94
148	38.71	9.97	248	23.10	9.92	397	14.43	4.97	596	9.61	4.94
149	38.45	9.97	250	22.92	9.92	400	14.32	4.97	600	9.55	4.94
150	38.20	9.97									

TABLE 9. ARC LENGTHS FOR CORRESPONDING CHORDS
(Arc definition)

D	30 ft	40 ft	50 ft	60 ft	70 ft	80 ft	90 ft	100 ft
1°	30	40	50	60	70	80	90	100.001
2	30	40	50	60	70	80	90	100.005
3	30	40	50	60	70	80.01	09.01	100.011
4	30	40	50	60	70.01	80.01	09.01	100.020
5	30	40	50	60.01	70.01	80.02	90.02	100.032
6	30	40	50.01	60.01	70.02	80.02	90.03	100.046
7	30	40	50.01	60.01	70.02	80.03	90.05	100.062
8	30	40.01	50.01	60.02	70.03	80.04	90.06	100.081
9	30	40.01	50.01	60.02	70.04	80.05	90.08	100.103
10	30	40.01	50.02	60.03	70.04	80.07	90.09	100.127
11	30	40.01	50.02	60.03	70.05	80.08	90.11	100.154
12	30	40.01	50.02	60.04	70.06	80.09	90.13	100.184
13	30.01	40.01	50.03	60.05	70.07	80.11	90.16	100.216
14	30.01	40.02	50.03	60.05	70.09	80.13	90.18	100.250
15	30.01	40.02	50.04	60.06	70.10	80.15	90.21	100.288
16	30.01	40.02	50.04	60.07	70.11	80.17	90.24	100.328
17	30.01	40.02	50.05	60.08	70.13	80.19	90.27	100.370
18	30.01	40.03	50.05	60.09	70.14	80.21	90.30	100.416
19	30.01	40.03	50.06	60.10	70.16	80.24	90.34	100.464
20	30.01	40.03	50.06	60.11	70.18	80.26	90.37	100.515
21	30.02	40.04	50.07	60.12	70.19	80.29	90.41	100.568
22	30.02	40.04	50.08	60.13	70.21	80.32	90.45	100.625
23	30.02	40.04	50.08	60.15	70.23	80.35	90.50	100.684
24	30.02	40.05	50.09	60.16	70.25	80.38	90.54	100.746
25	30.02	40.05	50.10	60.17	70.27	80.41	90.59	100.811
26	30.02	40.06	50.11	60.19	70.30	80.45	90.64	100.879
27	30.03	40.06	50.12	60.20	70.32	80.48	90.69	100.949
28	30.03	40.06	50.13	60.22	70.35	80.52	90.74	101.023
29	30.03	40.07	50.13	60.23	70.37	80.56	90.80	101.099
30	30.03	40.07	50.14	60.25	70.40	80.60	90.85	101.179
31	30.03	40.08	50.15	60.27	70.43	80.64	90.91	101.262
32	30.04	40.08	50.16	60.28	70.45	80.68	90.98	101.348
33	30.04	40.09	50.17	60.30	70.48	80.73	91.04	101.436
34	30.04	40.09	50.19	60.32	70.51	80.77	91.11	101.529
35	30.04	40.10	50.20	60.34	70.54	80.82	91.17	101.624
36	30.04	40.11	50.21	60.36	70.58	80.87	91.24	101.723
37	30.05	40.11	50.22	60.38	70.61	80.92	91.32	101.825
38	30.05	40.12	50.23	60.40	70.64	80.97	91.39	101.930
39	30.05	40.12	50.24	60.43	70.68	81.02	91.47	102.039
40	30.06	40.13	50.26	60.45	70.72	81.08	91.55	102.151
41	30.06	40.14	50.27	60.47	70.75	81.13	91.63	102.267
42	30.06	40.14	50.28	60.49	70.79	81.19	91.72	102.386
43	30.06	40.15	50.30	60.52	70.83	81.25	91.81	102.509
44	30.07	40.16	50.31	60.54	70.87	81.31	91.90	102.636
45	30.07	40.17	50.33	60.57	70.91	81.38	91.99	102.767
46	30.07	40.17	50.34	60.60	70.96	81.44	92.08	102.901
47	30.08	40.18	50.36	60.62	71.00	81.51	92.18	103.040
48	30.08	40.19	50.37	60.65	71.04	81.58	92.28	103.183
49	30.08	40.20	50.39	60.68	71.09	81.65	92.38	103.329
50	30.09	40.21	50.41	60.71	71.14	81.72	92.49	103.480

TABLE 10. TRUE CHORDS FOR NOMINAL CHORDS

D	(Chord definition)							
	30 ft	40 ft	50 ft	60 ft	70 ft	80 ft	90 ft	100 ft
1°	30	40	50	60	70	80	90	100
2	30	40	50	60	70	80	90	100
3	30	40.01	50.01	60.01	70	80	90	100
4	30.01	40.01	50.01	60.01	70	80.01	90	100
5	30.01	40.01	50.01	60.01	70	80.01	90.01	100
6	30.01	40.02	50.02	60.02	70.01	80.01	90.01	100
7	30.02	40.02	50.02	60.02	70.02	80.02	90.01	100
8	30.02	40.03	50.03	60.03	70.03	80.02	90.01	100
9	30.03	40.04	50.04	60.04	70.04	80.03	90.02	100
10	30.03	40.04	50.05	60.05	70.05	80.04	90.02	100
11	30.04	40.05	50.06	60.06	70.06	80.05	90.03	100
12	30.05	40.06	50.07	60.07	70.07	80.05	90.03	100
13	30.06	40.07	50.08	60.08	70.08	80.06	90.04	100
14	30.07	40.08	50.09	60.10	70.09	80.07	90.04	100
15	30.08	40.10	50.11	60.11	70.10	80.08	90.05	100
16	30.09	40.11	50.12	60.13	70.12	80.09	90.06	100
17	30.10	40.12	50.14	60.14	70.13	80.11	90.06	100
18	30.11	40.14	50.15	60.16	70.15	80.12	90.07	100
19	30.13	40.15	50.17	60.18	70.16	80.13	90.08	100
20	30.14	40.17	50.19	60.20	70.18	80.15	90.09	100
21	30.15	40.19	50.21	60.22	70.20	80.16	90.10	100
22	30.17	40.21	50.23	60.24	70.22	80.18	90.11	100
23	30.18	40.23	50.25	60.26	70.24	80.19	90.12	100
24	30.20	40.25	50.28	60.28	70.26	80.21	90.13	100
25	30.22	40.27	50.30	60.31	70.28	80.23	90.14	100
26	30.24	40.29	50.32	60.33	70.31	80.25	90.15	100
27	30.25	40.31	50.35	60.36	70.33	80.27	90.16	100
28	30.27	40.34	50.38	60.38	70.36	80.29	90.17	100
29	30.29	40.36	50.40	60.41	70.38	80.31	90.18	100
30	30.31	40.39	50.43	60.44	70.41	80.33	90.20	100
31	30.34	40.41	50.46	60.47	70.44	80.35	90.21	100
32	30.36	40.44	50.49	60.50	70.47	80.38	90.22	100
33	30.38	40.47	50.52	60.54	70.50	80.40	90.24	100
34	30.41	40.50	50.56	60.57	70.53	80.43	90.25	100
35	30.43	40.53	50.59	60.60	70.56	80.45	90.27	100
36	30.46	40.56	50.62	60.64	70.59	80.48	90.28	100
37	30.48	40.59	50.66	60.68	70.63	80.51	90.30	100
38	30.51	40.62	50.70	60.71	70.66	80.53	90.32	100
39	30.53	40.66	50.73	60.75	70.70	80.56	90.33	100
40	30.56	40.69	50.77	60.79	70.74	80.59	90.35	100
41	30.59	40.73	50.81	60.83	70.77	80.62	90.37	100
42	30.62	40.76	50.85	60.87	70.81	80.65	90.39	100
43	30.65	40.80	50.89	60.91	70.85	80.68	90.41	100
44	30.68	40.84	50.94	60.96	70.89	80.72	90.43	100
45	30.71	40.88	50.98	61.00	70.93	80.75	90.45	100
46	30.75	40.92	51.02	61.05	70.97	80.78	90.47	100
47	30.78	40.96	51.07	61.10	71.02	80.82	90.49	100
48	30.81	41.00	51.12	61.14	71.06	80.85	90.51	100
49	30.85	41.01	51.17	61.19	71.11	80.89	90.53	100
50	30.89	41.09	51.21	61.24	71.15	80.93	90.55	100

TABLE 11. ACTUAL ARCS FOR TRUE CHORDS

D	(Chord definition)							
	30 ft	40 ft	50 ft	60 ft	70 ft	80 ft	90 ft	100 ft
1°	30	40	50	60	70	80	90	100.001
2	30	40	50	60	70	80	90	100.005
3	30	40	50	60	70	80.01	90.01	100.011
4	30	40	50	60	70.01	80.01	90.01	100.020
5	30	40	50	60.01	70.01	80.02	90.02	100.032
6	30	40	50.01	60.01	70.02	80.02	90.03	100.046
7	30	40	50.01	60.01	70.02	80.03	90.05	100.062
8	30	40	50.01	60.02	70.03	80.04	90.06	100.081
9	30	40.01	50.01	60.02	70.04	80.05	90.07	100.103
10	30	40.01	50.02	60.03	70.04	80.06	90.09	100.127
11	30	40.01	50.02	60.03	70.05	80.08	90.11	100.154
12	30	40.01	50.02	60.04	70.06	80.09	90.13	100.183
13	30.01	40.01	50.03	60.05	70.07	80.11	90.16	100.215
14	30.01	40.02	50.03	60.05	70.09	80.13	90.18	100.249
15	30.01	40.02	50.04	60.06	70.10	80.15	90.21	100.286
16	30.01	40.02	50.04	60.07	70.11	80.17	90.24	100.326
17	30.01	40.02	50.05	60.08	70.13	80.19	90.27	100.368
18	30.01	40.03	50.05	60.09	70.14	80.21	90.30	100.412
19	30.01	40.03	50.06	60.10	70.16	80.23	90.33	100.460
20	30.01	40.03	50.06	60.11	70.17	80.26	90.37	100.510
21	30.01	40.03	50.07	60.12	70.19	80.29	90.41	100.562
22	30.02	40.04	50.08	60.13	70.21	80.31	90.45	100.617
23	30.02	40.04	50.08	60.14	70.23	80.34	90.49	100.675
24	30.02	40.05	50.09	60.16	70.25	80.37	90.53	100.735
25	30.02	40.05	50.10	60.17	70.27	80.41	90.58	100.798
26	30.02	40.05	50.11	60.18	70.29	80.44	90.63	100.863
27	30.02	40.06	50.11	60.20	70.32	80.47	90.68	100.931
28	30.03	40.06	50.12	60.21	70.34	80.51	90.73	101.002
29	30.03	40.07	50.13	60.23	70.36	80.54	90.78	101.075
30	30.03	40.07	50.14	60.24	70.39	80.58	90.83	101.152
31	30.03	40.08	50.15	60.26	70.41	80.62	90.89	101.230
32	30.03	40.08	50.16	60.28	70.44	80.66	90.95	101.312
33	30.04	40.09	50.17	60.29	70.47	80.70	91.01	101.396
34	30.04	40.09	50.18	60.31	70.50	80.75	91.07	101.482
35	30.04	40.10	50.19	60.33	70.53	80.79	91.14	101.572
36	30.04	40.10	50.20	60.35	70.56	80.84	91.20	101.664
37	30.04	40.11	50.21	60.37	70.59	80.89	91.27	101.759
38	30.05	40.11	50.22	60.39	70.62	80.93	91.34	101.857
39	30.05	40.12	50.23	60.41	70.65	80.98	91.41	101.957
40	30.05	40.13	50.25	60.43	70.69	81.03	91.49	102.060
41	30.05	40.13	50.26	60.45	70.72	81.09	91.56	102.166
42	30.06	40.14	50.27	60.47	70.76	81.14	91.64	102.275
43	30.06	40.14	50.28	60.49	70.79	81.19	91.72	102.386
44	30.06	40.15	50.30	60.52	70.83	81.25	91.80	102.500
45	30.06	40.16	50.31	60.54	70.87	81.31	91.88	102.617
46	30.07	40.16	50.32	60.56	70.90	81.36	91.97	102.737
47	30.07	40.17	50.34	60.59	70.94	81.42	92.05	102.860
48	30.07	40.18	50.35	60.61	70.98	81.48	92.14	102.985
49	30.08	40.19	50.36	60.64	71.02	81.55	92.23	103.114
50	30.08	40.19	50.38	60.66	71.06	81.61	92.32	103.245

Table 12. Actual Length of Arc Subtended by a Chord of 100 Ft

D	Values of *D* (chord definition)					
	0′	10′	20′	30′	40′	50′
0°		100.000	100.000	100.000	100.001	100.001
1	100.001	.002	.002	.003	.003	.004
2	100.005	.006	.007	.008	.009	.010
3	100.011	.013	.014	.015	.017	.019
4	100.020	.022	.024	.026	.028	.030
5	100.032	.034	.036	.038	.041	.043
6	100.046	.048	.051	.054	.056	.059
7	100.062	.065	.068	.071	.075	.078
8	100.081	.085	.088	.092	.095	.099
9	100.103	.107	.111	.115	.119	.123
10	100.127	.131	.136	.140	.145	.149
11	100.154	.158	.163	.168	.173	.178
12	100.183	.188	.193	.199	.204	.209
13	100.215	.220	.226	.232	.237	.243
14	100.249	.255	.261	.267	.274	.280
15	100.286	.293	.299	.306	.312	.319
16	100.326	.332	.339	.346	.353	.361
17	100.368	.375	.382	.390	.397	.405
18	100.412	.420	.428	.436	.444	.452
19	100.460	.468	.476	.484	.493	.501
20	100.510	.518	.527	.535	.544	.553
21	100.562	.571	.580	.589	.598	.608
22	100.617	.626	.636	.645	.655	.665
23	100.675	.684	.694	.704	.714	.725
24	100.735	.745	.756	.766	.776	.787
25	100.798	.808	.819	.830	.841	.852
26	100.863	.874	.886	.897	.908	.920
27	100.931	.943	.955	.966	.978	.990
28	101.002	101.014	101.026	101.038	101.051	101.063
29	101.075	.088	.100	.113	.126	.139
30	101.152	.164	.177	.190	.204	.217

TABLE 13. ARC LENGTHS FOR RADIUS = 1

Deg.	Length	Deg.	Length	Min.	Length	Sec.	Length
1	0.0174533	61	1.0646508	1	0.0002909	1	0.0000048
2	0.0349066	62	1.0821041	2	0.0005818	2	0.0000097
3	0.0523599	63	1.0995574	3	0.0008727	3	0.0000145
4	0.0698132	64	1.1170107	4	0.0011636	4	0.0000194
5	0.0872665	65	1.1344640	5	0.0014544	5	0.0000242
6	0.1047198	66	1.1519173	6	0.0017453	6	0.0000291
7	0.1221730	67	1.1693706	7	0.0020362	7	0.0000339
8	0.1396263	68	1.1868239	8	0.0023271	8	0.0000388
9	0.1570796	69	1.2042772	9	0.0026180	9	0.0000436
10	0.1745329	70	1.2217305	10	0.0029089	10	0.0000485
11	0.1919862	71	1.2391838	11	0.0031998	11	0.0000533
12	0.2094395	72	1.2566371	12	0.0034907	12	0.0000582
13	0.2268928	73	1.2740904	13	0.0037815	13	0.0000630
14	0.2443461	74	1.2915436	14	0.0040724	14	0.0000679
15	0.2617994	75	1.3089969	15	0.0043633	15	0.0000727
16	0.2792527	76	1.3264502	16	0.0046542	16	0.0000776
17	0.2967060	77	1.3439035	17	0.0049451	17	0.0000824
18	0.3141593	78	1.3613568	18	0.0052360	18	0.0000873
19	0.3316126	79	1.3788101	19	0.0055269	19	0.0000921
20	0.3490659	80	1.3962634	20	0.0058178	20	0.0000970
21	0.3665191	81	1.4137167	21	0.0061087	21	0.0001018
22	0.3839724	82	1.4311700	22	0.0063995	22	0.0001067
23	0.4014257	83	1.4486233	23	0.0066904	23	0.0001115
24	0.4188790	84	1.4660766	24	0.0069813	24	0.0001164
25	0.4363323	85	1.4835299	25	0.0072722	25	0.0001212
26	0.4537856	86	1.5009832	26	0.0075631	26	0.0001261
27	0.4712389	87	1.5184364	27	0.0078540	27	0.0001309
28	0.4886922	88	1.5358897	28	0.0081449	28	0.0001357
29	0.5061455	89	1.5533430	29	0.0084358	29	0.0001406
30	0.5235988	90	1.5707963	30	0.0087266	30	0.0001454
31	0.5410521	91	1.5882496	31	0.0090175	31	0.0001503
32	0.5585054	92	1.6057029	32	0.0093084	32	0.0001551
33	0.5759587	93	1.6231562	33	0.0095993	33	0.0001600
34	0.5934119	94	1.6406095	34	0.0098902	34	0.0001648
35	0.6108652	95	1.6580628	35	0.0101811	35	0.0001697
36	0.6283185	96	1.6755161	36	0.0104720	36	0.0001745
37	0.6457718	97	1.6929694	37	0.0107629	37	0.0001794
38	0.6632251	98	1.7104227	38	0.0110538	38	0.0001842
39	0.6806784	99	1.7278760	39	0.0113446	39	0.0001891
40	0.6981317	100	1.7453293	40	0.0116355	40	0.0001939
41	0.7155850	101	1.7627825	41	0.0119264	41	0.0001988
42	0.7330383	102	1.7802358	42	0.0122173	42	0.0002036
43	0.7504916	103	1.7976891	43	0.0125082	43	0.0002085
44	0.7679449	104	1.8151424	44	0.0127991	44	0.0002133
45	0.7853982	105	1.8325957	45	0.0130900	45	0.0002182
46	0.8028515	106	1.8500490	46	0.0133809	46	0.0002230
47	0.8203047	107	1.8675023	47	0.0136717	47	0.0002279
48	0.8377580	108	1.8849556	48	0.0139626	48	0.0002327
49	0.8552113	109	1.9024089	49	0.0142535	49	0.0002376
50	0.8726646	110	1.9198622	50	0.0145444	50	0.0002424
51	0.8901179	111	1.9373155	51	0.0148353	51	0.0002473
52	0.9075712	112	1.9547688	52	0.0151262	52	0.0002521
53	0.9250245	113	1.9722221	53	0.0154171	53	0.0002570
54	0.9424778	114	1.9896753	54	0.0157080	54	0.0002618
55	0.9599311	115	2.0071286	55	0.0159989	55	0.0002666
56	0.9773844	116	2.0245819	56	0.0162897	56	0.0002715
57	0.9948377	117	2.0420352	57	0.0165806	57	0.0002763
58	1.0122910	118	2.0594885	58	0.0168715	58	0.0002812
59	1.0297443	119	2.0769418	59	0.0171624	59	0.0002860
60	1.0471976	120	2.0943951	60	0.0174533	60	0.0002909

TABLE 14. DEFLECTIONS AND CHORDS FOR EVEN-RADIUS CURVES

Radius (ft)	Deflections per ft of arc (minutes)	Deflections for arcs of			Chords for arcs of		
		10 ft	25 ft	50 ft	10 ft	25 ft	50 ft
30	57.296	9°32.96'	23°52.39'	47°44.79'	9.95	24.28	44.41
40	42.972	7°09.72'	17°54.30'	35°48.59'	9.97	24.60	46.81
50	34.377	5°43.77'	14°19.44'	28°38.87'	9.98	24.74	47.94
60	28.648	4°46.48'	11°56.20'	23°52.39'	9.99	24.82	48.57
70	24.555	4°05.55'	10°13.88'	20°27.77'	9.99	24.87	48.94
80	21.486	3°34.86'	8°57.15'	17°54.30'	9.99	24.90	49.19
90	19.099	3°10.99'	7°57.46'	15°54.93'	10	24.92	49.36
100	17.189	2°51.89'	7°09.72'	14°19.44'	10	24.93	49.48
110	15.626	2°36.26'	6°30.65'	13°01.31'	10	24.95	49.57
120	14.324	2°23.24'	5°58.10'	11°56.20'	10	24.95	49.64
130	13.222	2°12.22'	5°30.55'	11°01.11'	10	24.96	49.69
140	12.278	2°02.78'	5°06.94'	10°13.88'	10	24.97	49.74
		25 ft	50 ft	100 ft	25 ft	50 ft	100 ft
150	11.459	4°46.48'	9°32.96'	19°05.92'	24.97	49.77	98.16
160	10.743	4°28.57'	8°57.15'	17°54.30'	24.97	49.80	98.38
170	10.111	4°12.78'	8°25.55'	16°51.10'	24.98	49.82	98.56
180	9.549	3°58.73'	7°57.46'	15°54.92'	24.98	49.84	98.72
190	9.047	3°46.17'	7°32.34'	15°04.67'	24.98	49.86	98.85
200	8.594	3°34.86'	7°09.72'	14°19.44'	24.98	49.87	98.96
210	8.185	3°24.63'	6°49.26'	13°38.51'	24.99	49.88	99.06
220	7.813	3°15.33'	6°30.65'	13°01.31'	24.99	49.89	99.14
230	7.473	3°06.83'	6°13.67'	12°27.34'	24.99	49.90	99.21
240	7.162	2°59.05'	5°58.10'	11°56.20'	24.99	49.91	99.28
250	6.876	2°51.89'	5°43.77'	11°27.55'	24.99	49.92	99.33
260	6.611	2°45.28'	5°30.55'	11°01.11'	24.99	49.92	99.39
270	6.366	2°39.15'	5°18.31'	10°36.62'	24.99	49.93	99.43
280	6.139	2°33.47'	5°06.94'	10°13.88'	24.99	49.93	99.47
290	5.927	2°28.18'	4°56.36'	9°52.71'	24.99	49.94	99.50
300	5.730	2°23.24'	4°46.48'	9°32.96'	24.99	49.94	99.54
310	5.545	2°18.62'	4°37.24'	9°14.48'	24.99	49.95	99.57
320	5.371	2°14.29'	4°28.57'	8°57.15'	24.99	49.95	99.59
330	5.209	2°10.22'	4°20.44'	8°40.87'	24.99	49.95	99.62
340	5.056	2°06.39'	4°12.78'	8°25.55'	24.99	49.96	99.64
350	4.911	2°02.78'	4°05.55'	8°11.11'	25	49.96	99.66
360	4.775	1°59.37'	3°58.73'	7°57.46'	25	49.96	99.68
370	4.646	1°56.14'	3°52.28'	7°44.56'	25	49.96	99.70
380	4.523	1°53.08'	3°46.17'	7°32.34'	25	49.96	99.71
390	4.407	1°50.19'	3°40.37'	7°20.74'	25	49.97	99.73
400	4.297	1°47.43'	3°34.86'	7°09.72'	25	49.97	99.74
410	4.193	1°44.81'	3°29.62'	6°59.24'	25	49.97	99.75
420	4.093	1°42.31'	3°24.63'	6°49.26'	25	49.97	99.76
430	3.997	1°39.93'	3°19.87'	6°39.74'	25	49.97	99.77
440	3.907	1°37.66'	3°15.33'	6°30.65'	25	49.97	99.78
450	3.820	1°35.49'	3°10.99'	6°21.97'	25	49.97	99.79
460	3.737	1°33.42'	3°06.83'	6°13.67'	25	49.98	99.80
470	3.657	1°31.43'	3°02.86'	6°05.72'	25	49.98	99.81
480	3.581	1°29.52'	2°59.05'	5°58.10'	25	49.98	99.82
490	3.508	1°27.70'	2°55.40'	5°50.79'	25	49.98	99.83
500	3.438	1°25.94'	2°51.89'	5°43.77'	25	49.98	99.83
510	3.370	1°24.26'	2°48.52'	5°37.03'	25	49.98	99.84
520	3.306	1°22.64'	2°45.28'	5°30.55'	25	49.98	99.85
530	3.243	1°21.08'	2°42.16'	5°24.32'	25	49.98	99.85
540	3.183	1°19.58'	2°39.15'	5°18.31'	25	49.98	99.86
550	3.125	1°18.13'	2°36.26'	5°12.52'	25	49.98	99.86
560	3.069	1°16.74'	2°33.47'	5°06.94'	25	49.98	99.87
570	3.016	1°15.39'	2°30.78'	5°01.56'	25	49.98	99.87
580	2.964	1°14.09'	2°28.18'	4°56.36'	25	49.99	99.88
590	2.913	1°12.84'	2°25.67'	4°51.33'	25	50	99.88

TABLE 14. DEFLECTIONS AND CHORDS FOR EVEN-RADIUS CURVES (*Continued*)

Radius (ft)	Deflections		Chords for arcs of 100 ft	Radius (ft)	Deflections per ft of arc (minutes)
	per ft of arc (minutes)	for arcs of 100 ft			
600	2.8648	4°46.48'	99.88	6400	0.268574
700	2.4555	4°05.55'	99.91	6500	0.264442
800	2.1486	3°34.86'	99.93	6600	0.260435
900	1.9099	3°11.00'	99.95	6700	0.256548
1000	1.7189	2°51.89'	99.96	6800	0.252775
1100	1.5626	2°36.26'	99.97	6900	0.249112
1200	1.4324	2°23.24'	99.97	7000	0.245553
1300	1.3222	2°12.22'	99.97	7100	0.242095
1400	1.2278	2°02.78'	99.98	7200	0.238732
1500	1.1459	1°54.59'	99.98	7300	0.235462
1600	1.0743	1°47.43'	99.98	7400	0.232280
1700	1.0111	1°41.11'	99.99	7500	0.229183
1800	0.9549	1°35.49'	99.99	7600	0.226168
1900	0.90467	1°30.47'	99.99	7700	0.223230
2000	0.85944	1°25.94'	99.99	7800	0.220368
2100	0.81851	1°21.85'	99.99	7900	0.217579
2200	0.78131	1°18.13'	99.99	8000	0.214859
2300	0.74734	1°14.73'	99.99	8100	0.212207
2400	0.71620	1°11.62'	99.99	8200	0.209619
2500	0.68755	1°08.75'	99.99	8300	0.207093
2600	0.66111	1°06.11'	99.99	8400	0.204628
2700	0.63662	1°03.66'	99.99	8500	0.202220
2800	0.61388	1°01.39'	99.99	8600	0.199869
2900	0.59271	0°59.27'	100	8700	0.197572
3000	0.57296	0°57.30'	100	8800	0.195327
3100	0.55448	0°55.45'	100	8900	0.193132
3200	0.53715	0°53.71'	100	9000	0.190986
3300	0.52087	0°52.09'	100	9100	0.188887
3400	0.50555	0°50.56'	100	9200	0.186834
3500	0.49111	0°49.11'	100	9400	0.182859
3600	0.47746	0°47.75'	100	9600	0.179049
3700	0.46456	0°46.46'	100	9800	0.175395
3800	0.45234	0°45.23'	100	10000	0.171887
3900	0.44074	0°44.07'	100	11000	0.156261
4000	0.42972	0°42.97'	100	12000	0.143239
4100	0.41924	0°41.92'	100	13000	0.132221
4200	0.40926	0°40.93'	100	14000	0.122777
4300	0.39974	0°39.97'	100	15000	0.114592
4400	0.39065	0°39.07'	100	16000	0.107430
4500	0.38197	0°38.20'	100	17000	0.101110
4600	0.37367	0°37.37'	100	18000	0.095492
4700	0.36572	0°36.57'	100	19000	0.090467
4800	0.35810	0°35.81'	100	20000	0.085944
4900	0.35079	0°35.08'	100	21000	0.081851
5000	0.34377	0°34.38'	100	22000	0.078131
5100	0.33703	0°33.70'	100	23000	0.074734
5200	0.33055	0°33.06'	100	24000	0.071620
5300	0.32432	0°32.43'	100	25000	0.068755
5400	0.31831	0°31.83'	100	26000	0.066111
5500	0.31252	0°31.25'	100	27000	0.063662
5600	0.30694	0°30.69'	100	28000	0.061388
5700	0.30156	0°30.16'	100	29000	0.059271
5800	0.29636	0°29.64'	100	30000	0.057296
5900	0.29133	0°29.13'	100	31000	0.055448
6000	0.28648	0°28.65'	100	32000	0.053715
6100	0.28178	0°28.18'	100	33000	0.052087
6200	0.27724	0°27.72'	100	34000	0.050555
6300	0.27284	0°27.28'	100	35000	0.049111

TABLE 15. FUNCTIONS OF UNIT SPIRAL LENGTH = 1

θ_s	x	y	k	p	L.T.	S.T	L.C.	θ_s
0°00′	1.000000	.000000	.500000	.000000	.666667	.333333	1.000000	0°00′
Diff, 1′	0.000001	.000097	.000000	.000024	.000000	.000000	0.000000	Diff. 1′
1°00′	0.999969	.005818	.499995	.001454	.666677	.333343	0.999986	1°00′
Diff. 1′	0.000002	.000097	.000000	.000024	.000001	.000001	0.000001	Diff. 1′
2°00′	0.999878	.011634	.499980	.002909	.666709	.333372	0.999946	2°00′
Diff. 1′	0.000003	.000097	.000000	.000024	.000001	.000001	0.000001	Diff. 1′
3°00′	0.999726	.017450	.499954	.004363	.666762	.333420	0.999878	3°00′
Diff. 1′	0.000004	.000097	.000001	.000024	.000001	.000001	0.000002	Diff. 1′
4°00′	0.999513	.023263	.499919	.005817	.666837	.333488	0.999783	4°00′
Diff. 1′	0.000005	.000097	.000001	.000024	.000001	.000001	0.000002	Diff. 1′
5°00′	0.999239	.029073	.499873	.007270	.666933	.333575	0.999662	5°00′
Diff. 1′	0.000006	.000097	.000001	.000024	.000002	.000002	0.000002	Diff. 1′
6°00′	0.998904	.034879	.499817	.008723	.667050	.333682	0.999513	6°00′
Diff. 1′	0.000007	.000097	.000001	.000024	.000002	.000002	0.000003	Diff. 1′
7°00′	0.998508	.040681	.499751	.010176	.667189	.333808	0.999337	7°00′
Diff. 1′	0.000008	.000097	.000001	.000024	.000003	.000002	0.000003	Diff. 1′
8°00′	0.998052	.046477	.499675	.011627	.667349	.333953	0.999134	8°00′
Diff. 1′	0.000009	.000096	.000001	.000024	.000003	.000003	0.000004	Diff. 1′
9°00′	0.997535	.052268	.499589	.013078	.667530	.334119	0.998904	9°00′
Diff. 1′	0.000010	.000096	.000002	.000024	.000003	.000003	0.000004	Diff. 1′
10°00′	0.996958	.058051	.499493	.014529	.667733	.334303	0.998647	10°00′
Diff. 1′	0.000011	.000096	.000002	.000024	.000004	.000003	0.000005	Diff. 1′
11°00′	0.996320	.063827	.499386	.015978	.667958	.334508	0.998363	11°00′
Diff. 1′	0.000012	.000096	.000002	.000024	.000004	.000004	0.000005	Diff. 1′
12°00′	0.995622	.069595	.499270	.017426	.668205	.334732	0.998052	12°00′
Diff. 1′	0.000013	.000096	.000002	.000024	.000004	.000004	0.000006	Diff. 1′
13°00′	0.994864	.075353	.499143	.018873	.668473	.334976	0.997714	13°00′
Diff. 1′	0.000014	.000096	.000002	.000024	.000005	.000004	0.000006	Diff. 1′
14°00′	0.994046	.081102	.499007	.020319	.668764	.335240	0.997349	14°00′
Diff. 1′	0.000015	.000096	.000002	.000024	.000005	.000005	0.000006	Diff. 1′
15°00′	0.993168	.086840	.498860	.021763	.669076	.335525	0.996957	15°00′
Diff. 1′	0.000016	.000095	.000003	.000024	.000006	.000005	0.000007	Diff. 1′
16°00′	0.992230	.092567	.498703	.023206	.669410	.335829	0.996538	16°00′
Diff. 1′	0.000017	.000095	.000003	.000024	.000006	.000005	0.000007	Diff. 1′
17°00′	0.991232	.098282	.498536	.024648	.669767	.336154	0.996093	17°00′
Diff. 1′	0.000018	.000095	.000003	.000024	.000006	.000006	0.000008	Diff. 1′
18°00′	0.990175	.103984	.498360	.026088	.670146	.336499	0.995620	18°00′
Diff. 1′	0.000019	.000095	.000003	.000024	.000007	.000006	0.000008	Diff. 1′
19°00′	0.989059	.109672	.498173	.027526	.670548	.336864	0.995121	19°00′
Diff. 1′	0.000020	.000095	.000003	.000024	.000007	.000006	0.000009	Diff. 1′
20°00′	0.987884	.115346	.497976	.028963	.670972	.337251	0.994595	20°00′
Diff. 1′	0.000021	.000094	.000003	.000024	.000007	.000007	0.000009	Diff. 1′
21°00′	0.986650	.121006	.497769	.030397	.671419	.337658	0.994042	21°00′
Diff. 1′	0.000021	.000094	.000004	.000024	.000008	.000007	0.000010	Diff. 1′
22°00′	0.985357	.126649	.497553	.031830	.671889	.338086	0.993463	22°00′
Diff. 1′	0.000022	.000094	.000004	.000024	.000008	.000007	0.000010	Diff. 1′
23°00′	0.984005	.132276	.497326	.033261	.672382	.338535	0.992856	23°00′
Diff. 1′	0.000023	.000093	.000004	.000024	.000009	.000008	0.000010	Diff. 1′
24°00′	0.982596	.137886	.497090	.034689	.672899	.339006	0.992223	24°00′
Diff. 1′	0.000024	.000093	.000004	.000024	.000009	.000008	0.000011	Diff. 1′
25°00′	0.981128	.143478	.496844	.036115	.673439	.339498	0.991564	25°00′
Diff. 1′	0.000025	.000093	.000004	.000024	.000009	.000009	0.000011	Diff. 1′
26°00′	0.979603	.149051	.496587	.037539	.674002	.340012	0.990878	26°00′
Diff. 1′	0.000026	.000093	.000004	.000024	.111110	.000009	0.000012	Diff. 1′
27°00′	0.978020	.154606	.496322	.038960	.674590	.340548	0.990165	27°00′
Diff. 1′	0.000027	.000092	.000005	.000024	.000010	.000009	0.000012	Diff. 1′
28°00′	0.976381	.160140	.496046	.040379	.675202	.341106	0.989426	28°00′
Diff. 1′	0.000028	.000092	.000005	.000024	.000010	.000010	0.000013	Diff. 1′
29°00′	0.974684	.165653	.495760	.041795	.675838	.341687	0.988660	29°00′
Diff. 1′	0.000029	.000091	.000005	.000024	.000011	.000010	0.000013	Diff. 1′

θ_s = centralangle of spiral

[illegible] ! All columns to be multiplied by spiral curve length to g the actu dat

TABLE 15. FUNCTIONS OF UNIT SPIRAL LENGTH = 1 (*Continued*)

θ_s	x	y	k	p	L.T.	S.T.	L.C.	θ_s
30°00′	.972930	.171145	.495465	.043208	.676499	.342290	.987868	30°00′
Diff. 1′	.000030	.000091	.000005	.000023	.000011	.000010	.000014	Diff. 1′
31°00′	.971120	.176614	.495160	.044619	.677184	.342915	.987050	31°00′
Diff. 1′	.000031	.000091	.000005	.000023	.000012	.000011	.000014	Diff. 1′
32°00′	.969254	.182061	.494846	.046027	.677895	.343564	.986205	32°00′
Diff. 1′	.000032	.000090	.000005	.000023	.000012	.000011	.000014	Diff. 1′
33°00′	.967333	.187485	.494522	.047432	.678631	.344237	.985334	33°00′
Diff. 1′	.000033	.000090	.000006	.000023	.000013	.000012	.000015	Diff. 1′
34°00′	.965356	.192884	.494188	.048834	.679393	.344933	.984437	34°00′
Diff. 1′	.000034	.000090	.000006	.000023	.000013	.000012	.000015	Diff. 1′
35°00′	.963323	.198258	.493845	.050232	.680181	.345653	.983513	35°00′
Diff. 1′	.000035	.000089	.000006	.000023	.000014	.000012	.000016	Diff. 1′
36°00′	.961237	.203607	.493492	.051628	.680995	.346397	.982564	36°00′
Diff. 1′	.000036	.000089	.000006	.000023	.000014	.000013	.000016	Diff. 1′
37°00′	.959095	.208930	.493130	.053020	.681836	.347166	.981588	37°00′
Diff. 1′	.000037	.000088	.000006	.000023	.000014	.000013	.000017	Diff. 1′
38°00′	.956900	.214225	.492758	.054409	.682704	.347960	.980587	38°00′
Diff. 1′	.000037	.000088	.000006	.000023	.000015	.000014	.000017	Diff. 1′
39°00′	.954651	.219494	.492377	.055794	.683599	.348779	.979559	39°00′
Diff. 1′	.000038	.000087	.000006	.000023	.000015	.000014	.000017	Diff. 1′
40°00′	.952349	.224734	.491986	.057175	.684521	.349623	.978506	40°00′
Diff. 1′	.000039	.000087	.000007	.000023	.000016	.000014	.000018	Diff. 1′
41°00′	.949993	.229945	.491586	.058553	.685472	.350494	.977426	41°00′
Diff. 1′	.000040	.000086	.000007	.000023	.000016	.000015	.000018	Diff. 1′
42°00′	.947586	.235126	.491177	.059927	.686451	.351391	.976321	42°00′
Diff. 1′	.000041	.000086	.000007	.000023	.000017	.000015	.000019	Diff. 1′
43°00′	.945126	.240278	.490758	.061298	.687459	.352315	.975191	43°00′
Diff. 1′	.000042	.000085	.000007	.000023	.000017	.000016	.000019	Diff. 1′
44°00′	.942614	.245399	.490330	.062664	.688496	.353265	.974034	44°00′
Diff. 1′	.000043	.000085	.000007	.000023	.000018	.000016	.000002	Diff. 1′
45°00′	.940052	.250488	.489893	.064028	.689563	.354244	.972852	45°00′
Diff. 1′	.000044	.000084	.000007	.000023	.000018	.000017	.000020	Diff. 1′
46°00′	.937438	.255546	.489447	.065385	.690660	.355250	.971645	46°00′
Diff. 1′	.000044	.000084	.000008	.000023	.000019	.000017	.000020	Diff. 1′
47°00′	.934774	.260571	.488992	.066739	.691781	.356285	.970412	47°00′
Diff. 1′	.000045	.000083	.000008	.000022	.000019	.000018	.000021	Diff. 1′
48°00′	.932060	.265562	.488528	.068089	.692947	.357349	.969154	48°00′
Diff. 1′	.000046	.000083	.000008	.000022	.000020	.000018	.000021	Diff. 1′
49°00′	.929296	.270520	.488055	.069435	.694137	.358442	.967870	49°00′
Diff. 1′	.000047	.000082	.000008	.000022	.000020	.000019	.000022	Diff. 1′
50°00′	.926484	.275443	.487573	.070776	.695359	.359566	.966562	50°00′
Diff. 1′	.000048	.000081	.000008	.000022	.000021	.000019	.000022	Diff. 1′
51°00′	.923623	.280332	.487082	.072112	.696614	.360720	.965228	51°00′
Diff. 1′	.000048	.000081	.000008	.000022	.000021	.000020	.000023	Diff. 1′
52°00′	.920713	.285185	.486582	.073444	.697903	.361905	.963869	52°00′
Diff. 1′	.000049	.000080	.000008	.000022	.000022	.000020	.000023	Diff. 1′
53°00′	.917756	.290001	.486073	.074772	.699225	.363121	.962485	53°00′
Diff. 1′	.000050	.000080	.000009	.000022	.000023	.000021	.000023	Diff. 1′
54°00′	.914752	.294781	.485556	.076095	.700581	.364370	.961076	54°00′
Diff. 1′	.000051	.000079	.000009	.000022	.000023	.000021	.000024	Diff. 1′
55°00′	.911702	.299524	.485029	.077412	.701973	.365651	.959643	55°00′
Diff. 1′	.000052	.000078	.000009	.000022	.000024	.000022	.000024	Diff. 1′
56°00′	.908605	.304229	.484495	.078725	.703400	.366966	.958185	56°00′
Diff. 1′	.000052	.000078	.000009	.000022	.000024	.000022	.000025	Diff. 1′
57°00′	.905462	.308895	.483951	.080033	.704863	.368316	.956702	57°00′
Diff. 1′	.000053	.000077	.000009	.000022	.000025	.000023	.000025	Diff. 1′
58°00′	.902275	.313523	.483399	.081336	.706364	.369700	.955194	58°00′
Diff. 1′	.000054	.000076	.000009	.000022	.000026	.000024	.000025	Diff. 1′
59°00′	.899043	.318111	.482839	.082634	.707902	.371119	.953663	59°00′
Diff. 1′	.000055	.000076	.000009	.000021	.000026	.000024	.000026	Diff. 1′

TABLE 15. FUNCTIONS OF UNIT SPIRAL LENGTH = 1 (*Continued*)

θ_s	x	y	k	p	L.T.	S.T.	L.C.	θ_s
60°00′	.895766	.322659	.482270	.083927	.709479	.372575	.952106	60°00′
Diff. 1′	.000055	.000075	.000010	.000021	.000027	.000025	.000026	Diff. 1′
61°00′	.892447	.327167	.481692	.085214	.711095	.374068	.950526	61°00′
Diff. 1′	.000056	.000074	.000010	.000021	.000028	.000026	.000027	Diff. 1′
62°00′	.889084	.331634	.481107	.086496	.712751	.375599	.948921	62°00′
Diff. 1′	.000057	.000074	.000010	.000021	.000028	.000026	.000027	Diff. 1′
63°00′	.885679	.336059	.480513	.087773	.714448	.377168	.947292	63°00′
Diff. 1′	.000057	.000073	.000010	.000021	.000029	.000027	.000027	Diff. 1′
64°00′	.882232	.340442	.479911	.089044	.716187	.378777	.945640	64°00′
Diff. 1′	.000058	.000072	.000010	.000021	.000030	.000027	.000028	Diff. 1′
65°00′	.878744	.344783	.479301	.090310	.717969	.380426	.943960	65°00′
Diff. 1′	.000059	.000072	.000010	.000021	.000030	.000028	.000028	Diff. 1′
66°00′	.875215	.349081	.478682	.091570	.719794	.382117	.942263	66°00′
Diff. 1′	.000059	.000071	.000010	.000021	.000031	.000029	.000029	Diff. 1′
67°00′	.871646	.353336	.478056	.092824	.721664	.383850	.940538	67°00′
Diff. 1′	.000060	.000070	.000011	.000021	.000032	.000030	.000029	Diff. 1′
68°00′	.868037	.357546	.477422	.094073	.723579	.385626	.938791	68°00′
Diff. 1′	.000061	.000069	.000011	.000021	.000033	.000030	.000030	Diff. 1′
69°00′	.864390	.361712	.476779	.095315	.725541	.387446	.937019	69°00′
Diff. 1′	.000061	.000069	.000011	.000021	.000033	.000031	.000030	Diff. 1′
70°00′	.860704	.365834	.476129	.096552	.727551	.389312	.935225	70°00′
Diff. 1′	.000062	.000068	.000011	.000020	.000034	.000032	.000030	Diff. 1′
71°00′	.856980	.369910	.475471	.097783	.729610	.391225	.933407	71°00′
Diff. 1′	.000063	.000067	.000011	.000020	.000035	.000033	.000031	Diff. 1′
72°00′	.853219	.373941	.474806	.099088	.731719	.393185	.931566	72°00′
Diff. 1′	.000063	.000066	.000011	.000020	.000036	.000033	.000031	Diff. 1′
73°00′	.849422	.377926	.474133	.100226	.733878	.395194	.929702	73°00′
Diff. 1′	.000064	.000066	.000011	.000020	.000037	.000034	.000031	Diff. 1′
74°00′	.845589	.381864	.473452	.101439	.736091	.397253	.927815	74°00′
Diff. 1′	.000064	.000065	.000011	.000020	.000038	.000035	.000032	Diff. 1′
75°00′	.841720	.385755	.472764	.102645	.738357	.399363	.925905	75°00′
Diff. 1′	.000065	.000064	.000012	.000020	.000039	.000036	.000032	Diff. 1′
76°00′	.837817	.389599	.472068	.103845	.740679	.401526	.923972	76°00′
Diff. 1′	.000066	.000063	.000012	.000020	.000040	.000037	.000033	Diff. 1′
77°00′	.833880	.393396	.471365	.105038	.743057	.403744	.922017	77°00′
Diff. 1′	.000066	.000062	.000012	.000020	.000041	.000038	.000033	Diff. 1′
78°00′	.839909	.397144	.470654	.106226	.745493	.406017	.920039	78°00′
Diff. 1′	.000067	.000062	.000012	.000020	.000042	.000039	.000033	Diff. 1′
79°00′	.825905	.400844	.469937	.107406	.747989	.408347	.918039	79°00′
Diff. 1′	.000067	.000061	.000012	.000020	.000043	.000040	.000034	Diff. 1′
80°00′	.821870	.404496	.469212	.108580	.750546	.410736	.916017	80°00′
Diff. 1′	.000068	.000060	.000012	.000020	.000044	.000041	.000034	Diff. 1′
81°00′	.817803	.408098	.468480	.109748	.753167	.413185	.913973	81°00′
Diff. 1′	.000068	.000059	.000012	.000019	.000045	.000042	.000034	Diff. 1′
82°00′	.813705	.411651	.467741	.110909	.755851	.415697	.911906	82°00′
Diff. 1′	.000069	.000058	.000012	.000019	.000046	.000043	.000035	Diff. 1′
83°00′	.809577	.415154	.466995	.112063	.758603	.418272	.909818	83°00′
Diff. 1′	.000069	.000058	.000012	.000019	.000047	.000044	.000035	Diff. 1′
84°00′	.805420	.418608	.466242	.113210	.761422	.420913	.907708	84°00′
Diff. 1′	.000070	.000057	.000013	.000019	.000048	.000045	.000035	Diff. 1′
85°00′	.801233	.422010	.465482	.114351	.764312	.423622	.905576	85°00′
Diff. 1′	.000070	.000056	.000013	.000019	.000049	.000046	.000036	Diff. 1′
86°00′	.797019	.425363	.464715	.115485	.767275	.426401	.903423	86°00′
Diff. 1′	.000071	.000055	.000013	.000019	.000051	.000047	.000036	Diff. 1′
87°00′	.792777	.428664	.463942	.116611	.770312	.429252	.901248	87°00′
Diff. 1′	.000071	.000054	.000013	.000019	.000052	.000049	.000037	Diff. 1′
88°00′	.788508	.431914	.463163	.117731	.773426	.432177	.899052	88°00′
Diff. 1′	.000072	.000053	.000013	.000018	.000053	.000050	.000037	Diff. 1′
89°00′	.784214	.435112	.462376	.118844	.776619	.435179	.896835	89°00′
Diff. 1′	.000072	.000052	.000013	.000018	.000055	.000051	.000037	Diff. 1′

Table 15. Functions of Unit Spiral Length = 1 (*Continued*)

θ_s	x	y	k	p	L.T.	S.T.	L.C.	θ_s
90°00′	.779893	.438259	.461583	.119949	.779893	.438259	.894598	90°00′
Diff. 1′	.000072	.000052	.000013	.000018	.000056	.000053	.000038	Diff. 1′
91°00′	.775548	.441354	.460784	.121048	.783252	.441421	.892339	91°00′
Diff. 1′	.000073	.000051	.000013	.000018	.000057	.000054	.000038	Diff. 1′
92°00′	.771179	.444396	.459979	.122139	.786698	.444667	.890059	92°00′
Diff. 1′	.000073	.000050	.000013	.000018	.000059	.000056	.000038	Diff. 1′
93°00′	.766787	.447386	.459167	.123223	.790233	.448000	.887749	93°00′
Diff. 1′	.000074	.000049	.000014	.000018	.000061	.000057	.000039	Diff. 1′
94°00′	.762372	.450324	.458349	.124299	.793861	.451423	.885439	94°00′
Diff. 1′	.000074	.000048	.000014	.000018	.000062	.000059	.000039	Diff. 1′
95°00′	.757935	.453208	.457525	.125369	.797585	.454939	.883098	95°00′
Diff. 1′	.000074	.000047	.000014	.000018	.000064	.000060	.000039	Diff. 1′
96°00′	.753476	.456039	.456696	.126431	.801408	.458551	.880737	96°00′
Diff. 1′	.000075	.000046	.000014	.000018	.000065	.000062	.000040	Diff. 1′
97°00′	.748997	.458817	.455860	.127485	.805333	.462263	.878357	97°00′
Diff. 1′	.000075	.000045	.000014	.000017	.000067	.000064	.000040	Diff. 1′
98°00′	.744499	.461541	.455018	.128532	.809364	.466077	.875956	98°00′
Diff. 1′	.000075	.000044	.000014	.000017	.000069	.000065	.000040	Diff. 1′
99°00′	.739981	.464212	.454171	.129572	.813505	.469999	.873535	99°00′
Diff. 1′	.000076	.000044	.000014	.000017	.000071	.000067	.000041	Diff. 1′
100°00′	.735444	.466829	.453317	.130604	.817759	.474031	.871095	100°00′
Diff. 1′	.000076	.000043	.000014	.000017	.000073	.000069	.000041	Diff. 1′
101°00′	.730890	.569392	.452459	.131628	.822130	.478177	.868636	101°00′
Diff. 1′	.000076	.000042	.000014	.000017	.000075	.000071	.000041	Diff. 1′
102°00′	.726319	.471901	.451594	.132645	.826624	.482443	.866158	102°00′
Diff. 1′	.000076	.000041	.000014	.000017	.000077	.000073	.000042	Diff. 1′
103°00′	.721731	.474355	.450725	.133654	.831244	.486833	.863660	103°00′
Diff. 1′	.000077	.000040	.000015	.000017	.000079	.000075	.000042	Diff. 1′
104°00′	.717128	.476755	.449850	.134655	.835996	.491350	.861143	104°00′
Diff. 1′	.000077	.000039	.000015	.000017	.000081	.000077	.000042	Diff. 1′
105°00′	.712510	.479101	.448969	.135648	.840884	.496001	.858608	105°00′
Diff. 1′	.000077	.000038	.000015	.000016	.000084	.000080	.000043	Diff. 1′
106°00′	.707877	.481392	.448084	.136634	.845914	.500791	.856054	106°00′
Diff. 1′	.000077	.000037	.000015	.000016	.000086	.000082	.000043	Diff. 1′
107°00′	.703231	.483628	.447193	.137612	.851091	.505725	.853481	107°00′
Diff. 1′	.000078	.000036	.000015	.000016	.000089	.000085	.000043	Diff. 1′
108°00′	.698573	.485809	.446297	.138581	.856422	.510810	.850890	108°00′
Diff. 1′	.000078	.000035	.000015	.000016	.000092	.000087	.000043	Diff. 1′
109°00′	.693902	.487935	.445396	.139543	.861912	.516051	.848281	109°00′
Diff. 1′	.000078	.000034	.000015	.000016	.000094	.000090	.000044	Diff. 1′
110°00′	.689220	.490007	.444491	.140498	.867568	.521455	.845654	110°00′
Diff. 1′	.000078	.000034	.000015	.000016	.000097	.000093	.000044	Diff. 1′
111°00′	.684527	.492024	.443580	.141444	.873398	.527029	.843009	111°00′
Diff. 1′	.000078	.000033	.000015	.000016	.000100	.000096	.000044	Diff. 1′
112°00′	.679825	.493985	.442665	.142382	.879408	.532780	.840347	112°00′
Diff. 1′	.000078	.000032	.000015	.000015	.000103	.000099	.000045	Diff. 1′
113°00′	.675113	.495891	.441746	.143312	.885606	.538717	.837667	113°00′
Diff. 1′	.000079	.000031	.000015	.000015	.000107	.000102	.000045	Diff. 1′
114°00′	.670393	.497743	.440821	.144234	.892002	.544847	.834970	114°00′
Diff. 1′	.000079	.000030	.000015	.000015	.000110	.000105	.000045	Diff. 1′
115°00′	.665665	.499539	.439893	.145147	.898604	.551180	.832255	115°00′
Diff. 1′	.000079	.000029	.000016	.000015	.000114	.000109	.000045	Diff. 1′
116°00′	.660930	.501280	.438959	.146053	.905420	.557725	.829524	116°00′
Diff. 1′	.000079	.000028	.000016	.000015	.000117	.000113	.000046	Diff. 1′
117°00′	.656188	.502966	.438022	.146951	.912462	.564492	.826775	117°00′
Diff. 1′	.000079	.000027	.000016	.000015	.000121	.000117	.000046	Diff. 1′
118°00′	.651441	.504596	.437080	.147840	.919740	.571491	.824010	118°00′
Diff. 1′	.000079	.000026	.000016	.000015	.000125	.000121	.000046	Diff. 1′
119°00′	.646689	.506172	.436135	.148721	.927265	.578734	.821229	119°00′
Diff. 1′	.000079	.000025	.000016	.000014	.000130	.000125	.000047	Diff. 1′

TABLE 15. FUNCTIONS OF UNIT SPIRAL LENGTH = 1 (*Continued*)

θ_s	x	y	k	p	L.T.	S.T.	L.C.	θ_s
120°00′	.641933	.507693	.435185	.149594	.935050	.586233	.818431	120°00′
Diff. 1′	.000079	.000024	.000016	.000014	.000134	.000129	.000047	Diff. 1′
121°00′	.637173	.509158	.434231	.150459	.943106	.594001	.815618	121°00′
Diff. 1′	.000079	.000023	.000016	.000014	.000139	.000134	.000047	Diff. 1′
122°00′	.632411	.510569	.433273	.151315	.951450	.602052	.812788	122°00′
Diff. 1′	.000079	.000023	.000016	.000014	.000144	.000139	.000047	Diff. 1′
123°00′	.627646	.511924	.432312	.152163	.960094	.610400	.809942	123°00′
Diff. 1′	.000079	.000022	.000016	.000014	.000149	.000144	.000048	Diff. 1′
124°00′	.622880	.513225	.431347	.153003	.969055	.619061	.807081	124°00′
Diff. 1′	.000079	.000021	.000016	.000014	.000155	.000150	.000048	Diff. 1′
125°00′	.618114	.514471	.430378	.153834	.978350	.628053	.804205	125°00′
Diff. 1′	.000079	.000020	.000016	.000014	.000161	.000156	.000048	Diff. 1′
126°00′	.613347	.515663	.429406	.154657	.987998	.637394	.801313	126°00′
Diff. 1′	.000079	.000019	.000016	.000014	.000167	.000162	.000048	Diff. 1′
127°00′	.608581	.516800	.428430	.155472	.998018	.647103	.798406	127°00′
Diff. 1′	.000079	.000018	.000016	.000013	.000173	.000168	.000049	Diff. 1′
128°00′	.603817	.517882	.427451	.156278	1.00843	.657202	.795485	128°00′
Diff. 1′	.000079	.000017	.000016	.000013	.000180	.000175	.000049	Diff. 1′
129°00′	.599054	.518910	.426468	.157076	1.01926	.667712	.792549	129°00′
Diff. 1′	.000079	.000016	.000016	.000013	.000188	.000182	.000049	Diff. 1′
130°00′	.594295	.519884	.425483	.157865	1.03053	.678660	.789592	130°00′
Diff. 1′	.000079	.000015	.000016	.000013	.000196	.000190	.000049	Diff. 1′
131°00′	.589538	.520804	.424494	.158646	1.04227	.690071	.786633	131°00′
Diff. 1′	.000079	.000014	.000016	.000013	.000204	.000198	.000050	Diff. 1′
132°00′	.584786	.521669	.423502	.159419	1.05450	.701975	.783654	132°00′
Diff. 1′	.000079	.000013	.000017	.000013	.000213	.000207	.000050	Diff. 1′
133°00′	.580039	.522481	.422507	.160183	1.06726	.714403	.780662	133°00′
Diff. 1′	.000079	.000013	.000017	.000013	.000222	.000216	.000050	Diff. 1′
134°00′	.575298	.523240	.421510	.160938	1.08058	.727389	.777655	134°00′
Diff. 1′	.000079	.000012	.000017	.000012	.000232	.000226	.000050	Diff. 1′
135°00′	.570562	.523945	.420510	.161686	1.09451	.740970	.774635	135°00′
Diff. 1′	.000079	.000011	.000017	.000012	.000243	.000237	.000051	Diff. 1′
136°00′	.565834	.524597	.419506	.162424	1.10907	.755187	.771602	136°00′
Diff. 1′	.000079	.000010	.000017	.000012	.000254	.000248	.000051	Diff. 1′
137°00′	.561113	.525195	.418501	.163154	1.12432	.770083	.768556	137°00′
Diff. 1′	.000078	.000009	.000017	.000012	.000266	.000260	.000051	Diff. 1′
138°00′	.556400	.525741	.417493	.163876	1.14030	.785708	.765496	138°00′
Diff. 1′	.000078	.000008	.000017	.000012	.000279	.000273	.000051	Diff. 1′
139°00′	.551696	.526235	.416482	.164589	1.15706	.802115	.762425	139°00′
Diff. 1′	.000078	.000007	.000017	.000012	.000293	.000287	.000051	Diff. 1′

EXAMPLE. **Given Spiral Angle θ = 120°40′ and Length of Spiral = 250 feet. (1) To find Long Chord (L.C.) and Deflection Angle ϕ_c.**

SOLUTION (page 463): L.C. = $[0.818431 - 0.000047 \times 40] \times 250 = 0.816551 \times 250 = 204.138$ feet; ϕ_c (page 465) = $38°20'23.3'' + 0.2881' \times 40 = 38°31'54.7''$. (2) To find Long Chord and Deflection Angle from the T.S. to the mid-point of this spiral, where $l = 125$ feet and $\theta = (\frac{1}{2})^2 \times (120°40') = 30°10'$, PROCEED AS FOLLOWS: From page 460, Long Chord = $[0.987868 - 0.000014 \times 10] \times 125 = 0.987728 \times 125 = 123.466$ feet; from page 464, Deflection Angle $\phi = 9°58'35.9'' + 0.3309' \times 10 = 10°01'54.4''$.

TABLE 16. DEFLECTION ANGLES TO ANY POINT ON SPIRAL

θ	φ	θ	φ	θ	φ
0°00′	0°00′00.0″	25°00′	8°19′11.4″	50°00′	16°33′25.9″
Diff. 1′	0.3333′	Diff. 1′	0.3316′	Diff. 1′	0.3266′
1°00′	0°20′00.0″	26°00′	8°39′05.3″	51°00′	16°53′01.5″
Diff. 1′	0.3333′	Diff. 1′	0.3315′	Diff. 1′	0.3263′
2°00′	0°40′00.0″	27°00′	8°58′58.8″	52°00′	17°12′36 0″
Diff. 1′	0.3333′	Diff. 1′	0.3314′	Diff. 1′	0.3260
3°00′	0°59′59.9″	28°00′	9°18′51.7″	53°00′	17°32′09.6″
Diff. 1′	0.3333′	Diff. 1′	0.3312′	Diff. 1′	0.3257′
4°00′	1°19′59.8″	29°00′	9°38′44.1″	54°00′	17°51′42.0″
Diff. 1′	0.3333′	Diff. 1′	0.3311′	Diff. 1′	0.3254′
5°00′	1°39′59.6″	30°00′	9°58′35.9″	55°00′	18°11′13.4″
Diff. 1′	0.3333′	Diff. 1′	0.3333′	Diff. 1′	0.3251′
6°00′	1°59′59.3″	31°00′	10°18′27.2″	56°00′	18°30′43.7″
Diff. 1	0.3332′	Diff. 1′	0.3307′	Diff. 1′	0.3248′
7°00′	2°19′58.9″	32°00′	10°38′17.8″	57°00′	18°50′12.9″
Diff. 1′	0.3332′	Diff. 1′	0.3306′	Diff. 1′	0.3244′
8°00′	2°39′58.4″	33°00′	10°58′07.9″	58°00′	19°09′40.9″
Diff. 1′	0.3332′	Diff. 1′	0.3304′	Diff. 1′	0.3241′
9°00′	2°59′57.7″	34°00′	11°17′57.3″	59°00′	19°29′07.7″
Diff. 1′	0.3331′	Diff. 1′	0.3302′	Diff. 1′	0.3238′
10°00′	3°19′56.9″	35°00′	11°37′46.1″	60°00′	19°48′33.4″
Diff. 1′	0.3331′	Diff. 1′	0.3300′	Diff. 1′	0.3234′
11°00′	3°39′55.9″	36°00′	11°57′34.2″	61°00′	20°07′57.8″
Diff. 1′	0.3330′	Diff. 1′	0.3298′	Diff. 1′	0.3231′
12°00′	3°59′54.6″	37°00′	12°17′21.7″	62°00′	20°27′21.0″
Diff. 1′	0.3329′	Diff. 1′	0.3296′	Diff. 1′	0.3228′
13°00′	4°19′53.2″	38°00′	12°37′08.4″	63°00	20°46′42.9″
Diff. 1′	0.3329′	Diff. 1′	0.3294′	Diff. 1′	0.3224′
14°00′	4°39′51.5″	39°00′	12°56′54.4″	64°00′	21°06′03.5″
Diff. 1′	0.3328′	Diff. 1′	0.3292′	Diff. 1′	0.3220′
15°00′	4°59′49.5″	40°00′	13°16′39.6″	65°00′	21°25′22.8″
Diff. 1′	0.3327′	Diff. 1′	0.3290′	Diff. 1′	0.3216′
16°00′	5°19′47.3″	41°00′	13°36′24.0″	66°00′	21°44′40.9″
Diff. 1′	0.3326′	Diff. 1′	0.3288′	Diff. 1′	0.3213′
17°00′	5°39′44.8″	42°00′	13°56′07.7″	67°00′	22°03′57.4″
Diff. 1′	0.3325′	Diff. 1′	0.3286′	Diff. 1′	0.3209′
18°00′	5°59′41.9″	43°00′	14°15′50.6″	68°00′	22°23′12.7″
Diff. 1′	0.3324′	Diff. 1′	0.3283′	Diff. 1′	0.3205′
19°00′	6°19′38.7″	44°00′	14°35′32.6″	69°00′	22°42′26.5″
Diff. 1′	0.3323′	Diff. 1′	0.3281′	Diff. 1′	0.3201′
20°00′	6°39′35.2″	45°00′	14°55′13.7″	70°00′	23°01′38.8″
Diff. 1′	0.3322′	Diff. 1′	0.3279′	Diff. 1′	0.3197′
21°00′	6°59′31.2″	46°00′	15°14′54.0″	71°00′	23°20′49.7″
Diff. 1′	0.3321′	Diff. 1′	0.3276′	Diff. 1′	0.3193′
22°00′	7°19′26.9″	47°00′	15°34′33.4″	72°00′	23°39′59.1″
Diff. 1′	0.3320′	Diff. 1′	0.3274′	Diff. 1′	0.3189′
23°00′	7°39′22.2″	48°00′	15°54′11.9″	73°00′	23°59′07.0″
Diff. 1′	0.3319′	Diff. 1′	0.3271′	Diff. 1′	0.3184′
24°00′	7°59′17.0″	49°00′	16°13′49.4″	74°00′	24°18′13.4″
Diff. 1′	0.3318′	Diff. 1′	0.3268′	Diff. 1′	0.3180′

TABLE 16. DEFLECTION ANGLES TO ANY POINT ON SPIRAL (*Continued*)

θ	φ	θ	φ	θ	φ
75°00'	24°37'18.1"	100°00'	32°24'20.4"	125°00'	39°46'17.2"
Diff. 1'	0.3176'	Diff. 1'	0.3037'	Diff. 1'	0.2835'
76°00'	24°56'21.3"	101°00'	32°42'33.8"	126°00'	40°03'17.8"
Diff. 1'	0.3171'	Diff. 1'	0.3030'	Diff. 1'	0.2825'
77°00'	25°15'22.8"	102°00'	33°00'44.7"	127°00'	40°20'15.0"
Diff. 1'	0.3166'	Diff. 1'	0.3024'	Diff. 1'	0.2815'
78°00'	25°34'22.7"	103°00'	33°18'53.2"	128°00'	40°37'08.6"
Diff. 1'	0.3161'	Diff. 1'	0.3017'	Diff. 1'	0.2805'
79°00'	25°53'20.8"	104°00'	33°36'59.2"	129°00'	40°53'58.6"
Diff. 1'	0.3157'	Diff. 1'	0.3009'	Diff. 1'	0.2795'
80°00'	26°12'17.3"	105°00'	33°55'02.6"	130°00'	41°10'44.9"
Diff. 1'	0.3152'	Diff. 1'	0.3002'	Diff. 1'	0.2784'
81°00'	26°31'12.0"	106°00'	34°13'03.5"	131°00'	41°27'27.6"
Diff. 1'	0.3147'	Diff. 1'	0.2995'	Diff. 1'	0.2774'
82°00'	26°50'05.0"	107°00'	34°31'01.8"	132°00'	41°44'06.5"
Diff. 1'	0.3142'	Diff. 1'	0.2988'	Diff. 1'	0.2763'
83°00'	27°08'56.2"	108°00'	34°48'57.4"	133°00'	42°00'41.6"
Diff. 1'	0.3137'	Diff. 1'	0.2980'	Diff. 1'	0.2753'
84°00'	27°27'45.5"	109°00'	35°06'50.3"	134°00'	42°17'12.8"
Diff. 1'	0.3132'	Diff. 1'	0.2973'	Diff. 1'	0.2742'
85°00'	27°46'32.9"	110°00'	35°24'40.5"	135°00'	42°33'40.0"
Diff. 1'	0.3127'	Diff. 1'	0.2965'	Diff. 1'	0.2731'
86°00'	28°05'18.5"	111°00'	35°42'28.0"	136°00'	42°50'03.4"
Diff. 1'	0.3121'	Diff. 1'	0.2957'	Diff. 1'	0.2719'
87°00'	28°24'02.2"	112°00'	36°00'12.6"	137°00'	43°06'22.6"
Diff. 1'	0.3116'	Diff. 1'	0.2949'	Diff. 1'	0.2708'
88°00'	28°42'43.9"	113°00'	36°17'54.4"	138°00'	43°22'37.8"
Diff. 1'	0.3110'	Diff. 1'	0.2941'	Diff. 1'	0.2696'
89°00'	29°01'23.6"	114°00'	36°35'33.3"	139°00'	43°38'48.8"
Diff. 1	0.3105'	Diff. 1'	0.2933'	Diff. 1'	0.2684'
90°00'	29°20'01.3"	115°00'	36°53'09.2"	140°00'	43°54'55.6"
Diff. 1'	0.3100'	Diff. 1'	0.2925'	Diff. 1'	0.2672'
91°00'	29°38'37.0"	116°00'	37°10'42.2"	141°00'	44°10'58.1"
Diff. 1'	0.3093'	Diff. 1'	0.2916'	Diff. 1'	0.2660'
92°00'	29°57'10.5"	117°00'	37°28'12.1"	142°00'	44°26'56.3"
Diff. 1'	0.3087'	Diff. 1'	0.2908'	Diff. 1'	0.2648'
93°00'	31°15'42.0"	118°00'	37°45'38.9"	143°00'	44°42'50.1"
Diff. 1'	0.3081'	Diff. 1'	0.2899'	Diff. 1'	0.2635'
94°00'	30°34'11.3"	119°00'	38°03'02.7"	144°00'	44°58 39.4"
Diff. 1'	0.3076'	Diff. 1'	0.2891'	Diff. 1'	0.2622'
95°00'	30°52'38.5"	120°00'	38°20'23.3"	145°00'	45°14'24.1"
Diff. 1'	0.3069'	Diff. 1'	0.2881'	Diff. 1'	0.2610'
96°00	31°11'03.4"	121°00'	38°37'40.7"	146°00'	
Diff. 1'	0.3063'	Diff. 1'	0.2872'		
97°00'	31°29'26.1"	122°00'	38°54'54.8"		
Diff. 1'	0.3057'	Diff. 1'	0.2863'		
98°00'	31°47'46.6"	123°00'	39°12'05.6"		
Diff. 1'	0.3050'	Diff. 1'	0.2854'		
99°00'	32°06'05.0"	124°00'	39°29'13.1"		
Diff. 1'	0.3044'	Diff. 1'	0.2844'		

TABLE 17. FUNCTIONS OF THE 10-CHORD SPIRAL*

θ_s	$\frac{y_c}{l_s}$	$\frac{x_c}{l_s}$	$p = al_s - bD_c$		$k = el_s - fD_c$		θ_s
			a	b	e	f	
0°	.000 000	1.000 000	.000 000	.000 00	.500 000	.000 00	0°
30′	.002 909	.999 993	.000 727	.000 00	.499 999	.000 64	30′
1°	.005 818	.999 970	.001 454	.000 01	.499 995	.001 27	1°
30′	.008 726	.999 932	.002 182	.000 03	.499 989	.001 91	30′
2°	.011 635	.999 879	.002 909	.000 04	.499 980	.002 55	2°
30′	.014 542	.999 811	.003 636	.000 07	.499 969	.003 18	30′
3°	.017 450	.999 727	.004 363	.000 10	.499 956	.003 82	3°
30′	.020 357	.999 629	.005 090	.000 14	.499 940	.004 46	30′
4°	.023 263	.999 515	.005 817	.000 18	.499 922	.005 09	4°
30′	.026 169	.999 387	.006 544	.000 23	.499 901	.005 73	30′
5°	.029 073	.999 243	.007 270	.000 28	.499 877	.006 36	5°
30′	.031 977	.999 084	.007 997	.000 34	.499 852	.007 00	30′
6°	.034 880	.998 910	.008 724	.000 40	.499 824	.007 63	6°
30′	.037 781	.998 721	.009 450	.000 47	.499 793	.008 26	30′
7°	.040 681	.998 517	.010 176	.000 54	.499 760	.008 90	7°
30′	.043 581	.998 298	.010 902	.000 62	.499 724	.009 53	30′
8°	.046 478	.998 063	.011 628	.000 71	.499 686	.010 16	8°
30′	.049 374	.997 814	.012 354	.000 80	.499 646	.010 79	30′
9°	.052 269	.997 549	.013 080	.000 90	.499 603	.011 42	9°
30′	.055 162	.997 270	.013 805	.001 00	.499 558	.012 05	30′
10°	.058 053	.996 975	.014 530	.001 11	.499 510	.012 68	10°
30′	.060 942	.996 666	.015 255	.001 22	.499 460	.013 30	30′
11°	.063 829	.996 341	.015 980	.001 34	.499 407	.013 93	11°
30′	.066 714	.996 002	.016 704	.001 47	.499 352	.014 55	30′
12°	.069 598	.995 647	.017 429	.001 60	.499 294	.015 18	12°
30′	.072 478	.995 278	.018 153	.001 73	.499 234	.015 80	30′
13°	.075 357	.994 893	.018 877	.001 87	.499 172	.016 42	13°
30′	.078 233	.994 494	.019 600	.002 02	.499 107	.017 04	30′
14°	.081 106	.994 079	.020 323	.002 17	.499 040	.017 66	14°
30′	.083 977	.993 650	.021 046	.002 33	.498 970	.018 28	30′
15°	.086 846	.993 206	.021 769	.002 49	.498 898	.018 89	15°
30′	.089 711	.992 747	.022 491	.002 65	.498 824	.019 51	30′
16°	.092 574	.992 273	.023 213	.002 83	.498 747	.020 12	16°
30′	.095 433	.991 785	.023 935	.003 01	.498 667	.020 73	30′
17°	.098 290	.991 281	.024 656	.003 19	.498 585	.021 34	17°
30′	.101 143	.990 763	.025 377	.003 38	.498 501	.021 95	30′
18°	.103 993	.990 230	.026 097	.003 57	.498 414	.022 56	18°
30′	.106 840	.989 682	.026 817	.003 77	.498 325	.023 16	30′
19°	.109 683	.989 120	.027 537	.003 98	.498 233	.023 77	19°
30′	.112 523	.988 543	.028 257	.004 19	.498 139	.024 37	30′
20°	.115 360	.987 951	.028 976	.004 40	.498 043	.024 97	20°
30′	.118 192	.987 344	.029 694	.004 62	.497 944	.025 57	30′
21°	.121 021	.986 723	.030 412	.004 85	.497 843	.026 16	21°
30′	.123 846	.986 088	.031 130	.005 08	.497 739	.026 75	30′
22°	.126 667	.985 437	.031 847	.005 32	.497 633	.027 35	22°
30′	.129 483	.984 772	.032 564	.005 56	.497 525	.027 94	30′
23°	.132 296	.984 093	.033 280	.005 80	.497 414	.028 52	23°
30′	.135 105	.985 399	.033 996	.006 05	.497 300	.029 11	30′
24°	.137 909	.982 691	.034 711	.006 31	.497 185	.029 69	24°
30′	.140 708	.981 968	.035 426	.006 57	.497 067	.030 27	30′

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* Applies only to chord definition.

TABLE 17. FUNCTIONS OF THE 10-CHORD SPIRAL* (*Continued*)

θ_s	$\frac{y_c}{l_s}$	$\frac{x_c}{l_s}$	$p = al_s - bD_c$		$k = el_s - fD_c$		θ_s
			a	b	e	f	
25°	.143 504	.981 231	.036 140	.006 84	.496 946	.030 85	**25°**
30′	.146 294	.980 479	.036 854	.007 11	.496 823	.031 43	**30′**
26°	.149 080	.979 714	.037 567	.007 39	.496 698	.032 00	**26°**
30′	.151 861	.978 933	.038 280	.007 67	.496 570	.032 57	**30′**
27°	.154 638	.978 139	.038 992	.007 96	.496 440	.033 14	**27°**
30′	.157 409	.977 330	.039 704	.008 25	.496 308	.033 71	**30′**
28°	.160 176	.976 508	.040 415	.008 54	.496 173	.034 27	**28°**
30′	.162 937	.975 670	.041 125	.008 85	.496 036	.034 83	**30′**
29°	.165 693	.974 819	.041 835	.009 15	.495 896	.035 39	**29°**
30′	.168 444	.973 954	.042 544	.009 46	.495 754	.035 95	**30′**
30°	.171 189	.973 074	.043 253	.009 78	.495 610	.036 50	**30°**
30′	.173 929	.972 181	.043 961	.010 10	.495 463	.037 05	**30′**
31°	.176 664	.971 273	.044 668	.010 43	.495 314	.037 60	**31°**
30′	.179 392	.970 352	.045 375	.010 76	.495 162	.038 14	**30′**
32°	.182 116	.969 417	.046 081	.011 09	.495 008	.038 68	**32°**
30′	.184 833	.968 468	.046 787	.011 43	.494 852	.039 22	**30′**
33°	.187 544	.967 504	.047 491	.011 78	.494 694	.039 76	**33°**
30′	.190 250	.966 528	.048 195	.012 13	.494 533	.040 29	**30′**
34°	.192 949	.965 537	.048 899	.012 48	.494 369	.040 82	**34°**
30′	.195 643	.964 532	.049 602	.012 84	.494 204	.041 35	**30′**
35°	.198 330	.963 515	.050 304	.013 20	.494 036	.041 87	**35°**
30′	.201 010	.962 483	.051 005	.013 57	.493 866	.042 39	**30′**
36°	.203 685	.961 438	.051 705	.013 94	.493 693	.042 91	**36°**
30′	.206 353	.960 379	.052 405	.014 32	.493 518	.043 42	**30′**
37°	.209 014	.959 306	.053 104	.014 70	.493 341	.043 93	**37°**
30′	.211 669	.958 221	.053 803	.015 09	.493 161	.044 44	**30′**
38°	.214 317	.957 121	.054 500	.015 48	.492 979	.044 94	**38°**
30′	.216 959	.956 009	.055 197	.015 87	.492 795	.045 44	**30′**
39°	.219 593	.954 883	.055 893	.016 27	.492 608	.045 94	**39°**
30′	.222 221	.953 744	.056 589	.016 67	.492 420	.046 43	**30′**
40°	.224 841	.952 591	.057 283	.017 08	.492 229	.046 92	**40°**
30′	.227 455	.951 426	.057 977	.017 49	.492 035	.047 41	**30′**
41°	.230 061	.950 247	.058 670	.017 91	.491 839	.047 89	**41°**
30′	.232 660	.949 055	.059 362	.018 33	.491 641	.048 37	**30′**
42°	.235 252	.947 850	.060 053	.018 75	.491 441	.048 85	**42°**
30′	.237 836	.946 632	.060 743	.019 18	.491 239	.049 32	**30′**
43°	.240 413	.945 402	.061 433	.019 61	.491 034	.049 79	**43°**
30′	.242 982	.944 158	.062 122	.020 05	.490 827	.050 25	**30′**
44°	.245 544	.942 901	.062 809	.020 49	.490 617	.050 71	**44°**
30′	.248 098	.941 632	.063 496	.020 93	.490 406	.051 17	**30′**
45°	.250 644	.940 350	.064 182	.021 38	.490 192	.051 62	**45°**

* Applies only to chord definition.

TABLE 17. FUNCTIONS OF THE 10-CHORD SPIRAL* (*Continued*)

Note: These corrections (frequently negligible) are to be *subtracted* from the calculated values of p and k.

Corrections for determining p

θ_s	$D =$												θ_s
	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	
8°	.00	.00	.00	.00	.00	.00	.01	.01	.01	.01	.01	.01	8°
10	.00	.00	.00	.01	.01	.01	.01	.01	.01	.01	.01	.01	10
12	...	.01	.01	.01	.01	.01	.01	.01	.02	.02	.02	.02	12
14	...	.01	.01	.01	.01	.01	.02	.02	.02	.02	.03	.03	14
16	...	...	.01	.01	.02	.02	.02	.03	.03	.03	.03	.04	16
18	...	...	.01	.02	.02	.02	.02	.03	.03	.03	.03	.05	18
20	...	...	...	.02	.02	.03	.03	.03	.03	.05	.05	.05	20
22	...	...	...	.02	.02	.03	.03	.04	.04	.05	.05	.06	22
24	...	...	...	.01	.02	.03	.04	.05	.05	.06	.07	.07	24
26	...	...	...	.01	.02	.03	.04	.05	.06	.07	.08	.09	26

θ_s	$D =$												θ_s
	14°	15°	16°	17°	18°	19°	20°	21°	22°	23°	24°	25°	
6°	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	6°
8	.01	.01	.01	.01	.01	.01	.01	.01	.02	.02	.02	.02	8
10	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02	.03	.02	10
12	.02	.02	.03	.03	.03	.03	.03	.03	.03	.03	.04	.04	12
14	.03	.03	.03	.04	.04	.04	.04	.05	.05	.05	.05	.05	14
16	.04	.04	.05	.05	.05	.05	.06	.06	.06	.07	.07	.07	16
18	.05	.05	.06	.06	.06	.07	.07	.07	.08	.08	.09	.09	18
20	.06	.07	.07	.07	.08	.08	.09	.09	.10	.10	.11	.11	20
22	.07	.07	.08	.09	.10	.10	.11	.11	.12	.12	.13	.13	22
24	.08	.08	.09	.09	.10	.11	.12	.12	.14	.15	.15	.16	24
26	.09	.10	.11	.12	.12	.13	.14	.15	.15	.16	.17	.17	26

Corrections for determining k

θ_s	$D =$												θ_s
	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	
2°	.00	.01	.01	.01	.02	.02	.02	.02	.03	.03	.03	.03	2°
4	.01	.02	.02	.03	.03	.04	.04	.05	.05	.06	.06	.07	4
6	.01	.02	.03	.04	.05	.05	.06	.07	.08	.08	.09	.10	6
8	.01	.02	.04	.05	.06	.07	.08	.09	.10	.11	.12	.13	8
10	.01	.03	.04	.05	.07	.09	.10	.11	.13	.14	.15	.16	10
12	...	.03	.05	.07	.09	.10	.12	.13	.15	.16	.18	.19	12
14	...	.02	.05	.07	.10	.11	.13	.15	.17	.18	.21	.22	14
16	...	.01	.05	.07	.10	.12	.14	.16	.19	.21	.23	.25	16
18	...	.01	.05	.07	.11	.13	.16	.18	.21	.23	.26	.28	18
20	...	...	...	.07	.11	.13	.17	.19	.22	.25	.28	.30	20
22	...	...	...	.07	.11	.14	.17	.21	.24	.27	.30	.33	22
24	..	...	...	.06	.11	.14	.18	.22	.25	.29	.32	.35	24
26	...	...	...	.05	.09	.14	.19	.23	.26	.30	.33	.37	26

θ_s	$D =$												θ_s
	14°	15°	16°	17°	18°	19°	20°	21°	22°	23°	24°	25°	
2°	.04	.04	.04	.04	.05	.05	.05	.05	.06	.06	.06	.06	2°
4	.07	.08	.08	.09	.09	.10	.10	.11	.11	.12	.12	.13	4
6	.11	.11	.12	.13	.14	.14	.15	.16	.17	.18	.18	.19	6
8	.14	.15	.16	.17	.18	.19	.20	.21	.22	.23	.24	.25	8
10	.18	.19	.20	.22	.23	.24	.25	.27	.28	.29	.30	.32	10
12	.21	.23	.24	.26	.27	.29	.30	.32	.33	.35	.36	.38	12
14	.24	.25	.28	.29	.32	.34	.35	.37	.39	.41	.42	.44	14
16	.27	.29	.31	.33	.36	.37	.40	.41	.44	.45	.48	.49	16
18	.31	.33	.35	.37	.40	.42	.44	.46	.49	.51	.54	.55	18
20	.33	.35	.38	.40	.44	.46	.49	.51	.54	.56	.59	.61	20
22	.35	.39	.42	.44	.47	.50	.53	.55	.58	.61	.65	.67	22
24	.38	.42	.44	.47	.51	.54	.57	.60	.63	.66	.69	.72	24
26	.41	.44	.46	.51	.55	.58	.61	.64	.67	.71	.75	.78	26

[illegible] l_s =
Simple Curve Deg. of Curvature

means For 1° [illegible] of [illegible] Simple Curve.

TABLE 18. T_s (FOR 1° CIRCULAR CURVE,) SPIRALED

θ_s	1°	2°	3°	4°	5°	6°	7°	8°	θ_s
Ignore ~~l_s~~	~~200~~	~~400~~	~~600~~	~~800~~	~~1000~~	~~1200~~	~~1400~~	~~1600~~	~~l_s~~
Δ									Δ
4°	300.09	400.11							4°
5	350.17	450.20							5
6	400.29	500.33	600.38						6
7	450.45	550.50	650.57						7
8	500.67	600.72	700.81	800.91					8
9	550.95	651.01	751.11	851.23					9
10	601.30	701.37	801.48	901.62	1001.78				10
11	651.72	751.80	851.92	952.08	1052.27				11
12	702.23	802.32	902.45	1002.63	1102.84	1203.08			12
13	752.83	852.93	953.07	1053.27	1153.51	1253.78			13
14	803.54	903.64	1003.80	1104.01	1204.27	1304.57	1404.90		14
15	854.35	954.46	1054.63	1154.86	1255.14	1355.47	1455.84		15
16	905.28	1005.40	1105.58	1205.83	1306.13	1406.49	1506.89	1607.33	16
17	956.33	1056.46	1156.65	1256.92	1357.25	1457.64	1558.07	1658.55	17
18	1007.52	1107.65	1207.86	1308.15	1408.50	1508.92	1609.38	1709.90	18
19	1058.85	1158.99	1259.21	1359.52	1459.89	1560.34	1660.84	1761.40	19
20	1110.33	1210.48	1310.71	1411.04	1511.43	1611.91	1712.44	1813.04	20
21	1161.97	1262.12	1362.37	1462.71	1563.13	1663.64	1764.21	1864.84	21
22	1213.77	1313.93	1414.20	1514.56	1615.00	1715.53	1816.14	1916.81	22
23	1265.75	1365.92	1466.20	1566.58	1667.05	1767.61	1868.25	1968.96	23
24	1317 92	1418 10	1518 39	1618.78	1719.28	1819.87	1920.54	2021.29	24
25	1370.28	1470.47	1570.77	1671.18	1771.70	1872.32	1973.03	2073.82	25
26	1422.84	1523.04	1623.36	1723.79	1824.33	1924.97	2025.72	2126.55	26
27	1475.62	1575.82	1676.15	1776.60	1877.17	1977.84	2078.62	2179.49	27
28	1528.62	1628.83	1729.17	1829.64	1930.23	2030.93	2131.75	2232.66	28
29	1581.84	1682.06	1782.42	1882.91	1983.52	2084.26	2185.11	2286.06	29
30	1635.31	1735.54	1835.91	1936.42	2037.06	2137.82	2238.71	2339.70	30
31	1689.03	1789.27	1889.65	1990.18	2090.84	2191.64	2292.56	2393.59	31
32	1743.01	1843.26	1943.65	2044.20	2144.89	2245.71	2346.67	2447.75	32
33	1797.26	1897.51	1997.92	2098.49	2199.21	2300.06	2401.05	2502.17	33
34	1851.80	1952.06	2052.48	2153.06	2253.80	2354.69	2455.71	2556.88	34
35	1906.62	2006.89	2107.33	2207.93	2308.69	2409.61	2510.67	2611.88	35
36	1961.75	2062.03	2162.48	2263.10	2363.89	2464.83	2565.93	2667.18	36
37	2017.19	2117.47	2217.94	2318.58	2419.40	2520.37	2621.51	2722.80	37
38	2072.95	2173.24	2273.73	2374.39	2475.23	2576.24	2677.41	2778.74	38
39	2129.05	2229.35	2329.85	2430.53	2531.40	2632.44	2733.65	2835.02	39
40	2185.50	2285.81	2386.32	2487.02	2587.91	2688.99	2790.23	2891.65	40
41	2242.31	2342.63	2443.16	2543.88	2644.79	2745.90	2847.18	2948.64	41
42	2299.49	2399.82	2500.36	2601.10	2702.04	2803.18	2904.50	3006.00	42
43	2357.06	2457.39	2557.94	2658.71	2759.68	2860.85	2962.21	3063.75	43
44	2415.02	2515.36	2615.93	2716.71	2817.71	2918.91	3020.31	3121.90	44
45	2473.39	2573.74	2674.33	2775.13	2876.15	2977.39	3078.82	3180.46	45

θs = central angle of spiral arc x
l_s = total length of spiral from TS to SC

TABLE 18. T_s FOR 1° CIRCULAR CURVE, SPIRALED (*Continued*)

more

θ_s	1°	2°	3°	4°	5°	6°	7°	8°	θ_s
l_s	200	400	600	800	1000	1200	1400	1600	l_s
Δ									Δ
45°	2473.39	2573.74	2674.33	2775.13	2876.15	2977.39	3078.82	3180.46	45°
46	2532.18	2632.55	2733.15	2833.97	2935.02	3036.29	3137.76	3239.44	46
47	2591.42	2691.79	2792.40	2893.25	2994.32	3095.62	3197.14	3298.86	47
48	2651.10	2751.48	2852.11	2952.98	3054.08	3155.41	3256.97	3358.74	48
49	2711.25	2811.64	2912.28	3013.17	3114.30	3215.67	3317.27	3419.08	49
50	2771.88	2872.28	2972.94	3073.85	3175.01	3276.41	3378.04	3479.90	50
51	2833.01	2933.42	3034.09	3135.02	3236.21	3337.64	3439.31	3541.22	51
52	2894.64	2995.06	3095.75	3196.71	3297.92	3399.39	3501.10	3603.06	52
53	2956.81	3057.23	3157.94	3258.92	3360.16	3461.66	3563.42	3665.42	53
54	3019.51	3119.95	3220.67	3321.67	3422.94	3524.48	3626.28	3728.33	54
55	3082.78	3183.23	3283.97	3384.99	3486.29	3587.86	3689.70	3791.79	55
56	3146.62	3247.08	3347.83	3448.88	3550.21	3651.82	3753.70	3855.84	56
57	3211.06	3311.53	3412.30	3513.37	3614.73	3716.38	3818.30	3920.49	57
58	3276.12	3376.59	3477.38	3578.47	3679.86	3781.54	3883.51	3985.75	58
59	3341.80	3442.29	3543.10	3644.21	3745.63	3847.34	3949.35	4051.64	59
60	3408.14	3508.64	3609.46	3710.59	3812.04	3913.80	4015.85	4118.19	60
61	3475.15	3575.66	3676.49	3777.65	3879.13	3980.92	4083.02	4185.42	61
62	3542.85	3643.37	3744.22	3845.41	3946.92	4048.75	4150.89	4253.34	62
63	3611.27	3711.80	3812.67	3913.88	4015.42	4117.29	4219.47	4321.97	63
64	3680.42	3780.96	3881.85	3983.08	4084.65	4186.56	4288.79	4391.34	64
65	3750.33	3850.88	3951.79	4053.04	4154.65	4256.59	4358.87	4461.48	65
66	3821.02	3921.58	4022.51	4123.79	4225.43	4327.41	4429.74	4532.40	66
67	3892.52	3993.09	4094.03	4195.34	4297.01	4399.03	4501.41	4604.12	67
68	3964.85	4065.43	4166.39	4267.72	4369.43	4471.49	4573.91	4676.68	68
69	4038.03	4138.62	4239.60	4340.96	4442.70	4544.80	4647.27	4750.10	69
70	4112.10	4212.70	4313.70	4415.09	4516.86	4619.00	4721.52	4824.40	70
71	4187.07	4287.69	4388.71	4490.12	4591.93	4694.12	4796.68	4899.62	71
72	4262.99	4363.62	4464.66	4566.10	4667.94	4770.17	4872.79	4975.78	72
73	4339.88	4440.52	4541.57	4643.04	4744.92	4847.19	4949.86	5052.91	73
74	4417.76	4518.41	4619.49	4720.99	4822.90	4925.22	5027.94	5131.05	74
75	4496.68	4597.34	4698.44	4799.97	4901.91	5004.27	5107.04	5210.22	75
76	4576.66	4677.34	4778.46	4880.01	4981.99	5084.40	5187.22	5290.45	76
77	4657.74	4758.43	4859.57	4961.15	5063.17	5165.62	5268.50	5371.79	77
78	4739.95	4840.65	4941.81	5043.42	5145.48	5247.98	5350.91	5454.27	78
79	4823.34	4924.05	5025.23	5126.87	5228.97	5331.51	5434.49	5537.92	79
80	4907.93	5008.66	5109.86	5211.53	5313.66	5416.25	5519.29	5622.78	80
81	4993.77	5094.51	5195.73	5297.43	5399.60	5502.24	5605.34	5708.89	81
82	5080.90	5181.65	5282.90	5384.63	5486.84	5589.53	5692.68	5796.30	82
83	5169.36	5270.12	5371.39	5473.16	5575.41	5678.15	5781.36	5885.05	83
84	5259.20	5359.97	5461.26	5563.06	5665.35	5768.14	5871.41	5975.17	84
85	5350.46	5451.25	5552.56	5654.39	5756.73	5859.56	5962.90	6066.72	85

TABLE 18. T_s FOR 1° CIRCULAR CURVE, SPIRALED (*Continued*)

θ_s	1°	2°	3°	4°	5°	6°	7°	8°	θ_s
Ignore l_s	200	400	600	800	1000	1200	1400	1600	l_s
Δ									Δ
85°	5350.46	5451.25	5552.56	5654.39	5756.73	5859.56	5962.90	6066.72	85°
86	5443.19	5543.99	5645.33	5747.19	5849.57	5952.46	6055.85	6159.75	86
87	5537.44	5638.26	5739.62	5841.52	5943.94	6046.88	6150.34	6254.30	87
88	5633.27	5734.10	5835.49	5937.42	6039.88	6142.88	6246.40	6350.43	88
89	5730.73	5831.58	5932.99	6034.95	6137.46	6240.51	6344.09	6448.20	89
90	5829.87	5930.73	6032.17	6134.17	6236.72	6339.83	6443.48	6547.66	90
91	5930.76	6031.64	6133.10	6235.13	6337.73	6440.89	6544.61	6648.87	91
92	6033.45	6134.35	6235.84	6337.91	6440.55	6543.77	6647.56	6751.90	92
93	6138.02	6238.93	6340.45	6442.55	6545.25	6648.53	6752.38	6856.80	93
94	6244.53	6345.45	6447.00	6549.15	6651.89	6755.23	6859.15	6963.65	94
95	6353.05	6454.00	6555.57	6657.75	6760.54	6863.94	6967.94	7072.52	95
96	6463.66	6564.62	6666.22	6768.44	6871.29	6974.75	7078.81	7183.48	96
97	6576.44	6677.42	6779.04	6881.30	6984.20	7087.72	7191.86	7296.62	97
98	6691.46	6792.46	6894.11	6996.41	7099.36	7202.95	7307.17	7412.01	98
99	6808.81	6909.83	7011.51	7113.86	7216.86	7320.51	7424.81	7529.74	99
100	6928.59	7029.62	7131.34	7233.73	7336.78	7440.50	7544.87	7649.90	100
101	7050.89	7151.94	7253.68	7356.11	7459.23	7563.01	7667.47	7772.58	101
102	7175.80	7276.87	7378.64	7481.12	7584.29	7688.15	7792.68	7897.89	102
103	7303.43	7404.52	7506.33	7608.85	7712.08	7816.01	7920.63	8025.94	103
104	7433.90	7535.01	7636.85	7739.42	7842.70	7946.70	8051.41	8156.82	104
105	7567.31	7668.44	7770.32	7872.93	7976.28	8080.35	8185.15	8290.66	105
106	7703.79	7804.94	7906.85	8009.52	8112.93	8217.08	8321.96	8427.58	106
107	7843.47	7944.64	8046.59	8149.30	8252.78	8357.01	8461.89	8567.70	107
108	7986.49	8087.68	8189.66	8292.43	8395.97	8500.28	8605.35	8711.17	108
109	8132.98	8234.20	8336.21	8439.03	8542.64	8647.03	8752.20	8858.13	109
110	8283.10	8384.34	8486.40	8589.27	8692.94	8797.42	8902.68	9008.73	110
111	8437.01	8538.27	8640.37	8743.29	8847.04	8951.60	9056.97	9163.14	111
112	8594.88	8696.17	8798.30	8901.28	9005.10	9109.75	9215.22	9321.51	112
113	8756.89	8858.20	8960.38	9063.41	9167.30	9272.04	9377.62	9484.04	113
114	8923.22	9024.56	9126.78	9229.88	9333.84	9438.68	9544.37	9650.90	114
115	9094.09	9195.45	9297.72	9400.88	9504.92	9609.85	9715.65	9822.32	115
116	9269.71	9371.10	9473.40	9576.62	9680.75	9785.77	9891.69	9998.49	116
117	9450.30	9551.71	9654.07	9757.35	9861.56	9966.68	10072.72	10179.66	117
118	9636.09	9737.55	9839.95	9943.30	10047.59	10152.82	10258.98	10366.06	118
119	9827.39	9928.86	10031.31	10134.73	10239.11	10344.44	10450.73	10557.96	119
120	10024.42	10125.93	10228.43	10331.92	10436.39	10541.83	10648.25	10755.62	120
121	10227.51	10329.04	10431.59	10535.16	10639.72	10745.28	10851.83	10959.36	121
122	10436.96	10538.52	10641.13	10744.76	10849.42	10955.10	11061.78	11169.48	122
123	10653.11	10754.70	10857.36	10961.08	11065.83	11171.63	11278.46	11386.31	123
124	10876.32	10977.95	11080.66	11184.46	11289.32	11395.24	11502.21	11610.24	124
125	11106.97	11208.64	11311.42	11415.29	11520.26	11626.31	11733.43	11841.63	125

Table 18. T_s for 1° Circular Curve, Spiraled (*Continued*)

note

θ_s	9°	10°	11°	12°	13°	14°	15°	16°	θ_s
l_s	1800	2000	2200	2400	2600	2800	3000	3200	l_s
Δ									Δ
18°	1810.46								18°
19	1862.00								19
20	1913.69	2014.39							20
21	1965.54	2066.29							21
22	2017.55	2118.35	2219.20						22
23	2069.75	2170.59	2271.50						23
24	2122.12	2223.02	2323.98	2425.00					24
25	2174.70	2275.64	2376.66	2477.74					25
26	2227.47	2328.47	2429.54	2530.68	2631.88				26
27	2280.46	2381.51	2482.64	2583.84	2685.10				27
28	2333.67	2434.77	2535.96	2637.22	2738.55	2839.95			28
29	2387.12	2488.27	2589.51	2690.83	2792.23	2893.70			29
30	2440.80	2542.01	2643.30	2744.69	2846.16	2947.70	3049.31		30
31	2494.74	2596.00	2697.35	2798.80	2900.33	3001.95	3103.64		31
32	2548.94	2650.25	2751.66	2853.17	2954.77	3056.46	3158.23	3260.07	32
33	2603.41	2704.77	2806.24	2907.82	3009.49	3101.25	3213.10	3315.03	33
34	2658.17	2759.58	2861.10	2962.74	3064.48	3166.32	3268.25	3370.26	34
35	2713.21	2814.68	2916.26	3017.96	3119.77	3221.68	3323.69	3425.79	35
36	2768.56	2870.08	2971.72	3073.49	3175.37	3277.36	3379.45	3481.63	36
37	2824.23	2925.80	3027.50	3129.33	3231.28	3333.35	3435.52	3537.79	37
38	2880.22	2981.84	3083.61	3185.50	3287.52	3389.66	3491.91	3594.27	38
39	2936.55	3038.23	3140.05	3242.01	3344.10	3446.32	3548.65	3651.10	39
40	2993.22	3094.96	3196.84	3298.87	3401.03	3503.32	3605.74	3708.28	40
41	3050.26	3152.05	3253.99	3356.09	3458.32	3560.69	3663.19	3765.82	41
42	3107.68	3209.52	3311.52	3413.68	3515.99	3618.44	3721.02	3823.74	42
43	3165.47	3267.37	3369.44	3471.66	3574.04	3676.57	3779.24	3882.04	43
44	3223.67	3325.62	3427.75	3530.04	3632.50	3735.10	3837.86	3940.75	44
45	3282.28	3384.29	3486.48	3588.84	3691.37	3794.05	3896.89	3999.88	45
46	3341.31	3443.38	3545.63	3648.06	3750.66	3853.43	3956.36	4059.44	46
47	3400.79	3502.91	3605.22	3707.72	3810.40	3913.25	4016.26	4119.43	47
48	3460.71	3562.89	3665.27	3767.84	3870.59	3973.52	4076.62	4179.88	48
49	3521.11	3623.35	3725.79	3828.43	3931.25	4034.26	4137.45	4240.81	49
50	3581.99	3684.28	3786.79	3889.50	3992.40	4095.49	4198.77	4302.23	50
51	3643.36	3745.71	3848.28	3951.06	4054.04	4157.22	4260.59	4364.14	51
52	3705.24	3807.66	3910.30	4013.15	4116.21	4219.47	4322.93	4426.57	52
53	3767.66	3870.13	3972.84	4075.76	4178.90	4282.25	4385.80	4489.54	53
54	3830.62	3933.16	4035.93	4138.92	4242.14	4345.57	4449.21	4553.05	54
55	3894.14	3996.74	4099.58	4202.64	4305.95	4409.46	4513.20	4617.14	55
56	3958.25	4060.91	4163.81	4266.95	4370.34	4473.94	4577.77	4681.81	56
57	4022.95	4125.67	4228.64	4331.86	4435.32	4539.01	4642.94	4747.08	57
58	4088.27	4191.05	4294.09	4397.39	4500.93	4604.71	4708.73	4812.97	58

TABLE 18. T_s FOR 1° CIRCULAR CURVE, SPIRALED *(Continued)*

gnore

θ_s	9°	10°	11°	12°	13°	14°	15°	16°	θ_s
l_s	1800	2000	2200	2400	2600	2800	3000	3200	l_s
Δ									Δ
58°	4088.27	4191.05	4294.09	4397.39	4500.93	4604.71	4708.73	4812.97	58°
59	4154.22	4257.07	4360.18	4463.55	4567.17	4671.04	4775.16	4879.50	59
60	4220.83	4323.74	4426.92	4530.37	4634.08	4738.04	4842.25	4946.70	60
61	4288.11	4391.08	4494.33	4597.86	4701.65	4805.71	4910.02	5014.57	61
62	4356.08	4459.12	4562.45	4666.06	4769.93	4874.08	4978.49	5083.15	62
63	4424.78	4527.88	4631.28	4734.97	4838.93	4943.17	5047.68	5152.45	63
64	4494.21	4597.38	4700.85	4804.62	4908.67	5013.01	5117.62	5222.49	64
65	4564.40	4667.64	4771.19	4875.04	4979.18	5083.61	5188.32	5293.30	65
66	4635.38	4738.69	4842.31	4946.24	5050.47	5155.00	5259.81	5364.91	66
67	4707.17	4810.54	4914.24	5018.25	5122.57	5227.20	5332.12	5437.33	67
68	4779.79	4883.23	4987.01	5091.11	5195.52	5300.24	5405.27	5510.59	68
69	4853.27	4956.78	5060.64	5164.82	5269.32	5374.15	5479.28	5584.72	69
70	4927.64	5031.22	5135.16	5239.43	5344.02	5448.95	5554.19	5659.74	70
71	5002.92	5106.58	5210.59	5314.95	5419.64	5524.67	5630.02	5735.69	71
72	5079.15	5182.88	5286.97	5391.42	5496.21	5601.34	5706.80	5812.59	72
73	5156.34	5260.15	5364.32	5468.86	5573.75	5678.98	5784.56	5890.47	73
74	5234.55	5338.43	5442.68	5547.31	5652.30	5757.64	5863.33	5969.36	74
75	5313.78	5417.74	5522.08	5626.80	5731.89	5837.33	5943.14	6049.29	75
76	5394.09	5498.12	5602.55	5707.36	5812.55	5918.10	6024.03	6130.30	76
77	5475.50	5579.61	5684.12	5789.03	5894.32	5999.99	6106.03	6212.43	77
78	5558.05	5662.24	5766.84	5871.84	5977.23	6083.01	6189.17	6295.71	78
79	5641.77	5746.04	5850.73	5955.82	6061.32	6167.22	6273.50	6380.17	79
80	5726.70	5831.05	5935.83	6041.03	6146.63	6252.64	6359.05	6465.85	80
81	5812.89	5917.32	6022.19	6127.49	6233.20	6339.33	6445.87	6552.80	81
82	5900.37	6004.89	6109.85	6215.25	6321.07	6427.32	6533.98	6641.05	82
83	5989.19	6093.80	6198.85	6304.35	6410.29	6516.66	6623.45	6730.65	83
84	6079.39	6184.08	6289.23	6394.84	6500.89	6607.38	6714.30	6821.65	84
85	6171.02	6275.80	6381.05	6486.76	6592.93	6699.54	6806.60	6914.09	85
86	6264.13	6369.00	6474.35	6580.17	6686.45	6793.19	6900.38	7008.02	86
87	6358.77	6463.73	6569.17	6675.10	6781.50	6888.37	6995.70	7103.49	87
88	6454.98	6560.03	6665.58	6771.63	6878.15	6985.15	7092.62	7200.55	88
89	6552.84	6657.98	6763.63	6869.79	6976.43	7083.57	7191.18	7299.27	89
90	6652.38	6757.62	6863.38	6969.65	7076.42	7183.69	7291.45	7399.69	90
91	6753.68	6859.02	6964.88	7071.27	7178.17	7285.57	7393.48	7501.88	91
92	6856.79	6962.23	7068.20	7174.71	7281.74	7389.28	7497.34	7605.90	92
93	6961.78	7067.32	7173.41	7280.04	7387.20	7494.89	7603.10	7711.82	93
94	7068.73	7174.37	7280.57	7387.32	7494.61	7602.45	7710.82	7819.71	94
95	7177.69	7283.43	7389.75	7496.63	7604.06	7712.04	7820.57	7929.63	95
96	7288.75	7394.60	7501.03	7608.04	7715.61	7823.74	7932.43	8041.67	96
97	7401.98	7507.94	7614.49	7721.63	7829.35	7937.63	8046.49	8155.90	97
98	7517.47	7623.54	7730.21	7837.49	7945.35	8053.79	8162.81	8272.40	98

TABLE 18. T_s FOR 1° CIRCULAR CURVE, SPIRALED (*Continued*)

θ_s	17°	18°	19°	20°	21°	22°	23°	24°	θ_s
l_s	3400	3600	3800	4000	4200	4400	4600	4800	l_s
Δ									Δ
34°	3472.35								34°
35	3527.98								35
36	3583.91	3686.26							36
37	3640.15	3742.61							37
38	3696.73	3799.28	3901.93						38
39	3753.65	3856.30	3959.05						39
40	3810.92	3913.67	4016.52	4119.47					40
41	3868.56	3971.41	4074.37	4177.42					41
42	3926.57	4029.52	4132.59	4235.75	4339.02				42
43	3984.98	4088.03	4191.20	4294.48	4397.86				43
44	4043.78	4146.94	4250.22	4353.61	4457.11	4560.72			44
45	4103.01	4206.27	4309.65	4413.16	4516.78	4620.51			45
46	4162.66	4266.02	4369.52	4473.14	4576.88	4680.74	4784.71		46
47	4222.75	4326.22	4429.83	4533.57	4637.44	4741.42	4845.52		47
48	4283.31	4386.88	4490.60	4594.46	4698.45	4802.56	4906.79	5011.14	48
49	4344.34	4448.01	4551.84	4655.82	4759.93	4864.18	4968.55	5073.03	49
50	4405.85	4509.63	4613.58	4717.67	4821.91	4926.29	5030.79	5135.42	50
51	4467.86	4571.76	4675.82	4780.03	4884.39	4988.90	5093.55	5198.32	51
52	4530.40	4634.40	4738.58	4842.91	4947.40	5052.04	5156.83	5261.74	52
53	4593.47	4697.58	4801.87	4906.33	5010.95	5115.72	5220.65	5325.71	53
54	4657.09	4761.31	4865.72	4970.30	5075.05	5179.96	5285.02	5390.24	54
55	4721.28	4825.61	4930.14	5034.84	5139.72	5244.77	5349.98	5455.34	55
56	4786.05	4890.50	4995.14	5099.97	5204.98	5310.17	5415.52	5521.03	56
57	4851.43	4955.99	5060.76	5165.71	5270.86	5376.18	5481.68	5587.34	57
58	4917.43	5022.11	5126.99	5232.08	5337.36	5442.82	5548.47	5654.28	58
59	4984.08	5088.87	5193.88	5299.09	5404.50	5510.11	5615.90	5721.87	59
60	5051.38	5156.29	5261.42	5366.76	5472.31	5578.06	5684.01	5790.14	60
61	5119.37	5224.40	5329.65	5435.12	5540.81	5646.71	5752.80	5859.09	61
62	5188.06	5293.20	5398.58	5504.19	5610.02	5716.06	5822.31	5928.76	62
63	5257.47	5362.74	5468.25	5573.99	5679.96	5786.15	5892.55	5999.16	63
64	5327.63	5433.02	5538.66	5644.53	5750.65	5856.98	5963.54	6070.31	64
65	5398.56	5504.07	5609.84	5715.85	5822.11	5928.60	6035.31	6142.25	65
66	5470.28	5575.92	5681.82	5787.97	5894.37	6001.01	6107.89	6214.99	66
67	5542.82	5648.58	5754.62	5860.91	5967.46	6074.25	6181.29	6288.56	67
68	5616.20	5722.09	5828.26	5934.69	6041.39	6148.35	6255.55	6362.99	68
69	5690.45	5796.47	5902.78	6009.36	6116.21	6223.32	6330.68	6438.30	69
70	5765.60	5871.75	5978.19	6084.92	6191.92	6299.19	6406.72	6514.51	70
71	5841.67	5947.95	6054.54	6161.41	6268.56	6376.00	6483.70	6591.67	71
72	5918.69	6025.11	6131.84	6238.86	6346.17	6453.77	6561.64	6669.79	72
73	5996.70	6103.25	6210.12	6317.29	6424.76	6532.53	6640.58	6748.90	73
74	6075.72	6182.41	6289.42	6396.75	6504.38	6612.32	6720.54	6829.05	74

TABLE 18. T_s FOR 1° CIRCULAR CURVE, SPIRALED (*Continued*)

θ_s	17°	18°	19°	20°	21°	22°	23°	24°	θ_s
Ignore l_s	3400	3600	3800	4000	4200	4400	4600	4800	l_s
Δ									Δ
75°	6155.79	6262.62	6369.78	6477.26	6585.05	6693.16	6801.56	6910.26	75°
76	6236.93	6343.91	6451.22	6558.85	6666.81	6775.09	6883.67	6992.56	76
77	6319.20	6426.31	6533.77	6641.57	6749.70	6858.15	6966.91	7075.99	77
78	6402.61	6509.87	6617.48	6725.44	6833.74	6942.36	7051.32	7160.59	78
79	6487.21	6594.61	6702.38	6810.50	6918.97	7027.78	7136.92	7246.39	79
80	6573.03	6680.59	6788.51	6896.80	7005.44	7114.44	7223.77	7333.43	80
81	6660.12	6767.83	6875.92	6984.37	7093.19	7202.37	7311.90	7421.76	81
82	6748.52	6856.38	6964.63	7073.26	7182.26	7291.62	7401.34	7511.42	82
83	6838.27	6946.29	7054.70	7163.50	7272.68	7382.24	7492.16	7602.45	83
84	6929.41	7037.59	7146.17	7255.15	7364.52	7474.27	7584.39	7694.89	84
85	7022.01	7130.34	7239.09	7348.25	7457.81	7567.76	7678.09	7788.80	85
86	7116.08	7224.59	7333.52	7442.85	7552.60	7662.75	7773.29	7884.22	86
87	7211.72	7320.39	7429.49	7539.01	7648.95	7759.30	7870.06	7981.21	87
88	7308.94	7417.78	7527.06	7636.77	7746.91	7857.47	7968.44	8079.81	88
89	7407.82	7516.83	7626.29	7736.19	7846.53	7957.30	8068.49	8180.10	89
90	7508.41	7617.59	7727.23	7837.33	7947.87	8058.86	8170.28	8282.11	90
91	7610.76	7720.13	7829.96	7940.26	8051.01	8162.21	8273.85	8385.93	91
92	7714.96	7824.50	7934.52	8045.02	8155.99	8267.41	8379.28	8491.60	92
93	7821.05	7930.78	8041.00	8151.70	8262.88	8374.53	8486.64	8599.21	93
94	7929.11	8039.03	8149.45	8260.36	8371.76	8483.64	8595.99	8708.81	94
95	8039.22	8149.32	8259.94	8371.07	8482.69	8594.81	8707.41	8820.48	95
96	8151.44	8261.74	8372.57	8483.91	8595.76	8708.11	8820.96	8934.29	96
97	8265.86	8376.36	8487.40	8598.96	8711.04	8823.64	8936.74	9050.34	97
98	8382.55	8493.26	8604.51	8716.30	8828.62	8941.47	9054.83	9168.70	98
99	8501.62	8612.53	8724.00	8836.02	8948.58	9061.68	9175.31	9289.46	99
100	8623.14	8734.27	8845.96	8958.21	9071.02	9184.38	9298.28	9412.71	100
101	8747.22	8858.56	8970.48	9082.98	9196.04	9309.66	9423.83	9538.55	101
102	8873.95	8985.51	9097.67	9210.41	9323.73	9437.62	9552.08	9667.09	102
103	9003.44	9115.23	9227.62	9340.61	9454.20	9568.37	9683.11	9798.42	103
104	9135.81	9247.83	9360.46	9473.71	9587.56	9702.01	9817.05	9932.67	104
105	9271.17	9383.42	9496.30	9609.81	9723.94	9838.68	9954.02	10070.96	105
106	9409.64	9522.13	9635.27	9749.05	9863.46	9978.49	10094.14	10210.40	106
107	9551.36	9664.10	9777.50	9891.55	10006.24	10121.58	10237.54	10354.13	107
108	9696.46	9809.45	9923.11	10037.45	10152.44	10268.08	10384.37	10501.29	108
109	9845.08	9958.33	10072.27	10186.89	10302.19	10418.15	10534.77	10652.04	109
110	9997.39	10110.91	10225.13	10340.04	10455.64	10571.93	10688.89	10806.51	110
111	10153.55	10267.33	10381.84	10497.06	10612.98	10729.60	10846.90	10964.89	111
112	10313.72	10427.78	10542.58	10658.11	10774.35	10891.31	11008.98	11127.33	112
113	10478.08	10592.44	10707.54	10823.38	10939.96	11057.27	11175.30	11294.04	113
114	10646.85	10761.49	10876.90	10993.07	11110.00	11227.67	11346.07	11465.20	114

Simple curve Deg. of
Curvature.

TABLE 19. E_s FOR 1° CIRCULAR CURVE, SPIRALED

θ_s	1°	2°	3°	4°	5°	6°	7°	8°	θ_s
ignore l_s	200	400	600	800	1000	1200	1400	1600	l_s
Δ									Δ
4°	3.8	4.7							4°
5	5.8	6.6							5
6	8.2	9.0	10.5						6
7	11.0	11.9	13.3						7
8	14.3	15.2	16.6	18.7					8
9	18.0	18.9	20.3	22.4					9
10	22.2	23.1	24.5	26.6	29.2				10
11	26.8	27.7	29.1	31.2	33.8				11
12	31.9	32.7	34.2	36.2	38.9	42.1			12
13	37.4	38.2	39.7	41.8	44.4	47.6			13
14	43.3	44.2	45.7	47.7	50.4	53.6	57.4		14
15	49.7	50.6	52.1	54.1	56.8	60.0	63.8		15
16	56.6	57.5	59.0	61.0	63.6	66.9	70.7	75.1	16
17	63.9	64.8	66.3	68.3	71.0	74.2	78.0	82.4	17
18	71.7	72.6	74.1	76.1	78.8	82.0	85.8	90.3	18
19	80.0	80.9	82.3	84.4	87.0	90.3	94.1	98.5	19
20	88.7	89.6	91.0	93.1	95.8	99.0	102.9	107.3	20
21	97.9	98.8	100.2	102.3	105.0	108.2	112.1	116.5	21
22	107.5	108.4	109.9	112.0	114.6	117.9	121.8	126.2	22
23	117.7	118.6	120.1	122.1	124.8	128.1	131.9	136.4	23
24	128.3	129.2	130.7	132.8	135.4	138.7	142.6	147.0	24
25	139.4	140.3	141.8	143.9	146.6	149.8	153.7	158.2	25
26	151.0	151.9	153.4	155.5	158.2	161.5	165.3	169.8	26
27	163.1	164.0	165.5	167.6	170.3	173.6	177.5	181.9	27
28	175.7	176.6	178.1	180.2	182.9	186.2	190.1	194.6	28
29	188.8	189.7	191.2	193.3	196.0	199.3	203.2	207.7	29
30	202.4	203.3	204.8	206.9	209.6	213.0	216.9	221.4	30
31	216.6	217.5	219.0	221.1	223.8	227.1	231.0	235.6	31
32	231.2	232.1	233.6	235.7	238.5	241.8	245.7	250.3	32
33	246.4	247.3	248.8	250.9	253.7	257.0	260.9	265.5	33
34	262.1	263.0	264.5	266.7	269.4	272.7	276.7	281.2	34
35	278.4	279.3	280.8	282.9	285.7	289.0	293.0	297.6	35
36	295.2	296.1	297.6	299.7	302.5	305.9	309.8	314.4	36
37	312.6	313.4	315.0	317.1	319.9	323.3	327.2	331.8	37
38	330.4	331.4	332.9	335.1	337.8	341.2	345.2	349.8	38
39	348.9	349.9	351.4	353.6	356.3	359.7	363.8	368.4	39
40	368.0	368.9	370.5	372.7	375.4	378.9	382.9	387.5	40
41	387.7	388.6	390.2	392.3	395.1	398.5	402.6	407.2	41
42	407.9	408.9	410.4	412.6	415.4	418.8	422.9	427.6	42
43	428.8	429.7	431.3	433.5	436.3	439.7	443.8	448.5	43
44	450.3	451.2	452.8	455.0	457.8	461.3	465.3	470.0	44
45	472.4	473.3	474.9	477.1	479.9	483.4	487.5	492.2	45

TABLE 19. E_s FOR 1° CIRCULAR CURVE, SPIRALED (*Continued*)

Ignore

θ_s	1°	2°	3°	4°	5°	6°	7°	8°	θ_s
l_s	200	400	600	800	1000	1200	1400	1600	l_s
Δ									Δ
45°	472.4	473.3	474.9	477.1	479.9	483.4	487.5	492.2	45°
46	495.1	496.1	497.7	499.9	502.7	506.2	510.3	515.0	46
47	518.5	519.5	521.0	523.3	526.1	529.6	533.7	538.5	47
48	542.5	543.5	545.1	547.3	550.2	553.7	557.8	562.6	48
49	567.2	568.2	569.8	572.0	574.9	578.4	582.6	587.4	49
50	592.6	593.6	595.2	597.4	600.3	603.9	608.0	612.8	50
51	618.7	619.7	621.3	623.5	626.4	630.0	634.2	639.0	51
52	645.5	646.5	648.1	650.3	653.2	656.8	661.0	665.9	52
53	673.0	674.0	675.6	677.9	680.8	684.4	688.6	693.4	53
54	701.1	702.2	703.8	706.1	709.0	712.6	716.9	721.8	54
55	730.2	731.2	732.8	735.1	738.0	741.6	745.9	750.8	55
56	759.9	760.9	762.5	764.8	767.8	771.4	775.7	780.6	56
57	790.4	791.4	793.0	795.4	798.3	802.0	806.3	811.2	57
58	821.7	822.7	824.4	826.7	829.7	833.3	837.6	842.6	58
59	853.8	854.8	856.5	858.8	861.8	865.5	869.8	874.8	59
60	886.7	887.7	889.4	891.7	894.8	898.5	902.8	907.9	60
61	920.5	921.5	923.2	925.5	928.6	932.3	936.7	941.7	61
62	955.1	956.1	957.8	960.2	963.2	967.0	971.4	976.4	62
63	990.6	991.6	993.3	995.7	998.8	1002.5	1006.9	1012.0	63
64	1027.0	1028.0	1029.7	1032.1	1035.2	1039.0	1043.4	1048.6	64
65	1064.3	1065.3	1067.0	1069.4	1072.5	1076.3	1080.8	1086.0	65
66	1102.5	1103.5	1105.3	1107.7	1110.8	1114.6	1119.1	1124.3	66
67	1141.7	1142.7	1144.5	1146.9	1150.1	1153.9	1158.4	1163.7	67
68	1181.9	1182.9	1184.7	1187.2	1190.3	1194.2	1198.7	1204.0	68
69	1223.1	1224.1	1225.9	1228.4	1231.5	1235.4	1240.0	1245.3	69
70	1265.3	1266.4	1268.1	1270.6	1273.8	1277.7	1282.3	1287.7	70
71	1308.6	1309.6	1311.4	1313.9	1317.1	1321.1	1325.7	1331.1	71
72	1352.9	1354.0	1355.8	1358.3	1361.6	1365.5	1370.2	1375.6	72
73	1398.4	1399.5	1401.3	1403.8	1407.1	1411.1	1415.8	1421.2	73
74	1445.0	1446.1	1447.9	1450.5	1453.7	1457.7	1462.5	1467.9	74
75	1492.8	1493.9	1495.7	1498.3	1501.6	1505.6	1510.4	1515.8	75
76	1541.7	1542.8	1544.7	1547.3	1550.6	1554.6	1559.5	1565.0	76
77	1591.9	1593.0	1594.9	1597.5	1600.8	1604.9	1609.8	1615.3	77
78	1643.4	1644.5	1646.4	1649.0	1652.4	1656.5	1661.3	1666.9	78
79	1696.1	1697.3	1699.2	1701.8	1705.2	1709.3	1714.2	1719.9	79
80	1750.2	1751.4	1753.3	1755.9	1759.4	1763.5	1768.4	1774.1	80
81	1805.7	1806.8	1808.8	1811.4	1814.9	1819.1	1824.0	1829.8	81
82	1862.6	1863.7	1865.7	1868.3	1871.8	1876.0	1881.0	1886.8	82
83	1920.9	1922.1	1924.0	1926.7	1930.2	1934.5	1939.5	1945.4	83
84	1980.7	1981.9	1983.9	1986.6	1990.1	1994.4	1999.5	2005.4	84
85	2042.1	2043.3	2045.2	2048.0	2051.5	2055.9	2061.0	2066.9	85

TABLE 19. E_s FOR 1° CIRCULAR CURVE, SPIRALED (*Continued*)

θ_s	1°	2°	3°	4°	5°	6°	7°	8°	θ_s
ignore l_s	200	400	600	800	1000	1200	1400	1600	l_s
Δ									Δ
85°	2042.1	2043.3	2045.2	2048.0	2051.5	2055.9	2061.0	2066.9	85°
86	2105.0	2106.2	2108.2	2111.0	2114.6	2118.9	2124.1	2130.1	86
87	2169.6	2170.8	2172.8	2175.6	2179.2	2183.6	2188.8	2194.9	87
88	2235.9	2237.1	2239.1	2241.9	2245.6	2250.0	2255.3	2261.3	88
89	2303.9	2305.1	2307.1	2310.0	2313.7	2318.2	2323.4	2329.6	89
90	2373.7	2374.9	2377.0	2379.8	2383.6	2388.1	2393.4	2399.6	90
91	2445.3	2446.6	2448.6	2451.5	2455.3	2459.8	2465.2	2471.5	91
92	2518.9	2520.1	2522.2	2525.1	2528.9	2533.5	2539.0	2545.3	92
93	2594.4	2595.7	2597.8	2600.7	2604.6	2609.2	2614.7	2621.0	93
94	2672.0	2673.3	2675.4	2678.4	2682.2	2686.9	2692.5	2698.9	94
95	2751.7	2753.0	2755.1	2758.1	2762.0	2766.8	2772.4	2778.8	95
96	2833.6	2834.9	2837.1	2840.1	2844.0	2848.8	2854.4	2860.9	96
97	2917.7	2919.0	2921.2	2924.3	2928.2	2933.1	2938.8	2945.4	97
98	3004.2	3005.5	3007.7	3010.8	3014.8	3019.7	3025.5	3032.1	98
99	3093.1	3094.4	3096.7	3099.8	3103.8	3108.8	3114.6	3121.3	99
100	3184.5	3185.9	3188.1	3191.3	3195.4	3200.3	3206.2	3213.0	100
101	3278.5	3279.9	3282.2	3285.4	3289.5	3294.5	3300.5	3307.3	101
102	3375.3	3376.7	3379.0	3382.2	3386.4	3391.4	3397.4	3404.4	102
103	3474.8	3476.2	3478.6	3481.8	3486.0	3491.2	3497.2	3504.2	103
104	3577.3	3578.7	3581.1	3584.3	3588.6	3593.8	3599.9	3607.0	104
105	3682.8	3684.2	3686.6	3689.9	3694.2	3699.5	3705.7	3712.8	105
106	3791.4	3792.9	3795.3	3798.6	3803.0	3808.3	3814.6	3821.8	106
107	3903.3	3904.8	3907.2	3910.6	3915.1	3920.4	3926.8	3934.1	107
108	4018.7	4020.1	4022.6	4026.1	4030.5	4036.0	4042.4	4049.8	108
109	4137.6	4139.0	4141.6	4145.0	4149.6	4155.1	4161.6	4169.1	109
110	4260.1	4261.7	4264.2	4267.7	4272.3	4277.9	4284.5	4292.1	110
111	4386.6	4388.2	4390.7	4394.3	4398.9	4404.6	4411.2	4418.9	111
112	4517.1	4518.7	4521.3	4524.9	4529.6	4535.3	4542.1	4549.8	112
113	4651.8	4653.4	4656.0	4659.7	4664.4	4670.2	4677.1	4685.0	113
114	4790.9	4792.5	4795.2	4798.9	4803.7	4809.6	4816.5	4824.5	114
115	4934.6	4936.2	4938.9	4942.7	4947.6	4953.6	4960.6	4968.7	115
116	5083.1	5084.8	5087.5	5091.4	5096.3	5102.3	5109.5	5117.7	116
117	5236.7	5238.4	5241.2	5245.1	5250.1	5256.2	5263.4	5271.8	117
118	5395.6	5397.3	5400.1	5404.0	5409.1	5415.3	5422.7	5431.1	118
119	5560.0	5561.7	5564.5	5568.5	5573.7	5580.0	5587.4	5596.0	119
120	5730.2	5731.9	5734.8	5738.9	5744.1	5750.5	5758.1	5766.8	120
121	5906.5	5908.2	5911.2	5915.4	5920.7	5927.1	5934.8	5943.7	121
122	6089.2	6091.0	6094.0	6098.2	6103.6	6110.2	6118.0	6127.0	122
123	6278.8	6280.6	6283.6	6287.9	6293.4	6300.1	6308.0	6317.1	123
124	6475.4	6477.2	6480.3	6484.6	6490.2	6497.0	6505.1	6514.4	124
125	6679.5	6681.4	6684.5	6688.9	6694.6	6701.5	6709.7	6719.1	125

TABLE 19. E_s FOR 1° CIRCULAR CURVE, SPIRALED *(Continued)*

θ_s	9°	10°	11°	12°	13°	14°	15°	16°	θ_s
gnore l_s	1800	2000	2200	2400	2600	2800	3000	3200	l_s
Δ									Δ
18°	95.2								18°
19	103.5								19
20	112.3	117.9							20
21	121.5	127.1							21
22	131.2	136.8	143.0						22
23	141.4	147.0	153.2						23
24	152.1	157.7	163.9	170.8					24
25	163.2	168.9	175.1	181.9					25
26	174.9	180.5	186.8	193.6	201.1				26
27	187.0	192.7	199.0	205.8	213.3				27
28	199.7	205.3	211.6	218.5	226.0	234.0			28
29	212.8	218.5	224.8	231.7	239.2	247.3			29
30	226.5	232.2	238.5	245.4	252.9	261.0	269.7		30
31	240.7	246.4	252.7	259.6	267.2	275.3	284.0		31
32	255.4	261.1	267.4	274.4	281.9	290.1	298.8	308.1	32
33	270.6	276.4	282.7	289.7	297.3	305.4	314.1	323.5	33
34	286.4	292.2	298.5	305.5	313.1	321.3	330.1	339.5	34
35	302.7	308.5	314.9	321.9	329.5	337.7	346.5	355.9	35
36	319.6	325.4	331.8	338.8	346.5	354.7	363.5	372.9	36
37	337.0	342.9	349.3	356.3	364.0	372.2	381.1	390.5	37
38	355.0	360.9	367.3	374.4	382.0	390.3	399.2	408.7	38
39	373.6	379.5	385.9	393.0	400.7	409.0	417.9	427.4	39
40	392.8	398.6	405.1	412.2	419.9	428.3	437.2	446.7	40
41	412.5	418.4	424.9	432.0	439.7	448.1	457.1	466.7	41
42	432.8	438.8	445.3	452.4	460.2	468.6	477.6	487.2	42
43	453.8	459.7	466.3	473.4	481.2	489.6	498.7	508.3	43
44	475.4	481.3	487.9	495.1	502.9	511.3	520.4	530.1	44
45	497.6	503.5	510.1	517.3	525.2	533.7	542.7	552.5	45
46	520.4	526.4	533.0	540.2	548.1	556.6	565.7	575.5	46
47	543.9	549.9	556.5	563.8	571.7	580.2	589.4	599.2	47
48	568.0	574.0	580.7	588.0	595.9	604.5	613.7	623.5	48
49	592.8	598.9	605.6	612.9	620.9	629.5	638.7	648.5	49
50	618.3	624.4	631.1	638.5	646.5	655.1	664.4	674.2	50
51	644.5	650.6	657.3	664.7	672.8	681.4	690.7	700.7	51
52	671.4	677.5	684.3	691.7	699.8	708.5	717.8	727.8	52
53	699.0	705.1	711.9	719.4	727.5	736.2	745.6	755.6	53
54	727.3	733.5	740.3	747.8	755.9	764.7	774.2	784.2	54
55	756.4	762.6	769.5	777.0	785.2	794.0	803.5	813.6	55
56	786.2	792.5	799.4	806.9	815.1	824.0	833.5	843.7	56
57	816.9	823.1	830.1	837.7	845.9	854.8	864.4	874.6	57
58	848.3	854.6	861.5	869.2	877.5	886.4	896.0	906.3	58

TABLE 19. E_s FOR 1° CIRCULAR CURVE, SPIRALED (*Continued*)

θ_s	9°	10°	11°	12°	13°	14°	15°	16°	θ_s
Ignore l_s	1800	2000	2200	2400	2600	2800	3000	3200	l_s
Δ									Δ
58°	848.3	854.6	861.5	869.2	877.5	886.4	896.0	906.3	58°
59	880.5	886.8	893.8	901.5	909.8	918.8	928.5	938.8	59
60	913.6	919.9	927.0	934.7	943.0	952.1	961.8	972.1	60
61	947.5	953.9	960.9	968.7	977.1	986.2	995.9	1006.3	61
62	982.2	988.6	995.7	1003.5	1012.0	1021.1	1030.9	1041.4	62
63	1017.8	1024.3	1031.5	1039.3	1047.8	1057.0	1066.8	1077.3	63
64	1054.4	1060.9	1068.1	1075.9	1084.5	1093.7	1103.6	1114.2	64
65	1091.8	1098.4	1105.6	1113.5	1122.1	1131.4	1141.3	1152.0	65
66	1130.2	1136.8	1144.1	1152.0	1160.7	1170.0	1180.0	1190.7	66
67	1169.6	1176.2	1183.5	1191.5	1200.2	1209.6	1219.7	1230.4	67
68	1209.9	1216.6	1223.9	1232.0	1240.7	1250.2	1260.3	1271.1	68
69	1251.3	1258.0	1265.4	1273.5	1282.3	1291.8	1302.0	1312.8	69
70	1293.7	1300.4	1307.9	1316.0	1324.8	1334.4	1344.7	1355.6	70
71	1337.1	1343.9	1351.4	1359.6	1368.5	1378.1	1388.4	1399.4	71
72	1381.7	1388.5	1396.0	1404.3	1413.2	1422.9	1433.3	1444.4	72
73	1427.3	1434.2	1441.8	1450.1	1459.1	1468.8	1479.3	1490.4	73
74	1474.1	1481.0	1488.6	1497.0	1506.1	1515.9	1526.4	1537.6	74
75	1522.1	1529.0	1536.7	1545.1	1554.2	1564.1	1574.7	1586.0	75
76	1571.2	1578.2	1586.0	1594.4	1603.6	1613.6	1624.2	1635.6	76
77	1621.6	1628.7	1636.5	1645.0	1654.3	1664.3	1675.0	1686.4	77
78	1673.3	1680.4	1688.2	1696.8	1706.2	1716.2	1727.0	1738.5	78
79	1726.3	1733.4	1741.3	1750.0	1759.4	1769.5	1780.4	1792.0	79
80	1780.6	1787.8	1795.7	1804.5	1813.9	1824.1	1835.1	1846.8	80
81	1836.3	1843.5	1851.5	1860.3	1869.8	1880.1	1891.2	1903.0	81
82	1893.4	1900.7	1908.8	1917.6	1927.2	1937.6	1948.7	1960.6	82
83	1951.9	1959.3	1967.4	1976.3	1986.0	1996.5	2007.7	2019.7	83
84	2012.0	2019.4	2027.6	2036.6	2046.4	2056.9	2068.2	2080.3	84
85	2073.6	2081.1	2089.4	2098.4	2108.2	2118.8	2130.2	2142.4	85
86	2136.8	2144.4	2152.7	2161.8	2171.7	2182.4	2193.9	2206.2	86
87	2201.7	2209.3	2217.7	2226.9	2236.9	2247.6	2259.2	2271.6	87
88	2268.2	2275.9	2284.3	2293.6	2303.7	2314.5	2326.2	2338.7	88
89	2336.5	2344.2	2352.7	2362.1	2372.3	2383.2	2395.0	2407.6	89
90	2406.6	2414.4	2423.0	2432.4	2442.7	2453.7	2465.6	2478.3	90
91	2478.5	2486.4	2495.1	2504.6	2514.9	2526.1	2538.1	2550.9	91
92	2552.4	2560.3	2569.1	2578.7	2589.1	2600.4	2612.5	2625.4	92
93	2628.2	2636.2	2645.1	2654.8	2665.3	2676.7	2688.9	2701.9	93
94	2706.1	2714.2	2723.1	2732.9	2743.5	2755.0	2767.3	2780.5	94
95	2786.1	2794.3	2803.3	2813.2	2823.9	2835.5	2847.9	2861.2	95
96	2868.3	2876.6	2885.7	2895.0	2906.5	2918.2	2930.7	2944.1	96
97	2952.8	2961.1	2970.3	2980.4	2991.3	3003.1	3015.8	3029.3	97
98	3039.6	3048.0	3057.3	3067.5	3078.5	3090.5	3103.3	3116.9	98

TABLE 19. E_s FOR 1° CIRCULAR CURVE, SPIRALED (*Continued*)

Ignore

θ_s	17°	18°	19°	20°	21°	22°	23°	24°	θ_s
l_s	3400	3600	3800	4000	4200	4400	4600	4800	l_s
Δ									Δ
34°	349.4								34°
35	365.9								35
36	383.0	393.6							36
37	400.6	411.3							37
38	418.8	429.5	440.8						38
39	437.6	448.3	459.6						39
40	456.9	467.7	479.0	491.0					40
41	476.8	487.6	499.0	511.1					41
42	497.4	508.2	519.7	531.7	544.4				42
43	518.6	529.4	540.9	553.0	565.7				43
44	540.4	551.3	562.8	574.9	587.7	601.0			44
45	562.8	573.7	585.3	597.5	610.3	623.7			45
46	585.8	596.8	608.4	620.7	633.5	647.0	661.0		46
47	609.6	620.6	632.2	644.5	657.4	670.9	685.0		47
48	634.0	645.0	656.7	669.0	682.0	695.5	709.7	724.5	48
49	659.0	670.1	681.9	694.2	707.2	720.8	735.1	749.9	49
50	684.8	695.9	707.7	720.1	733.2	746.8	761.1	776.0	50
51	711.2	722.4	734.3	746.7	759.8	773.6	787.9	802.9	51
52	738.4	749.6	761.5	774.0	787.2	801.0	815.4	830.4	52
53	766.3	777.6	789.5	802.1	815.3	829.1	843.6	858.7	53
54	794.9	806.3	818.3	830.9	844.2	858.1	872.6	887.8	54
55	824.3	835.7	847.8	860.5	873.8	887.7	902.3	917.6	55
56	854.5	865.9	878.0	890.8	904.2	918.2	932.8	948.1	56
57	885.4	896.9	909.1	921.9	935.3	949.4	964.2	979.5	57
58	917.2	928.7	940.9	953.8	967.3	981.5	996.3	1011.7	58
59	949.7	961.3	973.6	986.5	1000.1	1014.4	1029.2	1044.8	59
60	983.1	994.8	1007.1	1020.1	1033.8	1048.1	1063.0	1078.6	60
61	1017.4	1029.1	1041.5	1054.6	1068.3	1082.7	1097.7	1113.4	61
62	1052.5	1064.3	1076.8	1089.9	1103.7	1118.1	1133.2	1149.0	62
63	1088.5	1100.4	1112.9	1126.1	1140.0	1154.5	1169.7	1185.5	63
64	1125.4	1137.4	1150.0	1163.2	1177.2	1191.8	1207.0	1223.0	64
65	1163.3	1175.3	1187.9	1201.3	1215.3	1230.0	1245.3	1261.3	65
66	1202.1	1214.1	1226.9	1240.3	1254.4	1269.2	1284.6	1300.7	66
67	1241.9	1254.0	1266.8	1280.3	1294.5	1309.3	1324.8	1341.0	67
68	1282.6	1294.8	1307.7	1321.3	1335.5	1350.5	1366.1	1382.4	68
69	1324.4	1336.7	1349.7	1363.3	1377.6	1392.7	1408.4	1424.8	69
70	1367.3	1379.6	1392.6	1406.4	1420.8	1435.9	1451.7	1468.2	70
71	1411.2	1423.6	1436.6	1450.5	1465.0	1480.2	1496.1	1512.7	71
72	1456.2	1468.7	1481.9	1495.8	1510.4	1525.7	1541.7	1558.4	72
73	1502.3	1514.9	1528.2	1542.1	1556.9	1572.3	1588.4	1605.2	73
74	1549.6	1562.2	1575.6	1589.7	1604.5	1620.0	1636.2	1653.1	74

TABLE 19. E_s FOR 1° CIRCULAR CURVE, SPIRALED (*Continued*)

ignore

θ_s	17°	18°	19°	20°	21°	22°	23°	24°	θ_s
l_s	3400	3600	3800	4000	4200	4400	4600	4800	l_s
Δ									Δ
75°	1598.1	1610.8	1624.2	1638.4	1653.3	1668.9	1685.2	1702.3	**75°**
76	1647.7	1660.5	1674.1	1688.4	1703.4	1719.1	1735.5	1752.7	**76**
77	1698.6	1711.5	1725.2	1739.6	1754.7	1770.5	1787.0	1804.3	**77**
78	1750.8	1763.9	1777.6	1792.1	1807.3	1823.2	1839.9	1857.3	**78**
79	1804.4	1817.5	1831.3	1845.9	1861.2	1877.3	1894.1	1911.6	**79**
80	1859.3	1872.5	1886.4	1901.1	1916.5	1932.7	1949.6	1967.2	**80**
81	1915.5	1928.8	1942.9	1957.7	1973.2	1989.5	2006.5	2024.3	**81**
82	1973.2	1986.6	2000.8	2015.7	2031.4	2047.8	2064.9	2082.8	**82**
83	2032.4	2045.9	2060.2	2075.2	2091.0	2107.5	2124.8	2142.8	**83**
84	2093.1	2106.7	2121.1	2136.2	2152.1	2168.8	2186.2	2204.4	**84**
85	2155.4	2169.1	2183.6	2198.8	2214.8	2231.6	2249.2	2267.5	**85**
86	2219.2	2233.0	2247.6	2263.0	2279.2	2296.1	2313.8	2332.3	**86**
87	2284.8	2298.7	2313.4	2328.9	2345.2	2362.3	2380.1	2398.7	**87**
88	2352.0	2366.0	2380.9	2396.5	2412.9	2430.2	2448.2	2466.9	**88**
89	2421.0	2435.1	2450.1	2465.9	2482.5	2499.9	2518.0	2536.9	**89**
90	2491.8	2506.1	2521.2	2537.1	2553.8	2571.3	2589.6	2608.7	**90**
91	2564.5	2578.9	2594.1	2610.2	2627.1	2644.7	2663.2	2682.5	**91**
92	2639.1	2653.7	2669.1	2685.3	2702.3	2720.1	2738.7	2758.2	**92**
93	2715.7	2730.4	2746.0	2762.3	2779.5	2797.5	2816.3	2835.9	**93**
94	2794.5	2809.3	2825.0	2841.5	2858.8	2876.9	2895.9	2915.7	**94**
95	2875.3	2890.3	2906.1	2922.8	2940.3	2958.6	2977.7	2997.7	**95**
96	2958.4	2973.5	2989.5	3006.3	3024.0	3042.5	3061.8	3082.0	**96**
97	3043.7	3059.0	3075.1	3092.1	3109.9	3128.6	3148.2	3168.6	**97**
98	3131.5	3146.9	3163.2	3180.3	3198.3	3217.2	3237.0	3257.6	**98**
99	3221.7	3237.3	3253.7	3271.0	3289.2	3308.3	3328.2	3349.0	**99**
100	3314.4	3330.2	3346.8	3364.3	3382.7	3402.0	3422.1	3443.1	**100**
101	3409.8	3425.7	3442.5	3460.2	3478.8	3498.3	3518.6	3539.8	**101**
102	3508.0	3524.0	3541.0	3558.9	3577.7	3597.3	3617.9	3639.4	**102**
103	3609.0	3625.2	3642.4	3660.5	3679.4	3699.3	3720.1	3741.8	**103**
104	3712.9	3729.3	3746.7	3765.0	3784.2	3804.3	3825.3	3847.2	**104**
105	3819.9	3836.5	3854.1	3872.6	3892.0	3912.3	3933.6	3955.8	**105**
106	3930.2	3947.0	3964.7	3983.4	4003.0	4023.6	4045.1	4067.6	**106**
107	4043.7	4060.7	4078.7	4097.6	4117.5	4138.3	4160.0	4182.7	**107**
108	4160.7	4177.9	4196.1	4215.3	4235.4	4256.5	4278.5	4301.4	**108**
109	4281.4	4298.8	4317.2	4336.6	4356.9	4378.2	4400.5	4423.8	**109**
110	4405.7	4423.4	4442.0	4461.6	4482.2	4503.8	4526.4	4549.9	**110**
111	4534.0	4551.9	4570.8	4590.7	4611.5	4633.3	4656.2	4680.1	**111**
112	4666.4	4684.5	4703.6	4723.7	4744.9	4767.0	4790.2	4814.3	**112**
113	4803.1	4821.4	4840.8	4861.2	4882.6	4905.0	4928.5	4953.0	**113**
114	4944.2	4962.8	4982.4	5003.1	5024.8	5047.5	5071.3	5096.1	**114**

TABLE 20. VALUES OF THE CORRECTION C_s FOR LARGE SPIRAL DEFLECTIONS

[*Formula:* C_s (in seconds) $= 0.0031\theta_s^3 + 0.0023\theta_s^5(10)^{-5}$, where θ_s = spiral angle (in degrees)]

θ_s	C_s	θ_s	C_s	θ_s	C_s	θ_s	C_s
5°	0.4″	14°	8.5″	23°	0′37.9″	32°	1′42.4″
6	0.7	15	10.5	24	43.0″	33	1′52.3″
7	1.1	16	12.7	25	48.7″	34	2′02.9″
8	1.6	17	15.3	26	54.8″	35	2′14.1″
9	2.3	18	18.1	27	1′01.3″	36	2′26.0″
10	3.1	19	21.3	28	1′08.4″	37	2′38.0″
11	4.1	20	24.9	29	1′16.1″	38	2′51.9″
12	5.4	21	28.8	30	1′24.3″	39	3′06.0″
13	6.8	22	33.1	31	1′33.0″	40	3′20.8″

TABLE 21. VALUES OF THE CORRECTION C_a FOR SPIRALED COMPOUND CURVES FOR VARIOUS VALUES OF THE PRODUCT $\Delta_2\Delta_1(\Delta_2 - \Delta_1)$

(Δ's in degrees)

[*Formula:* C_a (in seconds) $= 0.01218\Delta_2\Delta_1(\Delta_2 - \Delta_1)$]

$\Delta_2\Delta_1(\Delta_2 - \Delta_1)$	C_a	$\Delta_2\Delta_1(\Delta_2 - \Delta_1)$	C_a	$\Delta_2\Delta_1(\Delta_2 - \Delta_1)$	C_a	$\Delta_2\Delta_1(\Delta_2 - \Delta_1)$	C_a
10	0.1″	500	6.1″	1800	21.9″	3100	37.8″
20	0.2	600	7.3	1900	23.1	3200	39.0
30	0.4	700	8.5	2000	24.4	3300	40.2
40	0.5	800	9.7	2100	25.6	3400	41.4
50	0.6	900	11.0	2200	26.8	3500	42.6
60	0.7	1000	12.2	2300	28.0	3600	43.8
70	0.9	1100	13.4	2400	29.2	3700	45.1
80	1.0	1200	14.6	2500	30.4	3800	46.3
90	1.1	1300	15.8	2600	31.7	3900	47.5
100	1.2	1400	17.1	2700	32.9	4000	48.7
200	2.4	1500	18.3	2800	34.1	4100	49.9
300	3.7	1600	19.5	2900	35.3	4200	51.2
400	4.9	1700	20.7	3000	36.5	4300	52.4

Note: C_a applies to the spiral combining two circular curves of different radii (where D_1 and D_2 = degree of curve of the flatter and sharper curves). L_s = total length of the spiral; θ_s = nominal spiral angle $= \frac{L_s}{200}(D_2 - D_1)$; $\Delta_2 = \frac{L_s}{200} \times D_2$; $\Delta_1 = \frac{L_s}{200} \times D_1$; $\theta_s = \Delta_2 - \Delta_1$; $I = \Delta_1 + \Delta_2$. Deflection from A to end of sharper curve $= \phi_1 = \frac{L_s}{200} \times D_1 + \frac{1}{3}\theta_s - C$, and deflection from B to end of flatter curve $= \phi_2 = \frac{L_s}{200} \times D_2 - \frac{1}{3}\theta_s + C$, where $C = C_a$ (from Table 21) $+ C_s$ (from Table 20).

Example. Given $D_1 = 5°$, $D_2 = 11°$, $L_s = 300$ ft; then $\Delta_2 = 16.5°$, $\Delta_1 = 7.5°$, $\theta_s = 9°$, and the product $\Delta_2\Delta_1(\Delta_2 - \Delta_1) = 16.5(7.5)(9) = 1{,}113.75$. Hence, C_a (by Table 21) $= 13.5''$ and C_s (by Table 20) $= 2.3''$, giving $C = 15.8''$. Next, $\phi_1 = \frac{L_s}{200} \times 5 + \frac{9}{3} - 15.8'' = 10°29'44''$. Also $\tan \phi_1 = \frac{Y}{X}$ = (by Table 22) $\frac{54.24558}{292.80912} = 0.1852592$, from which $\phi_1 = 10°29'44''$ (check).

TABLE 22. VALUES OF *A*, *B*, *C*, *D*, *E*, *F* GIVING *Y* AND *X* FOR SPIRALS COMBINING CURVES OF DIFFERENT RADII

$$Y = \frac{L_s}{100}\left[\theta_s \times A - \left(\frac{\theta_s}{10}\right)^3 \times B + \left(\frac{\theta_s}{10}\right)^5 \times C\right];\quad X = \frac{L_s}{100}\left[100 - \left(\frac{\theta_s}{10}\right)^2 \times D + \left(\frac{\theta_s}{10}\right)^4 \times E - \left(\frac{\theta_s}{10}\right)^6 \times F\right];$$

$$N = \frac{2D_1}{D_2 - D_1};\ \theta_s = \frac{L_s}{200}(D_2 - D_1);\ D_1 \text{ and } D_2 = \text{degree of flatter and sharper curve}$$

N	*A*	*B*	*C*	*D*	*E*	*F*	*N*
0.000	0.581776	0.012658	0.000012	0.30462	0.000430	0.000000	0.000
0.200	0.756309	0.023823	0.000033	0.47723	0.000971	0.000001	0.200
2/9	0.775702	0.025373	0.000037	0.49892	0.001053	0.000001	2/9
0.250	0.799942	0.027406	0.000041	0.52673	0.001163	0.000001	0.250
0.300	0.843576	0.031336	0.000051	0.57877	0.001384	0.000002	0.300
1/3	0.872664	0.034155	0.000059	0.61488	0.001547	0.000002	1/3
0.350	0.887209	0.035627	0.000063	0.63335	0.001634	0.000002	0.350
0.375	0.909026	0.037919	0.000070	0.66159	0.001772	0.000002	0.375
0.400	0.930842	0.040305	0.000077	0.69047	0.001918	0.000003	0.400
0.425	0.952659	0.042792	0.000084	0.71998	0.002073	0.000003	0.425
4/9	0.969627	0.044796	0.000091	0.74337	0.002200	0.000003	4/9
0.450	0.974475	0.045381	0.000093	0.75012	0.002237	0.000003	0.450
0.500	1.018109	0.050875	0.000111	0.81231	0.002595	0.000004	0.500
0.550	1.061742	0.056792	0.000133	0.87704	0.002995	0.000005	0.550
5/9	1.066590	0.057480	0.000135	0.88439	0.003042	0.000005	5/9
0.600	1.105375	0.063166	0.000157	0.94431	0.003439	0.000006	0.600
0.625	1.127192	0.066527	0.000171	0.97890	0.003679	0.000007	0.625
0.650	1.149008	0.070002	0.000186	1.01412	0.003932	0.000008	0.650
2/3	1.163553	0.072388	0.000196	1.03796	0.004108	0.000008	2/3
0.675	1.170825	0.073603	0.000201	1.04998	0.004198	0.000008	0.675
0.700	1.192641	0.077325	0.000218	1.08647	0.004476	0.000009	0.700
0.750	1.236275	0.085141	0.000254	1.16135	0.005076	0.000011	0.750
7/9	1.260515	0.089703	0.000276	1.20405	0.005434	0.000012	7/9
0.800	1.279908	0.093470	0.000295	1.23878	0.005733	0.000013	0.800
0.850	1.323541	0.102331	0.000342	1.31874	0.006454	0.000016	0.850
0.875	1.345358	0.106971	0.000367	1.35967	0.006839	0.000017	0.875
8/9	1.357478	0.109606	0.000382	1.38269	0.007059	0.000018	8/9
0.900	1.367174	0.111749	0.000394	1.40124	0.007240	0.000018	0.900
0.950	1.410808	0.121724	0.000452	1.48628	0.008097	0.000022	0.950
1.000	1.454441	0.132281	0.000517	1.57386	0.009028	0.000026	1.000
1.125	1.563524	0.161333	0.000713	1.80391	0.011708	0.000037	1.125
1.200	1.628974	0.180662	0.000857	1.94955	0.013579	0.000047	1.200
1.250	1.672607	0.194378	0.000964	2.04982	0.014946	0.000054	1.250
1.300	1.716240	0.208777	0.001083	2.15263	0.016415	0.000061	1.300
1 1/3	1.745329	0.218758	0.001168	2.22258	0.017452	0.000067	1 1/3
1.350	1.759873	0.223866	0.001213	2.25798	0.017990	0.000070	1.350
1.375	1.781690	0.231683	0.001282	2.31160	0.018818	0.000075	1.375
1.400	1.803507	0.239676	0.001355	2.36586	0.019674	0.000080	1.400
1.450	1.847140	0.256215	0.001510	2.47629	0.021477	0.000091	1.450
1.500	1.890773	0.273506	0.001679	2.58925	0.023400	0.000104	1.500
1.550	1.934406	0.291557	0.001863	2.70475	0.025450	0.000117	1.550
1.600	1.978039	0.310387	0.002063	2.82279	0.027632	0.000132	1.600
1.625	1.999856	0.320102	0.002169	2.88276	0.028774	0.000141	1.625
1.650	2.021673	0.330020	0.002279	2.94337	0.029952	0.000149	1.650
1 2/3	2.036217	0.336740	0.002355	2.98412	0.030757	0.000155	1 2/3
1.700	2.065306	0.350464	0.002514	3.06648	0.032415	0.000168	1.700
1.750	2.108939	0.371736	0.002767	3.19214	0.035028	0.000188	1.750
1.800	2.152572	0.393857	0.003040	3.32033	0.037796	0.000210	1.800
1.850	2.196206	0.416845	0.003335	3.45106	0.040725	0.000235	1.850
1.875	2.218022	0.428668	0.003491	3.51738	0.042253	0.000248	1.875
1.900	2.239839	0.440713	0.003652	3.58433	0.043822	0.000262	1.900

TABLE 22. VALUES OF A, B, C, D, E, F GIVING Y AND X FOR SPIRALS COMBINING CURVES OF DIFFERENT RADII (*Continued*)

$$Y = \frac{L_s}{100}\left[\theta_s \times A - \left(\frac{\theta_s}{10}\right)^3 \times B + \left(\frac{\theta_s}{10}\right)^5 \times C\right];\quad X = \frac{L_s}{100}\left[100 - \left(\frac{\theta_s}{10}\right)^2 \times D + \left(\frac{\theta_s}{10}\right)^4 \times E - \left(\frac{\theta_s}{10}\right)^6 \times F\right];$$

$N = \frac{2D_1}{D_2 - D_1}$; $\theta_s = \frac{L_s}{200}(D_2 - D_1)$; D_1 and D_2 = degree of flatter and sharper curve

N	A	B	C	D	E	F	N
1.950	2.283472	0.465473	0.003993	3.72014	0.047093	0.000291	1.950
2.000	2.327105	0.491151	0.004359	3.85849	0.050544	0.000323	2.000
2.050	2.370739	0.517759	0.004751	3.99937	0.054183	0.000358	2.050
2.100	2.414372	0.545316	0.005171	4.14278	0.058013	0.000396	2.100
2.125	2.436188	0.559450	0.005392	4.21546	0.060003	0.000417	2.125
2.150	2.458005	0.573830	0.005620	4.28876	0.062044	0.000438	2.150
2.200	2.501638	0.603330	0.006100	4.43726	0.066282	0.000483	2.200
2.250	2.545271	0.633825	0.006612	4.58830	0.070735	0.000532	2.250
2.300	2.588905	0.665338	0.007159	4.74188	0.075406	0.000584	2.300
2 1/3	2.617994	0.686912	0.007542	4.84567	0.078649	0.000622	2 1/3
2.350	2.632538	0.697877	0.007740	4.89799	0.080309	0.000641	2.350
2.400	2.676171	0.731460	0.008359	5.05665	0.085445	0.000703	2.400
2.450	2.719804	0.766113	0.009017	5.21784	0.090824	0.000770	2.450
2.500	2.763438	0.801841	0.009716	5.38157	0.096456	0.000841	2.500
2.550	2.807071	0.838665	0.010458	5.54784	0.102343	0.000919	2.550
2.600	2.850704	0.876602	0.011244	5.71665	0.108498	0.001002	2.600
2.650	2.894337	0.915673	0.012077	5.88800	0.114927	0.001091	2.650
2 2/3	2.908882	0.928948	0.012366	5.94568	0.117131	0.001122	2 2/3
2.700	2.937970	0.955889	0.012959	6.06189	0.121637	0.001187	2.700
2.750	2.981604	0.997263	0.013891	6.23831	0.128638	0.001289	2.750
3.000	3.199770	1.222180	0.019393	7.15851	0.168283	0.001921	3.000
3.050	3.243403	1.270884	0.020678	7.35016	0.177202	0.002074	3.050
3.100	3.287036	1.320872	0.022031	7.54436	0.186471	0.002237	3.100
3.125	3.308853	1.346347	0.022733	7.64241	0.191239	0.002322	3.125
3.150	3.330669	1.372152	0.023453	7.74109	0.196098	0.002410	3.150
3.200	3.374303	1.424742	0.024947	7.94036	0.206095	0.002594	3.200
3.250	3.417936	1.478667	0.026517	8.14217	0.216469	0.002791	3.250
3.300	3.461569	1.533934	0.028166	8.34652	0.227232	0.003000	3.300
3 1/3	3.490658	1.571532	0.029309	8.48416	0.234625	0.003146	3 1/3
3.350	3.505202	1.590562	0.029895	8.55340	0.238390	0.003221	3.350
3.400	3.548836	1.648570	0.031708	8.76283	0.249954	0.003456	3.400
3.450	3.592469	1.707976	0.033608	8.97479	0.261936	0.003705	3.450
3.500	3.636102	1.768794	0.035598	9.18929	0.274344	0.003969	3.500
3.550	3.679735	1.831036	0.037682	9.40633	0.287185	0.004248	3.550
3.600	3.723368	1.894727	0.039862	9.62591	0.300476	0.004543	3.600
3.650	3.767002	1.959881	0.042142	9.84803	0.314222	0.004856	3.650
3 2/3	3.781546	1.981926	0.042924	9.92263	0.318606	0.004964	3 2/3
3.700	3.810635	2.026515	0.044525	10.07268	0.328434	0.005186	3.700
3.750	3.854268	2.094643	0.047014	10.29988	0.343124	0.005534	3.750
3.800	3.897901	2.164278	0.049614	10.52961	0.358302	0.005902	3.800
3.850	3.941535	2.235444	0.052328	10.76188	0.373976	0.006290	3.850
3.900	3.985168	2.308161	0.055159	10.99669	0.390163	0.006699	3.900
3.950	4.028801	2.382435	0.058112	11.23404	0.406868	0.007130	3.950
4.000	4.072434	2.458285	0.061189	11.47392	0.424105	0.007584	4.000
4.100	4.159701	2.614794	0.067737	11.96131	0.460216	0.008564	4.100
4.200	4.246967	2.777811	0.074833	12.45885	0.498590	0.009647	4.200
4.250	4.290600	2.861807	0.078597	12.71143	0.518652	0.010230	4.250
4.300	4.334234	2.947479	0.082512	12.96655	0.539315	0.010843	4.300
4 1/3	4.363323	3.005531	0.085208	13.13804	0.553429	0.011268	4 1/3
4.350	4.377867	3.034842	0.086581	13.22420	0.560590	0.011485	4.350
4.400	4.421500	3.123920	0.090809	13.48440	0.582488	0.012159	4.400

6 min.

Interpolations.

10 Even stations -

TABLE 23. DEFLECTION ANGLES TO CHORD POINTS OF SPIRAL

θ_s	1	2	3	4	5	6	7	8	9	10
0.0°	0°00.00′	0°00.0′	0°00.0′	0°00.0′	0°00.0′	0°00.0′	0°00.0′	0°00.0′	0°00.0′	0°00.0′
.1	00.02	00.1	00.2	00.3	00.5	00.7	01.0	01.3	01.6	02.0
.2	00.04	00.2	00.4	00.6	01.0	01.4	02.0	02.6	03.2	04.0
.3	00.06	00.2	00.5	01.0	01.5	02.2	02.9	03.8	04.9	06.0
.4	00.08	00.3	00.7	01.3	02.0	02.9	03.9	05.1	06.5	08.0
0.5	0 00.10	0 00.4	0 00.9	0 01.6	0 02.5	0 03.6	0 04.9	0 06.4	0 08.1	0 10.0
.6	00.12	00.5	01.1	01.9	03.0	04.3	05.9	07.7	09.7	12.0
.7	00.14	00.6	01.3	02.2	03.5	05.0	06.9	09.0	11.3	14.0
.8	00.16	00.6	01.4	02.6	04.0	05.8	07.8	10.2	13.0	16.0
.9	00.18	00.7	01.6	02.9	04.5	06.5	08.8	11.5	14.6	18.0
1.0	0 00.20	0 00.8	0 01.8	0 03.2	0 05.0	0 07.2	0 09.8	0 12.8	0 16.2	0 20.0
.1	00.22	00.9	02.0	03.5	05.5	07.9	10.8	14.1	17.8	22.0
.2	00.24	01.0	02.2	03.8	06.0	08.6	11.8	15.4	19.4	24.0
.3	00.26	01.0	02.3	04.2	06.5	09.4	12.7	16.6	21.1	26.0
.4	00.28	01.1	02.5	04.5	07.0	10.1	13.7	17.9	22.7	28.0
1.5	0 00.30	0 01.2	0 02.7	0 04.8	0 07.5	0 10.8	0 14.7	0 19.2	0 24.3	0 30.0
.6	00.32	01.3	02.9	05.1	08.0	11.5	15.7	20.5	25.9	32.0
.7	00.34	01.4	03.1	05.4	08.5	12.2	16.7	21.8	27.5	34.0
.8	00.36	01.4	03.2	05.8	09.0	13.0	17.6	23.0	29.2	36.0
.9	00.38	01.5	03.4	06.1	09.5	13.7	18.6	24.3	30.8	38.0
2.0	0 00.40	0 01.6	0 03.6	0 06.4	0 10.0	0 14.4	0 19.6	0 25.6	0 32.4	0 40.0
.1	00.42	01.7	03.8	06.7	10.5	15.1	20.6	26.9	34.0	42.0
.2	00.44	01.8	04.0	07.0	11.0	15.8	21.6	28.2	35.6	44.0
.3	00.46	01.8	04.1	07.4	11.5	16.6	22.5	29.4	37.3	46.0
.4	00.48	01.9	04.3	07.7	12.0	17.3	23.5	30.7	38.9	48.0
2.5	0 00.50	0 02.0	0 04.5	0 08.0	0 12.5	0 18.0	0 24.5	0 32.0	0 40.5	0 50.0
.6	00.52	02.1	04.7	08.3	13.0	18.7	25.5	33.3	42.1	52.0
.7	00.54	02.2	04.9	08.6	13.5	19.4	26.5	34.6	43.7	54.0
.8	00.56	02.2	05.0	09.0	14.0	20.2	27.4	35.8	45.4	56.0
.9	00.58	02.3	05.2	09.3	14.5	20.9	28.4	37.1	47.0	58.0
3.0	0 00.60	0 02.4	0 05.4	0 09.6	0 15.0	0 21.6	0 29.4	0 38.4	0 48.6	1 00.0
.1	00.62	02.5	05.6	09.9	15.5	22.3	30.4	39.7	50.2	02.0
.2	00.64	02.6	05.8	10.2	16.0	23.0	31.4	41.0	51.8	04.0
.3	00.66	02.6	05.9	10.6	16.5	23.8	32.3	42.2	53.5	06.0
.4	00.68	02.7	06.1	10.9	17.0	24.5	33.3	43.5	55.1	08.0
3.5	0 00.70	0 02.8	0 06.3	0 11.2	0 17.5	0 25.2	0 34.3	0 44.8	0 56.7	1 10.0
.6	00.72	02.9	06.5	11.5	18.0	25.9	35.3	46.1	58.3	12.0
.7	00.74	03.0	06.7	11.8	18.5	26.6	36.3	47.4	59.9	14.0
.8	00.76	03.0	06.8	12.2	19.0	27.4	37.2	48.6	1 01.6	16.0
.9	00.78	03.1	07.0	12.5	19.5	28.1	38.2	49.9	03.2	18.0
4.0	0 00.80	0 03.2	0 07.2	0 12.8	0 20.0	0 28.8	0 39.2	0 51.2	1 04.8	1 20.0
.1	00.82	03.3	07.4	13.1	20.5	29.5	40.2	52.5	06.4	22.0
.2	00.84	03.4	07.6	13.4	21.0	30.2	41.2	53.8	08.0	24.0
.3	00.86	03.4	07.7	13.8	21.5	31.0	42.1	55.0	09.7	26.0
.4	00.88	03.5	07.9	14.1	22.0	31.7	43.1	56.3	11.3	28.0
4.5	0 00.90	0 03.6	0 08.1	0 14.4	0 22.5	0 32.4	0 44.1	0 57.6	1 12.9	1 30.0
.6	00.92	03.7	08.3	14.7	23.0	33.1	45.1	58.9	14.5	32.0
.7	00.94	03.8	08.5	15.0	23.5	33.8	46.1	1 00.2	16.1	34.0
.8	00.96	03.8	08.6	15.4	24.0	34.6	47.0	01.4	17.8	36.0
.9	00.98	03.9	08.8	15.7	24.5	35.3	48.0	02.7	19.4	38.0
5.0	0 01.00	0 04.0	0 09.0	0 16.0	0 25.0	0 36.0	0 49.0	1 04.0	1 21.0	1 40.0
.1	01.02	04.1	09.2	16.3	25.5	36.7	50.0	05.3	22.6	42.0
.2	01.04	04.2	09.4	16.6	26.0	37.4	51.0	06.6	24.2	44.0
.3	01.06	04.2	09.5	17.0	26.5	38.2	51.9	07.8	25.9	46.0
.4	01.08	04.3	09.7	17.3	27.0	38.9	52.9	09.1	27.5	48.0
5.5	0 01.10	0 04.4	0 09.9	0 17.6	0 27.5	0 39.6	0 53.9	1 10.4	1 29.1	1 50.0
.6	01.12	04.5	10.1	17.9	28.0	40.3	54.9	11.7	30.7	52.0
.7	01.14	04.6	10.3	18.2	28.5	41.0	55.9	13.0	32.3	54.0
.8	01.16	04.6	10.4	18.6	29.0	41.8	56.8	14.2	34.0	56.0
.9	01.18	04.7	10.6	18.9	29.5	42.5	57.8	15.5	35.6	58.0
6.0	0 01.20	0 04.8	0 10.8	0 19.2	0 30.0	0 43.2	0 58.8	1 16.8	1 37.2	2 00.0
	1	2	3	4	5	6	7	8	9	10

TABLE 23. DEFLECTION ANGLES TO CHORD POINTS OF SPIRAL (*Continued*)

θ_s	1	2	3	4	5	6	7	8	9	10
6.0°	0°01.20′	0°04.8′	0°10.8′	0°19.2′	0°30.0′	0°43.2′	0°58.8′	1°16.8′	1°37.2′	2°00.0′
.1	01.22	04.9	11.0	19.5	30.5	43.9	59.8	18.1	38.8	02.0
.2	01.24	05.0	11.2	19.8	31.0	44.6	1 00.8	19.4	40.4	04.0
.3	01.26	05.0	11.3	20.2	31.5	45.4	01.7	20.6	42.1	06.0
.4	01.28	05.1	11.5	20.5	32.0	46.1	02.7	21.9	43.7	08.0
.5	0 01.30	0 05.2	0 11.7	0 20.8	0 32.5	0 46.8	1 03.7	1 23.2	1 45.3	2 10.0
.6	01.32	05.3	11.9	21.1	33.0	47.5	04.7	24.5	46.9	12.0
.7	01.34	05.4	12.1	21.4	33.5	48.2	05.7	25.8	48.5	14.0
.8	01.36	05.4	12.2	21.8	34.0	49.0	06.6	27.0	50.2	16.0
.9	01.38	05.5	12.4	22.1	34.5	49.7	07.6	28.3	51.8	18.0
7.0	0 01.40	0 05.6	0 12.6	0 22.4	0 35.0	0 50.4	1 08.6	1 29.6	1 53.4	2 20.0
.1	01.42	05.7	12.8	22.7	35.5	51.1	09.6	30.9	55.0	22.0
.2	01.44	05.8	13.0	23.0	36.0	51.8	10.6	32.2	56.6	24.0
.3	01.46	05.8	13.1	23.4	36.5	52.6	11.5	33.4	58.3	26.0
.4	01.48	05.9	13.3	23.7	37.0	53.3	12.5	34.7	59.9	28.0
.5	0 01.50	0 06.0	0 13.5	0 24.0	0 37.5	0 54.0	1 13.5	1 36.0	2 01.5	2 30.0
.6	01.52	06.1	13.7	24.3	38.0	54.7	14.5	37.3	03.1	32.0
.7	01.54	06.2	13.9	24.6	38.5	55.4	15.5	38.6	04.7	34.0
.8	01.56	06.2	14.0	25.0	39.0	56.2	16.4	39.8	06.4	36.0
.9	01.58	06.3	14.2	25.3	39.5	56.9	17.4	41.1	08.0	38.0
8.0	0 01.60	0 06.4	0 14.4	0 25.6	0 40.0	0 57.6	1 18.4	1 42.4	2 09.6	2 40.0
.1	01.62	06.5	14.6	25.9	40.5	58.3	19.4	43.7	11.2	42.0
.2	01.64	06.6	14.8	26.2	41.0	59.0	20.4	45.0	12.8	44.0
.3	01.66	06.6	14.9	26.6	41.5	59.8	21.3	46.2	14.5	46.0
.4	01.68	06.7	15.1	26.9	42.0	1 00.5	22.3	47.5	16.1	48.0
.5	0 01.70	0 06.8	0 15.3	0 27.2	0 42.5	1 01.2	1 23.3	1 48.8	2 17.7	2 50.0
.6	01.72	06.9	15.5	27.5	43.0	01.9	24.3	50.1	19.3	52.0
.7	01.74	07.0	15.7	27.8	43.5	02.6	25.3	51.4	20.9	54.0
.8	01.76	07.0	15.8	28.2	44.0	03.4	26.2	52.6	22.6	56.0
.9	01.78	07.1	16.0	28.5	44.5	04.1	27.2	53.9	24.2	58.0
9.0	0 01.80	0 07.2	0 16.2	0 28.8	0 45.0	1 04.8	1 28.2	1 55.2	2 25.8	3 00.0
.1	01.82	07.3	16.4	29.1	45.5	05.5	29.2	56.5	27.4	02.0
.2	01.84	07.4	16.6	29.4	46.0	06.2	30.2	57.8	29.0	04.0
.3	01.86	07.4	16.7	29.8	46.5	07.0	31.1	59.0	30.7	06.0
.4	01.88	07.5	16.9	30.1	47.0	07.7	32.1	2 00.3	32.3	08.0
.5	0 01.90	0 07.6	0 17.1	0 30.4	0 47.5	1 08.4	1 33.1	2 01.6	2 33.9	3 10.0
.6	01.92	07.7	17.3	30.7	48.0	09.1	34.1	02.9	35.5	12.0
.7	01.94	07.8	17.5	31.0	48.5	09.8	35.1	04.2	37.1	14.0
.8	01.96	07.8	17.6	31.4	49.0	10.6	36.0	05.4	38.8	16.0
.9	01.98	07.9	17.8	31.7	49.5	11.3	37.0	06.7	40.4	18.0
10.0	0 02.00	0 08.0	0 18.0	0 32.0	0 50.0	1 12.0	1 38.0	2 08.0	2 42.0	3 20.0
.1	02.02	08.1	18.2	32.3	50.5	12.7	39.0	09.3	43.6	21.9
.2	02.04	08.2	18.4	32.6	51.0	13.4	40.0	10.6	45.2	23.9
.3	02.06	08.2	18.5	33.0	51.5	14.2	40.9	11.8	46.8	25.9
.4	02.08	08.3	18.7	33.3	52.0	14.9	41.9	13.1	48.5	27.9
.5	0 02.10	0 08.4	0 18.9	0 33.6	0 52.5	1 15.6	1 42.9	2 14.4	2 50.1	3 29.9
.6	02.12	08.5	19.1	33.9	53.0	16.3	43.9	15.7	51.7	31.9
.7	02.14	08.6	19.3	34.2	53.5	17.0	44.9	17.0	53.3	33.9
.8	02.16	08.6	19.4	34.6	54.0	17.8	45.8	18.2	54.9	35.9
.9	02.18	08.7	19.6	34.9	54.5	18.5	46.8	19.5	56.6	37.9
11.0	0 02.20	0 08.8	0 19.8	0 35.2	0 55.0	1 19.2	1 47.8	2 20.8	2 58.2	3 39.9
.1	02.22	08.9	20.0	35.5	55.5	19.9	48.8	22.1	59.8	41.9
.2	02.24	09.0	20.2	35.8	56.0	20.6	49.8	23.4	3 01.4	43.9
.3	02.26	09.0	20.3	36.2	56.5	21.4	50.7	24.6	03.0	45.9
.4	02.28	09.1	20.5	36.5	57.0	22.1	51.7	25.9	04.7	47.9
.5	0 02.30	0 09.2	0 20.7	0 36.8	0 57.5	1 22.8	1 52.7	2 27.2	3 06.3	3 49.9
.6	02.32	09.3	20.9	37.1	58.0	23.5	53.7	28.5	07.9	51.9
.7	02.34	09.4	21.1	37.4	58.5	24.2	54.7	29.7	09.5	53.9
.8	02.36	09.4	21.2	37.8	59.0	25.0	55.6	31.0	11.1	55.9
.9	02.38	09.5	21.4	38.1	59.5	25.7	56.6	32.3	12.7	57.9
12.0	0 02.40	0 09.6	0 21.6	0 38.4	1 00.0	1 26.4	1 57.6	2 33.6	3 14.4	3 59.9
	1	2	3	4	5	6	7	8	9	10

TABLE 23. DEFLECTION ANGLES TO CHORD POINTS OF SPIRAL (*Continued*)

θ_S	1	2	3	4	5	6	7	8	9	10
12.0°	0°02.40′	0°09.6′	0°21.6′	0°38.4′	1°00.0′	1°26.4′	1°57.6′	2°33.6′	3°14.4′	3°59.9′
.1	02.42	09.7	21.8	38.7	00.5	27.1	58.6	34.9	16.0	4 01.9
.2	02.44	09.8	22.0	39.0	01.0	27.8	59.5	36.1	17.6	03.9
.3	02.46	09.8	22.1	39.4	01.5	28.6	2 00.5	37.4	19.2	05.9
.4	02.48	09.9	22.3	39.7	02.0	29.3	01.5	38.7	20.8	07.9
.5	0 02.50	0 10.0	0 22.5	0 40.0	1 02.5	1 30.0	2 02.5	2 40.0	3 22.5	4 09.9
.6	02.52	10.1	22.7	40.3	03.0	30.7	03.5	41.3	24.1	11.9
.7	02.54	10.2	22.9	40.6	03.5	31.4	04.4	42.5	25.7	13.9
.8	02.56	10.2	23.0	41.0	04.0	32.2	05.4	43.8	27.3	15.9
.9	02.58	10.3	23.2	41.3	04.5	32.9	06.4	45.1	28.9	17.9
13.0	0 02.60	0 10.4	0 23.4	0 41.6	1 05.0	1 33.6	2 07.4	2 46.4	3 30.5	4 19.9
.1	02.62	10.5	23.6	41.9	05.5	34.3	08.4	47.7	32.2	21.9
.2	02.64	10.6	23.8	42.2	06.0	35.0	09.3	48.9	33.8	23.9
.3	02.66	10.6	23.9	42.6	06.5	35.8	10.3	50.2	35.4	25.9
.4	02.68	10.7	24.1	42.9	07.0	36.5	11.3	51.5	37.0	27.9
.5	0 02.70	0 10.8	0 24.3	0 43.2	1 07.5	1 37.2	2 12.3	2 52.8	3 38.6	4 29.9
.6	02.72	10.9	24.5	43.5	08.0	37.9	13.3	54.0	40.3	31.9
.7	02.74	11.0	24.7	43.8	08.5	38.6	14.2	55.3	41.9	33.9
.8	02.76	11.0	24.8	44.2	09.0	39.4	15.2	56.6	43.5	35.9
.9	02.78	11.1	25.0	44.5	09.5	40.1	16.2	57.9	45.1	37.9
14.0	0 02.80	0 11.2	0 25.2	0 44.8	1 10.0	1 40.8	2 17.2	2 59.2	3 46.7	4 39.9
.1	02.82	11.3	25.4	45.1	10.5	41.5	18.2	3 00.4	48.4	41.9
.2	02.84	11.4	25.6	45.4	11.0	42.2	19.1	01.7	50.0	43.9
.3	02.86	11.4	25.7	45.8	11.5	43.0	20.1	03.0	51.6	45.9
.4	02.88	11.5	25.9	46.1	12.0	43.7	21.1	04.3	53.2	47.9
.5	0 02.90	0 11.6	0 26.1	0 46.4	1 12.5	1 44.4	2 22.1	3 05.6	3 54.8	4 49.9
.6	02.92	11.7	26.3	46.7	13.0	45.1	23.1	06.8	56.4	51.9
.7	02.94	11.8	26.5	47.0	13.5	45.8	24.0	08.1	58.1	53.9
.8	02.96	11.8	26.6	47.4	14.0	46.6	25.0	09.4	59.7	55.8
.9	02.98	11.9	26.8	47.7	14.5	47.3	26.0	10.7	4 01.3	57.8
15.0	0 03.00	0 12.0	0 27.0	0 48.0	1 15.0	1 48.0	2 27.0	3 12.0	4 02.9	4 59.8
.1	03.02	12.1	27.2	48.3	15.5	48.7	28.0	13.2	04.5	5 01.8
.2	03.04	12.2	27.4	48.6	16.0	49.4	28.9	14.5	06.1	03.8
.3	03.06	12.2	27.5	49.0	16.5	50.2	29.9	15.8	07.8	05.8
.4	03.08	12.3	27.7	49.3	17.0	50.9	30.9	17.1	09.4	07.8
.5	0 03.10	0 12.4	0 27.9	0 49.6	1 17.5	1 51.6	2 31.9	3 18.4	4 11.0	5 09.8
.6	03.12	12.5	28.1	49.9	18.0	52.3	32.9	19.6	12.6	11.8
.7	03.14	12.6	28.3	50.2	18.5	53.0	33.8	20.9	14.2	13.8
.8	03.16	12.6	28.4	50.6	19.0	53.7	34.8	22.2	15.9	15.8
.9	03.18	12.7	28.6	50.9	19.5	54.5	35.8	23.5	17.5	17.8
16.0	0 03.20	0 12.8	0 28.8	0 51.2	1 20.0	1 55.2	2 36.8	3 24.8	4 19.1	5 19.8
.1	03.22	12.9	29.0	51.5	20.5	55.9	37.8	26.0	20.7	21.8
.2	03.24	13.0	29.2	51.8	21.0	56.6	38.7	27.3	22.3	23.8
.3	03.26	13.0	29.3	52.2	21.5	57.3	39.7	28.6	23.9	25.8
.4	03.28	13.1	29.5	52.5	22.0	58.1	40.7	29.9	25.6	27.8
.5	0 03.30	0 13.2	0 29.7	0 52.8	1 22.5	1 58.8	2 41.7	3 31.1	4 27.2	5 29.8
.6	03.32	13.3	29.9	53.1	23.0	59.5	42.7	32.4	28.8	31.8
.7	03.34	13.4	30.1	53.4	23.5	2 00.2	43.6	33.7	30.4	33.8
.8	03.36	13.4	30.2	53.8	24.0	00.9	44.6	35.0	32.0	35.8
.9	03.38	13.5	30.4	54.1	24.5	01.7	45.6	36.3	33.6	37.8
17.0	0 03.40	0 13.6	0 30.6	0 54.4	1 25.0	2 02.4	2 46.6	3 37.5	4 35.3	5 39.8
.1	03.42	13.7	30.8	54.7	25.5	03.1	47.5	38.8	36.9	41.8
.2	03.44	13.8	31.0	55.0	26.0	03.8	48.5	40.1	38.5	43.7
.3	03.46	13.8	31.1	55.4	26.5	04.5	49.5	41.4	40.1	45.7
.4	03.48	13.9	31.3	55.7	27.0	05.3	50.5	42.7	41.7	47.7
.5	0 03.50	0 14.0	0 31.5	0 56.0	1 27.5	2 06.0	2 51.5	3 43.9	4 43.4	5 49.7
.6	03.52	14.1	31.7	56.3	28.0	06.7	52.4	45.2	45.0	51.7
.7	03.54	14.2	31.9	56.6	28.5	07.4	53.4	46.5	46.6	53.7
.8	03.56	14.2	32.0	57.0	29.0	08.1	54.4	47.8	48.2	55.7
.9	03.58	14.3	32.2	57.3	29.5	08.9	55.4	49.0	49.8	57.7
18.0	0 03.60	0 14.4	0 32.4	0 57.6	1 30.0	2 09.6	2 56.4	3 50.3	4 51.5	5 59.7
	1	2	3	4	5	6	7	8	9	10

TABLE 23. DEFLECTION ANGLES TO CHORD POINTS OF SPIRAL (*Continued*)

θ_s	1	2	3	4	5	6	7	8	9	10
18.0°	0°03.60′	0°14.4′	0°32.4′	0°57.6′	1°30.0′	2°09.6′	2°56.4′	3°50.3′	4°51.5′	5°59.7′
.1	03.62	14.5	32.6	57.9	30.5	10.3	57.3	51.6	53.1	6 01.7
.2	03.64	14.6	32.8	58.2	31.0	11.0	58.3	52.9	54.7	03.7
.3	03.66	14.6	32.9	58.6	31.5	11.7	59.3	54.2	56.3	05.7
.4	03.68	14.7	33.1	58.9	32.0	12.5	3 00.3	55.4	57.9	07.7
.5	0 03.70	0 14.8	0 33.3	0 59.2	1 32.5	2 13.2	3 01.3	3 56.7	4 59.5	6 09.7
.6	03.72	14.9	33.5	59.5	33.0	13.9	02.2	58.0	5 01.2	11.7
.7	03.74	15.0	33.7	59.8	33.5	14.6	03.2	59.3	02.8	13.7
.8	03.76	15.0	33.8	1 00.2	34.0	15.3	04.2	4 00.6	04.4	15.7
.9	03.78	15.1	34.0	00.5	34.5	16.1	05.2	01.8	06.0	17.7
19.0	0 03.80	0 15.2	0 34.2	1 00.8	1 35.0	2 16.8	3 06.2	4 03.1	5 07.6	6 19.7
.1	03.82	15.3	34.4	01.1	35.5	17.5	07.1	04.4	09.2	21.7
.2	03.84	15.4	34.6	01.4	36.0	18.2	08.1	05.7	10.9	23.7
.3	03.86	15.4	34.7	01.8	36.5	18.9	09.1	06.9	12.5	25.6
.4	03.88	15.5	34.9	02.1	37.0	19.7	10.1	08.2	14.1	27.6
.5	0 03.90	0 15.6	0 35.1	1 02.4	1 37.5	2 20.4	3 11.1	4 09.5	5 15.7	6 29.6
.6	03.92	15.7	35.3	02.7	38.0	21.1	12.0	10.8	17.3	31.6
.7	03.94	15.8	35.5	03.0	38.5	21.8	13.0	12.1	18.9	33.6
.8	03.96	15.8	35.6	03.4	39.0	22.5	14.0	13.3	20.6	35.6
.9	03.98	15.9	35.8	03.7	39.5	23.3	15.0	14.6	22.2	37.6
20.0	0 04.00	0 16.0	0 36.0	1 04.0	1 40.0	2 24.0	3 16.0	4 15.9	5 23.8	6 39.6
.1	04.02	16.1	36.2	04.3	40.5	24.7	16.9	17.2	25.4	41.6
.2	04.04	16.2	36.4	04.6	41.0	25.4	17.9	18.5	27.0	43.6
.3	04.06	16.2	36.5	05.0	41.5	26.1	18.9	19.7	28.6	45.6
.4	04.08	16.3	36.7	05.3	42.0	26.9	19.9	21.0	30.3	47.6
.5	0 04.10	0 16.4	0 36.9	1 05.6	1 42.5	2 27.6	3 20.9	4 22.3	5 31.9	6 49.6
.6	04.12	16.5	37.1	05.9	43.0	28.3	21.8	23.6	33.5	51.6
.7	04.14	16.6	37.3	06.2	43.5	29.0	22.8	24.8	35.1	53.6
.8	04.16	16.6	37.4	06.6	44.0	29.7	23.8	26.1	36.7	55.6
.9	04.18	16.7	37.6	06.9	44.5	30.5	24.8	27.4	38.3	57.5
21.0	0 04.20	0 16.8	0 37.8	1 07.2	1 45.0	2 31.2	3 25.8	4 28.7	5 40.0	6 59.5
.2	04.24	17.0	38.2	07.8	46.0	32.6	27.7	31.2	43.2	7 03.5
.4	04.28	17.1	38.5	08.5	47.0	34.1	29.7	33.8	46.4	07.5
.6	04.32	17.3	38.9	09.1	48.0	35.5	31.6	36.4	49.7	11.5
.8	04.36	17.4	39.2	09.8	49.0	37.0	33.6	38.9	52.9	15.5
22.0	0 04.40	0 17.6	0 39.6	1 10.4	1 50.0	2 38.4	3 35.5	4 41.5	5 56.1	7 19.5
.2	04.44	17.8	40.0	11.0	51.0	39.8	37.5	44.0	59.3	23.5
.4	04.48	17.9	40.3	11.7	52.0	41.3	39.5	46.6	6 02.6	27.5
.6	04.52	18.1	40.7	12.3	53.0	42.7	41.4	49.1	05.8	31.4
.8	04.56	18.2	41.0	13.0	54.0	44.2	43.4	51.7	09.0	35.4
23.0	0 04.60	0 18.4	0 41.4	1 13.6	1 55.0	2 45.6	3 45.3	4 54.2	6 12.3	7 39.4
.2	04.64	18.6	41.8	14.2	56.0	47.0	47.3	56.8	15.5	43.4
.4	04.68	18.7	42.1	14.9	57.0	48.5	49.2	59.4	18.7	47.4
.6	04.72	18.9	42.5	15.5	58.0	49.9	51.2	5 01.9	22.0	51.4
.8	04.76	19.0	42.8	16.2	59.0	51.4	53.2	04.5	25.2	55.3
24.0	0 04.80	0 19.2	0 43.2	1 16.8	2 00.0	2 52.8	3 55.1	5 07.0	6 28.4	7 59.3
.2	04.84	19.4	43.6	17.4	01.0	54.2	57.1	09.6	31.7	8 03.3
.4	04.88	19.5	43.9	18.1	02.0	55.7	59.0	12.1	34.9	07.3
.6	04.92	19.7	44.3	18.7	03.0	57.1	4 01.0	14.7	38.1	11.3
.8	04.96	19.8	44.6	19.4	04.0	58.6	03.0	17.2	41.3	15.3
25.0	0 05.00	0 20.0	0 45.0	1 20.0	2 05.0	3 00.0	4 04.9	5 19.8	6 44.6	8 19.2
.2	05.04	20.2	45.4	20.6	06.0	01.4	06.9	22.4	47.8	23.2
.4	05.08	20.3	45.7	21.3	07.0	02.9	08.8	24.9	51.0	27.2
.6	05.12	20.5	46.1	21.9	08.0	04.3	10.8	27.5	54.3	31.2
.8	05.16	20.6	46.4	22.6	09.0	05.8	12.8	30.0	57.5	35.2
26.0	0 05.20	0 20.8	0 46.8	1 23.2	2 10.0	3 07.2	4 14.7	5 32.6	7 00.7	8 39.1
.2	05.24	21.0	47.2	23.8	11.0	08.6	16.7	35.1	04.0	43.1
.4	05.28	21.1	47.5	24.5	12.0	10.1	18.6	37.7	07.2	47.1
.6	05.32	21.3	47.9	25.1	13.0	11.5	20.6	40.2	10.4	51.1
.8	05.36	21.4	48.2	25.8	14.0	13.0	22.5	42.8	13.7	55.1
27.0	0 05.40	0 21.6	0 48.6	1 26.4	2 15.0	3 14.4	4 24.5	5 45.4	7 16.9	8 59.0
	1	2	3	4	5	6	7	8	9	10

TABLE 24. COEFFICIENTS FOR DEFLECTIONS FROM INTERMEDIATE SETUP ON SPIRAL

Deflection angle to chord point number	Transit at chord point number										
	0(TS)	1	2	3	4	5	6	7	8	9	10(SC)
0(TS)	0	2	8	18	32	50	72	98	128	162	200
1	1	0	5	14	27	44	65	90	119	152	189
2	4	4	0	8	20	36	56	80	108	140	176
3	9	10	7	0	11	26	45	68	95	126	161
4	16	18	16	10	0	14	32	54	80	110	144
5	25	28	27	22	13	0	17	38	63	92	125
6	36	40	40	36	28	16	0	20	44	72	104
7	49	54	55	52	45	34	19	0	23	50	81
8	64	70	72	70	64	54	40	22	0	26	56
9	81	88	91	90	85	76	63	46	25	0	29
10(SC)	100	108	112	112	108	100	88	72	52	28	0

Note: The coefficients in Table 24 are to be multiplied by $\frac{\theta_s}{300}$ to obtain the deflection angle to any forward (or backward) chord point.

A transit is assumed to be set at one of the 10 chord points and oriented so that the vernier reads 0°00′ on the tangent to spiral at this point.

Example. Given spiral angle $\theta_s = 5°00'$. Required: Deflection angles to various chord points when transit is set at Point 6, vernier reading 0°00′ on the tangent to spiral at point 6.

Solution: $\frac{\theta_s}{300} = \frac{5 \times 60}{300}$ (minutes) $= 01.0'$

Forward deflection to pt 7 = 01 × 19 = 0°19′
Forward deflection to pt 8 = 01 × 40 = 0°40′
Forward deflection to pt 9 = 01 × 63 = 1°03′
Forward deflection to pt 10 = 01 × 88 = 1°28′
Backward deflection to pt 5 = 01 × 17 = 0°17′, etc.

The results are slightly approximate when central angle from transit to the point sighted exceeds the central angle from the T.S. to the point sighted by more than 15°.

TABLE 25. TANGENTS AND EXTERNALS TO THE UNIT DOUBLE-SPIRAL CURVE

Length of each unit spiral = 100 ft; spiral angle = $\theta_s = \frac{1}{2}\Delta$

$D_c = 200\theta_s/l_s = 100\Delta/l_s$

Δ	T_s	E_s	Δ	T_s	E_s	Δ	T_s	E_s
0°00′	100.000	0.000	6°00′	100.064	1.747	12°00′	100.257	3.507
10	.000	.048	10	.068	.796	10	.264	.556
20	.001	.097	20	.071	.844	20	.272	.605
30	.001	.145	30	.075	.893	30	.279	.655
40	.001	.194	40	.079	.942	40	.286	.704
50	.002	.242	50	.083	.991	50	.294	.753
1 00	.002	.291	7 00	.087	2.039	13 00	.302	.803
10	.003	.339	10	.091	.088	10	.310	.852
20	.003	.388	20	.096	.137	20	.318	.901
30	.004	.436	30	.100	.186	30	.326	.951
40	.005	.485	40	.105	.234	40	.334	4.000
50	.006	.533	50	.109	.283	50	.342	.049
2 00	.007	.582	8 00	.114	.332	14 00	.350	.099
10	.008	.630	10	.119	.381	10	.359	.148
20	.010	.679	20	.124	.430	20	.367	.198
30	.011	.727	30	.129	.478	30	.376	.247
40	.012	.776	40	.134	.527	40	.385	.297
50	.014	.824	50	.139	.576	50	.394	.346
3 00	.016	.873	9 00	.144	.625	15 00	.403	.396
10	.018	.921	10	.150	.674	10	.412	.445
20	.020	.970	20	.155	.723	20	.421	.495
30	.022	1.018	30	.161	.272	30	.430	.544
40	.024	.067	40	.167	.821	40	.439	.594
50	.026	.116	50	.172	.869	50	.449	.644
4 00	.028	.164	10 00	.178	.918	16 00	.458	.693
10	.031	.213	10	.184	.967	10	.468	.743
20	.033	.261	20	.190	3.016	20	.478	.793
30	.036	.310	30	.197	.065	30	.488	.843
40	.039	.358	40	.203	.114	40	.498	.892
50	.042	.407	50	.209	.163	50	.508	.942
5 00	.044	.456	11 00	.216	.212	17 00	.518	.992
10	.047	.504	10	.222	.262	10	.528	5.042
20	.051	.553	20	.229	.311	20	.539	.092
30	.054	.601	30	.236	.360	30	.549	.142
40	.057	.650	40	.243	.409	40	.560	.192
50	.061	.699	50	.250	.458	50	.571	.242

TABLE 25. TANGENTS AND EXTERNALS TO THE UNIT DOUBLE-SPIRAL CURVE (*Continued*)

Length of each spiral = 100 ft; spiral angle = $\theta_s = \frac{1}{2}\Delta$

$D_c = 200\theta_s/l_s = 100\Delta/l_s$

Δ	T_s	E_s	Δ	T_s	E_s	Δ	T_s	E_s
18°00′	100.581	5.292	24°00′	101.041	7.115	30°00′	101.644	8.990
10	.592	.342	10	.056	.166	10	.663	9.043
20	.603	.392	20	.071	.218	20	.682	.096
30	.615	.442	30	.085	.269	30	.701	.149
40	.626	.492	40	.101	.321	40	.720	.203
50	.637	.543	50	.116	.372	50	.739	.256
19 00	.649	.593	25 00	.131	.424	31 00	.758	.309
10	.660	.643	10	.146	.476	10	.777	.362
20	.672	.693	20	.162	.527	20	.797	.416
30	.684	.744	30	.177	.579	30	.817	.469
40	.695	.794	40	.193	.631	40	.837	.523
50	.707	.844	50	.209	.682	50	.857	.576
20 00	.719	.894	26 00	.225	.734	32 00	.877	.629
10	.731	.944	10	.241	.786	10	.897	.683
20	.744	.995	20	.257	.837	20	.918	.737
30	.756	6.045	30	.274	.889	30	.938	.790
40	.769	.096	40	.290	.941	40	.959	.844
50	.781	.146	50	.307	.993	50	.980	.898
21 00	.794	.197	27 00	.324	8.045	33 00	102.001	.952
10	.807	.247	10	.341	.097	10	.022	10.006
20	.820	.298	20	.358	.149	20	.043	.060
30	.833	.348	30	.375	.201	30	.064	.114
40	.846	.399	40	.392	.253	40	.085	.168
50	.860	.450	50	.410	.306	50	.106	.223
22 00	.873	.502	28 00	.427	.358	34 00	.128	.277
10	.886	.553	10	.444	.410	10	.150	.331
20	.900	.604	20	.462	.463	20	.171	.386
30	.913	.655	30	.480	.515	30	.193	.440
40	.927	.706	40	.497	.568	40	.215	.495
50	.941	.757	50	.515	.620	50	.238	.549
23 00	.955	.808	29 00	.534	.673	35 00	.260	.604
10	.969	.859	10	.552	.726	10	.282	.659
20	.983	.910	20	.570	.778	20	.305	.713
30	.997	.961	30	.588	.831	30	.327	.768
40	101.012	7.013	40	.607	.884	40	.350	.823
50	.026	.064	50	.625	.937	50	.373	.878

TABLE 25. TANGENTS AND EXTERNALS TO THE UNIT DOUBLE-SPIRAL CURVE (*Continued*)

Length of each spiral = 100 ft; spiral angle = $\theta_s = \frac{1}{2}\Delta$

$D_c = 200\theta_s/l_s = 100\Delta/l_s$

Δ	T_s	E_s	Δ	T_s	E_s	Δ	T_s	E_s
36°00′	102.396	10.933	42°00′	103.310	12.962	48°00′	104.399	15.094
10	.419	.988	10	.338	13.020	10	.432	.155
20	.442	11.043	20	.366	.077	20	.465	.216
30	.465	.098	30	.394	.135	30	.498	.277
40	.489	.153	40	.422	.193	40	.531	.338
50	.513	.209	50	.450	.251	50	.565	.400
37 00	.537	.264	43 00	.479	.309	49 00	.598	.461
10	.561	.319	10	.508	.367	10	.632	.523
20	.585	.375	20	.536	.425	20	.666	.584
30	.609	.431	30	.565	.484	30	.700	.646
40	.633	.486	40	.594	.542	40	.735	.708
50	.657	.542	50	.624	.600	50	.769	.770
38 00	.682	.598	44 00	.653	.659	50 00	.803	.832
10	.707	.654	10	.682	.718	10	.838	.894
20	.731	.710	20	.712	.777	20	.873	.956
30	.756	.766	30	.742	.836	30	.908	16.018
40	.781	.822	40	.772	.895	40	.943	.081
50	.807	.879	50	.802	.954	50	.979	.143
39 00	.832	.935	45 00	.832	14.013	51 00	105.014	.206
10	.857	.991	10	.862	.072	10	.050	.269
20	.882	12.048	20	.892	.132	20	.085	.332
30	.908	.105	30	.923	.191	30	.121	.394
40	.934	.161	40	.953	.251	40	.158	.458
50	.960	.218	50	.984	.310	50	.194	.521
40 00	.987	.275	46 00	104.015	.370	52 00	.230	.584
10	103.013	.332	10	.046	.430	10	.267	.647
20	.039	.389	20	.077	.490	20	.304	.711
30	.065	.446	30	.108	.549	30	.340	.774
40	.092	.503	40	.140	.610	40	.377	.838
50	.119	.560	50	.171	.670	50	.415	.902
41 00	.146	.617	47 00	.203	.730	53 00	.452	.966
10	.173	.674	10	.235	.790	10	.490	17.030
20	.200	.732	20	.267	.851	20	.527	.094
30	.227	.789	30	.299	.911	30	.565	.158
40	.255	.847	40	.332	.972	40	.603	.223
50	.282	.904	50	.365	15.033	50	.642	.287

TABLE 25. TANGENTS AND EXTERNALS TO THE UNIT DOUBLE-SPIRAL CURVE (*Continued*)

Length of each spiral = 100 ft; spiral angle = $\theta_s = \frac{1}{2}\Delta$

$D_c = 200\theta_s/l_s = 100\Delta/l_s$

Δ	T_s	E_s	Δ	T_s	E_s	Δ	T_s	E_s
54°00′	105.680	17.352	60°00′	107.174	19.761	66°00′	108.909	22.355
10	.718	.417	10	.218	.830	10	.961	.430
20	.757	.481	20	.263	.900	20	109.013	.505
30	.796	.546	30	.309	.970	30	.065	.580
40	.835	.611	40	.354	20.040	40	.118	.656
50	.874	.677	50	.400	.110	50	.170	.731
55 00	.913	.742	61 00	.446	.180	67 00	.223	.807
10	.953	.807	10	.491	.250	10	.277	.883
20	.992	.872	20	.537	.321	20	.330	.959
30	106.032	.938	30	.584	.391	30	.384	23.036
40	.072	18.004	40	.630	.462	40	.438	.112
50	.113	.070	50	.677	.533	50	.492	.189
56 00	.153	.137	62 00	.724	.604	68 00	.546	.266
10	.194	.203	10	.771	.675	10	.601	.343
20	.234	.269	20	.818	.746	20	.655	.420
30	.275	.336	30	.865	.818	30	.710	.498
40	.316	.420	40	.913	.889	40	.765	.575
50	.358	.469	50	.961	.961	50	.821	.653
57 00	.399	.536	63 00	108.009	21.033	69 00	.876	.731
10	.441	.603	10	.057	.105	10	.932	.809
20	.482	.670	20	.105	.177	20	.988	.888
30	.524	.737	30	.154	.250	30	110.044	.966
40	.566	.805	40	.203	.322	40	.100	24.045
50	.609	.872	50	.252	.395	50	.157	.124
58 00	.651	.940	64 00	.301	.468	70 00	.214	.203
10	.694	19.008	10	.351	.541	10	.272	.282
20	.737	.075	20	.400	.614	20	.329	.362
30	.780	.143	30	.450	.688	30	.387	.442
40	.823	.211	40	.501	.761	40	.445	.522
50	.866	.280	50	.551	.835	50	.503	.602
59 00	.909	.348	65 00	.602	.909	71 00	.561	.682
10	.953	.416	10	.653	.983	10	.620	.762
20	.997	.485	20	.704	22.057	20	.679	.843
30	107.041	.554	30	.755	.131	30	.738	.924
40	.085	.623	40	.807	.206	40	.798	25.005
50	.129	.692	50	.858	.280	50	.857	.086

TABLE 26. FUNCTIONS OF THE SPIRAL WHICH IS APPLIED TO AN EXISTING 1° CIRCULAR CURVE

θ_s	l_s	a	D_c	x_c	y_c	L.T.	S.T.
0° 40.00′	**100.0**	33.33	1.3333°	99.99	0.39	66.67	33.33
1 00.00	150.0	50.00	1.3333	149.99	0.87	100.00	50.00
1 20.00	**200.0**	66.67	1.3333	199.98	1.55	133.34	66.67
2 00.00	**300.0**	100.00	1.3333	299.96	3.49	200.01	100.01
2 40.00	**400.0**	133.33	1.3333	399.90	6.20	266.69	133.36
3 00.00	450.0	150.00	1.3333	449.86	7.85	300.03	150.03
3 20.00	**500.0**	166.67	1.3334	499.81	9.69	333.38	166.71
4 00.00	**600.0**	200.00	1.3334	599.68	13.96	400.08	200.08
4 40.00	**700.0**	233.34	1.3334	699.50	19.06	466.81	233.47
5 00.00	750.0	250.02	1.3334	749.36	21.80	500.18	250.16
5 20.03	**800.0**	266.68	1.3334	799.28	24.81	533.54	266.86
6 00.00	**900.0**	300.05	1.3334	899.01	31.39	600.34	300.29
6 04.05	**910.0**	303.39	1.3335	908.98	32.09	607.03	303.64
08.05	**920.0**	306.71	1.3335	918.95	32.80	613.70	306.99
12.05	**930.0**	310.04	1.3335	928.91	33.52	620.38	310.33
16.06	**940.0**	313.37	1.3335	938.88	34.25	627.06	313.68
20.06	**950.0**	316.69	1.3335	948.84	34.98	633.72	317.03
24.06	**960.0**	320.02	1.3335	958.80	35.72	640.41	320.38
28.06	**970.0**	323.36	1.3335	968.76	36.47	647.09	323.73
32.06	**980.0**	326.70	1.3335	978.73	37.22	563.77	327.08
36.06	**990.0**	330.03	1.3335	988.69	37.98	660.46	330.42
40.06	**1000.0**	333.37	1.3336	998.65	38.75	667.14	333.76
44.07	**1010.0**	336.71	1.3336	1008.60	39.53	673.82	337.11
48.07	**1020.0**	340.05	1.3336	1018.56	40.32	680.50	340.46
52.07	**1030.0**	343.39	1.3336	1028.52	41.11	687.19	343.80
6 56.07	**1040.0**	346.72	1.3336	1038.48	41.91	693.87	347.15
7 00.00	1049.8	349.99	1.3336	1048.25	42.71	700.42	350.43
7 00.07	**1050.0**	350.04	1.3336	1048.43	42.72	700.54	350.49
04.08	**1060.0**	353.37	1.3336	1058.39	43.54	707.22	353.84
08.08	**1070.0**	356.71	1.3336	1068.34	44.37	713.90	357.18
12.08	**1080.0**	360.05	1.3336	1078.30	45.20	720.59	360.53
16.08	**1090.0**	363.39	1.3336	1088.25	46.04	727.28	363.88
20.08	**1100.0**	366.73	1.3336	1098.20	46.88	733.96	367.24
24.09	**1110.0**	370.07	1.3336	1108.15	47.74	740.65	370.60
28.09	**1120.0**	373.40	1.3336	1118.10	48.60	747.34	373.95
32.09	**1130.0**	376.74	1.3336	1128.05	49.47	754.02	377.29
36.09	**1140.0**	380.07	1.3336	1137.99	50.35	760.70	380.64
40.10	**1150.0**	383.40	1.3336	1147.94	51.24	767.39	383.99
44.10	**1160.0**	386.73	1.3336	1157.89	52.13	774.08	387.34
48.10	**1170.0**	390.07	1.3336	1167.83	53.03	780.76	390.69
52.10	**1180.0**	393.40	1.3336	1177.78	53.95	787.44	394.04
7 56.11	**1190.0**	396.73	1.3336	1187.72	54.86	794.12	397.39
8 00.00	1199.7	399.98	1.3336	1197.38	55.76	800.63	400.65
8 00.11	**1200.0**	400.06	1.3336	1197.66	55.79	800.80	400.73
04.11	**1210.0**	403.40	1.3337	1207.60	56.72	807.49	404.08
08.12	**1220.0**	406.74	1.3337	1217.54	57.66	814.19	407.44
12.12	**1230.0**	410.08	1.3337	1227.48	58.61	820.88	410.88
16.12	**1240.0**	413.42	1.3337	1237.42	59.56	827.57	414.16
20.12	**1250.0**	416.76	1.3337	1247.36	60.52	834.26	417.51
24.13	**1260.0**	420.10	1.3337	1257.29	61.50	840.96	420.87
28.13	**1270.0**	423.44	1.3337	1267.23	62.48	847.65	424.22

TABLE 26. FUNCTIONS OF THE SPIRAL WHICH IS APPLIED TO AN EXISTING 1° CIRCULAR CURVE (*Continued*)

θ_s	l_s	a	D_c	x_c	y_c	L.T.	S.T.
8° 32.13′	**1280.0**	426.77	1.3337°	1277.2	63.46	854.33	427.57
36.14	**1290.0**	430.10	1.3337	1287.1	64.45	861.02	430.92
40.14	**1300.0**	433.44	1.3337	1297.0	65.46	867.71	434.28
44.14	**1310.0**	436.77	1.3337	1307.0	66.47	874.40	437.63
48.15	**1320.0**	440.10	1.3337	1316.9	67.49	881.09	440.99
52.15	**1330.0**	443.44	1.3337	1326.8	68.51	887.78	444.35
8 56.16	**1340.0**	446.77	1.3337	1336.7	69.54	894.48	447.70
9 00.00	1349.6	449.97	1.3337	1346.3	70.54	900.90	450.93
9 00.16	**1350.0**	450.11	1.3337	1346.7	70.58	901.17	451.06
04.16	**1360.0**	453.44	1.3337	1356.6	71.63	907.86	454.42
08.17	**1370.0**	456.78	1.3337	1366.5	72.69	914.56	457.77
12.17	**1380.0**	460.12	1.3337	1376.4	73.75	921.25	461.13
16.18	**1390.0**	463.45	1.3337	1386.4	74.82	927.94	464.49
20.18	**1400.0**	466.78	1.3338	1396.3	75.90	934.63	467.85
24.18	**1410.0**	470.12	1.3338	1406.2	76.99	941.33	471.21
28.19	**1420.0**	473.46	1.3338	1416.1	78.08	948.03	474.57
32.19	**1430.0**	476.80	1.3338	1426.0	79.18	954.72	477.93
36.20	**1440.0**	480.13	1.3338	1436.0	80.29	961.42	481.29
40.20	**1450.0**	483.47	1.3338	1445.9	81.41	968.12	484.65
44.21	**1460.0**	486.81	1.3338	1455.8	82.53	974.81	488.01
48.21	**1470.0**	490.14	1.3338	1465.7	83.66	981.51	491.37
52.22	**1480.0**	493.48	1.3338	1475.6	84.80	988.21	494.73
9 56.22	**1490.0**	496.82	1.3338	1485.5	85.95	994.91	498.09
10 00.00	1499.5	499.96	1.3338	1494.9	87.05	1001.23	501.27
00.22	**1500.0**	500.16	1.3338	1495.4	87.11	1001.61	501.45
04.23	**1510.0**	503.49	1.3338	1505.3	88.27	1008.31	504.81
08.23	**1520.0**	506.83	1.3338	1515.2	89.44	1015.01	508.18
12.24	**1530.0**	510.17	1.3338	1525.2	90.62	1021.71	511.54
16.24	**1540.0**	513.50	1.3338	1535.1	91.81	1028.41	514.90
20.25	**1550.0**	516.84	1.3338	1545.0	93.00	1035.11	518.27
24.25	**1560.0**	520.18	1.3338	1554.9	94.20	1041.81	521.63
28.26	**1570.0**	523.52	1.3338	1564.8	95.41	1048.51	524.99
32.26	**1580.0**	526.85	1.3338	1574.7	96.63	1055.21	528.36
36.27	**1590.0**	530.19	1.3339	1584.6	97.85	1061.92	531.72
40.27	**1600.0**	533.53	1.3339	1594.5	99.08	1068.62	535.09
44.28	**1610.0**	536.87	1.3339	1604.4	100.32	1075.33	538.46
48.28	**1620.0**	540.21	1.3339	1614.3	101.57	1082.03	541.82
52.29	**1630.0**	543.54	1.3339	1624.1	102.82	1088.73	545.19
10 56.29	**1640.0**	546.88	1.3339	1634.0	104.09	1095.44	548.56
11 00.00	1649.3	549.95	1.3339	1643.2	105.27	1101.65	551.70
00.30	**1650.0**	550.21	1.3339	1643.9	105.36	1102.14	551.93
04.30	**1660.0**	553.54	1.3339	1653.8	106.64	1108.84	555.30
08.31	**1670.0**	556.87	1.3339	1663.7	107.92	1115.55	558.67
12.31	**1680.0**	560.21	1.3339	1673.6	109.22	1122.26	562.04
16.32	**1690.0**	563.55	1.3339	1683.5	110.52	1128.96	565.41
20.32	**1700.0**	566.88	1.3339	1693.3	111.83	1135.67	568.78
24.33	**1710.0**	570.22	1.3340	1703.2	113.15	1142.38	572.16
28.33	**1720.0**	573.56	1.3340	1713.1	114.47	1149.09	575.53
32.34	**1730.0**	576.90	1.3340	1723.0	115.80	1155.80	578.90
36.34	**1740.0**	580.23	1.3340	1732.9	117.14	1162.51	582.27
40.35	**1750.0**	583.57	1.3340	1742.7	118.48	1169.22	585.64

TABLE 26. FUNCTIONS OF THE SPIRAL WHICH IS APPLIED TO AN EXISTING 1° CIRCULAR CURVE (*Continued*)

θ_s	l_s	a	D_c	x_c	y_c	L.T.	S.T.
11° 44.36′	**1760.0**	586.91	1.3340°	1752.6	119.84	1175.93	589.02
48.36	**1770.0**	590.24	1.3340	1762.5	121.20	1182.64	592.39
52.37	**1780.0**	593.58	1.3340	1772.4	122.57	1189.35	595.77
11 56.37	**1790.0**	596.91	1.3340	1782.2	123.95	1196.06	599.15
12 00.00	1799.1	599.94	1.3340	1791.2	125.21	1202.14	602.20
00.39	**1800.0**	600.25	1.3340	1792.1	125.34	1202.77	602.52
04.39	**1810.0**	603.59	1.3340	1802.0	126.73	1209.49	605.89
08.39	**1820.0**	606.93	1.3340	1811.9	128.13	1216.20	609.27
12.40	**1830.0**	610.27	1.3341	1821.7	129.54	1222.91	612.65
16.40	**1840.0**	613.61	1.3341	1831.6	130.95	1229.63	616.02
20.41	**1850.0**	616.94	1.3341	1841.4	132.37	1236.35	619.40
24.42	**1860.0**	620.28	1.3341	1851.3	133.80	1243.06	622.78
28.42	**1870.0**	623.62	1.3341	1861.2	135.24	1249.78	626.16
32.43	**1880.0**	626.95	1.3341	1871.0	136.69	1256.60	629.54
36.43	**1890.0**	630.29	1.3341	1880.9	138.14	1263.21	632.92
40.44	**1900.0**	633.63	1.3341	1890.7	139.60	1269.93	636.30
44.45	**1910.0**	636.97	1.3341	1900.6	141.07	1276.65	639.68
48.46	**1920.0**	640.31	1.3341	1910.4	142.55	1283.37	643.06
52.46	**1930.0**	643.65	1.3341	1920.3	144.04	1290.09	646.44
12 56.47	**1940.0**	646.99	1.3341	1930.1	145.53	1296.81	649.82
13 00.00	1948.8	649.92	1.3342	1938.8	146.85	1302.72	652.80
00.48	**1950.0**	650.33	1.3342	1940.0	147.03	1303.53	653.20
04.49	**1960.0**	653.67	1.3342	1949.8	148.54	1310.25	656.59
08.50	**1970.0**	657.00	1.3342	1959.7	150.05	1316.97	659.97
12.50	**1980.0**	660.34	1.3342	1969.5	151.57	1323.70	663.35
16.51	**1990.0**	663.68	1.3342	1979.3	153.10	1330.42	666.74
20.52	**2000.0**	667.02	1.3342	1989.2	154.64	1337.15	670.12
24.53	**2010.0**	670.36	1.3342	1999.0	156.18	1343.87	673.51
28.54	**2020.0**	673.70	1.3342	2008.8	157.74	1350.59	676.89
32.54	**2030.0**	677.04	1.3342	2018.7	159.30	1357.32	680.28
36.55	**2040.0**	680.38	1.3342	2028.5	160.87	1364.05	683.67
40.56	**2050.0**	683.71	1.3342	2038.3	162.44	1370.77	687.06
44.57	**2060.0**	687.05	1.3343	2048.2	164.02	1377.50	690.45
48.58	**2070.0**	690.39	1 3343	2058.0	165.61	1384.23	693.84
52.58	**2080.0**	693.73	1.3343	2067.8	167.21	1390.95	697.23
13 56.59	**2090.0**	697.07	1.3343	2077.7	168.82	1397.68	700.62
14 00.00	2098.5	699.90	1.3343	2086.0	170.19	1403.40	703.50
00.60	**2100.0**	700.41	1.3343	2087.5	170.43	1404.41	704.01
04.61	**2110.0**	703.74	1.3343	2097.3	172.05	1411.14	707.40
08.62	**2120.0**	707.08	1.3343	2107.1	173.68	1417.87	710.79
12.63	**2130.0**	710.42	1.3343	2116.9	175.32	1424.60	714.18
16.64	**2140.0**	713.76	1.3343	2126.8	176.96	1431.34	717.57
20.64	**2150.0**	717.10	1.3343	2136.6	178.61	1438.07	720.97
24.65	**2160.0**	720.44	1.3343	2146.4	180.27	1444.80	724.36
28.66	**2170.0**	723.78	1.3344	2156.2	181.94	1451.53	727.76
32.67	**2180.0**	727.12	1.3344	2166.0	183.62	1458.27	731.15
36.68	**2190.0**	730.46	1.3344	2175.8	185.30	1465.01	734.55
40.69	**2200.0**	733.80	1.3344	2185.6	186.99	1471.74	737.95
44.70	**2210.0**	737.14	1.3344	2195.4	188.68	1478.48	741.34
48.71	**2220.0**	740.48	1.3344	2205.2	190.39	1485.22	744.74
52.72	**2230.0**	743.82	1.3344	2215.0	192.10	1491.95	748.14

TABLE 26. FUNCTIONS OF THE SPIRAL WHICH IS APPLIED TO AN EXISTING 1° CIRCULAR CURVE (*Continued*)

θ_s	l_s	a	D_c	x_c	y_c	L.T.	S.T.
14° 56.73′	**2240.0**	747.16	1.3344°	2224.8	193.82	1498.69	751.54
15 00.00	2248.2	749.88	1.3344	2232.8	195.23	1504.19	754.31
00.74	**2250.0**	750.50	1.3344	2234.6	195.55	1505.43	754.94
04.75	**2260.0**	753.84	1.3344	2244.4	197.28	1512.17	758.34
08.76	**2270.0**	757.18	1.3344	2254.2	199.02	1518.91	761.73
12.77	**2280.0**	760.52	1.3345	2264.0	200.77	1525.66	765.13
16.78	**2290.0**	763.86	1.3345	2273.8	202.53	1532.40	768.53
20.79	**2300.0**	767.20	1.3345	2283.5	204.30	1539.14	771.94
24.80	**2310.0**	770.54	1.3345	2293.3	206.07	1545.88	775.34
28.81	**2320.0**	773.88	1.3345	2303.1	207.85	1552.62	778.75
32.82	**2330.0**	777.22	1.3345	2312.9	209.64	1559.37	782.15
36.83	**2340.0**	780.56	1.3345	2322.7	211.44	1566.11	785.56
40.85	**2350.0**	783.90	1.3345	2332.5	213.24	1572.86	788.96
44.86	**2360.0**	787.24	1.3345	2342.2	215.05	1579.60	792.37
48.87	**2370.0**	790.58	1.3346	2352.0	216.87	1586.35	795.78
52.88	**2380.0**	793.92	1.3346	2361.8	218.69	1593.10	799.18
15 56.89	**2390.0**	797.26	1.3346	2371.6	220.52	1599.85	802.59
16 00.00	2397.8	799.85	1.3346	2379.1	221.95	1605.09	805.24
00.90	**2400.0**	800.60	1.3346	2381.3	222.37	1606.60	806.00
04.91	**2410.0**	803.94	1.3346	2391.1	224.22	1613.35	809.41
08.92	**2420.0**	807.28	1.3346	2400.9	226.07	1620.10	812.82
12.93	**2430.0**	810.62	1.3346	2410.6	227.93	1626.85	816.23
16.94	**2440.0**	813.96	1.3346	2420.4	229.80	1633.60	819.64
20.96	**2450.0**	817.30	1.3346	2430.1	231.68	1640.35	823.05
24.97	**2460.0**	820.64	1.3346	2439.9	233.57	1647.10	826.47
28.98	**2470.0**	823.98	1.3347	2449.6	235.46	1653.86	829.88
32.99	**2480.0**	827.32	1.3347	2459.4	237.36	1660.62	833.29
37.00	**2490.0**	830.66	1.3347	2469.1	239.27	1667.37	836.71
41.01	**2500.0**	834.00	1.3347	2478.9	241.19	1674.13	840.12
45.02	**2510.0**	837.35	1.3347	2488.6	243.11	1680.88	843.54
49.04	**2520.0**	840.69	1.3347	2498.4	245.04	1687.64	846.96
53.05	**2530.0**	844.03	1.3347	2508.1	246.98	1694.41	850.37
16 57.06	**2540.0**	847.37	1.3347	2517.9	248.93	1701.16	853.79
17 00.00	2547.3	849.82	1.3347	2525.0	250.36	1706.11	856.29
01.08	**2550.0**	850.71	1.3347	2527.6	250.88	1707.92	857.21
05.09	**2560.0**	854.05	1.3348	2537.3	252.84	1714.68	860.63
09.10	**2570.0**	857.39	1.3348	2547.1	254.81	1721.44	864.05
13.11	**2580.0**	860.73	1.3348	2556.8	256.79	1728.21	867.47
17.13	**2590.0**	864.07	1.3348	2566.5	258.77	1734.97	870.89
21.14	**2600.0**	867.42	1.3348	2576.2	260.76	1741.73	874.31
25.15	**2610.0**	870.76	1.3348	2586.0	262.76	1748.49	877.74
29.17	**2620.0**	874.10	1.3348	2595.7	264.77	1755.26	881.16
33.18	**2630.0**	877.45	1.3348	2605.4	266.78	1762.03	884.58
37.20	**2640.0**	880.79	1.3348	2615.1	268.80	1768.80	888.01
41.21	**2650.0**	884.13	1.3349	2624.9	270.83	1775.57	891.43
45.22	**2660.0**	887.47	1.3349	2634.6	272.87	1782.33	894.86
49.24	**2670.0**	890.82	1.3349	2644.3	274.91	1789.10	898.28
53.25	**2680.0**	894.16	1.3349	2654.0	276.96	1795.87	901.71
17 57.27	**2690.0**	897.50	1.3349	2663.7	279.02	1802.64	905.14
18 00.00	2696.8	899.79	1.3349	2670.3	280.43	1807.26	907.47
01.28	**2700.0**	900.84	1.3349	2673.4	281.09	1809.41	908.57

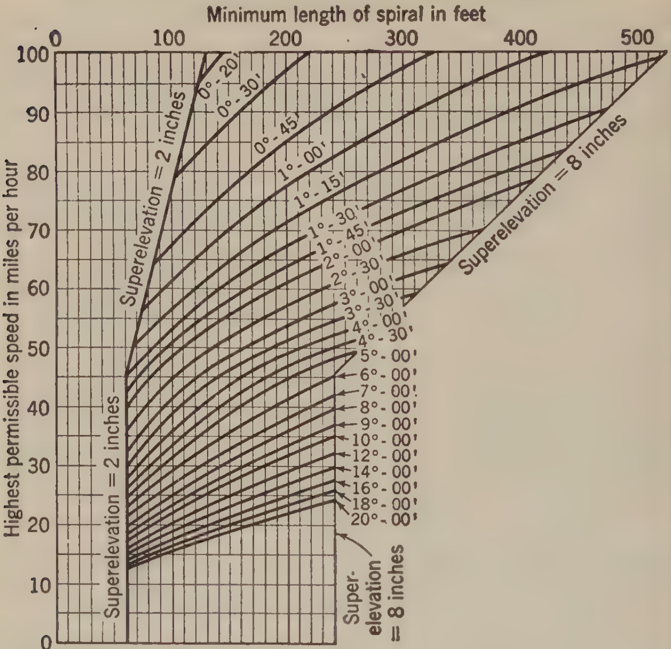
TABLE 26. FUNCTIONS OF THE SPIRAL WHICH IS APPLIED TO AN EXISTING 1° CIRCULAR CURVE (*Continued*)

θ_s	l_s	a	D_c	x_c	y_c	L.T.	S.T.
18° 05.30′	**2710.0**	904.19	1.3349°	2683.1	283.16	1816.18	912.00
09.31	**2720.0**	907.53	1.3349	2692.8	285.24	1822.96	915.43
13.33	**2730.0**	910.88	1.3350	2702.5	287.33	1829.74	918.86
17.34	**2740.0**	914.22	1.3350	2712.2	289.42	1836.50	922.28
21.36	**2750.0**	917.56	1.3350	2721.9	291.53	1843.27	925.71
25.37	**2760.0**	920.91	1.3350	2731.6	293.64	1850.07	929.15
29.39	**2770.0**	924.25	1.3350	2741.3	295.76	1856.84	932.59
33.40	**2780.0**	927.60	1.3350	2751.0	297.88	1863.62	936.02
37.42	**2790.0**	930.94	1.3350	2760.7	300.01	1870.40	939.46
18 41.43	**2800.0**	934.29	1.3350	2770.4	302.15	1877.18	942.89
19 00.00	2846.3	949.75	1.3351	2815.1	312.16	1908.05	958.80
21.58	**2900.0**	967.73	1.3352	2867.1	323.96	1945.03	977.30
20 00.00	2995.6	999.71	1.3353	2959.3	345.54	2009.98	1010.28
01.75	**3000.0**	1001.17	1.3353	2963.5	346.53	2012.96	1011.78
41.94	**3100.0**	1034.62	1.3354	3059.8	369.84	2080.97	1046.36
21 00.00	3144.9	1049.66	1.3355	3102.9	380.56	2111.58	1061.92
22.13	**3200.0**	1068.09	1.3356	3155.8	393.87	2149.09	1081.00
22 00.00	3294.2	1099.61	1.3357	3245.9	417.21	2213.33	1113.72
02.34	**3300.0**	1101.55	1.3357	3251.5	418.66	2217.30	1115.74
42.56	**3400.0**	1135.04	1.3358	3347.0	444.19	2285.60	1150.60
23 00.00	3443.4	1149.55	1.3359	3388.3	455.47	2315.25	1165.70
22.80	**3500.0**	1168.52	1.3360	3442.2	470.44	2354.01	1185.50
24 00.00	3592.4	1199.49	1.3361	3529.9	495.35	2417.35	1217.86
03.04	**3600.0**	1202.03	1.3361	3537.1	497.40	2422.53	1220.51
43.31	**3700.0**	1235.52	1.3363	3631.7	525.11	2491.15	1255.63
25 00.00	3741.5	1249.42	1.3364	3670.8	536.82	2519.64	1270.22
23.58	**3800.0**	1269.06	1.3365	3726.0	553.53	2559.90	1290.85
26 00.00	3890.4	1299.35	1.3366	3811.0	579.87	2622.13	1322.78
03.88	**3900.0**	1302.57	1.3367	3820.1	582.69	2628.75	1326.18
44.19	**4000.0**	1336.10	1.3368	3913.8	612.57	2697.73	1361.62
27 00.00	4039.2	1349.27	1.3369	3950.4	624.49	2724.82	1375.55
24.51	**4100.0**	1369.66	1.3370	4007.2	643.15	2766.83	1397.17
28 00.00	4188.0	1399.18	1.3372	4089.1	670.66	2827.72	1428.54
04.85	**4200.0**	1403.22	1.3372	4100.2	674.46	2836.06	1432.84
45.21	**4300.0**	1436.79	1.3374	4193.0	706.46	2905.42	1468.63
29 00.00	4336.6	1449.09	1.3375	4226.9	718.38	2930.85	1481.77
25.59	**4400.0**	1470.39	1.3376	4285.3	733.45	2974.91	1504.54
30 00.00	4485.2	1498.99	1.3377	4363.8	767.62	3034.23	1535.24
05.98	**4500.0**	1503.97	1.3378	4377.4	772.59	3044.55	1540.58
46.40	**4600.0**	1537.56	1.3380	4469.1	806.74	3114.32	1576.75
31 00.00	4633.7	1548.89	1.3381	4499.8	818.37	3137.84	1588.96
26.83	**4700.0**	1571.19	1.3382	4560.4	841.53	3184.24	1613.06
32 00.00	4782.0	1598.77	1.3383	4635.0	870.62	3241.70	1642.93
07.28	**4800.0**	1604.82	1.3384	4651.3	877.04	3254.14	1649.32
47.75	**4900.0**	1638.47	1.3386	4741.9	913.26	3324.55	1686.08
33 00.00	4930.3	1648.65	1.3387	4769.2	924.35	3345.83	1697.18
28.24	**5000.0**	1672.12	1.3388	4832.0	950.13	3394.93	1722.81
34 00.00	5078.4	1698.52	1.3390	4902.5	979.54	3450.23	1751.71
08.75	**5100.0**	1705.80	1.3391	4921.8	987.71	3465.49	1759.68
49.29	**5200.0**	1739.49	1.3393	5011.2	1025.95	3536.21	1796.72
35 00.00	5226.4	1748.39	1.3394	5034.7	1036.18	3554.91	1806.53

TABLE 26. FUNCTIONS OF THE SPIRAL WHICH IS APPLIED TO AN EXISTING 1° CIRCULAR CURVE (*Continued*)

θ_s	l_s	a	D_c	x_c	y_c	L.T.	S.T.
35° 29.85′	**5300.0**	1773.17	1.3396°	5100.1	1064.89	3607.08	1833.91
36 00.00	5374.3	1798.24	1.3397	5166.0	1094.25	3659.89	1861.65
10.42	**5400.0**	1806.89	1.3398	5188.7	1104.47	3678.15	1871.26
51.02	**5500.0**	1840.62	1.3400	5276.8	1114.74	3749.40	1908.77
37 00.00	5522.1	1848.08	1.3401	5296.2	1153.73	3765.18	1917.09
31.64	**5600.0**	1874.37	1.3403	5364.5	1185.65	3820.83	1946.45
38 00.00	5669.8	1897.92	1.3404	5425.4	1214.61	3870.78	1972.85
12.29	**5700.0**	1908.13	1.3405	5451.7	1227.23	3892.43	1984.30
52.96	**5800.0**	1941.91	1.3408	5538.5	1269.46	3964.26	2022.35
39 00.00	5817.3	1947.75	1.3408	5553.5	1276.86	3976.70	2028.95
33.66	**5900.0**	1975.69	1.3411	5624.9	1306.63	4036.27	2060.57
40 00.00	5964.7	1997.56	1.3412	5680.5	1340.47	4082.96	2085.40
14.38	**6000.0**	2009.51	1.3413	5710.7	1355.88	4108.48	2098.98
55.13	**6100.0**	2043.33	1.3416	5796.1	1400.06	4180.90	2137.58
41 00.00	6111.9	2047.37	1.3416	5806.3	1405.41	4189.57	2142.20
35.91	**6200.0**	2077.16	1.3419	5881.1	1444.89	4253.54	2176.38
42 00.00	6259.1	2097.16	1.3421	5931.0	1471.67	4296.54	2199.38
16.71	**6300.0**	2111.02	1.3422	5965.5	1490.33	4326.39	2215.37
57.54	**6400.0**	2144.90	1.3425	6049.5	1536.42	4399.47	2254.57
43 00.00	6406.0	2146.95	1.3425	6054.5	1539.23	4403.89	2256.94
38.39	**6500.0**	2178.80	1.3428	6132.9	1583.11	4472.78	2293.98
44 00.00	6552.9	2196.72	1.3429	6176.8	1608.06	4511.62	2314.90
19.28	**6600.0**	2212.70	1.3431	6215.9	1630.43	4546.15	2333.45
45 00.00	6699.5	2246.48	1.3434	6297.9	1678.15	4619.75	2373.27
00.19	**6700.0**	2246.64	1.3434	6298.3	1678.36	4620.10	2373.45
41.14	**6800.0**	2280.59	1.3437	6380.2	1726.91	4694.12	2413.53
46 00.00	6846.0	2296.23	1.3438	6417.7	1749.48	4728.29	2432.06
22.11	**6900.0**	2314.56	1.3440	6461.6	1776.04	4768.40	2453.84
47 00.00	6992.4	2345.97	1.3443	6536.3	1822.02	4837.26	2491.29
03.12	**7000.0**	2348.55	1.3443	6542.4	1825.81	4842.93	2494.38
44.15	**7100.0**	2382.57	1.3447	6622.8	1876.15	4917.73	2535.16
48 00.00	7138.6	2395.70	1.3448	6653.6	1895.74	4946.67	2550.97
25.22	**7200.0**	2416.60	1.3450	6702.5	1927.07	4992.79	2576.20
49 00.00	7284.6	2445.41	1.3453	6769.6	1970.64	5056.53	2611.12
06.32	**7300.0**	2450.65	1.3454	6781.7	1978.58	5068.13	2617.48
47.46	**7400.0**	2484.76	1.3457	6860.4	2030.67	5143.79	2659.03
50 00.00	7430.5	2495.11	1.3458	6884.2	2046.68	5166.86	2671.75
28.63	**7500.0**	2518.80	1.3461	6938.4	2083.32	5219.65	2700.85
51 00.00	7576.2	2544.79	1.3463	6997.5	2123.84	5277.66	2732.87
09.83	**7600.0**	2552.92	1.3464	7015.9	2136.55	5295.85	2742.93
51.07	**7700.0**	2587.06	1.3468	7092.9	2190.37	5372.36	2785.30
52 00.00	7721.7	2594.46	1.3469	7109.4	2202.10	5388.96	2794.50
32.34	**7800.0**	2621.23	1.3472	7169.2	2244.71	5449.17	2827.94
53 00.00	7867.0	2644.12	1.3474	7220.0	2281.43	5500.78	2856.66
13.65	**7900.0**	2655.41	1.3475	7244.9	2299.63	5526.29	2870.88
55.00	**8000.0**	2689.61	1.3479	7320.0	2355.08	5603.73	2914.12
54 00.00	8012.1	2693.80	1.3480	7329.1	2361.82	5613.12	2919.37
36.38	**8100.0**	2723.85	1.3483	7394.6	2411.02	5681.51	2957.66
55 00.00	8157.0	2743.38	1.3485	7436.8	2443.23	5726.01	2982.63
17.81	**8200.0**	2758.10	1.3487	7468.5	2467.56	5759.62	3001.52
59.27	**8300.0**	2792.38	1.3491	7541.7	2524.62	5837.89	3045.51

TABLE 27. MINIMUM LENGTH OF SPIRALS FOR RAILROADS*



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* From the A.R.E.A. Manual of 1929.

Limiting Curves. For all curves that are liable to limit the speed of trains, the length of spiral should equal that indicated on the line marked "superelevation = 8 inches." Longer spirals may be used provided the increased length does not adversely affect the degree of curve or seriously affect the cost of construction.

Minor Curves. For minor curves the length of spiral should never be less than that indicated by the diagram. An increase of about 50 per cent over the indicated length may be desirable where cost is not seriously affected.

Spirals need not be used when superelevation required for highest permissible speed is less than 2 in.

TABLE 28 (PART 1). EXTRA AREA OF THE TWO SPIRALED PORTIONS OF WIDENED PAVEMENT

Area of both portions $= \frac{l_s f}{9} - B$ (sq. yd.),

where l_s = length of each ℄ spiral;

f = maximum central widening.

Based on normal width of pavement = **16 ft.**

Values of B:

θ_s	$f =$								θ_s
	1.0′	1.5′	2.0′	2.5′	3.0′	3.5′	4.0′	4.5′	
5°	0.11	0.17	0.23	0.29	0.35	0.42	0.49	0.56	5°
10	0.22	0.33	0.45	0.58	0.71	0.84	0.98	1.13	10
15	0.33	0.50	0.68	0.87	1.06	1.26	1.47	1.69	15
20	0.43	0.66	0.91	1.16	1.42	1.69	1.97	2.25	20
25	0.54	0.83	1.13	1.44	1.77	2.11	2.46	2.82	25
30	0.65	1.00	1.36	1.73	2.12	2.53	2.95	3.38	30
35	0.76	1.16	1.58	2.02	2.48	2.95	3.44	3.95	35
40	0.87	1.33	1.81	2.31	2.83	3.37	3.93	4.51	40
45	0.97	1.49	2.04	2.60	3.18	3.79	4.42	5.07	45
50	1.08	1.66	2.26	2.89	3.54	4.21	4.91	5.64	50
55	1.19	1.83	2.49	3.18	3.89	4.64	5.40	6.20	55
60	1.30	1.99	2.71	3.47	4.25	5.06	5.90	6.76	60
65	1.41	2.16	2.94	3.76	4.60	5.48	6.39	7.33	65
70	1.52	2.32	3.17	4.04	4.96	5.90	6.88	7.89	70

θ_s	$f =$								θ_s
	5.0′	5.5′	6.0′	6.5′	7.0′	8.0′	9.0′	10.0′	
5°	0.64	0.72	0.80	0.88	0.96	1.14	1.32	1.52	5°
10	1.28	1.43	1.59	1.75	1.92	2.27	2.65	3.04	10
15	1.91	2.15	2.39	2.63	2.88	3.41	3.97	4.56	15
20	2.55	2.86	3.18	3.51	3.85	4.55	5.29	6.08	20
25	3.19	3.58	3.97	4.39	4.81	5.69	6.62	7.60	25
30	3.83	4.29	4.77	5.26	5.77	6.83	7.94	9.11	30
35	4.47	5.01	5.57	6.14	6.73	7.96	9.26	10.63	35
40	5.11	5.72	6.36	7.02	7.69	9.10	10.59	12.15	40
45	5.75	6.44	7.16	7.89	8.65	10.24	11.91	13.67	45
50	6.38	7.16	7.95	8.77	9.61	11.38	13.23	15.19	50
55	7.02	7.87	8.75	9.65	10.58	12.51	14.56	16.71	55
60	7.66	8.59	9.54	10.53	11.54	13.65	15.88	18.23	60
65	8.30	9.30	10.34	11.40	12.50	14.79	17.21	19.75	65
70	8.94	10.02	11.13	12.28	13.46	15.93	18.53	21.27	70

Area of intermediate circular portion is given in Table 29.

TABLE 28 (PART 2). EXTRA AREA OF THE TWO SPIRALED PORTIONS OF WIDENED PAVEMENT

Area of both portions $= \frac{l_s f}{9} - B$ (sq. yd.),

where l_s = length of each ℄ spiral;

f = maximum central widening.

Based on normal width of pavement = **18 ft.**

Values of B:

θ_s	$f =$								θ_s
	1.0′	1.5′	2.0′	2.5′	3.0′	3.5′	4.0′	4.5′	
5°	0.12	0.19	0.25	0.32	0.39	0.47	0.54	0.62	5°
10	0.24	0.37	0.50	0.64	0.78	0.93	1.09	1.24	10
15	0.36	0.56	0.76	0.96	1.18	1.40	1.63	1.87	15
20	0.48	0.74	1.01	1.28	1.57	1.87	2.17	2.49	20
25	0.61	0.93	1.26	1.61	1.96	2.33	2.72	3.11	25
30	0.73	1.11	1.51	1.93	2.36	2.80	3.26	3.73	30
35	0.85	1.30	1.76	2.25	2.75	3.27	3.80	4.35	35
40	0.97	1.48	2.02	2.57	3.14	3.73	4.34	4.97	40
45	1.09	1.67	2.27	2.89	3.53	4.20	4.89	5.60	45
50	1.21	1.85	2.52	3.21	3.93	4.67	5.43	6.22	50
55	1.33	2.04	2.77	3.53	4.32	5.13	5.97	6.84	55
60	1.54	2.33	3.02	3.85	4.71	5.60	6.52	7.46	60
65	1.66	2.41	3.27	4.18	5.10	6.07	7.06	8.08	65
70	1.78	2.60	3.52	4.50	5.49	6.53	7.60	8.70	70

θ_s	$f =$								θ_s
	5.0′	5.5′	6.0′	6.5′	7.0′	8.0′	9.0′	10.0′	
5°	0.70	0.79	0.87	0.96	1.05	1.24	1.44	1.65	5°
10	1.41	1.57	1.74	1.92	2.10	2.48	2.88	3.30	10
15	2.11	2.36	2.62	2.88	3.16	3.72	4.32	4.94	15
20	2.81	3.15	3.49	3.84	4.21	4.96	5.76	6.59	20
25	3.51	3.93	4.36	4.81	5.26	6.21	7.20	8.24	25
30	4.22	4.72	5.24	5.77	6.31	7.45	8.64	9.89	30
35	4.92	5.51	6.11	6.73	7.36	8.69	10.08	11.54	35
40	5.62	6.29	6.98	7.69	8.42	9.93	11.52	13.19	40
45	6.33	7.08	7.85	8.65	9.47	11.17	12.96	14.83	45
50	7.03	7.87	8.73	9.61	10.52	12.41	14.40	16.48	50
55	7.73	8.65	9.60	10.57	11.57	13.65	15.84	18.13	55
60	8.44	9.44	10.47	11.53	12.62	14.89	17.28	19.78	60
65	9.14	10.23	11.34	12.49	13.67	15.13	18.72	21.43	65
70	9.84	11.01	12.21	13.46	14.72	16.37	19.16	23.08	70

Area of intermediate circular portion is given in Table 29.

TABLE 28 (PART 3). EXTRA AREA OF THE TWO SPIRALED PORTIONS OF WIDENED PAVEMENT

Area of both portions $= \frac{l_s f}{9} - B$ (sq. yd.),

where l_s = length of each ℄ spiral;

f = maximum central widening.

Based on normal width of pavement = **20 ft.**

Values of B:

θ_s	$f =$								θ_s
	1.0′	1.5′	2.0′	2.5′	3.0′	3.5′	4.0′	4.5′	
5°	0.13	0.20	0.28	0.35	0.43	0.51	0.60	0.68	5°
10	0.27	0.41	0.56	0.71	0.86	1.02	1.19	1.36	10
15	0.40	0.61	0.83	1.06	1.29	1.54	1.78	2.04	15
20	0.54	0.82	1.11	1.41	1.73	2.05	2.38	2.72	20
25	0.67	1.02	1.39	1.77	2.16	2.56	2.97	3.40	25
30	0.80	1.23	1.67	2.12	2.59	3.07	3.57	4.08	30
35	0.94	1.43	1.95	2.47	3.02	3.58	4.16	4.76	35
40	1.07	1.64	2.22	2.83	3.45	4.10	4.76	5.44	40
45	1.21	1.84	2.50	3.18	3.88	4.61	5.35	6.12	45
50	1.34	2.05	2.78	3.54	4.31	5.12	5.95	6.80	50
55	1.47	2.25	3.06	3.89	4.75	5.63	6.54	7.48	55
60	1.61	2.46	3.34	4.24	5.18	6.14	7.14	8.16	60
65	1.74	2.66	3.61	4.60	5.61	6.65	7.73	8.84	65
70	1.88	2.87	3.89	4.95	6.04	7.17	8.33	9.52	70

θ_s	$f =$								θ_s
	5.0′	5.5′	6.0′	6.5′	7.0′	8.0′	9.0′	10.0′	
5°	0.77	0.86	0.95	1.05	1.14	1.34	1.56	1.79	5°
10	1.53	1.72	1.90	2.09	2.28	2.69	3.11	3.55	10
15	2.30	2.57	2.85	3.14	3.43	4.03	4.67	5.33	15
20	3.07	3.43	3.80	4.18	4.57	5.38	6.22	7.11	20
25	3.84	4.29	4.75	5.23	5.71	6.72	7.78	8.89	25
30	4.61	5.15	5.70	6.27	6.86	8.07	9.34	10.67	30
35	5.37	6.00	6.65	7.32	8.00	9.41	10.89	12.44	35
40	6.14	6.86	7.60	8.36	9.14	10.76	12.45	14.22	40
45	6.91	7.72	8.55	9.41	10.28	12.10	14.01	16.00	45
50	7.68	8.58	9.50	10.45	11.42	13.44	15.56	17.78	50
55	8.44	9.43	10.45	11.50	12.57	14.79	17.12	19.55	55
60	9.21	10.29	11.40	12.54	13.71	16.13	18.67	21.33	60
65	9.98	11.15	12.35	13.59	14.85	17.48	20.23	23.11	65
70	10.75	12.01	13.30	14.63	16.00	18.82	21.79	24.89	70

Area of intermediate circular portion is given in Table 29.

TABLE 28 (PART 4). EXTRA AREA OF THE TWO SPIRALED PORTIONS OF WIDENED PAVEMENT

$$\text{Area of both portions} = \frac{l_s f}{9} - B \text{ (sq. yd.)},$$

where l_s = length of each ℄ spiral;

f = maximum central widening.

Based on normal width of pavement = **22 ft.**

Values of B:

θ_s	$f =$ 1.0′	1.5′	2.0′	2.5′	3.0′	3.5′	4.0′	4.5′	θ_s
5°	0.15	0.22	0.30	0.39	0.47	0.56	0.65	0.74	5°
10	0.29	0.45	0.61	0.77	0.94	1.11	1.29	1.48	10
15	0.44	0.67	0.91	1.16	1.41	1.67	1.94	2.21	15
20	0.59	0.90	1.22	1.54	1.88	2.23	2.59	2.95	20
25	0.74	1.12	1.52	1.93	2.35	2.79	3.23	3.69	25
30	0.88	1.35	1.82	2.32	2.82	3.34	3.88	4.43	30
35	1.03	1.57	2.13	2.70	3.29	3.90	4.52	5.17	35
40	1.18	1.79	2.43	3.09	3.76	4.46	5.17	5.91	40
45	1.32	2.02	2.73	3.47	4.23	5.01	5.82	6.64	45
50	1.47	2.24	3.04	3.86	4.70	5.57	6.46	7.38	50
55	1.62	2.47	3.34	4.24	5.17	6.13	7.11	8.12	55
60	1.76	2.69	3.65	4.63	5.64	6.69	7.76	8.86	60
65	1.91	2.91	3.95	5.02	6.11	7.24	8.40	9.60	65
70	2.06	3.14	4.25	5.40	6.58	7.80	9.05	10.33	70

θ_s	$f =$ 5.0′	5.5′	6.0′	6.5′	7.0′	8.0′	9.0′	10.0′	θ_s
5°	0.83	0.93	1.05	1.13	1.23	1.45	1.67	1.91	5°
10	1.66	1.86	2.09	2.26	2.47	2.90	3.35	3.81	10
15	2.50	2.79	3.14	3.39	3.70	4.34	5.02	5.72	15
20	3.33	3.72	4.18	4.52	4.93	5.79	6.69	7.63	20
25	4.16	4.64	5.23	5.65	6.17	7.24	8.36	9.53	25
30	4.99	5.57	6.27	6.78	7.40	8.69	10.04	11.44	30
35	5.83	6.50	7.32	7.90	8.63	10.14	11.71	13.35	35
40	6.66	7.43	8.36	9.03	9.86	11.58	13.38	15.26	40
45	7.49	8.36	9.41	10.16	11.10	13.03	15.05	17.16	45
50	8.32	9.29	10.45	11.29	12.33	14.48	16.73	19.07	50
55	9.15	10.22	11.50	12.42	13.56	15.93	18.40	20.98	55
60	9.99	11.15	12.54	13.55	14.80	17.38	20.07	22.88	60
65	10.82	12.07	13.59	14.68	16.03	18.82	21.74	24.79	65
70	11.65	13.00	14.13	15.81	17.26	20.27	23.42	26.70	70

Area of intermediate circular portion is given in Table 29.

TABLE 29 (PART 1). EXTRA AREA OF THE INTERMEDIATE CIRCULAR PORTION OF WIDENED PAVEMENT (IN SQUARE YARDS) PER LINEAL FOOT OF NOMINAL ℄ CURVE

	Deg. of curve *D*	*f* = constant central widening =										Deg. of curve *D*
		1′	2′	3′	4′	5′	6′	7′	8′	9′	10′	
16-ft. pavement	5°	.110	.220	.331	.441	.550	.660	.770	.880	.989	1.099	5°
	6	.110	.220	.330	.440	.549	.659	.768	.878	.987	1.096	6
	7	.110	.220	.329	.439	.548	.658	.767	.876	.985	1.093	7
	8	.110	.219	.329	.438	.547	.656	.765	.874	.983	1.091	8
	9	.110	.219	.328	.437	.546	.655	.764	.872	.980	1.088	9
	10	.109	.219	.328	.437	.545	.654	.762	.870	.978	1.086	10
	11	.109	.218	.327	.436	.544	.652	.761	.868	.976	1.083	11
	12	.109	.218	.327	.435	.543	.651	.759	.867	.974	1.081	12
	13	.109	.218	.326	.434	.542	.650	.757	.865	.972	1.078	13
	14	.109	.217	.326	.434	.541	.649	.756	.863	.969	1.076	14
	15	.109	.217	.325	.433	.540	.647	.754	.861	.967	1.073	15
	16	.108	.217	.324	.432	.539	.646	.753	.859	.965	1.071	16
	17	.108	.216	.324	.431	.538	.645	.751	.857	.963	1.068	17
	18	.108	.216	.323	.430	.537	.643	.750	.855	.961	1.066	18
	19	.108	.216	.323	.430	.536	.642	.748	.854	.959	1.063	19
	20	.108	.215	.322	.429	.535	.641	.747	.852	.956	1.061	20
	30	.106	.212	.317	.421	.525	.628	.731	.833	.935	1.035	30
	40	.105	.208	.311	.413	.515	.615	.715	.814	.913	1.010	40
	50	.103	.205	.306	.406	.505	.602	.700	.796	.891	.985	50
	60	.101	.201	.300	.398	.494	.589	.684	.777	.869	.960	60
	70	.100	.198	.295	.390	.484	.576	.669	.759	.847	.935	70
	80	.098	.194	.289	.382	.474	.563	.653	.740	.825	.909	80
	90	.096	.191	.284	.375	.464	.551	.637	.721	.804	.884	90
	100	.095	.187	.278	.367	.454	.538	.622	.703	.782	.859	100
		f = constant central widening =										
		1′	2′	3′	4′	5′	6′	7′	8′	9′	10′	
18-ft. pavement	5°	.110	.220	.330	.440	.550	.660	.769	.879	.988	1.098	5°
	6	.110	.220	.330	.439	.549	.658	.768	.877	.986	1.095	6
	7	.110	.220	.329	.438	.548	.657	.766	.875	.984	1.092	7
	8	.110	.219	.328	.438	.547	.655	.764	.873	.981	1.089	8
	9	.109	.219	.328	.437	.546	.654	.763	.871	.979	1.087	9
	10	.109	.218	.327	.436	.544	.653	.761	.869	.976	1.084	10
	11	.109	.218	.327	.435	.543	.651	.759	.867	.974	1.081	11
	12	.109	.218	.326	.434	.542	.650	.757	.865	.972	1.079	12
	13	.109	.217	.325	.433	.541	.649	.756	.863	.969	1.076	13
	14	.109	.217	.325	.432	.540	.647	.754	.861	.967	1.073	14
	15	.108	.216	.324	.432	.539	.646	.752	.859	.965	1.070	15
	16	.108	.216	.324	.431	.538	.644	.751	.857	.962	1.068	16
	17	.108	.216	.323	.430	.537	.643	.749	.855	.960	1.065	17
	18	.108	.215	.322	.429	.535	.642	.747	.853	.958	1.062	18
	19	.108	.215	.322	.428	.534	.640	.746	.851	.955	1.060	19
	20	.107	.214	.321	.427	.533	.639	.744	.849	.953	1.057	20
	30	.106	.211	.315	.419	.522	.625	.727	.828	.929	1.030	30
	40	.104	.207	.309	.410	.511	.611	.710	.808	.906	1.003	40
	50	.102	.203	.303	.402	.500	.597	.693	.788	.882	.975	50
	60	.100	.199	.297	.393	.489	.583	.676	.768	.859	.948	60
	70	.098	.195	.291	.385	.478	.569	.659	.748	.835	.921	70
	80	.096	.191	.284	.376	.466	.555	.642	.728	.812	.894	80
	90	.095	.187	.278	.368	.455	.541	.625	.707	.788	.867	90
	100	.093	.183	.272	.359	.444	.527	.608	.687	.764	.840	100

TABLE 29 (PART 2). EXTRA AREA OF THE INTERMEDIATE CIRCULAR PORTION OF WIDENED PAVEMENT (IN SQUARE YARDS) PER LINEAL FOOT OF NOMINAL ℄ CURVE

	Deg. of curve *D*	*f* = constant central widening =										Deg. of curve *D*
		1′	2′	3′	4′	5′	6′	7′	8′	9′	10′	
20-ft. pavement	5°	.111	.220	.330	.440	.549	.659	.769	.878	.987	1.097	5°
	6	.110	.220	.329	.439	.548	.658	.767	.876	.985	1.094	6
	7	.110	.219	.329	.438	.547	.656	.765	.874	.982	1.091	7
	8	.109	.219	.328	.437	.546	.655	.763	.872	.980	1.088	8
	9	.109	.218	.327	.436	.545	.653	.761	.869	.977	1.085	9
	10	.109	.218	.327	.435	.543	.652	.759	.867	.975	1.082	10
	11	.109	.218	.326	.434	.542	.650	.758	.865	.972	1.079	11
	12	.109	.217	.325	.433	.541	.649	.756	.863	.970	1.076	12
	13	.108	.217	.325	.432	.540	.647	.754	.861	.967	1.073	13
	14	.108	.216	.324	.431	.539	.645	.752	.858	.965	1.070	14
	15	.108	.216	.323	.430	.537	.644	.750	.856	.962	1.067	15
	16	.108	.215	.323	.430	.536	.642	.748	.854	.960	1.065	16
	17	.108	.215	.322	.429	.535	.641	.747	.852	.957	1.062	17
	18	.107	.215	.321	.428	.534	.639	.745	.850	.954	1.059	18
	19	.107	.214	.321	.427	.533	.638	.743	.848	.952	1.056	19
	20	.107	.214	.320	.426	.531	.636	.741	.845	.949	1.053	20
	30	.105	.209	.313	.417	.519	.621	.723	.824	.924	1.024	30
	40	.103	.205	.307	.407	.507	.606	.704	.802	.899	.995	40
	50	.101	.201	.300	.398	.495	.591	.686	.780	.873	.966	50
	60	.099	.197	.293	.389	.483	.576	.668	.759	.848	.937	60
	70	.097	.192	.287	.379	.471	.561	.649	.737	.823	.907	70
	80	.095	.188	.280	.370	.459	.546	.631	.715	.798	.878	80
	90	.093	.184	.273	.361	.446	.531	.613	.693	.772	.849	90
	100	.091	.180	.266	.351	.434	.515	.595	.672	.747	.820	100
		f = constant central widening =										
		1′	2′	3′	4′	5′	6′	7′	8′	9′	10′	
22-ft. pavement	5°	.110	.220	.330	.439	.549	.659	.768	.877	.986	1.096	5°
	6	.110	.219	.329	.438	.548	.657	.766	.875	.984	1.092	6
	7	.110	.219	.328	.437	.546	.655	.764	.873	.981	1.089	7
	8	.109	.218	.328	.436	.545	.654	.762	.870	.978	1.086	8
	9	.109	.218	.327	.435	.544	.652	.760	.868	.976	1.083	9
	10	.109	.218	.326	.434	.542	.650	.758	.866	.973	1.080	10
	11	.109	.217	.325	.433	.541	.649	.756	.863	.970	1.077	11
	12	.108	.217	.325	.432	.540	.647	.754	.861	.968	1.074	12
	13	.108	.216	.324	.431	.539	.645	.752	.859	.965	1.071	13
	14	.108	.216	.323	.430	.537	.644	.750	.856	.962	1.068	14
	15	.108	.215	.322	.429	.536	.642	.748	.854	.959	1.065	15
	16	.108	.215	.322	.428	.535	.641	.746	.852	.957	1.061	16
	17	.107	.214	.321	.427	.533	.639	.744	.849	.954	1.058	17
	18	.107	.214	.320	.426	.532	.637	.742	.847	.951	1.055	18
	19	.107	.213	.320	.425	.531	.636	.740	.845	.949	1.052	19
	20	.107	.213	.319	.424	.529	.634	.738	.842	.946	1.049	20
	30	.104	.208	.312	.414	.516	.618	.719	.819	.919	1.018	30
	40	.102	.204	.304	.404	.503	.602	.699	.796	.892	.987	40
	50	.100	.199	.297	.394	.490	.585	.679	.773	.865	.956	50
	60	.098	.194	.290	.384	.477	.569	.660	.749	.838	.925	60
	70	.095	.190	.282	.374	.464	.553	.640	.726	.811	.894	70
	80	.093	.185	.275	.364	.451	.536	.620	.703	.784	.863	80
	90	.091	.180	.268	.354	.438	.520	.601	.679	.757	.832	90
	100	.089	.176	.261	.344	.425	.504	.581	.656	.729	.801	100

TABLE 30. FACTORS A, B, AND C GIVING THE EXTRA AREA OF WIDENED PAVEMENT, WHERE SURVEYED CENTER LINE IS A CIRCULAR CURVE

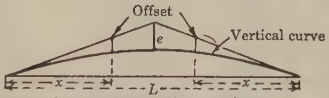
Formula: Area (square yards) $= f\left(\frac{A}{D} - Bw + Cf\right)$, in which D = ℄ degree of curve; w = normal width of pavement (ft), f = maximum widening, ft (see Art. 114).

θ_c*	A	B	C
1°	15.10	0.0013	8.648
2	29.60	0.0026	4.238
3	44.42	0.0039	2.825
4	59.24	0.0052	2.117
5	74.05	0.0065	1.692
6	88.86	0.0078	1.408
7	103.69	0.0090	1.205
8	118.49	0.0103	1.052
9	133.28	0.0116	0.933
10	148.07	0.0129	0.838
11	162.86	0.0142	0.759
12	177.65	0.0155	0.694
13	192.43	0.0168	0.638
14	207.20	0.0181	0.591
15	221.97	0.0194	0.549
16	236.73	0.0207	0.513
17	251.48	0.0219	0.480
18	266.23	0.0232	0.451
19	280.96	0.0245	0.426
20	295.69	0.0258	0.402
21	310.41	0.0271	0.381
22	325.12	0.0284	0.361
23	339.82	0.0297	0.343
24	354.51	0.0309	0.327
25	369.19	0.0322	0.312
26	383.85	0.0335	0.298
27	398.51	0.0348	0.284
28	413.15	0.0361	0.272
29	427.78	0.0373	0.260
30	442.39	0.0386	0.250
35	515.25	0.0450	0.204
40	587.69	0.0513	0.168
45	659.66	0.0576	0.139
50	731.08	0.0638	0.115
55	801.88	0.0700	0.094
60	871.98	0.0761	0.076

* θ_c = angle subtending arc AB (Fig. 92).

TABLE 31. COEFFICIENTS *C* GIVING VERTICAL OFFSET FROM TANGENT TO VERTICAL CURVE

x	*L* = 200	*L* = 250	*L* = 300	*L* = 400	*L* = 450	*L* = 500	*L* = 550	*L* = 600	*L* = 700	*L* = 800	*L* = 900
25	.0625	.04000	.02778	.01562	.01235	.01000	.00826	.00694	.00510	.00391	.00309
50	.2500	.16000	.11111	.06250	.04938	.04000	.03306	.02778	.02041	.01562	.01235
75	.5625	.36000	.25000	.14062	.11111	.09000	.07438	.06250	.04592	.03516	.02778
100	1.0	.64000	.44444	.25000	.19753	.16000	.13223	.11111	.08163	.06250	.04938
125		1.0	.69444	.39062	.30864	.25000	.20661	.17361	.12755	.09766	.07716
150			1.0	.56250	.44444	.36000	.29752	.25000	.18367	.14062	.11111
175				.76562	.60494	.49000	.40496	.34028	.25000	.19141	.15123
200				1.0	.79012	.64000	.52893	.44444	.32653	.25000	.19753
225					1.0	.81000	.66942	.56250	.41327	.31641	.25000
250						1.0	.82645	.69444	.51020	.39062	.30864
275							1.0	.84028	.61735	.47266	.37346
300								1.0	.73469	.56250	.44444
325									.86224	.66016	.52160
350									1.0	.76562	.60494
375										.87891	.69444
400										1.0	.79012
425											.89198
450											1.0



x	*L* = 1000	*L* = 1100	*L* = 1200	*L* = 1300	*L* = 1400	*L* = 1500	*L* = 1600	*L* = 1700	*L* = 1800	*L* = 1900	*L* = 2000
25	.0025	.00207	.00174	.00148	.00128	.00111	.00098	.00087	.00077	.00069	.00062
50	.0100	.00827	.00694	.00592	.00510	.00444	.00391	.00346	.00309	.00277	.00250
75	.0225	.01860	.01562	.01331	.01148	.01000	.00879	.00779	.00694	.00623	.00562
100	.0400	.03306	.02778	.02367	.02041	.01778	.01562	.01384	.01235	.01108	.01000
125	.0625	.05165	.04340	.03698	.03189	.02778	.02441	.02163	.01929	.01731	.01562
150	.0900	.07438	.06250	.05325	.04592	.04000	.03516	.03114	.02778	.02493	.02250
175	.1225	.10124	.08507	.07249	.06250	.05444	.04785	.04239	.03781	.03393	.03062
200	.1600	.13223	.11111	.09467	.08163	.07111	.06250	.05536	.04938	.04432	.04000
225	.2025	.16736	.14062	.11982	.10332	.09000	.07910	.07007	.06250	.05609	.05062
250	.2500	.20661	.17361	.14793	.12755	.11111	.09766	.08651	.07716	.06925	.06250
275	.3025	.25000	.21007	.17899	.15434	.13444	.11816	.10467	.09336	.08380	.07562
300	.3600	.29752	.25000	.21302	.18367	.16000	.14062	.12457	.11111	.09972	.09000
325	.4225	.34917	.29340	.25000	.21556	.18778	.16504	.14619	.13040	.11704	.10562
350	.4900	.40496	.34028	.28994	.25000	.21778	.19141	.16955	.15123	.13573	.12250
375	.5625	.46488	.39062	.33284	.28699	.25000	.21973	.19464	.17361	.15582	.14062
400	.6400	.52893	.44444	.37870	.32653	.28444	.25000	.22145	.19753	.17729	.16000
425	.7225	.59711	.50174	.42751	.36862	.32111	.28223	.25000	.22299	.20014	.18062
450	.8100	.66942	.56250	.47929	.41327	.36000	.31641	.28028	.25000	.22438	.20250
475	.9025	.74587	.62674	.53402	.46046	.40111	.35254	.31228	.27855	.25000	.22562
500	1.0	.82645	.69444	.59172	.51020	.44444	.39062	.34602	.30864	.27701	.25000
525		.91116	.76562	.65237	.56250	.49000	.43066	.38149	.34028	.30540	.27562
550		1.0	.84028	.71598	.61735	.53778	.47266	.41869	.37346	.33518	.30250
575			.91840	.78254	.67474	.58778	.51660	.45761	.40818	.36634	.33062
600			1.0	.85207	.73469	.64000	.56250	.49827	.44444	.39889	.36000
625				.92456	.79719	.69444	.61035	.54066	.48225	.43282	.39062
650				1.0	.86224	.75111	.66016	.58478	.52160	.46814	.42250
675					.92985	.81000	.71191	.63062	.56250	.50485	.45562
700					1.0	.87111	.76562	.67820	.60494	.54294	.49000
725						.93444	.82129	.72751	.64892	.58241	.52562
750						1.0	.87891	.77854	.69444	.62327	.56250
775							.93848	.83131	.74151	.66551	.60062
800							1.0	.88581	.79012	.70914	.64000
825								.94204	.84028	.75416	.68062
850								1.0	.89198	.80055	.72250
875									.94522	.84834	.76562
900									1.0	.89751	.81000
925										.94896	.85562
950										1.0	.90250
975											.95062
1000											1.0

TABLE 32. MINUTES IN DECIMALS OF A DEGREE

′	0″	10″	15″	20″	30″	40″	45″	50″	′
0	.00000	.00278	.00417	.00556	.00833	.01111	.01250	.01389	0
1	.01667	.01944	.02083	.02222	.02500	.02778	.02917	.03055	1
2	.03333	.03611	.03750	.03889	.04167	.04444	.04583	.04722	2
3	.05000	.05278	.05417	.05556	.05833	.06111	.06250	.06389	3
4	.06667	.06944	.07083	.07222	.07500	.07778	.07917	.08056	4
5	.08333	.08611	.08750	.08889	.09167	.09444	.09583	.09722	5
6	.10000	.10278	.10417	.10556	.10833	.11111	.11250	.11389	6
7	.11667	.11944	.12083	.12222	.12500	.12778	.12917	.13056	7
8	.13333	.13611	.13750	.13889	.14167	.14444	.14583	.14722	8
9	.15000	.15278	.15417	.15556	.15833	.16111	.16250	.16389	9
10	.16667	.16944	.17083	.17222	.17500	.17778	.17917	.18056	10
11	.18333	.18611	.18750	.18889	.19167	.19444	.19583	.19722	11
12	.20000	.20278	.20417	.20556	.20833	.21111	.21250	.21389	12
13	.21667	.21944	.22083	.22222	.22500	.22778	.22917	.23056	13
14	.23333	.23611	.23750	.23889	.24167	.24444	.24583	.24722	14
15	.25000	.25278	.25417	.25556	.25833	.26111	.26250	.26389	15
16	.26667	.26944	.27083	.27222	.27500	.27778	.27917	.28056	16
17	.28333	.28611	.28750	.28889	.29167	.29444	.29583	.29722	17
18	.30000	.30278	.30417	.30556	.30833	.31111	.31250	.31389	18
19	.31667	.31944	.32083	.32222	.32500	.32778	.32917	.33056	19
20	.33333	.33611	.33750	.33889	.34167	.34444	.34583	.34722	20
21	.35000	.35278	.35417	.35556	.35833	.36111	.36250	.36389	21
22	.36667	.36944	.37083	.37222	.37500	.37778	.37917	.38056	22
23	.38333	.38611	.38750	.38889	.39167	.39444	.39583	.39722	23
24	.40000	.40278	.40417	.40556	.40833	.41111	.41250	.41389	24
25	.41667	.41944	.42083	.42222	.42500	.42778	.42917	.43056	25
26	.43333	.43611	.43750	.43889	.44167	.44444	.44583	.44722	26
27	.45000	.45278	.45417	.45556	.45833	.46111	.46250	.46389	27
28	.46667	.46944	.47083	.47222	.47500	.47778	.47917	.48056	28
29	.48333	.48611	.48750	.48889	.49167	.49444	.49583	.49722	29
30	.50000	.50278	.50417	.50556	.50833	.51111	.51250	.51389	30
31	.51667	.51944	.52083	.52222	.52500	.52778	.52917	.53056	31
32	.53333	.53611	.53750	.53889	.54167	.54444	.54583	.54722	32
33	.55000	.55278	.55417	.55556	.55833	.56111	.56250	.56389	33
34	.56667	.56944	.57083	.57222	.57500	.57778	.57917	.58056	34
35	.58333	.58611	.58750	.58889	.59167	.59444	.59583	.59722	35
36	.60000	.60278	.60417	.60556	.60833	.61111	.61250	.61389	36
37	.61667	.61944	.62083	.62222	.62500	.62778	.62917	.63056	37
38	.63333	.63611	.63750	.63889	.64167	.64444	.64583	.64722	38
39	.65000	.65278	.65417	.65556	.65833	.66111	.66250	.66389	39
40	.66667	.66944	.67083	.67222	.67500	.67778	.67917	.68056	40
41	.68333	.68611	.68750	.68889	.69167	.69444	.69583	.69722	41
42	.70000	.70278	.70417	.70556	.70833	.71111	.71250	.71389	42
43	.71667	.71944	.72083	.72222	.72500	.72778	.72917	.73056	43
44	.73333	.73611	.73750	.73889	.74167	.74444	.74583	.74722	44
45	.75000	.75278	.75417	.75556	.75833	.76111	.76250	.76389	45
46	.76667	.76944	.77083	.77222	.77500	.77778	.77917	.78056	46
47	.78333	.78611	.78750	.78889	.79167	.79444	.79583	.79722	47
48	.80000	.80278	.80417	.80556	.80833	.81111	.81250	.81389	48
49	.81667	.81944	.82083	.82222	.82500	.82778	.82917	.83056	49
50	.83333	.83611	.83750	.83889	.84167	.84444	.84583	.84722	50
51	.85000	.85278	.85417	.85556	.85833	.86111	.86250	.86389	51
52	.86667	.86944	.87083	.87222	.87500	.87778	.87917	.88056	52
53	.88333	.88611	.88750	.88889	.89167	.89444	.89583	.89722	53
54	.90000	.90278	.90417	.90556	.90833	.91111	.91250	.91389	54
55	.91667	.91944	.92083	.92222	.92500	.92778	.92917	.93056	55
56	.93333	.93611	.93750	.93889	.94167	.94444	.94583	.94722	56
57	.95000	.95278	.95417	.95556	.95833	.96111	.96250	.96389	57
58	.96667	.96944	.97083	.97222	.97500	.97778	.97917	.98056	58
59	.98333	.98611	.98750	.98889	.99167	.99444	.99583	.99722	59
′	0″	10″	15″	20″	30″	40″	45″	50″	′

From "Field Engineering" by Prof. W. H. Searles. By permission of the publishers, John Wiley & Sons, Inc., New York.

TABLE 33. INCHES IN DECIMALS OF A FOOT

In.	0	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$	$\frac{15}{16}$
0	000	005	010	016	021	026	031	036	042	047	052	057	062	068	073	078
1	083	089	094	099	104	109	115	120	125	130	135	141	146	151	156	161
2	167	172	177	182	188	193	198	203	208	214	219	224	229	234	240	245
3	250	255	260	266	271	276	281	286	292	297	302	307	313	318	323	328
4	333	339	344	349	354	359	365	370	375	380	385	391	396	401	406	411
5	417	422	427	432	438	433	448	453	458	464	469	474	479	484	490	495
6	500	505	510	516	521	526	531	536	542	547	552	557	562	568	573	578
7	583	589	594	599	604	609	615	620	625	630	635	641	646	651	656	661
8	667	672	677	682	688	693	698	703	708	714	719	724	729	734	740	745
9	750	755	760	766	771	776	781	786	792	797	802	807	813	818	823	828
10	833	839	844	849	854	859	865	870	875	880	885	891	896	901	906	911
11	917	922	927	932	938	943	948	953	958	964	969	974	979	984	990	995

From the New Jersey State Highway Department.

TABLE 34. MAP SCALES AND CONVERSION FACTORS

Map scale	Miles per inch	Feet per inch	Inches per mile	Kilometers per centimeter	Meters per centimeter	Centimeters per kilometer
1:1000,000	15.782828	83333.33	0.06336	10.0	10000	0.1
1:500,000	7.891414	41667.67	0.12672	5.0	5000	0.2
1:250,000	3.945707	20833.33	0.25344	2.5	2500	0.4
1:126,720	2.0	10560.00	0.5	1.2672	1267.2	0.789
1:125,000	1.972854	10416.67	0.50688	1.25	1250	0.8
1:100,000	1.578283	8333.33	0.63360	1.0	1000	1.0
1:63,360	1.0	5280	1.0	0.6336	633.6	1.578
1:62,500	0.986427	5208.33	1.01376	0.625	625	1.6
1:50,000	0.789141	4166.67	1.2672	0.5	500	2.0
1:48,000	0.757576	4000	1.32	0.48	480	2.083
1:40,000	0.631313	3333.33	1.584	0.4	400	2.5
1:36,000	0.568182	3000	1.76	0.36	360	2.778
1:31,680	0.5	2640	2.0	0.3168	316.8	3.157
1:25,000	0.394571	2083.33	2.5344	0.25	250	4.0
1:24,000	0.378788	2000	2.64	0.24	240	4.167
1:20,000	0.315657	1666.67	3.168	0.2	200	5.0
1:18,000	0.284091	1500	3.52	0.18	180	5.555
1:14,400	0.227273	1200	4.4	0.144	144	6.944
1:12,000	0.189394	1000	5.28	0.12	120	8.333
1:10,000	0.157828	833.33	6.336	0.1	100	10.0
1:9,600	0.151515	800	6.6	0.096	96	10.417
1:7,200	0.113636	600	8.8	0.072	72	13.889
1:6,000	0.094697	500	10.56	0.06	60	16.667
1:4,800	0.075758	400	13.2	0.048	48	20.833
1:3,600	0.056818	300	17.6	0.036	36	27.778
1:3,000	0.047348	250	21.12	0.03	30	33.333
1:2,400	0.037879	200	26.4	0.024	24	41.667
1:1,800	0.028409	150	35.2	0.018	18	55.555
1:1,440	0.022727	120	44.0	0.0144	14.4	69.444
1:1,200	0.018939	100	52.8	0.012	12	83.333
1:960	0.015152	80	66.0	0.0096	9.6	104.167
1:720	0.011364	60	88.0	0.0072	7.2	138.889
1:600	0.009470	50	105.6	0.006	6	166.667
1:480	0.007576	40	132.0	0.0048	4.8	208.333
1:360	0.005682	30	176.0	0.0036	3.6	277.778
1:240	0.003788	20	264.0	0.0024	2.4	416.667

1 MILE = 5280 FEET = 1760 YARDS = 63360 INCHES = 1.609347219 KILOMETERS.

1 KILOMETER = 1000 METERS = 100,000 CENTIMETERS = 0.6213699495 MILE.

TABLE 35. HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS

	0°		1°		2°		3°	
Minutes	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	100.00	0.00	99.97	1.74	99.88	3.49	99.73	5.23
2	100.00	0.06	99.97	1.80	99.87	3.55	99.72	5.28
4	100.00	0.12	99.97	1.86	99.87	3.60	99.71	5.34
6	100.00	0.17	99.96	1.92	99.87	3.66	99.71	5.40
8	100.00	0.23	99.96	1.98	99.86	3.72	99.70	5.46
10	100.00	0.29	99.96	2.04	99.86	3.78	99.69	5.52
12	100.00	0.35	99.96	2.09	99.85	3.84	99.69	5.57
14	100.00	0.41	99.95	2.15	99.85	3.90	99.68	5.63
16	100.00	0.47	99.95	2.21	99.84	3.95	99.68	5.69
18	100.00	0.52	99.95	2.27	99.84	4.01	99.67	5.75
20	100.00	0.58	99.95	2.33	99.83	4.07	99.66	5.80
22	100.00	0.64	99.94	2.38	99.83	4.13	99.66	5.86
24	100.00	0.70	99.94	2.44	99.82	4.18	99.65	5.92
26	99.99	0.76	99.94	2.50	99.82	4.24	99.64	5.98
28	99.99	0.81	99.93	2.56	99.81	4.30	99.63	6.04
30	99.99	0.87	99.93	2.62	99.81	4.36	99.63	6.09
32	99.99	0.93	99.93	2.67	99.80	4.42	99.62	6.15
34	99.99	0.99	99.93	2.73	99.80	4.48	99.62	6.21
36	99.99	1.05	99.92	2.79	99.79	4.53	99.61	6.27
38	99.99	1.11	99.92	2.85	99.79	4.59	99.60	6.33
40	99.99	1.16	99.92	2.91	99.78	4.65	99.59	6.38
42	99.99	1.22	99.91	2.97	99.78	4.71	99.59	6.44
44	99.98	1.28	99.91	3.02	99.77	4.76	99.58	6.50
46	99.98	1.34	99.90	3.08	99.77	4.82	99.57	6.56
48	99.98	1.40	99.90	3.14	99.76	4.88	99.56	6.61
50	99.98	1.45	99.90	3.20	99.76	4.94	99.56	6.67
52	99.98	1.51	99.89	3.26	99.75	4.99	99.55	6.73
54	99.98	1.57	99.89	3.31	99.74	5.05	99.54	6.78
56	99.97	1.63	99.89	3.37	99.74	5.11	99.53	6.84
58	99.97	1.69	99.88	3.43	99.73	5.17	99.52	6.90
60	99.97	1.74	99.88	3.49	99.73	5.23	99.51	6.96
c = 0.75	0.75	0.01	0.75	0.02	0.75	0.03	0.75	0.05
c = 1.00	1.00	0.01	1.00	0.03	1.00	0.04	1.00	0.06
c = 1.25	1.25	0.02	1.25	0.03	1.25	0.05	1.25	0.08

From "Theory and Practice of Surveying," by Prof. J. B. Johnson. By permission of the publishers, John Wiley & Sons, Inc., New York.

TABLE 35. HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS (*Continued*)

Minutes	4°		5°		6°		7°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	99.51	6.96	99.24	8.68	98.91	10.40	98.51	12.10
2	99.51	7.02	99.23	8.74	98.90	10.45	98.50	12.15
4	99.50	7.07	99.22	8.80	98.88	10.51	98.48	12.21
6	99.49	7.13	99.21	8.85	98.87	10.57	98.47	12.26
8	99.48	7.19	99.20	8.91	98.86	10.62	98.46	12.32
10	99.47	7.25	99.19	8.97	98.85	10.68	98.44	12.38
12	99.46	7.30	99.18	9.03	98.83	10.74	98.43	12.43
14	99.46	7.36	99.17	9.08	98.82	10.79	98.41	12.49
16	99.45	7.42	99.16	9.14	98.81	10.85	98.40	12.55
18	99.44	7.48	99.15	9.20	98.80	10.91	98.39	12.60
20	99.43	7.53	99.14	9.25	98.78	10.96	98.37	12.66
22	99.42	7.59	99.13	9.31	98.77	11.02	98.36	12.72
24	99.41	7.65	99.11	9.37	98.76	11.08	98.34	12.77
26	99.40	7.71	99.10	9.43	98.74	11.13	98.33	12.83
28	99.39	7.76	99.09	9.48	98.73	11.19	98.31	12.88
30	99.38	7.82	99.08	9.54	98.72	11.25	98.29	12.94
32	99.38	7.88	99.07	9.60	98.71	11.30	98.28	13.00
34	99.37	7.94	99.06	9.65	98.69	11.36	98.27	13.05
36	99.36	7.99	99.05	9.71	98.68	11.42	98.25	13.11
38	99.35	8.05	99.04	9.77	98.67	11.47	98.24	13.17
40	99.34	8.11	99.03	9.83	98.65	11.53	98.22	13.22
42	99.33	8.17	99.01	9.88	98.64	11.59	98.20	13.28
44	99.32	8.22	99.00	9.94	98.63	11.64	98.19	13.33
46	99.31	8.28	98.99	10.00	98.61	11.70	98.17	13.39
48	99.30	8.34	98.98	10.05	98.60	11.76	98.16	13.45
50	99.29	8.40	98.97	10.11	98.58	11.81	98.14	13.50
52	99.28	8.45	98.96	10.17	98.57	11.87	98.13	13.56
54	99.27	8.51	98.94	10.22	98.56	11.93	98.11	13.61
56	99.26	8.57	98.93	10.28	98.54	11.98	98.10	13.67
58	99.25	8.63	98.92	10.34	98.53	12.04	98.08	13.73
60	99.24	8.68	98.91	10.40	98.51	12.10	98.06	13.78
c = 0.75.	0.75	0.06	0.75	0.07	0.75	0.08	0.74	0.10
c = 1.00.	1.00	0.08	0.99	0.09	0.99	0.11	0.99	0.13
c = 1.25.	1.25	0.10	1.24	0.11	1.24	0.14	1.24	0.16

TABLE 35. HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS (*Continued*)

	8°		9°		10°		11°	
Minutes	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	98.06	13.78	97.55	15.45	96.98	17.10	96.36	18.73
2	98.05	13.84	97.53	15.51	96.96	17.16	96.34	18.78
4	98.03	13.89	97.52	15.56	96.94	17.21	96.32	18.84
6	98.01	13.95	97.50	15.62	96.92	17.26	96.29	18.89
8	98.00	14.01	97.48	15.67	96.90	17.32	96.27	18.95
10	97.98	14.06	97.46	15.73	96.88	17.37	96.25	19.00
12	97.97	14.12	97.44	15.78	96.86	17.43	96.23	19.05
14	97.95	14.17	97.43	15.84	96.84	17.48	96.21	19.11
16	97.93	14.23	97.41	15.89	96.82	17.54	96.18	19.16
18	97.92	14.28	97.39	15.95	96.80	17.59	96.16	19.21
20	97.90	14.34	97.37	16.00	96.78	17.65	96.14	19.27
22	97.88	14.40	97.35	16.06	96.76	17.70	96.12	19.32
24	97.87	14.45	97.33	16.11	96.74	17.76	96.09	19.38
26	97.85	14.51	97.31	16.17	96.72	17.81	96.07	19.43
28	97.83	14.56	97.29	16.22	96.70	17.86	96.05	19.48
30	97.82	14.62	97.28	16.28	96.68	17.92	96.03	19.54
32	97.80	14.67	97.26	16.33	96.66	17.97	96.00	19.59
34	97.78	14.73	97.24	16.39	96.64	18.03	95.98	19.64
36	97.76	14.79	97.22	16.44	96.62	18.08	95.96	19.70
38	97.75	14.84	97.20	16.50	96.60	18.14	95.93	19.75
40	97.73	14.90	97.18	16.55	96.57	18.19	95.91	19.80
42	97.71	14.95	97.16	16.61	96.55	18.24	95.89	19.86
44	97.69	15.01	97.14	16.66	96.53	18.30	95.86	19.91
46	97.68	15.06	97.12	16.72	96.51	18.35	95.84	19.96
48	97.66	15.12	97.10	16.77	96.49	18.41	95.82	20.02
50	97.64	15.17	97.08	16.83	96.47	18.46	95.79	20.07
52	97.62	15.23	97.06	16.88	96.45	18.51	95.77	20.12
54	97.61	15.28	97.04	16.94	96.42	18.57	95.75	20.18
56	97.59	15.34	97.02	16.99	96.40	18.62	95.72	20.23
58	97.57	15.40	97.00	17.05	96.38	18.68	95.70	20.28
60	97.55	15.45	96.98	17.10	96.36	18.73	95.68	20.34
c = 0.75.	0.74	0.11	0.74	0.12	0.74	0.14	0.73	0.15
c = 1.00.	0.99	0.15	0.99	0.16	0.98	0.18	0.98	0.20
c = 1.25.	1.23	0.18	1.23	0.21	1.23	0.23	1.22	0.25

TABLE 35. HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS (*Continued*)

	12°		13°		14°		15°	
Minutes	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	95.68	20.34	94.94	21.92	94.15	23.47	93.30	25.00
2	95.65	20.39	94.91	21.97	94.12	23.52	93.27	25.05
4	95.63	20.44	94.89	22.02	94.09	23.58	93.24	25.10
6	95.61	20.50	94.86	22.08	94.07	23.63	93.21	25.15
8	95.58	20.55	94.84	22.13	94.04	23.68	93.18	25.20
10	95.56	20.60	94.81	22.18	94.01	23.73	93.16	25.25
12	95.53	20.66	94.79	22.23	93.98	23.78	93.13	25.30
14	95.51	20.71	94.76	22.28	93.95	23.83	93.10	25.35
16	95.49	20.76	94.73	22.34	93.93	23.88	93.07	25.40
18	95.46	20.81	94.71	22.39	93.90	23.93	93.04	25.45
20	95.44	20.87	94.68	22.44	93.87	23.99	93.01	25.50
22	95.41	20.92	94.66	22.49	93.84	24.04	92.98	25.55
24	95.39	20.97	94.63	22.54	93.81	24.09	92.95	25.60
26	95.36	21.03	94.60	22.60	93.79	24.14	92.92	25.65
28	95.34	21.08	94.58	22.65	93.76	24.19	92.89	25.70
30	95.32	21.13	94.55	22.70	93.73	24.24	92.86	25.75
32	95.29	21.18	94.52	22.75	93.70	24.29	92.83	25.80
34	95.27	21.24	94.50	22.80	93.67	24.34	92.80	25.85
36	95.24	21.29	94.47	22.85	93.65	24.39	92.77	25.90
38	95.22	21.34	94.44	22.91	93.62	24.44	92.74	25.95
40	95.19	21.39	94.42	22.96	93.59	24.49	92.71	26.00
42	95.17	21.45	94.39	23.01	93.56	24.55	92.68	26.05
44	95.14	21.50	94.36	23.06	93.53	24.60	92.65	26.10
46	95.12	21.55	94.34	23.11	93.50	24.65	92.62	26.15
48	95.09	21.60	94.31	23.16	93.47	24.70	92.59	26.20
50	95.07	21.66	94.28	23.22	93.45	24.75	92.56	26.25
52	95.04	21.71	94.26	23.27	93.42	24.80	92.53	26.30
54	95.02	21.76	94.23	23.32	93.39	24.85	92.49	26.35
56	94.99	21.81	94.20	23.37	93.36	24.90	92.46	26.40
58	94.97	21.87	94.17	23.42	93.33	24.95	92.43	26.45
60	94.94	21.92	94.15	23.47	93.30	25.00	92.40	26.50
c = 0.75	0.73	0.16	0.73	0.17	0.73	0.19	0.72	0.20
c = 1.00	0.98	0.22	0.97	0.23	0.97	0.25	0.96	0.27
c = 1.25	1.22	0.27	1.21	0.29	1.21	0.31	1.20	0.34

TABLE 35. HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS (*Continued*)

	16°		17°		18°		19°	
Minutes	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	92.40	26.50	91.45	27.96	90.45	29.39	89.40	30.78
2	92.37	26.55	91.42	28.01	90.42	29.44	89.36	30.83
4	92.34	26.59	91.39	28.06	90.38	29.48	89.33	30.87
6	92.31	26.64	91.35	28.10	90.35	29.53	89.29	30.92
8	92.28	26.69	91.32	28.15	90.31	29.58	89.26	30.97
10	92.25	26.74	91.29	28.20	90.28	29.62	89.22	31.01
12	92.22	26.79	91.26	28.25	90.24	29.67	89.18	31.06
14	92.19	26.84	91.22	28.30	90.21	29.72	89.15	31.10
16	92.15	26.89	91.19	28.34	90.18	29.76	89.11	31.15
18	92.12	26.94	91.16	28.39	90.14	29.81	89.08	31.19
20	92.09	26.99	91.12	28.44	90.11	29.86	89.04	31.24
22	92.06	27.04	91.09	28.49	90.07	29.90	89.00	31.28
24	92.03	27.09	91.06	28.54	90.04	29.95	88.96	31.33
26	92.00	27.13	91.02	28.58	90.00	30.00	88.93	31.38
28	91.97	27.18	90.99	28.63	89.97	30.04	88.89	31.42
30	91.93	27.23	90.96	28.68	89.93	30.09	88.86	31.47
32	91.90	27.28	90.92	28.73	89.90	30.14	88.82	31.51
34	91.87	27.33	90.89	28.77	89.86	30.19	88.78	31.56
36	91.84	27.38	90.86	28.82	89.83	30.23	88.75	31.60
38	91.81	27.43	90.82	28.87	89.79	30.28	88.71	31.65
40	91.77	27.48	90.79	28.92	89.76	30.32	88.67	31.69
42	91.74	27.52	90.76	28.96	89.72	30.37	88.64	31.74
44	91.71	27.57	90.72	29.01	89.69	30.41	88.60	31.78
46	91.68	27.62	90.69	29.06	89.65	30.46	88.56	31.83
48	91.65	27.67	90.66	29.11	89.61	30.51	88.53	31.87
50	91.61	27.72	90.62	29.15	89.58	30.55	88.49	31.92
52	91.58	27.77	90.59	29.20	89.54	30.60	88.45	31.96
54	91.55	27.81	90.55	29.25	89.51	30.65	88.41	32.01
56	91.52	27.86	90.52	29.30	89.47	30.69	88.38	32.05
58	91.48	27.91	90.48	29.34	89.44	30.74	88.34	32.09
60	91.45	27.96	90.45	29.39	89.40	30.78	88.30	32.14
c = 0.75.	0.72	0.21	0.72	0.23	0.71	0.24	0.71	0.25
c = 1.00.	0.96	0.28	0.95	0.30	0.95	0.32	0.94	0.33
c = 1.25.	1.20	0.35	1.19	0.38	1.19	0.40	1.18	0.42

TABLE 35. HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS (*Continued*)

	20°		21°		22°		23°	
Minutes	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	88.30	32.14	87.16	33.46	85.97	34.73	84.73	35.97
2	88.26	32.18	87.12	33.50	85.93	34.77	84.69	36.01
4	88.23	32.23	87.08	33.54	85.89	34.82	84.65	36.05
6	88.19	32.27	87.04	33.59	85.85	34.86	84.61	36.09
8	88.15	32.32	87.00	33.63	85.80	34.90	84.57	36.13
10	88.11	32.36	86.96	33.67	85.76	34.94	84.52	36.17
12	88.08	32.41	86.92	33.72	85.72	34.98	84.48	36.21
14	88.04	32.45	86.88	33.76	85.68	35.02	84.44	36.25
16	88.00	32.49	86.84	33.80	85.64	35.07	84.40	36.29
18	87.96	32.54	86.80	33.84	85.60	35.11	84.35	36.33
20	87.93	32.58	86.77	33.89	85.56	35.15	84.31	36.37
22	87.89	32.63	86.73	33.93	85.52	35.19	84.27	36.41
24	87.85	32.67	86.69	33.97	85.48	35.23	84.23	36.45
26	87.81	32.72	86.65	34.01	85.44	35.27	84.18	36.49
28	87.77	32.76	86.61	34.06	85.40	35.31	84.14	36.53
30	87.74	32.80	86.57	34.10	85.36	35.36	84.10	36.57
32	87.70	32.85	86.53	34.14	85.31	35.40	84.06	36.61
34	87.66	32.89	86.49	34.18	85.27	35.44	84.01	36.65
36	87.62	32.93	86.45	34.23	85.23	35.48	83.97	36.69
38	87.58	32.98	86.41	34.27	85.19	35.52	83.93	36.73
40	87.54	33.02	86.37	34.31	85.15	35.56	83.89	36.77
42	87.51	33.07	86.33	34.35	85.11	35.60	83.84	36.80
44	87.47	33.11	86.29	34.40	85.07	35.64	83.80	36.84
46	87.43	33.15	86.25	34.44	85.02	35.68	83.76	36.88
48	87.39	33.20	86.21	34.48	84.98	35.72	83.72	36.92
50	87.35	33.24	86.17	34.52	84.94	35.76	83.67	36.96
52	87.31	33.28	86.13	34.57	84.90	35.80	83.63	37.00
54	87.27	33.33	86.09	34.61	84.86	35.85	83.59	37.04
56	87.24	33.37	86.05	34.65	84.82	35.89	83.54	37.08
58	87.20	33.41	86.01	34.69	84.77	35.93	83.50	37.12
60	87.16	33.46	85.97	34.73	84.73	35.97	83.46	37.16
c = 0.75	0.70	0.26	0.70	0.27	0.69	0.29	0.69	0.30
c = 1.00	0.94	0.35	0.93	0.37	0.92	0.38	0.92	0.40
c = 1.25	1.17	0.44	1.16	0.46	1.15	0.48	1.15	0.50

TABLE 35. HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS (*Continued*)

	24°		25°		26°		27°	
Minutes	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff Elev.	Hor. Dist.	Diff. Elev.
0	83.46	37.16	82.14	38.30	80.78	39.40	79.39	40.45
2	83.41	37.20	82.09	38.34	80.74	39.44	79.34	40.49
4	83.37	37.23	82.05	38.38	80.69	39.47	79.30	40.52
6	83.33	37.27	82.01	38.41	80.65	39.51	79.25	40.55
8	83.28	37.31	81.96	38.45	80.60	39.54	79.20	40.59
10	83.24	37.35	81.92	38.49	80.55	39.58	79.15	40.62
12	83.20	37.39	81.87	38.53	80.51	39.61	79.11	40.66
14	83.15	37.43	81.83	38.56	80.46	39.65	79.06	40.69
16	83.11	37.47	81.78	38.60	80.41	39.69	79.01	40.72
18	83.07	37.51	81.74	38.64	80.37	39.72	78.96	40.76
20	83.02	37.54	81.69	38.67	80.32	39.76	78.92	40.79
22	82.98	37.58	81.65	38.71	80.28	39.79	78.87	40.82
24	82.93	37.62	81.60	38.75	80.23	39.83	78.82	40.86
26	82.89	37.66	81.56	38.78	80.18	39.86	78.77	40.89
28	82.85	37.70	81.51	38.82	80.14	39.90	78.73	40.92
30	82.80	37.74	81.47	38.86	80.09	39.93	78.68	40.96
32	82.76	37.77	81.42	38.89	80.04	39.97	78.63	40.99
34	82.72	37.81	81.38	38.93	80.00	40.00	78.58	41.02
36	82.67	37.85	81.33	38.97	79.95	40.04	78.54	41.06
38	82.63	37.89	81.28	39.00	79.90	40.07	78.49	41.09
40	82.58	37.93	81.24	39.04	79.86	40.11	78.44	41.12
42	82.54	37.96	81.19	39.08	79.81	40.14	78.39	41.16
44	82.49	38.00	81.15	39.11	79.76	40.18	78.34	41.19
46	82.45	38.04	81.10	39.15	79.72	40.21	78.30	41.22
48	82.41	38.08	81.06	39.18	79.67	40.24	78.25	41.26
50	82.36	38.11	81.01	39.22	79.62	40.28	78.20	41.29
52	82.32	38.15	80.97	39.26	79.58	40.31	78.15	41.32
54	82.27	38.19	80.92	39.29	79.53	40.35	78.10	41.35
56	82.23	38.23	80.87	39.33	79.48	40.38	78.06	41.39
58	82.18	38.26	80.83	39.36	79.44	40.42	78.01	41.42
60	82.14	38.30	80.78	39.40	79.39	40.45	77.96	41.45
$c = 0.75$	0.68	0.31	0.68	0.32	0.67	0.33	0.66	0.35
$c = 1.00$	0.91	0.41	0.90	0.43	0.89	0.45	0.89	0.46
$c = 1.25$	1.14	0.52	1.13	0.54	1.12	0.56	1.11	0.58

TABLE 35. HORIZONTAL DISTANCES AND ELEVATIONS FROM STADIA READINGS (*Continued*)

	28°		29°		30°	
Minutes	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	77.96	41.45	76.50	42.40	75.00	43.30
2	77.91	41.48	76.45	42.43	74.95	43.33
4	77.86	41.52	76.40	42.46	74.90	43.36
6	77.81	41.55	76.35	42.49	74.85	43.39
8	77.77	41.58	76.30	42.53	74.80	43.42
10	77.72	41.61	76.25	42.56	74.75	43.45
12	77.67	41.65	76.20	42.59	74.70	43.47
14	77.62	41.68	76.15	42.62	74.65	43.50
16	77.57	41.71	76.10	42.65	74.60	43.53
18	77.52	41.74	76.05	42.68	74.55	43.56
20	77.48	41.77	76.00	42.71	74.49	43.59
22	77.42	41.81	75.95	42.74	74.44	43.62
24	77.38	41.84	75.90	42.77	74.39	43.65
26	77.33	41.87	75.85	42.80	74.34	43.67
28	77.28	41.90	75.80	42.83	74.29	43.70
30	77.23	41.93	75.75	42.86	74.24	43.73
32	77.18	41.97	75.70	42.89	74.19	43.76
34	77.13	42.00	75.65	42.92	74.14	43.79
36	77.09	42.03	75.60	42.95	74.09	43.82
38	77.04	42.06	75.55	42.98	74.04	43.84
40	76.99	42.09	75.50	43.01	73.99	43.87
42	76.94	42.12	75.45	43.04	73.93	43.90
44	76.89	42.15	75.40	43.07	73.88	43.93
46	76.84	42.19	75.35	43.10	73.83	43.95
48	76.79	42.22	75.30	43.13	73.78	43.98
50	76.74	42.25	75.25	43.16	73.73	44.01
52	76.69	42.28	75.20	43.18	73.68	44.04
54	76.64	42.31	75.15	43.21	73.63	44.07
56	76.59	42.34	75.10	43.24	73.58	44.09
58	76.55	42.37	75.05	43.27	73.52	44.12
60	76.50	42.40	75.00	43.30	73.47	44.15
c = 0.75	0.66	0.36	0.65	0.37	0.65	0.38
c = 1.00	0.88	0.48	0.87	0.49	0.86	0.51
c = 1.25	1.10	0.60	1.09	0.62	1.08	0.64

TABLE 36. AREA OF WATERWAY CALCULATED BY TALBOT'S FORMULA

a = area of waterway in square feet.

A = drainage area in acres.

$$a = C\sqrt[4]{A^3}.$$

Drainage area		Areas of waterway in square feet						
		Mountainous land	Hilly land		Rolling land		Flat land	
Acres	Square miles	$C=1.00$	$C=0.80$	$C=0.60$	$C=0.50$	$C=0.40$	$C=0.30$	$C=0.20$
1	0.0016	1.0	0.8	0.6	0.5	0.4	0.3	0.2
2	.0031	1.7	1.4	1.0	.8	.7	.5	.3
4	.0062	2.8	2.2	1.7	1.4	1.1	.8	.6
6	.0094	3.8	3.0	2.3	1.9	1.5	1.1	.8
8	.0125	4.8	3.8	2.9	2.4	1.9	1.4	1.0
10	.018	5.6	4.5	3.4	2.8	2.2	1.7	1.2
15	.023	7.6	5.1	4.6	3.8	3.0	2.3	1.5
20	.031	9.5	7.6	5.7	4.7	3.8	2.8	1.9
25	.039	11.2	9.0	6.7	5.6	4.5	3.4	2.2
30	.047	12.8	10.2	7.7	6.4	5.1	3.8	2.6
35	.05	14.4	11.5	8.6	7.2	5.8	4.3	2.9
40	.06	15.9	12.7	9.5	8.0	6.4	4.8	3.2
45	.07	17.4	13.9	10.4	8.7	7.0	5.2	3.5
50	.08	18.8	15.0	11.3	9.4	7.5	5.6	3.8
55	.09	20	16	12	10	8	6	4
60	.09	22	17	13	11	9	6	4
65	.10	23	18	14	11	10	7	5
70	.11	24	19	14	12	10	7	5
75	.12	26	21	16	13	10	8	5
80	.13	27	22	16	13	11	8	5
85	.13	28	22	17	14	11	8	6
90	.14	29	23	17	14	12	9	6
95	.15	30	24	18	15	12	9	6
100	.16	32	26	19	16	13	10	6
150	.23	43	34	26	21	17	13	9
200	.31	53	42	32	27	21	16	11
250	.39	63	50	38	32	25	19	13
300	.47	72	58	43	36	29	22	14
350	.54	81	65	49	41	32	24	16
400	.63	89	71	53	44	36	27	18
450	.70	98	78	59	49	39	29	20
500	.78	106	85	64	53	42	32	21
550	.86	114	91	68	57	46	34	23
600	.94	121	97	73	61	48	36	24
650	1.02	129	103	77	65	52	39	26
700	1.09	136	109	82	68	54	41	27
750	1.17	143	114	86	72	57	43	29
800	1.25	150	120	90	75	60	45	30
850	1.33	157	126	94	79	63	47	31
900	1.41	164	131	98	82	66	49	33
950	1.48	171	137	103	86	68	51	34
1,000	1.56	178	142	107	89	71	53	36
1,100	1.72	191	153	115	95	76	57	38
1,200	1.88	204	163	122	102	82	61	41
1,300	2.03	216	173	130	108	86	65	43
1,400	2.2	229	183	137	114	92	69	46
1,500	2.3	241	193	145	121	96	72	48
1,600	2.5	253	202	152	126	101	76	51
1,700	2.7	265	212	159	132	106	80	53
1,800	2.8	276	221	166	138	110	83	55
1,900	3.0	288	230	173	144	115	86	58
2,000	3.1	299	239	179	150	120	90	60

TABLE 36. AREA OF WATERWAY CALCULATED BY TALBOT'S FORMULA (*Continued*)

a = area of waterway in square feet.
A = drainage area in acres.

$$a = C\sqrt[4]{A^3}.$$

Drainage area		Area of waterway in square feet						
		Mountainous land	Hilly land		Rolling land		Flat land	
Acres	Square miles	$C = 1.00$	$C = 0.80$	$C = 0.60$	$C = 0.50$	$C = 0.40$	$C = 0.30$	$C = 0.20$
2,500	3.91	354	283	212	177	142	106	71
3,000	4.69	405	324	243	203	162	122	81
3,500	5.47	455	364	273	228	182	137	91
4,000	6.25	503	402	302	252	201	151	101
5,000	7.81	595	476	357	298	238	179	119
6,000	9.37	682	546	409	341	273	205	136
7,000	10.94	765	612	459	383	306	230	153
8,000	12.50	846	677	508	423	338	254	169
9,000	14.06	924	739	554	462	370	277	185
10,000	15.62	1,000	800	600	500	400	300	200
20,000	31.2	1,682	1,346	1,009	841	673	505	336
30,000	46.9	2,280	1,824	1,368	1,140	912	684	456
40,000	62.5	2,828	2,262	1,697	1,414	1,131	848	566
50,000	78.1	3,344	2,675	2,006	1,672	1,338	1,003	669
60,000	93.7	3,834	3,067	2,300	1,917	1,534	1,150	767
70,000	109	4,304	3,443	2,582	2,152	1,722	1,291	861
80,000	125	4,757	3,806	2,854	2,378	1,903	1,427	951
90,000	141	5,196	4,157	3,118	2,598	2,078	1,559	1,039
100,000	156	5,623	4,498	3,374	2,811	2,249	1,687	1,125
110,000	172	6,040	4,832	3,624	3,020	2,416	1,812	1,208
120,000	187	6,447	5,158	3,868	3,224	2,579	1,934	1,289
130,000	203	6,846	5,477	4,108	3,423	2,738	2,054	1,369
140,000	219	7,238	5,790	4,343	3,619	2,895	2,171	1,448
150,000	234	7,622	6,098	4,573	3,811	3,049	2,287	1,524
160,000	250	8,000	6,400	4,800	4,000	3,200	2,400	1,600
170,000	266	8,372	6,698	5,024	4,187	3,349	2,512	1,675
180,000	281	8,739	6,991	5,243	4,370	3,496	2,622	1,748
190,000	297	9,100	7,280	5,460	4,550	3,640	2,730	1,820
200,000	312	9,457	7,566	5,674	4,729	3,783	2,837	1,891
210,000	328	9,810	7,848	5,886	4,905	3,924	2,943	1,962
220,000	344	10,158	8,126	6,095	5,079	4,063	3,047	2,032
230,000	359	10,503	8,402	6,302	5,252	4,201	3,151	2,101
240,000	375	10,843	8,674	6,506	5,422	4,337	3,253	2,169
250,000	391	11,180	8,944	6,708	5,590	4,472	3,354	2,236
260,000	406	11,514	9,211	6,908	5,757	4,606	3,454	2,303
270,000	422	11,845	9,476	7,107	5,923	4,738	3,554	2,369
280,000	437	12,172	9,738	7,303	6,086	4,869	3,652	2,434
290,000	453	12,497	9,998	7,498	6,249	4,999	3,749	2,499
300,000	469	12,819	10,255	7,691	6,410	5,128	3,846	2,564
310,000	484	13,138	10,510	7,883	6,569	5,255	3,941	2,628
320,000	500	13,454	10,763	8,072	6,727	5,382	4,036	2,691
330,000	516	13,768	11,014	8,261	6,884	5,507	4,130	2,754
340,000	531	14,080	11,264	8,448	7,040	5,632	4,224	2,816
350,000	547	14,390	11,512	8,634	7,195	5,756	4,317	2,878
360,000	562	14,697	11,758	8,818	7,349	5,879	4,409	2,939
370,000	578	15,002	12,002	9,001	7,501	6,001	4,501	3,000
380,000	594	15,305	12,244	9,183	7,653	6,122	4,592	3,061
390,000	609	15,606	12,485	9,364	7,803	6,242	4,682	3,121
400,000	625	15,905	12,724	9,543	7,953	6,362	4,772	3,181
500,000	781	18,804	15,043	11,282	9,402	7,522	5,641	3,761

TABLE 37. VOLUME OF 100-FT EARTHWORK SOLIDS
(CUBIC YARDS)
A = Sum of end areas in square feet.

A	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	*A*
0	0.0	0.2	0.4	0.6	0.7	0.9	1.1	1.3	1.5	1.7	0
1	1.9	2.0	2.2	2.4	2.6	2.8	3.0	3.1	3.3	3.6	1
2	3.7	3.9	4.1	4.3	4.4	4.6	4.8	5.0	5.2	5.4	2
3	5.6	5.7	5.9	6.1	6.3	6.5	6.7	6.9	7.0	7.2	3
4	7.4	7.6	7.8	8.0	8.1	8.3	8.5	8.7	8.9	9.1	4
5	9.3	9.4	9.6	9.8	10.0	10.2	10.4	10.6	10.7	10.9	5
6	11.1	11.3	11.5	11.7	11.9	12.0	12.2	12.4	12.5	12.8	6
7	13 0	13.1	13.3	13.5	13.7	13.9	14.1	14.3	14.4	14.6	7
8	14.8	15.0	15.2	15.4	15.6	15.7	15.9	16.1	16.3	16.5	8
9	16.7	16.9	17.0	17.2	17.4	17.6	17.8	18.0	18.1	18.3	9
10	18.5	18.7	18.9	19.1	19.3	19.4	19.6	19.8	20.0	20.2	10
11	20.4	20.6	20.7	20.9	21.1	21.3	21.5	21.7	21.9	22.0	11
12	22.2	22.4	22.6	22.8	23.0	23.1	23.3	23.5	23.7	23.9	12
13	24.1	24.3	24.4	24.6	24.8	25.0	25.2	25.4	25.6	25.7	13
14	25.9	26.1	26.3	26.5	26.7	26.8	27.0	27.2	27.4	27.6	14
15	27.8	28.0	28.1	28.3	28.5	28.7	28.9	29.1	29.3	29.4	15
16	29.6	29.8	30.0	30.2	30.4	30.6	30.7	30.9	31.1	31.3	16
17	31.5	31.7	31.9	32.0	32.2	32.4	32.6	32.8	33.0	33.1	17
18	33.3	33.5	33.7	33.9	34.1	34.3	34.4	34.6	34.8	35.0	18
19	35.2	35.4	35.6	35.7	35.9	36.1	36.3	36.5	36.7	36.9	19
20	37.0	37.2	37.4	37.6	37.8	38.0	38.1	38.3	38 5	38.7	20
21	38.9	39.1	39.3	39.4	39.6	39.8	40.0	40.2	40.4	40.6	21
22	40.7	40.9	41.1	41.3	41.5	41.7	41.9	42.0	42.2	42.4	22
23	42.6	42.8	43.0	43.1	43.3	43.5	43.7	43.9	44.1	44.3	23
24	44.4	44.6	44.8	45.0	45.2	45.4	45.6	45.7	45.9	46.1	24
25	46.3	46.5	46.7	46.9	47.0	47.2	47.4	47.6	47.8	48.0	25
26	48.1	48.3	48.5	48.7	48.9	49.1	49.3	49.4	49.6	49.8	26
27	50.0	50.2	50.4	50.6	50.7	50.9	51.1	51.3	51.5	51.7	27
28	51.9	52.0	52.2	52.4	52.6	52.8	53.0	53.1	53.3	53.5	28
29	53.7	53.9	54.1	54.3	54.4	54.6	54.8	55.0	55.2	55.4	29
30	55.6	55.7	55.9	56.1	56.3	56.5	56.7	56.9	57.0	57.2	30
31	57.4	57.6	57.8	58.0	58.1	58.3	58.5	58.7	58.9	59.1	31
32	59.3	59.4	59.6	59.8	60.0	60.2	60.4	60.6	60.7	60.9	32
33	61.1	61.3	61.5	61.7	61.9	62.0	62.2	62.4	62.6	62.8	33
34	63.0	63.1	63.3	63.5	63.7	63.9	64.1	64.3	64.4	64.6	34
35	64.8	65.0	65.2	65.4	65.6	65.7	65.9	66.1	66.3	66.5	35
36	66.7	66.9	67.0	67.2	67.4	67.6	67.8	68.0	68.1	68.3	36
37	68.5	68.7	68.9	69.1	69.3	69.4	69.6	69.8	70.0	70.2	37
38	70.4	70.6	70.7	70.9	71.1	71.3	71.5	71.7	71.9	72.0	38
39	72.2	72.4	72.6	72.8	73.0	73.1	73.3	73.5	73.7	73.9	39
40	74.1	74.3	74.4	74.6	74.8	75.0	75.2	75.4	75.6	75.7	40
41	75.9	76.1	76.3	76.5	76.7	79.6	77.0	77.2	77.4	77.6	41
42	77.8	78.0	78.1	78.3	78.5	78.7	78.9	79.1	79.3	79.4	42
43	79.6	79.8	80.0	80.2	80.4	80.6	80.7	80.9	81.1	81.3	43
44	81.5	81.7	81.9	82.0	82.2	82.4	82.6	82.8	83.0	83.1	44
45	83.3	83.5	83.7	83.9	84.1	84.3	84.4	84.6	84.8	85.0	45
46	85.2	85.4	85.6	85.7	85.9	86.1	86.3	86.5	86.7	86.9	46
47	87.0	87.2	87.4	87.6	87.8	88.0	88.1	88.3	88.5	88.7	47
48	88.9	89.1	89.3	89.4	89.6	89.8	90.0	90.2	90.4	90.6	48
49	90.7	90.9	91.1	91.3	91.5	91.7	91.9	92.0	92.2	92.4	49
50	92.6	92.8	93.0	93.1	93.3	93.5	93.7	93.9	94.1	94.3	50

TABLE 37. VOLUME OF 100-FT EARTHWORK SOLIDS (CUBIC YARDS) (*Continued*)

A = Sum of end areas in square feet.

A	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	*A*
51	94.4	94.6	94.8	95.0	95.2	95.4	95.6	95.7	95.9	96.1	51
52	96.3	96.5	96.7	96.9	97.0	97.2	97.4	97.6	97.8	98.0	52
53	98.1	98.3	98.5	98.7	98.9	99.1	99.3	99.4	99.6	99.8	53
54	100.0	100.2	100.4	100.6	100.7	100.9	101.1	101.3	101.5	101.7	54
55	101.9	102.0	102.2	102.4	102.6	102.8	103.0	103.1	103.3	103.5	55
56	103.7	103.9	104.1	104.3	104.4	104.6	104.8	105.0	105.2	105.4	56
57	105.6	105.7	105.9	106.1	106.3	106.5	106.7	106.9	107.0	107.2	57
58	107.4	107.6	107.8	108.0	108.1	108.3	108.5	108.7	108.9	109.1	58
59	109.3	109.4	109.6	109.8	110.0	110.2	110.4	110.6	110.7	110.9	59
60	111.1	111.3	111.5	111.7	111.9	112.0	112.2	112.4	112.6	112.8	60
61	113.0	113.1	113.3	113.5	113.7	113.9	114.1	114.3	114.4	114.6	61
62	114.8	115.0	115.2	115.4	115.6	115.7	115.9	116.1	116.3	116.5	62
63	116.7	116.9	117.0	117.2	117.4	117.6	117.8	118.0	118.1	118.3	63
64	118.5	118.7	118.9	119.1	119.3	119.4	119.6	119.8	120.0	120.2	64
65	120.4	120.6	120.7	120.9	121.1	121.3	121.5	121.7	121.9	122.0	65
66	122.2	122.2	122.6	122.8	123.0	123.1	123.3	123.5	123.7	123.9	66
67	124.1	124.3	124.4	124.6	124.8	125.0	125.2	125.4	125.6	125.7	67
68	125.9	126.1	126.3	126.5	126.7	126.9	127.0	127.2	127.4	127.6	68
69	127.8	128.0	128.1	128.3	128.5	128.7	128.9	129.1	129.3	129.4	69
70	129.6	129.8	130.0	130.2	130.4	130.6	130.7	130.9	131.1	131.3	70
71	131.5	131.7	131.9	132.0	132.2	132.4	132.6	132.8	133.0	133.1	71
72	133.3	133.5	133.7	133.9	134.1	134.3	134.4	134.6	134.8	135.0	72
73	135.2	135.4	135.6	135.7	135.9	136.1	136.3	136.5	136.7	136.9	73
74	137.0	137.2	137.4	137.6	137.8	138.0	138.1	138.3	138.5	138.7	74
75	138.9	139.1	139.3	139.4	139.6	139.8	140.0	140.2	140.4	140.6	75
76	140.7	140.9	141.1	141.3	141.5	141.7	141.9	142.0	142.2	142.4	76
77	142.6	142.8	143.0	143.1	143.3	143.5	143.7	143.9	144.1	144.3	77
78	144.4	144.6	144.8	145.0	145.2	145.4	145.6	145.7	145.9	146.1	78
79	146.3	146.5	146.7	146.9	147.0	147.2	147.4	147.6	147.8	148.0	79
80	148.1	148.3	148.5	148.7	149.9	149.1	149.3	149.4	149.6	149.8	80
81	150.0	150.2	150.4	150.6	150.7	150.9	151.1	151.3	151.5	151.7	81
82	151.9	152.0	152.2	152.4	152.6	152.8	153.0	153.1	153.3	153.5	82
83	153.7	153.9	154.0	154.3	154.4	154.6	154.8	155.0	155.2	155.4	83
84	155.6	155.7	155.9	156.1	156.3	156.5	156.7	156.9	157.0	157.2	84
85	157.4	157.6	157.8	158.0	158.1	158.3	158.5	158.7	158.9	159.1	85
86	159.3	159.4	159.6	159.8	160.0	160.2	160.4	160.6	160.7	160.9	86
87	161.1	161.3	161.5	161.7	161.9	162.0	162.2	162.4	162.6	162.8	87
88	163.0	163.1	163.3	163.5	163.7	163.9	164.1	164.3	164.4	164.6	88
89	164.8	165.0	165.2	165.4	165.6	165.7	165.9	166.1	166.3	166.5	89
90	166.7	166.9	167.0	167.2	167.4	167.6	167.8	168.0	168.1	168.3	90
91	168.5	168.7	168.9	169.1	169.3	169.4	169.6	169.8	170.0	170.1	91
92	170.4	170.6	170.7	170.9	171.1	171.3	171.5	171.7	171.9	172.0	92
93	172.2	172.4	172.6	172.8	173.0	173.1	173.3	173.5	173.7	173.9	93
94	174.1	174.3	174.4	174.6	174.8	175.0	175.2	175.4	175.6	175.7	94
95	175.9	176.1	176.3	176.5	176.7	176.9	177.0	177.2	177.4	177.6	95
96	177.8	178.0	178.1	178.3	178.5	178.7	178.9	179.1	179.3	179.4	96
97	179.6	179.8	180.0	180.2	180.4	180.6	180.7	180.9	181.1	181.3	97
98	181.5	181.7	181.9	182.0	182.2	182.4	182.6	182.8	183.0	183.1	98
99	183.3	183.5	183.7	183.9	184.1	184.3	184.4	184.6	184.8	185.0	99
100	185.2	185.4	185.6	185.7	185.9	186.1	186.3	186.5	186.7	186.9	100

TABLE 37. VOLUME OF 100-FT EARTHWORK SOLIDS (CUBIC YARDS) (*Continued*)

A = Sum of end areas in square feet.

A	V	A	V	A	V	A	V	A	V
101	187.0	152	281.5	203	376.0	254	470.4	305	564.8
102	188.9	153	283.3	204	377.8	255	472.2	306	566.7
103	190.7	154	285.2	205	379.6	256	474.1	307	568.5
104	192.6	155	287.0	206	381.5	257	475.9	308	570.4
105	194.4	156	288.9	207	383.3	258	477.8	309	572.2
106	196.3	157	290.7	208	385.2	259	479.6	310	574.1
107	198.1	158	292.6	209	387.0	260	481.5	311	575.9
108	200.0	159	294.4	210	388.9	261	483.3	312	577.8
109	201.9	160	296.3	211	390.7	262	485.2	313	579.6
110	203.7	161	298.1	212	392.6	263	487.0	314	581.5
111	205.6	162	300.0	213	394.4	264	488.9	315	583.3
112	207.4	163	301.9	214	396.3	265	490.7	316	585.2
113	209.3	164	303.7	215	398.1	266	492.6	317	587.0
114	211.1	165	305.6	216	400.0	267	494.4	318	588.9
115	213.0	166	307.4	217	401.9	268	496.3	319	590.7
116	214.8	167	309.3	218	403.7	269	498.1	320	592.6
117	216.7	168	311.1	219	405.6	270	500.0	321	594.4
118	218.6	169	313.0	220	407.4	271	501 9	322	596.3
119	220.4	170	314.8	221	409.3	272	503.7	323	598.1
120	222.2	171	316.7	222	411.1	273	505.6	324	600.0
121	224.1	172	318.5	223	413.0	274	507.4	325	601.9
122	226.0	173	320.4	224	414.8	275	509.3	326	603.7
123	227.8	174	322.2	225	416.7	276	511.1	327	605.6
124	229.6	175	324.1	226	418.5	277	513.0	328	607.4
125	231.5	176	326.0	227	420.4	278	514.8	329	609.3
126	233.3	177	327.8	228	422.2	279	516.7	330	611.1
127	235.2	178	329.6	229	424.1	280	518.5	331	613.0
128	237.0	179	331.5	230	425.9	281	520.4	332	614.8
129	238.9	180	333.3	231	427.8	282	522.2	333	616.7
130	240.7	181	332.2	232	429.6	283	524.1	334	618.5
131	242.6	182	337.0	233	431.5	284	525.9	335	620.4
132	244.4	183	338.9	234	433.3	285	527.8	336	622.2
133	246.3	184	340.7	235	435.2	286	529.6	337	624.1
134	248.1	185	342.6	236	437.0	287	531.5	338	625.9
135	250.0	186	344.4	237	438.9	288	533.3	339	627.8
136	251.9	187	346.3	238	440.7	289	535.2	340	629.6
137	253.7	188	348.1	239	442.6	290	537.0	341	631.5
138	255.6	189	350.0	240	444.4	291	538.9	342	633.3
139	257.4	190	351.9	241	446.3	292	540.7	343	635.2
140	259.3	191	353.7	242	448.1	293	542.6	344	637.0
141	261.1	192	355.6	243	450.0	294	544.4	345	638.9
142	263.0	193	357.4	244	451.9	295	546.3	346	640.7
143	264.8	194	359.3	245	453.7	296	548.1	347	642.6
144	266.7	195	361.1	246	455.6	297	550.0	348	644.4
145	268.5	196	363.0	247	457.4	298	551.9	349	646.3
146	270.4	197	364.8	248	459.3	299	553.7	350	648.1
147	272.2	198	366.7	249	461.1	300	555.6	351	650.0
148	274.1	199	368.5	250	463.0	301	557.4	352	651.9
149	275.9	200	370.4	251	464.8	302	559.3	353	653.7
150	277.8	201	372.2	252	466.7	303	561.1	354	655.6
151	279.6	202	374.1	253	468.5	304	563.0	355	657.4

TABLE 37. VOLUME OF 100-FT EARTHWORK SOLIDS (CUBIC YARDS) (*Continued*)

A = Sum of end areas in square feet.

A	V	A	V	A	V	A	V	A	V
356	659.3	407	753.7	458	848.1	509	942.6	560	1,037.0
357	661.1	408	755.6	459	850.0	510	944.4	561	1,038.9
358	663.0	409	757.4	460	851.9	511	946.3	562	1,040.7
359	664.8	410	759.3	461	853.7	512	948.3	563	1,042.6
360	666.7	411	761.2	462	855.6	513	950.0	564	1,044.4
361	668.5	412	763.0	463	857.4	514	951.9	565	1,046.3
362	670.4	413	764.8	464	859.3	515	953.7	566	1,048.1
363	672.2	414	766.7	465	861.1	516	955.6	567	1,050.0
364	674.1	415	768.5	466	863.0	517	957.4	568	1,051.9
365	675.9	416	770.4	467	864.8	518	959.3	569	1,053.7
366	677.8	417	772.2	468	866.7	519	961.1	570	1,055.6
367	679.6	418	774.1	469	868.5	520	963.0	571	1,057.4
368	681.5	419	775.9	470	870.4	521	964.8	572	1,059.3
369	683.3	420	777.8	471	872.2	522	966.7	573	1,061.1
370	685.2	421	779.6	472	874.1	523	968.6	574	1,063.0
371	687.0	422	781.5	473	875.9	524	970.4	575	1,064.8
372	688.9	423	783.3	474	877.8	525	972.2	576	1,066.7
373	690.7	424	785.2	475	879.6	526	974.1	577	1,068.5
374	692.6	425	787.0	476	881.5	527	975.9	578	1,070.4
375	694.4	426	788.9	477	883.3	528	977.8	579	1,072.2
376	696.3	427	790.7	478	885.2	529	979.6	580	1,074.1
377	698.1	428	792.6	479	887.0	530	981.5	581	1,075.9
378	700.0	429	794.4	480	888.9	531	983.3	582	1,077.8
379	701.9	430	796.3	481	890.7	532	985.2	583	1,079.6
380	703.7	431	798.1	482	892.6	533	987.0	584	1,081.5
381	705.6	432	800.0	483	894.4	534	988.9	585	1,083.3
382	707.4	433	801.9	484	896.3	535	990.7	586	1,085.2
383	709.3	434	803.7	485	898.1	536	992.6	587	1,087.0
384	711.1	435	805.6	486	900.0	537	994.4	588	1,088.9
385	713.0	436	807.4	487	901.9	538	996.3	589	1,090.7
386	714.8	437	809.3	488	903.7	539	998.1	590	1,092.6
387	716.7	438	811.1	489	905.6	540	1,000.0	591	1,094.6
388	718.5	439	813.0	490	907.4	541	1,001.9	592	1,096.3
389	720.4	440	814.8	491	909.3	542	1,003.7	593	1,098.1
390	722.2	441	816.7	492	911.1	543	1,005.6	594	1,100.0
391	724.1	442	818.5	493	913.0	544	1,007.4	595	1,101.9
392	725.9	443	820.4	494	914.8	545	1,009.3	596	1,103.7
393	727.8	444	822.2	495	916.7	546	1,011.1	597	1,105.6
394	729.6	445	824.1	496	918.5	547	1,013.0	598	1,107.4
395	731.5	446	825.9	497	920.4	548	1,014.8	599	1,109.3
396	733.3	447	827.8	498	922.2	549	1,016.7	600	1,111.1
397	735.2	448	829.6	499	924.1	550	1,018.5	601	1,113.0
398	737.0	449	831.5	500	925.9	551	1,020.4	602	1,114.8
399	738.9	450	833.4	501	927.8	552	1,022.2	603	1,116.7
400	740.7	451	835.2	502	929.6	553	1,024.1	604	1,118.5
401	742.6	452	837.0	503	931.5	554	1,025.9	605	1,120.3
402	744.4	453	838.9	504	933.3	555	1,027.8	606	1,122.2
403	746.3	454	840.7	505	935.2	556	1,029.6	607	1,124.1
404	748.1	455	842.6	506	937.0	557	1,031.5	608	1,125.9
405	750.0	456	844.4	507	938.9	558	1,033.3	609	1,127.8
406	751.9	457	846.3	508	940.7	559	1,035.2	610	1,129.6

TABLE 38. PRISMOIDAL CORRECTION (CUBIC YARDS) FOR 100-FT EARTHWORK SOLIDS

$d - d'$	$X - X'$								
	1	2	3	4	5	6	7	8	9
0.2	0.06	0.12	0.19	0.25	0.31	0.37	0.43	0.49	0.56
0.4	0.12	0.25	0.37	0.49	0.62	0.74	0.86	0.99	1.11
0.6	0.19	0.37	0.56	0.74	0.93	1.11	1.30	1.48	1.67
0.8	0.25	0.49	0.74	0.99	1.23	1.48	1.73	1.98	2.22
1.0	0.31	0.62	0.93	1.23	1.54	1.85	2.16	2.47	2.78
1.2	0.37	0.74	1.11	1.48	1.85	2.22	2.59	2.96	3.33
1.4	0.43	0.86	1.30	1.73	2.16	2.59	3.02	3.46	3.89
1.6	0.49	0.99	1.48	1.98	2.47	2.96	3.46	3.95	4.44
1.8	0.56	1.11	1.67	2.22	2.78	3.33	3.89	4.44	5.00
2.0	0.62	1.23	1.85	2.47	3.09	3.70	4.32	4.94	5.56
2.2	0.68	1.36	2.04	2.72	3.40	4.07	4.75	5.43	6.11
2.4	0.74	1.48	2.22	2.96	3.70	4.44	5.19	5.93	6.67
2.6	0.80	1.60	2.41	3.21	4.01	4.81	5.62	6.42	7.22
2.8	0.86	1.73	2.59	3.46	4.32	5.19	6.05	6.91	7.78
3.0	0.93	1.85	2.78	3.70	4.63	5.56	6.48	7.41	8.33
3.2	0.99	1.98	2.96	3.95	4.94	5.93	6.91	7.90	8.89
3.4	1.05	2.10	3.15	4.20	5.25	6.30	7.35	8.40	9.44
3.6	1.11	2.22	3.33	4.44	5.56	6.67	7.78	8.89	10.00
3.8	1.17	2.35	3.52	4.69	5.86	7.04	8.21	9.38	10.56
4.0	1.23	2.47	3.70	4.94	6.17	7.41	8.64	9.88	11.11
4.2	1.30	2.59	3.89	5.19	6.48	7.78	9.07	10.37	11.67
4.4	1.36	2.72	4.07	5.43	6.79	8.15	9.51	10.86	12.22
4.6	1.42	2.84	4.26	5.68	7.10	8.52	9.94	11.36	12.78
4.8	1.48	2.96	4.44	5.93	7.41	8.89	10.37	11.85	13.33
5.0	1.54	3.09	4.63	6.17	7.72	9.26	10.80	12.35	13.89
5.2	1.60	3.21	4.81	6.42	8.02	9.63	11.23	12.84	14.44
5.4	1.67	3.33	5.00	6.67	8.33	10.00	11.67	13.33	15.00
5.6	1.73	3.46	5.19	6.91	8.64	10.37	12.10	13.83	15.56
5.8	1.79	3.58	5.37	7.16	8.95	10.74	12.53	14.32	16.11
6.0	1.85	3.70	5.56	7.41	9.26	11.11	12.96	14.81	16.67
6.2	1.91	3.83	5.74	7.65	9.57	11.48	13.40	15.31	17.22
6.4	1.98	3.95	5.93	7.90	9.88	11.85	13.83	15.80	17.78
6.6	2.04	4.07	6.11	8.15	10.19	12.22	14.26	16.30	18.33
6.8	2.10	4.20	6.30	8.40	10.49	12.59	14.69	16.79	18.89
7.0	2.16	4.32	6.48	8.64	10.80	12.96	15.12	17.28	19.44
7.2	2.22	4.44	6.67	8.89	11.11	13.33	15.56	17.78	20.00
7.4	2.28	4.57	6.85	9.14	11.42	13.70	15.99	18.27	20.56
7.6	2.35	4.69	7.04	9.38	11.73	14.07	16.42	18.77	21.11
7.8	2.41	4.81	7.22	9.63	12.04	14.44	16.85	19.26	21.67
8.0	2.47	4.94	7.41	9.88	12.35	14.81	17.28	19.75	22.22
8.2	2.53	5.06	7.59	10.12	12.65	15.19	17.72	20.25	22.78
8.4	2.59	5.19	7.78	10.37	12.96	15.56	18.15	20.74	23.33
8.6	2.65	5.31	7.96	10.62	13.27	15.93	18.58	21.23	23.89
8.8	2.72	5.43	8.15	10.86	13.58	16.30	19.01	21.73	24.44
9.0	2.78	5.56	8.33	11.11	13.89	16.67	19.44	22.22	25.00
9.2	2.84	5.68	8.52	11.36	14.20	17.04	19.88	22.72	25.56
9.4	2.90	5.80	8.70	11.60	14.51	17.41	20.31	23.21	26.11
9.6	2.96	5.93	8.89	11.85	14.81	17.78	20.74	23.70	26.67
9.8	3.02	6.05	9.07	12.10	15.12	18.15	21.17	24.20	27.22
10.0	3.09	6.17	9.26	12.35	15.43	18.52	21.60	24.69	27.78

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TABLE 39. NATURAL SINES AND COSINES

′	0°		1°		2°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	0.0000000	1.0000000	0.0174524	0.9998477	0.0348995	0.9993908	60
1	.0002909	1.0000000	.0177432	.9998426	.0351902	.9993806	59
2	.0005818	.9999998	.0180341	.9998374	.0354809	.9993704	58
3	.0008727	.9999996	.0183249	.9998321	.0357716	.9993600	57
4	.0011636	.9999993	.0186158	.9998267	.0360623	.9993495	56
5	.0014544	.9999989	.0189066	.9998213	.0363530	.9993390	55
6	.0017453	.9999985	.0191974	.9998157	.0366437	.9993284	54
7	.0020362	.9999979	.0194883	.9998101	.0369344	.9993177	53
8	.0023271	.9999973	.0197791	.9998044	.0372251	.9993069	52
9	.0026180	.9999966	.0200699	.9997986	.0375158	.9992960	51
10	0.0029089	0.9999958	0.0203608	0.9997927	0.0378065	0.9992851	50
11	.0031998	.9999949	.0206516	.9997867	.0380971	.9992740	49
12	.0034907	.9999939	.0209424	.9997807	.0383878	.9992629	48
13	.0037815	.9999928	.0212332	.9997745	.0386785	.9992517	47
14	.0040724	.9999917	.0215241	.9997683	.0389692	.9992404	46
15	.0043633	.9999905	.0218149	.9997620	.0392598	.9992290	45
16	.0046542	.9999892	.0221057	.9997556	.0395505	.9992176	44
17	.0049451	.9999878	.0223965	.9997492	.0398411	.9992060	43
18	.0052360	.9999863	.0226873	.9997426	.0401318	.9991944	42
19	.0055268	.9999847	.0229781	.9997360	.0404224	.9991827	41
20	0.0058177	0.9999831	0.0232690	0.9997292	0.0407131	0.9991709	40
21	.0061086	.9999813	.0235598	.9997224	.0410037	.9991590	39
22	.0063995	.9999795	.0238506	.9997155	.0412944	.9991470	38
23	.0066904	.9999776	.0241414	.9997086	.0415850	.9991350	37
24	.0069813	.9999756	.0244322	.9997015	.0418757	.9991228	36
25	.0072721	.9999736	.0247230	.9996943	.0421663	.9991106	35
26	.0075630	.9999714	.0250138	.9996871	.0424569	.9990983	34
27	.0078539	.9999692	.0253046	.9996798	.0427475	.9990859	33
28	.0081448	.9999668	.0255954	.9996724	.0430382	.9990734	32
29	.0084357	.9999644	.0258862	.9996649	.0433288	.9990609	31
30	0.0087265	0.9999619	0.0261769	0.9996573	0.0436194	0.9990482	30
31	.0090174	.9999593	.0264677	.9996497	.0439100	.9990355	29
32	.0093083	.9999567	.0267585	.9996419	.0442006	.9990227	28
33	.0095992	.9999539	.0270493	.9996341	.0444912	.9990098	27
34	.0098900	.9999511	.0273401	.9996262	.0447818	.9989968	26
35	.0101809	.9999482	.0276309	.9996182	.0450724	.9989837	25
36	.0104718	.9999452	.0279216	.9996101	.0453630	.9989706	24
37	.0107627	.9999421	.0282124	.9996020	.0456536	.9989573	23
38	.0110535	.9999389	.0285032	.9995937	.0459442	.9989440	22
39	.0113444	.9999357	.0287940	.9995854	.0462347	.9989306	21
40	0.0116353	0.9999323	0.0290847	0.9995770	0.0465253	0.9989171	20
41	.0119261	.9999289	.0293755	.9995684	.0468159	.9989035	19
42	.0122170	.9999254	.0296662	.9995599	.0471065	.9988899	18
43	.0125079	.9999218	.0299570	.9995512	.0473970	.9988761	17
44	.0127987	.9999181	.0302478	.9995424	.0476876	.9988623	16
45	.0130896	.9999143	.0305385	.9995336	.0479781	.9988484	15
46	.0133805	.9999105	.0308293	.9995247	.0482687	.9988344	14
47	.0136713	.9999065	.0311200	.9995157	.0485592	.9988203	13
48	.0139622	.9999025	.0314108	.9995066	.0488498	.9988061	12
49	.0142530	.9998984	.0317015	.9994974	.0491403	.9987919	11
50	0.0145439	0.9998942	0.0319922	0.9994881	0.0494308	0.9987775	10
51	.0148348	.9998900	.0322830	.9994788	.0497214	.9987631	9
52	.0151256	.9998856	.0325737	.9994693	.0500119	.9987486	8
53	.0154165	.9998812	.0328644	.9994598	.0503024	.9987340	7
54	.0157073	.9998766	.0331552	.9994502	.0505929	.9987194	6
55	.0159982	.9998720	.0334459	.9994405	.0508835	.9987046	5
56	.0162890	.9998673	.0337366	.9994308	.0511740	.9986898	4
57	.0165799	.9998625	.0340274	.9994209	.0514645	.9986748	3
58	.0168707	.9998577	.0343181	.9994110	.0517550	.9986598	2
59	.0171616	.9998527	.0346088	.9994009	.0520455	.9986447	1
60	0.0174524	0.9998477	0.0348995	0.9993908	0.0523360	0.9986295	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	89°		88°		87°		

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TABLE 39. NATURAL SINES AND COSINES (*Continued*)

′	3°		4°		5°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	0.0523360	0.9986295	0.0697565	0.9975641	0.0871557	0.9961947	60
1	.0526264	.9986143	.0700467	.9975437	.0874455	.9961693	59
2	.0529169	.9985989	.0703368	.9975233	.0877353	.9961438	58
3	.0532074	.9985835	.0706270	.9975028	.0880251	.9961183	57
4	.0534979	.9985680	.0709171	.9974822	.0883148	.9960926	56
5	.0537883	.9985524	.0712073	.9974615	.0886046	.9960669	55
6	.0540788	.9985367	.0714974	.9974408	.0888943	.9960411	54
7	.0543693	.9985209	.0717876	.9974199	.0891840	.9960152	53
8	.0546597	.9985050	.0720777	.9973990	.0894738	.9959892	52
9	.0549502	.9984891	.0723678	.9973780	.0897635	.9959631	51
10	0.0552406	0.9984731	0.0726580	0.9973569	0.0900532	0.9959370	50
11	.0555311	.9984570	.0729481	.9973357	.0903429	.9959107	49
12	.0558215	.9984408	.0732382	.9973145	.0906326	.9958844	48
13	.0561119	.9984245	.0735283	.9972931	.0909223	.9958580	47
14	.0564024	.9984081	.0738184	.9972717	.0912119	.9958315	46
15	.0566928	.9983917	.0741085	.9972502	.0915016	.9958049	45
16	.0569832	.9983751	.0743986	.9972286	.0917913	.9957783	44
17	.0572736	.9983585	.0746887	.9972069	.0920809	.9957515	43
18	.0575640	.9983418	.0749787	.9971851	.0923706	.9957247	42
19	.0578544	.9983250	.0752688	.9971633	.0926602	.9956978	41
20	0.0581448	0.9983082	0.0755589	0.9971413	0.0929499	0.9956708	40
21	.0584352	.9982912	.0758489	.9971193	.0932395	.9956437	39
22	.0587256	.9982742	.0761390	.9970972	.0935291	.9956165	38
23	.0590160	.9982570	.0764290	.9970750	.0938187	.9955893	37
24	.0593064	.9982398	.0767190	.9970528	.0941083	.9955620	36
25	.0595967	.9982225	.0770091	.9970304	.0943979	.9955345	35
26	.0598871	.9982052	.0772991	9970080	.0946875	.9955070	34
27	.0601775	.9981877	.0775891	.9969854	.0949771	.9954795	33
28	.0604678	.9981701	.0778791	.9969628	.0952666	.9954518	32
29	.0607582	.9981525	.0781691	.9969401	.0955562	.9954240	31
30	0.0610485	0.9981348	0.0784591	0.9969173	0.0958458	0.9953962	30
31	.0613389	.9981170	.0787491	.9968945	.0961353	.9953683	29
32	.0616292	.9980991	.0790391	.9968715	.0964248	.9953403	28
33	.0619196	.9980811	.0793290	.9968485	.0967144	.9953122	27
34	.0622099	.9980631	.0796190	.9968254	.0970039	.9952840	26
35	.0625002	.9980450	.0799090	.9968022	.0972934	.9952557	25
36	.0627905	.9980267	.0801989	.9967789	.0975829	.9952274	24
37	.0630808	.9980084	.0804889	.9967555	.0978724	.9951990	23
38	.0633711	.9979900	.0807788	.9967321	.0981619	.9951705	22
39	.0636614	.9979716	.0810687	.9967085	.0984514	.9951419	21
40	0.0639517	0.9979530	0.0813587	0.9966849	0.0987408	0.9951132	20
41	.0642420	.9979343	.0816486	.9966612	.0990303	.9950844	19
42	.0645323	.9979156	.0819385	.9966374	.0993197	.9950556	18
43	.0648226	.9978968	.0822284	.9966135	.0996092	.9950266	17
44	.0651129	.9978779	.0825183	.9965895	.0998986	.9949976	16
45	.0654031	.9978589	.0828082	.9965655	.1001881	.9949685	15
46	.0656934	.9978399	.0830981	.9965414	.1004775	.9949393	14
47	.0659836	.9978207	.0833880	.9965172	.1007669	.9949101	13
48	.0662739	.9978015	.0836778	.9964929	.1010563	.9948807	12
49	.0665641	.9977821	.0839677	.9964685	.1013457	.9948513	11
50	0.0668544	0.9977627	0.0842576	0.9964440	0.1016351	0.9948217	10
51	.0671446	.9977433	.0845474	.9964195	.1019245	.9947921	9
52	.0674349	.9977237	.0848373	.9963948	.1022138	.9947625	8
53	.0677251	.9977040	.0851271	.9963701	.1025032	.9947327	7
54	.0680153	.9976843	.0854169	.9963453	.1027925	.9947028	6
55	.0683055	.9976645	.0857067	.9963204	.1030819	.9946729	5
56	.0685957	.9976445	.0859966	.9962954	.1033712	.9946428	4
57	.0688859	.9976245	.0862864	.9962704	.1036605	.9946127	3
58	.0691761	.9976045	.0865762	.9962452	.1039499	.9945825	2
59	.0694663	.9975843	.0868660	.9962200	.1042392	.9945523	1
60	0.0697565	0.9975641	0.0871557	0.9961947	0.1045285	0.9945219	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	86°		85°		84°		

TABLE 39. NATURAL SINES AND COSINES (*Continued*)

′	6°		7°		8°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	0.1045285	0.9945219	0.1218693	0.9925462	0.1391731	0.9902681	60
1	.1048178	.9944914	.1221581	.9925107	.1394612	.9902275	59
2	.1051070	.9944609	.1224468	.9924751	.1397492	.9901869	58
3	.1053963	.9944303	.1227355	.9924394	.1400372	.9901462	57
4	.1056856	.9943996	.1230241	.9924037	.1403252	.9901055	56
5	.1059748	.9943688	.1233128	.9923679	.1406132	.9900646	55
6	.1062641	.9943379	.1236015	.9923319	.1409012	.9900237	54
7	.1065533	.9943070	.1238901	.9922959	.1411892	.9899826	53
8	.1068425	.9942760	.1241788	.9922599	.1414772	.9899415	52
9	.1071318	.9942448	.1244674	.9922237	.1417651	.9899003	51
10	0.1074210	0.9942136	0.1247560	0.9921874	0.1420531	0.9898590	50
11	.1077102	.9941823	.1250446	.9921511	.1423410	.9898177	49
12	.1079994	.9941510	.1253332	.9921147	.1426289	.9897762	48
13	.1082885	.9941195	.1256218	.9920782	.1429168	.9897347	47
14	.1085777	.9940880	.1259104	.9920416	.1432047	.9896931	46
15	.1088669	.9940563	.1261990	.9920049	.1434926	.9896514	45
16	.1091560	.9940246	.1264875	.9919682	.1437805	.9896096	44
17	.1094452	.9939928	.1267761	.9919314	.1440684	.9895677	43
18	.1097343	.9939610	.1270646	.9918944	.1443562	.9895258	42
19	.1100234	.9939290	.1273531	.9918574	.1446440	.9894838	41
20	0.1103126	0.9938969	0.1276416	0.9918204	0.1449319	0.9894416	40
21	.1106017	.9938648	.1279302	.9917832	.1452197	.9893994	39
22	.1108908	.9938326	.1282186	.9917459	.1455075	.9893572	38
23	.1111799	.9938003	.1285071	.9917086	.1457953	.9893148	37
24	.1114689	.9937679	.1287956	.9916712	.1460830	.9892723	36
25	.1117580	.9937355	.1290841	.9916337	.1463708	.9892298	35
26	.1120471	.9937029	.1293725	.9915961	.1466585	.9891872	34
27	.1123361	.9936703	.1296609	.9915584	.1469463	.9891445	33
28	.1126252	.9936375	.1299494	.9915206	.1472340	.9891017	32
29	.1129142	.9936047	.1302378	.9914828	.1475217	.9890588	31
30	0.1132032	0.9935719	0.1305262	0.9914449	0.1478094	0.9890159	30
31	.1134922	.9935389	.1308146	.9914069	.1480971	.9889728	29
32	.1137812	.9935058	.1311030	.9913688	.1483848	.9889297	28
33	.1140702	.9934727	.1313913	.9913306	.1486724	.9888865	27
34	.1143592	.9934395	.1316797	.9912923	.1489601	.9888432	26
35	.1146482	.9934062	.1319681	.9912540	.1492477	.9887998	25
36	.1149372	.9933728	.1322564	.9912155	.1495353	.9887564	24
37	.1152261	.9933393	.1325447	.9911770	.1498230	.9887128	23
38	.1155151	.9933057	.1328330	.9911384	.1501106	.9886692	22
39	.1158040	.9932721	.1331213	.9910997	.1503981	.9886255	21
40	0.1160929	0.9932384	0.1334096	0.9910610	0.1506857	0.9885817	20
41	.1163818	.9932045	.1336979	.9910221	.1509733	.9885378	19
42	.1166707	.9931706	.1339862	.9909832	.1512608	.9884939	18
43	.1169596	.9931367	.1342744	.9909442	.1515484	.9884498	17
44	.1172485	.9931026	.1345627	.9909051	.1518359	.9884057	16
45	.1175374	.9930685	.1348509	.9908659	.1521234	.9883615	15
46	.1178263	.9930342	.1351392	.9908266	.1524109	.9883172	14
47	.1181151	.9929999	.1354274	.9907873	.1526984	.9882728	13
48	.1184040	.9929655	.1357156	.9907478	.1529858	.9882284	12
49	.1186928	.9929310	.1360038	.9907083	.1532733	.9881838	11
50	0.1189816	0.9928965	0.1362919	0.9906687	0.1535607	0.9881392	10
51	.1192704	.9928618	.1365801	.9906290	.1538482	.9880945	9
52	.1195593	.9928271	.1368683	.9905893	.1541356	.9880497	8
53	.1198481	.9927922	.1371564	.9905494	.1544230	.9880048	7
54	.1201368	.9927573	.1374445	.9905095	.1547104	.9879599	6
55	.1204256	.9927224	.1377327	.9904694	.1549978	.9879148	5
56	.1207144	.9926873	.1380208	.9904293	.1552851	.9878697	4
57	.1210031	.9926521	.1383089	.9903891	.1555725	.9878245	3
58	.1212919	.9926169	.1385970	.9903489	.1558598	.9877792	2
59	.1215806	.9925816	.1388850	.9903085	.1561472	.9877338	1
60	0.1218693	0.9925462	0.1391731	0.9902681	0.1564345	0.9876883	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	83°		82°		81°		

TABLE 39. NATURAL SINES AND COSINES (*Continued*)

′	9°		10°		11°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	0.1564345	0.9876883	0.1736482	0.9848078	0.1908090	0.9816272	60
1	.1567218	.9876428	.1739346	.9847572	.1910945	.9815716	59
2	.1570091	.9875972	.1742211	.9847066	.1913801	.9815160	58
3	.1572963	.9875514	.1745075	.9846558	.1916656	.9814603	57
4	.1575836	.9875057	.1747939	.9846050	.1919510	.9814045	56
5	.1578708	.9874598	.1750803	.9845542	.1922365	.9813486	55
6	.1581581	.9874138	.1753667	.9845032	.1925220	.9812927	54
7	.1584453	.9873678	.1756531	.9844521	.1928074	.9812366	53
8	.1587325	.9873216	.1759395	.9844010	.1930928	.9811805	52
9	.1590197	.9872754	.1762258	.9843498	.1933782	.9811243	51
10	0.1593069	0.9872291	0.1765121	0.9842985	0.1936636	0.9810680	50
11	.1595940	.9871827	.1767984	.9842471	.1939490	.9810116	49
12	.1598812	.9871363	.1770847	.9841956	.1942344	.9809552	48
13	.1601683	.9870897	.1773710	.9841441	.1945197	.9808986	47
14	.1604555	.9870431	.1776573	.9840924	.1948050	.9808420	46
15	.1607426	.9869964	.1779435	.9840407	.1950903	.9807853	45
16	.1610297	.9869496	.1782298	.9839889	.1953756	.9807285	44
17	.1613167	.9869027	.1785160	.9839370	.1956609	.9806716	43
18	.1616038	.9868557	.1788022	.9838850	.1959461	.9806147	42
19	.1618909	.9868087	.1790884	.9838330	.1962314	.9805576	41
20	0.1621779	0.9867615	0.1793746	0.9837808	0.1965166	0.9805005	40
21	.1624650	.9867143	.1796607	.9837286	.1968018	.9804433	39
22	.1627520	.9866670	.1799469	.9836763	.1970870	.9803860	38
23	.1630390	.9866196	.1802330	.9836239	.1973722	.9803286	37
24	.1633260	.9865722	.1805191	.9835715	.1976573	.9802712	36
25	.1636129	.9865246	.1808052	.9835189	.1979425	.9802136	35
26	.1638999	.9864770	.1810913	.9834663	.1982276	.9801560	34
27	.1641868	.9864293	.1813774	.9834136	.1985127	.9800983	33
28	.1644738	.9863815	.1816635	.9833608	.1987978	.9800405	32
29	.1647607	.9853336	.1819495	.9833079	.1990829	.9799827	31
30	0.1650476	0.9862856	0.1822355	0.9832549	0.1993679	0.9799247	30
31	.1653345	.9862375	.1825215	.9832019	.1996530	.9798667	29
32	.1656214	.9861894	.1828075	.9831487	.1999380	.9798086	28
33	.1659082	.9861412	.1830935	.9830955	.2002230	.9797504	27
34	.1661951	.9860929	.1833795	.9830422	.2005080	.9796921	26
35	.1664819	.9860445	.1836654	.9829888	.2007930	.9796337	25
36	.1667687	.9859960	.1839514	.9829353	.2010779	.9795752	24
37	.1670556	.9859475	.1842373	.9828818	.2013629	.9795167	23
38	.1673423	.9858988	.1845232	.9828282	.2016478	.9794581	22
39	.1676291	.9858501	.1848091	.9827744	.2019327	.9793994	21
40	0.1679159	0.9858013	0.1850949	0.9827206	0.2022176	0.9793406	20
41	.1682026	.9857524	.1853808	.9826668	.2025024	.9792818	19
42	.1684894	.9857035	.1856666	.9826128	.2027873	.9792228	18
43	.1687761	.9856544	.1859524	.9825587	.2030721	.9791638	17
44	.1690628	.9856053	.1862382	.9825046	.2033569	.9791047	16
45	.1693495	.9855561	.1865240	.9824504	.2036418	.9790455	15
46	.1696362	.9855068	.1868098	.9823961	.2039265	.9789862	14
47	.1699228	.9854574	.1870956	.9823417	.2042113	.9789268	13
48	.1702095	.9854079	.1873813	.9822873	.2044961	.9788674	12
49	.1704961	.9853583	.1876670	.9822327	.2047808	.9788079	11
50	0.1707828	0.9853087	0.1879528	0.9821781	0.2050655	0.9787483	10
51	.1710694	.9852590	.1882385	.9821234	.2053502	.9786886	9
52	.1713560	.9852092	.1885241	.9820686	.2056349	.9786288	8
53	.1716425	.9851593	.1888098	.9820137	.2059195	.9785689	7
54	.1719291	.9851093	.1890954	.9819587	.2062042	.9785090	6
55	.1722156	.9850593	.1893811	.9819037	.2064888	.9784490	5
56	.1725022	.9850091	.1896667	.9818485	.2067734	.9783889	4
57	.1727887	.9849589	.1899523	.9817933	.2070580	.9783287	3
58	.1730752	.9849086	.1902379	.9817380	.2073426	.9782684	2
59	.1733617	.9848582	.1905234	.9816826	.2076272	.9782080	1
60	0.1736482	0.9848078	0.1908090	0.9816272	0.2079117	0.9781476	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	80°		79°		78°		

TABLE 39. NATURAL SINES AND COSINES (*Continued*)

′	12°		13°		14°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	0.2079117	0.9781476	0.2249511	0.9743701	0.2419219	0.9702957	60
1	.2081962	.9780871	.2252345	.9743046	.2422041	.9702253	59
2	.2084807	.9780265	.2255179	.9742390	.2424863	.9701548	58
3	.2087652	.9779658	.2258013	.9741734	.2427685	.9700842	57
4	.2090497	.9779050	.2260846	.9741077	.2430507	.9700136	56
5	.2093341	.9778442	.2263680	.9740419	.2433329	.9699428	55
6	.2096186	.9777832	.2266513	.9739760	.2436150	.9698720	54
7	.2099030	.9777222	.2269346	.9739100	.2438971	.9698011	53
8	.2101874	.9776611	.2272179	.9738439	.2441792	.9697301	52
9	.2104718	.9775999	.2275012	.9737778	.2444613	.9696591	51
10	0.2107561	0.9775387	0.2277844	0.9737116	0.2447433	0.9695879	50
11	.2110405	.9774773	.2280677	.9736453	.2450254	.9695167	49
12	2113248	.9774159	.2283509	.9735789	.2453074	.9694453	48
13	.2116091	.9773544	.2286341	.9735124	.2455894	.9693740	47
14	.2118934	.9772928	.2289172	.9734459	.2458713	.9693025	46
15	.2121777	.9772311	.2292004	.9733793	.2461533	.9692309	45
16	.2124619	.9771693	.2294835	.9733125	.2464352	.9691593	44
17	.2127462	.9771075	.2297666	.9732458	.2467171	.9690875	43
18	.2130304	.9770456	.2300497	.9731789	.2469990	.9690157	42
19	.2133146	.9769836	.2303328	.9731119	.2472809	.9689438	41
20	0.2135988	0.9769215	0.2306159	0.9730449	0.2475627	0.9688719	40
21	.2138829	.9768593	.2308989	.9729777	.2478445	.9687998	39
22	.2141671	.9767970	.2311819	.9729105	.2481263	.9687277	38
23	.2144512	.9767347	.2314649	.9728432	.2484081	.9686555	37
24	.2147353	.9766723	.2317479	.9727759	.2486899	.9685832	36
25	.2150194	.9766098	.2320309	.9727084	.2489716	.9685108	35
26	.2153035	.9765472	.2323138	.9726409	.2492533	.9684383	34
27	.2155876	.9764845	.2325967	.9725733	.2495350	.9683658	33
28	.2158716	.9764218	.2328796	.9725056	.2498167	.9682931	32
29	.2161556	.9763589	.2331625	.9724378	.2500984	.9682204	31
30	0.2164396	0.9762960	0.2334454	0.9723699	0.2503800	0.9681476	30
31	.2167236	.9762330	.2337282	.9723020	.2506616	.9680748	29
32	.2170076	.9761699	.2340110	.9722339	.2509432	.9680018	28
33	.2172915	.9761068	.2342938	.9721658	.2512248	.9679288	27
34	.2175754	.9760435	.2345766	.9720976	.2515063	.9678557	26
35	.2178593	.9759802	.2348594	.9720294	.2517879	.9677825	25
36	.2181432	.9759168	.2351421	.9719610	.2520694	.9677092	24
37	.2184271	.9758533	.2354248	.9718926	.2523508	.9676358	23
38	.2187110	.9757897	.2357075	.9718240	.2526323	.9675624	22
39	.2189948	.9757260	.2359902	.9717554	.2529137	.9674888	21
40	0.2192786	0.9756623	0.2362729	0.9716867	0.2531952	0.9674152	20
41	.2195624	.9755985	.2365555	.9716180	.2534766	.9673415	19
42	.2198462	.9755345	.2368381	.9715491	.2537579	.9672678	18
43	.2201300	.9754706	.2371207	.9714802	.2540393	.9671939	17
44	.2204137	.9754065	.2374033	.9714112	.2543206	.9671200	16
45	.2206974	.9753423	.2376859	.9713421	.2546019	.9670459	15
46	.2209811	.9752781	.2379684	.9712729	.2548832	.9669718	14
47	.2212648	.9752138	.2382510	.9712036	.2551645	.9668977	13
48	.2215485	.9751494	.2385335	.9711343	.2554458	.9668234	12
49	.2218321	.9750849	.2388159	.9710649	.2557270	.9667490	11
50	0.2221158	0.9750203	0.2390984	0.9709953	0.2560082	0.9666746	10
51	.2223994	.9749556	.2393808	.9709258	.2562894	.9666001	9
52	.2226830	.9748909	.2396633	.9708561	.2565705	.9665255	8
53	.2229666	.9748261	.2399457	.9707863	.2568517	.9664508	7
54	.2232501	.9747612	.2402280	.9707165	.2571328	.9663761	6
55	.2235337	.9746962	.2405104	.9706466	.2574139	.9663012	5
56	.2238172	.9746311	.2407927	.9705766	.2576950	.9662263	4
57	.2241007	.9745660	.2410751	.9705065	.2579760	.9661513	3
58	.2243842	.9745008	.2413574	.9704363	.2582570	.9660762	2
59	.2246676	.9744355	.2416396	.9703661	.2585381	.9660011	1
60	0.2249511	0.9743701	0.2419219	0.9702957	0.2588190	0.9659258	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	77°		76°		75°		

TABLE 39. NATURAL SINES AND COSINES (*Continued*)

′	15°		16°		17°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	0.2588190	0.9659258	0.2756374	0.9612617	0.2923717	0.9563048	60
1	.2591000	.9658505	.2759170	.9611815	.2926499	.9562197	59
2	.2593810	.9657751	.2761965	.9611012	.2929280	.9561345	58
3	.2596619	.9656996	.2764761	.9610208	.2932061	.9560492	57
4	.2599428	.9656240	.2767556	.9609403	.2934842	.9559639	56
5	.2602237	.9655484	.2770352	.9608598	.2937623	.9558785	55
6	.2605045	.9654726	.2773147	.9607792	.2940403	.9557930	54
7	.2607853	.9653968	.2775941	.9606984	.2943183	.9557074	53
8	.2610662	.9653209	.2778736	.9606177	.2945963	.9556218	52
9	.2613469	.9652449	.2781530	.9605368	.2948743	.9555361	51
10	0.2616277	0.9651689	0.2784324	0.9604558	0.2951522	0.9554502	50
11	.2619085	.9650927	.2787118	.9603748	.2954302	.9553643	49
12	.2621892	.9650165	.2789911	.9602937	.2957081	.9552784	48
13	.2624699	.9649402	.2792704	.9602125	.2959859	.9551923	47
14	.2627506	.9648638	.2795497	.9601312	.2962638	.9551062	46
15	.2630312	.9647873	.2798290	.9600499	.2965416	.9550199	45
16	.2633118	.9647108	.2801083	.9599684	.2968194	.9549336	44
17	.2635925	.9646341	.2803875	.9598869	.2970971	.9548473	43
18	.2638730	.9645574	.2806667	.9598053	.2973749	.9547608	42
19	.2641536	.9644806	.2809459	.9597236	.2976526	.9546743	41
20	0.2644342	0.9644037	0.2812251	0.9596418	0.2979303	0.9545876	40
21	.2647147	.9643268	.2815042	.9595600	.2982079	.9545009	39
22	.2649952	.9642497	.2817833	.9594781	.2984856	.9544141	38
23	.2652757	.9641726	.2820624	.9593961	.2987632	.9543273	37
24	.2655561	.9640954	.2823415	.9593140	.2990408	.9542403	36
25	.2658366	.9640181	.2826205	.9592318	.2993184	.9541533	35
26	.2661170	.9639407	.2828995	.9591496	.2995959	.9540662	34
27	.2663973	.9638633	.2831785	.9590672	.2998734	.9539790	33
28	.2666777	.9637858	.2834575	.9589848	.3001509	.9538917	32
29	.2669581	.9637081	.2837364	.9589023	.3004284	.9538044	31
30	0.2672384	0.9636305	0.2840153	0.9588197	0.3007058	0.9537170	30
31	.2675187	.9635527	.2842942	.9587371	.3009832	.9536294	29
32	.2677989	.9634748	.2845731	.9586543	.3012606	.9535418	28
33	.2680792	.9633969	.2848520	.9585715	.3015380	.9534542	27
34	.2683594	.9633189	.2851308	.9584886	.3018153	.9533664	26
35	.2686396	.9632408	.2854096	.9584056	.3020926	.9532786	25
36	.2689198	.9631626	.2856884	.9583226	.3023699	.9531907	24
37	.2692000	.9630843	.2859671	.9582394	.3026471	.9531027	23
38	.2694801	.9630060	.2862458	.9581562	.3029244	.9530146	22
39	.2697602	.9629275	.2865246	.9580729	.3032016	.9529264	21
40	0.2700403	0.9628490	0.2868032	0.9579895	0.3034788	0.9528382	20
41	.2703204	.9627704	.2870819	.9579060	.3037559	.9527499	19
42	.2706004	.9626917	.2873605	.9578225	.3040331	.9526615	18
43	.2708805	.9626130	.2876391	.9577389	.3043102	.9525730	17
44	.2711605	.9625342	.2879177	.9576552	.3045872	.9524844	16
45	.2714404	.9624552	.2881963	.9575714	.3048643	.9523958	15
46	.2717204	.9623762	.2884748	.9574875	.3051413	.9523071	14
47	.2720003	.9622972	.2887533	.9574035	.3054183	.9522183	13
48	.2722802	.9622180	.2890318	.9573195	.3056953	.9521294	12
49	.2725601	.9621387	.2893103	.9572354	.3059723	.9520404	11
50	0.2728400	0.9620594	0.2895887	0.9571512	0.3062492	0.9519514	10
51	.2731198	.9619800	.2898671	.9570669	.3065261	.9518623	9
52	.2733997	.9619005	.2901455	.9569825	.3068030	.9517731	8
53	.2736794	.9618210	.2904239	.9568981	.3070798	.9516838	7
54	.2739592	.9617413	.2907022	.9568136	.3073566	.9515944	6
55	.2742390	.9616616	.2909805	.9567290	.3076334	.9515050	5
56	.2745187	.9615818	.2912588	.9566443	.3079102	.9514154	4
57	.2747984	.9615019	.2915371	.9565595	.3081869	.9513258	3
58	.2750781	.9614219	.2918153	.9564747	.3084636	.9512361	2
59	.2753577	.9613418	.2920935	.9563898	.3087403	.9511464	1
60	0.2756374	0.9612617	0.2923717	0.9563048	0.3090170	0.9510565	0
	Cosine	Sine	Cosine	Sine	Cosine	Sine	
′	74°		73°		72°		′

Table 39. Natural Sines and Cosines (*Continued*)

′	18°		19°		20°		′
	Sine	**Cosine**	**Sine**	**Cosine**	**Sine**	**Cosine**	
0	0.3090170	0.9510565	0.3255682	0.9455186	0.3420201	0.9396926	60
1	.3092936	.9509666	.3258432	.9454238	.3422935	.9395931	59
2	.3095702	.9508766	.3261182	.9453290	.3425668	.9394935	58
3	.3098468	.9507865	.3263932	.9452341	.3428400	.9393938	57
4	.3101234	.9506963	.3266681	.9451391	.3431133	.9392940	56
5	.3103999	.9506061	.3269430	.9450441	.3433865	.9391942	55
6	.3106764	.9505157	.3272179	.9449489	.3436597	.9390943	54
7	.3109529	.9504253	.3274928	.9448537	.3439329	.9389942	53
8	.3112294	.9503348	.3277676	.9447584	.3442060	.9388942	52
9	.3115058	.9502443	.3280424	.9446630	.3444791	.9387940	51
10	0.3117822	0.9501536	0.3283172	0.9445675	0.3447521	0.9386938	50
11	.3120586	.9500629	.3285919	.9444720	.3450252	.9385934	49
12	.3123349	.9499721	.3288666	.9443764	.3452982	.9384930	48
13	.3126112	.9498812	.3291413	.9442807	.3455712	.9383925	47
14	.3128875	.9497902	.3294160	.9441849	.3458441	.9382920	46
15	.3131638	.9496991	.3296906	.9440890	.3461171	.9381913	45
16	.3134400	.9496080	.3299653	.9439931	.3463900	.9380906	44
17	.3137163	.9495168	.3302398	.9438971	.3466628	.9379898	43
18	.3139925	.9494255	.3305144	.9438010	.3469357	.9378889	42
19	.3142686	.9493341	.3307889	.9437048	.3472085	.9377880	41
20	0.3145448	0.9492426	0.3310634	0.9436085	0.3474812	0.9376869	40
21	.3148209	.9491511	.3313379	.9435122	.3477540	.9375858	39
22	.3150969	.9490595	.3316123	.9434157	.3480267	.9374846	38
23	.3153730	.9489678	.3318867	.9433192	.3482994	.9373833	37
24	.3156490	.9488760	.3321611	.9432227	.3485720	.9372820	36
25	.3159250	.9487842	.3324355	.9431260	.3488447	.9371806	35
26	.3162010	.9486922	.3327098	.9430293	.3491173	.9370790	34
27	.3164770	.9486002	.3329841	.9429324	.3493898	.9369774	33
28	.3167529	.9485081	.3332584	.9428355	.3496624	.9368758	32
29	.3170288	.9484159	.3335326	.9427386	.3499349	.9367740	31
30	0.3173047	0.9483237	0.3338069	0.9426415	0.3502074	0.9366722	30
31	.3175805	.9482313	.3340810	.9425444	.3504798	.9365703	29
32	.3178563	.9481389	.3343552	.9424471	.3507523	.9364683	28
33	.3181321	.9430464	.3346293	.9423498	.3510246	.9363662	27
34	.3184079	.9479538	.3349034	.9422525	.3512970	.9362641	26
35	.3186836	.9478612	.3351775	.9421550	.3515693	.9361618	25
36	.3189593	.9477684	.3354516	.9420575	.3518416	.9360595	24
37	.3192350	.9476756	.3357256	.9419598	.3521139	.9359571	23
38	.3195106	.9475827	.3359996	.9418621	.3523862	.9358547	22
39	.3197863	9474897	.3362735	.9417644	.3526584	.9357521	21
40	0.3200619	0.9473966	0.3365475	0.9416665	0.3529306	0.9356495	20
41	.3203374	.9473035	.3368214	.9415686	.3532027	.9355468	19
42	.3206130	.9472103	.3370953	.9414705	.3534748	.9354440	18
43	.3208885	.9471170	.3373691	.9413724	.3537469	.9353412	17
44	.3211640	.9470236	.3376429	.9412743	.3540190	.9352382	16
45	.3214395	.9469301	.3379167	.9411760	.3542910	.9351352	15
46	.3217149	.9468366	.3381905	.9410777	.3545630	.9350321	14
47	.3219903	.9467430	.3384642	.9409793	.3548350	.9349289	13
48	.3222657	.9466493	.3387379	.9408808	.3551070	.9348257	12
49	.3225411	.9465555	.3390116	.9407822	.3553789	.9347223	11
50	0.3228164	0.9464616	0.3392852	0.9406835	0.3556508	0.9346189	10
51	.3230917	.9463677	.3395589	.9405848	.3559226	.9345154	9
52	.3233670	.9462736	.3398325	.9404860	.3561944	.9344119	8
53	.3236422	.9461795	.3401060	.9403871	.3564662	.9343082	7
54	.3239174	.9460854	.3403796	.9402881	.3567380	.9342045	6
55	.3241926	.9459911	.3406531	.9401891	.3570097	.9341007	5
56	.3244678	.9458968	.3409265	.9400899	.3572814	.9339968	4
57	.3247429	.9458023	.3412000	.9399907	.3575531	.9338928	3
58	.3250180	.9457078	.3414734	.9398914	.3578248	.9337888	2
59	.3252931	.9456132	.3417468	.9397921	.3580964	.9336846	1
60	0.3255682	0.9455186	0.3420201	0.9396926	0.3583679	0.9335804	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	71°		**70°**		**69°**		

TABLE 39. NATURAL SINES AND COSINES *(Continued)*

′	21°		22°		23°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	0.3583679	0.9335804	0.3746066	0.9271839	0.3907311	0.9205049	60
1	.3586395	.9334761	.3748763	.9270748	.3909989	.9203912	59
2	.3589110	.9333718	.3751459	.9269658	.3912666	.9202774	58
3	.3591825	.9332673	.3754156	.9268566	.3915343	.9201635	57
4	.3594540	.9331628	.3756852	.9267474	.3918019	.9200496	56
5	.3597254	.9330582	.3759547	.9266380	.3920695	.9199356	55
6	.3599968	.9329535	.3762243	.9265286	.3923371	.9198215	54
7	.3602682	.9328488	.3764938	.9264192	.3926047	.9197073	53
8	.3605395	.9327439	.3767632	.9263096	.3928722	.9195931	52
9	.3608108	.9326390	.3770327	.9262000	.3931397	.9194788	51
10	0.3610821	0.9325340	0.3773021	0.9260902	0.3934071	0.9193644	50
11	.3613534	.9324290	.3775714	.9259805	.3936745	.9192499	49
12	.3616246	.9323238	.3778408	.9258706	.3939419	.9191353	48
13	.3618958	.9322186	.3781101	.9257606	.3942093	.9190207	47
14	.3621669	.9321133	.3783794	.9256506	.3944766	.9189060	46
15	.3624380	.9320079	.3786486	.9255405	.3947439	.9187912	45
16	.3627091	.9319024	.3789178	.9254303	.3950111	.9186763	44
17	.3629802	.9317969	.3791870	.9253201	.3952783	.9185614	43
18	.3632512	.9316912	.3794562	.9252097	.3955455	.9184464	42
19	.3635222	.9315855	.3797253	.9250993	.3958127	.9183313	41
20	0.3637932	0.9314797	0.3799944	0.9249888	0.3960798	0.9182161	40
21	.3640641	.9313739	.3802634	.9248782	.3963468	.9181009	39
22	.3643351	.9312679	.3805324	.9247676	.3966139	.9179855	38
23	.3646059	.9311619	.3808014	.9246568	.3968809	.9178701	37
24	.3648768	.9310558	.3810704	.9245460	.3971479	.9177546	36
25	.3651476	.9309496	.3813393	.9244351	.3974148	.9176391	35
26	.3654184	.9308434	.3816082	.9243242	.3976818	.9175234	34
27	.3656891	.9307370	.3818770	.9242131	.3979486	.9174077	33
28	.3659599	.9306306	.3821459	.9241020	.3982155	.9172919	32
29	.3662306	.9305241	.3824147	.9239908	.3984823	.9171760	31
30	0.3665012	0.9304176	0.3826834	0.9238795	0.3987491	0.9170601	30
31	.3667719	.9303109	.3829522	.9237682	.3990158	.9169440	29
32	.3670425	.9302042	.3832209	.9236567	.3992825	.9168279	28
33	.3673130	.9300974	.3834895	.9235452	.3995492	.9167118	27
34	.3675836	.9299905	.3837582	.9234336	.3998158	.9165955	26
35	.3678541	.9298835	.3840268	.9233220	.4000825	.9164791	25
36	.3681246	.9297765	.3842953	.9232102	.4003490	.9163627	24
37	.3683950	.9296694	.3845639	.9230984	.4006156	.9162462	23
38	.3686654	.9295622	.3848324	.9229865	.4008821	.9161297	22
39	.3689358	.9294549	.3851008	.9228745	.4011486	.9160130	21
40	0.3692061	0.9293475	0.3853693	0.9227624	0.4014150	0.9158963	20
41	.3694765	.9292401	.3856377	.9226503	.4016814	.9157795	19
42	.3697468	.9291326	.3859060	.9225381	.4019478	.9156626	18
43	.3700170	.9290250	.3861744	.9224258	.4022141	.9155456	17
44	.3702872	.9289173	.3864427	.9223134	.4024804	.9154286	16
45	.3705574	.9288096	.3867110	.9222010	.4027467	.9153115	15
46	.3708276	.9287017	.3869792	.9220884	.4030129	.9151943	14
47	.3710977	.9285938	.3872474	.9219758	.4032791	.9150770	13
48	.3713678	.9284858	.3875156	.9218632	.4035453	.9149597	12
49	.3716379	.9283778	.3877837	.9217504	.4038114	.9148422	11
50	0.3719079	0.9282696	0.3880518	0.9216375	0.4040775	0.9147247	10
51	.3721780	.9281614	.3883199	.9215246	.4043436	.9146072	9
52	.3724479	.9280531	.3885880	.9214116	.4046096	.9144895	8
53	.3727179	.9279447	.3888560	.9212986	.4048756	.9143718	7
54	.3729878	.9278363	.3891240	.9211854	.4051416	.9142540	6
55	.3732577	.9277277	.3893919	.9210722	.4054075	.9141361	5
56	.3735275	.9276191	.3896598	.9209589	.4056734	.9140181	4
57	.3737973	.9275104	.3899277	.9208455	.4059393	.9139001	3
58	.3740671	.9274016	.3901955	.9207320	.4062051	.9137819	2
59	.3743369	.9272928	.3904633	.9206185	.4064709	.9136637	1
60	0.3746066	0.9271839	0.3907311	0.9205049	0.4067366	0.9135455	0
	Cosine	Sine	Cosine	Sine	Cosine	Sine	
′	68°		67°		66°		′

TABLE 39. NATURAL SINES AND COSINES (*Continued*)

′	24°		25°		26°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	0.4067366	0.9135455	0.4226183	0.9063078	0.4383711	0.8987940	60
1	.4070024	.9134271	.4228819	.9061848	.4386326	.8986665	59
2	.4072681	.9133087	.4231455	.9060618	.4388940	.8985389	58
3	.4075337	.9131902	.4234090	.9059386	.4391553	.8984112	57
4	.4077993	.9130716	.4236725	.9058154	.4394166	.8982834	56
5	.4080649	.9129529	.4239360	.9056922	.4396779	.8981555	55
6	.4083305	.9128342	.4241994	.9055688	.4399392	.8980276	54
7	.4085960	.9127154	.4244628	.9054454	.4402004	.8978996	53
8	.4088615	.9125965	.4247262	.9053219	.4404615	.8977715	52
9	.4091269	.9124775	.4249895	.9051983	.4407227	.8976433	51
10	0.4093923	0.9123584	0.4252528	0.9050746	0.4409838	0.8975151	50
11	.4096577	.9122393	.4255161	.9049509	.4412448	.8973868	49
12	.4099230	.9121201	.4257793	.9048271	.4415059	.8972584	48
13	.4101883	.9120008	.4260425	.9047032	.4417668	.8971299	47
14	.4104536	.9118815	.4263056	.9045792	.4420278	.8970014	46
15	.4107189	.9117620	.4265687	.9044551	.4422887	.8968727	45
16	.4109841	.9116425	.4268318	.9043310	.4425496	.8967440	44
17	.4112492	.9115229	.4270949	.9042068	.4428104	.8966153	43
18	.4115144	.9114033	.4273579	.9040825	.4430712	.8964864	42
19	.4117795	.9112835	.4276208	.9039582	.4433319	.8963575	41
20	0.4120445	0.9111637	0.4278838	0.9038338	0.4435927	0.8962285	40
21	.4123096	.9110438	.4281467	.9037093	.4438534	.8960994	39
22	.4125745	.9109238	.4284095	.9035847	.4441140	.8959703	38
23	.4128395	.9108038	.4286723	.9034600	.4443746	.8958411	37
24	.4131044	.9106837	.4289351	.9033353	.4446352	.8957118	36
25	.4133693	.9105635	.4291979	.9032105	.4448957	.8955824	35
26	.4136342	.9104432	.4294606	.9030856	.4451562	.8954529	34
27	.4138990	.9103228	.4297233	.9029606	.4454167	.8953234	33
28	.4141638	.9102024	.4299859	.9028356	.4456771	.8951938	32
29	.4144285	.9100819	.4302485	.9027105	.4459375	.8950641	31
30	0.4146932	0.9099613	0.4305111	0.9025853	0.4461978	0.8949344	30
31	.4149579	.9098406	.4307736	.9024600	.4464581	.8948045	29
32	.4152226	.9097199	.4310361	.9023347	.4467184	.8946746	28
33	.4154872	.9095990	.4312986	.9022092	.4469786	.8945446	27
34	.4157517	.9094781	.4315610	.9020838	.4472388	.8944146	26
35	.4160163	.9093572	.4318234	.9019582	.4474990	.8942844	25
36	.4162808	.9092361	.4320857	.9018325	.4477591	.8941542	24
37	.4165453	.9091150	.4323481	.9017068	.4480192	.8940240	23
38	.4168097	.9089938	.4326103	.9015810	.4482792	.8938936	22
39	.4170741	.9088725	.4328726	.9014551	.4485392	.8937632	21
40	0.4173385	0.9087511	0.4331348	0.9013292	0.4487992	0.8936326	20
41	.4176028	.9086297	.4333970	.9012031	.4490591	.8935021	19
42	.4178671	.9085082	.4336591	.9010770	.4493190	.8933714	18
43	.4181313	.9083866	.4339212	.9009508	.4495789	.8932406	17
44	.4183956	.9082649	.4341832	.9008246	.4498387	.8931098	16
45	.4186597	.9081432	.4344453	.9006982	.4500984	.8929789	15
46	.4189239	.9080214	.4347072	.9005718	.4503582	.8928480	14
47	.4191880	.9078995	.4349692	.9004453	.4506179	.8927169	13
48	.4194521	.9077775	.4352311	.9003188	.4508775	.8925858	12
49	.4197161	.9076554	.4354930	.9001921	.4511372	.8924546	11
50	0.4199801	0.9075333	0.4357548	0.9000654	0.4513967	0.8923234	10
51	.4202441	.9074111	.4360166	.8999386	.4516563	.8921920	9
52	.4205080	.9072888	.4362784	.8998117	.4519158	.8920606	8
53	.4207719	.9071665	.4365401	.8996848	.4521753	.8919291	7
54	.4210358	.9070440	.4368018	.8995578	.4524347	.8917975	6
55	.4212996	.9069215	.4370634	.8994307	.4526941	.8916659	5
56	.4215634	.9067989	.4373251	.8993035	.4529535	.8915342	4
57	.4218272	.9066762	.4375866	.8991763	.4532128	.8914024	3
58	.4220909	.9065535	.4378482	.8990489	.4534721	.8912705	2
59	.4223546	.9064307	.4381097	.8989215	.4537313	.8911385	1
60	0.4226183	0.9063078	0.4383711	0.8987940	0.4539905	0.8910065	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	65°		64°		63°		

TABLE 39. NATURAL SINES AND COSINES (*Continued*)

′	27°		28°		29°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	0.4539905	0.8910065	0.4694716	0.8829476	0.4848096	0.8746197	60
1	.4542497	.8908744	.4697284	.8828110	.4850640	.8744786	59
2	.4545088	.8907423	.4699852	.8826743	.4853184	.8743375	58
3	.4547679	.8906100	.4702419	.8825376	.4855727	.8741963	57
4	.4550269	.8904777	.4704986	.8824007	.4858270	.8740550	56
5	.4552859	.8903453	.4707553	.8822638	.4860812	.8739137	55
6	.4555449	.8902128	.4710119	.8821269	.4863354	.8737722	54
7	.4558038	.8900803	.4712685	.8819898	.4865895	.8736307	53
8	.4560627	.8899476	.4715250	.8818527	.4868436	.8734891	52
9	.4563216	.8898149	.4717815	.8817155	.4870977	.8733475	51
10	0.4565804	0.8896822	0.4720380	0.8815782	0.4873517	0.8732058	50
11	.4568392	.8895493	.4722944	.8814409	.4876057	.8730640	49
12	.4570979	.8894164	.4725508	.8813035	.4878597	.8729221	48
13	.4573566	.8892834	.4728071	.8811660	.4881136	.8727801	47
14	.4576153	.8891503	.4730634	.8810284	.4883674	.8726381	46
15	.4578739	.8890171	.4733197	.8808907	.4886212	.8724960	45
16	.4581325	.8888839	.4735759	.8807530	.4888750	.8723538	44
17	.4583910	.8887506	.4738321	.8806152	.4891288	.8722116	43
18	.4586496	.8886172	.4740882	.8804774	.4893825	.8720693	42
19	.4589080	.8884838	.4743443	.8803394	.4896361	.8719269	41
20	0.4591665	0.8883503	0.4746004	0.8802014	0.4898897	0.8717844	40
21	.4594248	.8882166	.4748564	.8800633	.4901433	.8716419	39
22	.4596832	.8880830	.4751124	.8799251	.4903968	.8714993	38
23	.4599415	.8879492	.4753683	.8797869	.4906503	.8713566	37
24	.4601998	.8878154	.4756242	.8796486	.4909038	.8712138	36
25	.4604580	.8876815	.4758801	.8795102	.4911572	.8710710	35
26	.4607162	.8875475	.4761359	.8793717	.4914105	.8709281	34
27	.4609744	.8874134	.4763917	.8792332	.4916638	.8707851	33
28	.4612325	.8872793	.4766474	.8790946	.4919171	.8706420	32
29	.4614906	.8871451	.4769031	.8789559	.4921704	.8704989	31
30	0.4617486	0.8870108	0.4771588	0.8788171	0.4924236	0.8703557	30
31	.4620066	.8868765	.4774144	.8786783	.4926767	.8702124	29
32	.4622646	.8867420	.4776700	.8785394	.4929298	.8700691	28
33	.4625225	.8866075	.4779255	.8784004	.4931829	.8699256	27
34	.4627804	.8864730	.4781810	.8782613	.4934359	.8697821	26
35	.4630382	.8863383	.4784364	.8781222	.4936889	.8696386	25
36	.4632960	.8862036	.4786919	.8779830	.4939419	.8694949	24
37	.4635538	.8860688	.4789472	.8778437	.4941948	.8693512	23
38	.4638115	.8859339	.4792026	.8777043	.4944476	.8692074	22
39	.4640692	.8857989	.4794579	.8775649	.4947005	.8690636	21
40	0.4643269	0.8856639	0.4797131	0.8774254	0.4949532	0.8689196	20
41	.4645845	.8855288	.4799683	.8772858	.4952060	.8687756	19
42	.4648420	.8853936	.4802235	.8771462	.4954587	.8686315	18
43	.4650996	.8852584	.4804786	.8770064	.4957113	.8684874	17
44	.4653571	.8851230	.4807337	.8768666	.4959639	.8683431	16
45	.4656145	.8849876	.4809888	.8767268	.4962165	.8681988	15
46	.4658719	.8848522	.4812438	.8765868	.4964690	.8680544	14
47	.4661293	.8847166	.4814987	.8764468	.4967215	.8679100	13
48	.4663866	.8845810	.4817537	.8763067	.4969740	.8677655	12
49	.4666439	.8844453	.4820086	.8761665	.4972264	.8676209	11
50	0.4669012	0.8843095	0.4822634	0.8760263	0.4974787	0.8674762	10
51	.4671584	.8841736	.4825182	.8758859	.4977310	.8673314	9
52	.4674156	.8840377	.4827730	.8757455	.4979833	.8671866	8
53	.4676727	.8839017	.4830277	.8756051	.4982355	.8670417	7
54	.4679298	.8837656	.4832824	.8754645	.4984877	.8668967	6
55	.4681869	.8836295	.4835370	.8753239	.4987399	.8667517	5
56	.4684439	.8834933	.4837916	.8751832	.4989920	.8666066	4
57	.4687009	.8833569	.4840462	.8750425	.4992441	.8664614	3
58	.4689578	.8832206	.4843007	.8749016	.4994961	.8663161	2
59	.4692147	.8830841	.4845552	.8747607	.4997481	.8661708	1
60	0.4694716	0.8829476	0.4848096	0.8746197	0.5000000	0.8660254	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	62°		61°		60°		

TABLE 39. NATURAL SINES AND COSINES (*Continued*)

′	30°		31°		32°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	0.5000000	0.8660254	0.5150381	0.8571673	0.5299193	0.8480481	60
1	.5002519	.8658799	.5152874	.8570174	.5301659	.8478939	59
2	.5005037	.8657344	.5155367	.8568675	.5304125	.8477397	58
3	.5007556	.8655887	.5157859	.8567175	.5306591	.8475853	57
4	.5010073	.8654430	.5160351	.8565674	.5309057	.8474309	56
5	.5012591	.8652973	.5162842	.8564173	.5311521	.8472765	55
6	.5015107	.8651514	.5165333	.8562671	.5313986	.8471219	54
7	.5017624	.8650055	.5167824	.8561168	.5316450	.8469673	53
8	.5020140	.8648595	.5170314	.8559664	.5318913	.8468126	52
9	.5022655	.8647134	.5172804	.8558160	.5321376	.8466579	51
10	0.5025170	0.8645673	0.5175293	0.8556655	0.5323839	0.8465030	50
11	.5027685	.8644211	.5177782	.8555149	.5326301	.8463481	49
12	.5030199	.8642748	.5180270	.8553643	.5328763	.8461932	48
13	.5032713	.8641284	.5182758	.8552135	.5331224	.8460381	47
14	.5035227	.8639820	.5185246	.8550627	.5333685	.8458830	46
15	.5037740	.8638355	.5187733	.8549119	.5336145	.8457278	45
16	.5040252	.8636889	.5190219	.8547609	.5338605	.8455726	44
17	.5042765	.8635423	.5192705	.8546099	.5341065	.8454172	43
18	.5045276	.8633956	.5195191	.8544588	.5343523	.8452618	42
19	.5047788	.8632488	.5197676	.8543077	.5345982	.8451064	41
20	0.5050298	0.8631019	0.5200161	0.8541564	0.5348440	0.8449508	40
21	.5052809	.8629549	.5202646	.8540051	.5350898	.8447952	39
22	.5055319	.8628079	.5205130	.8538538	.5353355	.8446395	38
23	.5057828	.8626608	.5207613	.8537023	.5355812	.8444838	37
24	.5060338	.8625137	.5210096	.8535508	.5358268	.8443279	36
25	.5062846	.8623664	.5212579	.8533992	.5360724	.8441720	35
26	.5065355	.8622191	.5215061	.8532475	.5363179	.8440161	34
27	.5067863	.8620717	.5217543	.8530958	.5365634	.8438600	33
28	.5070370	.8619243	.5220024	.8529440	.5368089	.8437039	32
29	.5072877	.8617768	.5222505	.8527921	.5370543	.8435477	31
30	0.5075384	0.8616292	0.5224986	0.8526402	0.5372996	0.8433914	30
31	.5077890	.8614815	.5227466	.8524881	.5375449	.8432351	29
32	.5080396	.8613337	.5229945	.8523360	.5377902	.8430787	28
33	.5082901	.8611859	.5232424	.8521839	.5380354	.8429222	27
34	.5085406	.8610380	.5234903	.8520316	.5382806	.8427657	26
35	.5087910	.8608901	.5237381	.8518793	.5385257	.8426091	25
36	.5090414	.8607420	.5239859	.8517269	.5387708	.8424524	24
37	.5092918	.8605939	.5242336	.8515745	.5390158	.8422956	23
38	.5095421	.8604457	.5244813	.8514219	.5392608	.8421388	22
39	.5097924	.8602975	.5247290	.8512693	.5395058	.8419818	21
40	0.5100426	0.8601491	0.5249766	0.8511167	0.5397507	0.8418249	20
41	.5102928	.8600007	.5252241	.8509639	.5399955	.8416679	19
42	.5105429	.8598523	.5254717	.8508111	.5402403	.8415108	18
43	.5107930	.8597037	.5257191	.8506582	.5404851	.8413536	17
44	.5110431	.8595551	.5259665	.8505053	.5407298	.8411963	16
45	.5112931	.8594064	.5262139	.8503522	.5409745	.8410390	15
46	.5115431	.8592576	.5264613	.8501991	.5412191	.8408816	14
47	.5117930	.8591088	.5267085	.8500459	.5414637	.8407241	13
48	.5120429	.8589599	.5269558	.8498927	.5417082	.8405666	12
49	.5122927	.8588109	.5272030	.8497394	.5419527	.8404090	11
50	0.5125425	0.8586619	0.5274502	0.8495860	0.5421971	0.8402513	10
51	.5127923	.8585127	.5276973	.8494325	.5424415	.8400936	9
52	.5130420	.8583635	.5279443	.8492790	.5426859	.8399357	8
53	.5132916	.8582143	.5281914	.8491254	.5429302	.8397778	7
54	.5135413	.8580649	.5284383	.8489717	.5431744	.8396199	6
55	.5137908	.8579155	.5286853	.8488179	.5434187	.8394618	5
56	.5140404	.8577660	.5289322	.8486641	.5436628	.8393037	4
57	.5142899	.8576164	.5291790	.8485102	.5439069	.8391455	3
58	.5145393	.8574668	.5294258	.8483562	.5441510	.8389873	2
59	.5147887	.8573171	.5296726	.8482022	.5443951	.8388290	1
60	0.5150381	0.8571673	0.5299193	0.8480481	0.5446390	0.8386706	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	59°		58°		57°		

TABLE 39. NATURAL SINES AND COSINES (*Continued*)

′	33°		34°		35°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	0.5446390	0.8386706	0.5591929	0.8290376	0.5735764	0.8191520	60
1	.5448830	.8385121	.5594340	.8288749	.5738147	.8189852	59
2	.5451269	.8383536	.5596751	.8287121	.5740529	.8188182	58
3	.5453707	.8381950	.5599162	.8285493	.5742911	.8186512	57
4	.5456145	.8380363	.5601572	.8283864	.5745292	.8184841	56
5	.5458583	.8378775	.5603981	.8282234	.5747672	.8183169	55
6	.5461020	.8377187	.5606390	.8280603	.5750053	.8181497	54
7	.5463456	.8375598	.5608798	.8278972	.5752432	.8179824	53
8	.5465892	.8374009	.5611206	.8277340	.5754811	.8178151	52
9	.5468328	.8372418	.5613614	.8275708	.5757190	.8176476	51
10	0.5470763	0.8370827	0.5616021	0.8274074	0.5759568	0.8174801	50
11	.5473198	.8369236	.5618428	.8272440	.5761946	.8173125	49
12	.5475632	.8367643	.5620834	.8270806	.5764323	.8171449	48
13	.5478066	.8366050	.5623239	.8269170	.5766700	.8169772	47
14	.5480499	.8364456	.5625645	.8267534	.5769076	.8168094	46
15	.5482932	.8362862	.5628049	.8265897	.5771452	.8166416	45
16	.5485365	.8361266	.5630453	.8264260	.5773827	.8164736	44
17	.5487797	.8359670	.5632857	.8262622	.5776202	.8163056	43
18	.5490228	.8358074	.5635260	.8260983	.5778576	.8161376	42
19	.5492659	.8356476	.5637663	.8259343	.5780950	.8159695	41
20	0.5495090	0.8354878	0.5640066	0.8257703	0.5783323	0.8158013	40
21	.5497520	.8353279	.5642467	.8256062	.5785696	.8156330	39
22	.5499950	.8351680	.5644869	.8254420	.5788069	.8154647	38
23	.5502379	.8350080	.5647270	.8252778	.5790440	.8152963	37
24	.5504807	.8348479	.5649670	.8251135	.5792812	.8151278	36
25	.5507236	.8346877	.5652070	.8249491	.5795183	.8149593	35
26	.5509663	.8345275	.5654469	.8247847	.5797553	.8147906	34
27	.5512091	.8343672	.5656868	.8246202	.5799923	.8146220	33
28	.5514518	.8342068	.5659267	.8244556	.5802292	.8144532	32
29	.5516944	.8340463	.5661665	.8242909	.5804661	.8142844	31
30	0.5519370	0.8338858	0.5664062	0.8241262	0.5807030	0.8141155	30
31	.5521795	.8337252	.5666459	.8239614	.5809397	.8139466	29
32	.5524220	.8335646	.5668856	.8237965	.5811765	.8137775	28
33	.5526645	.8334038	.5671252	.8236316	.5814132	.8136084	27
34	.5529069	.8332430	.5673648	.8234666	.5816498	.8134393	26
35	.5531492	.8330822	.5676043	.8233015	.5818864	.8132701	25
36	.5533915	.8329212	.5678437	.8231364	.5821230	.8131008	24
37	.5536338	.8327602	.5680832	.8229712	.5823595	.8129314	23
38	.5538760	.8325991	.5683225	.8228059	.5825959	.8127620	22
39	.5541182	.8324380	.5685619	.8226405	.5828323	.8125925	21
40	0.5543603	0.8322768	0.5688011	0.8224751	0.5830687	0.8124229	20
41	.5546024	.8321155	.5690403	.8223096	.5833050	.8122532	19
42	.5548444	.8319541	.5692795	.8221440	.5835412	.8120835	18
43	.5550864	.8317927	.5695187	.8219784	.5837774	.8119137	17
44	.5553283	.8316312	.5697577	.8218127	.5840136	.8117439	16
45	.5555702	.8314696	.5699968	.8216469	.5842497	.8115740	15
46	.5558121	.8313080	.5702357	.8214811	.5844857	.8114040	14
47	.5560539	.8311463	.5704747	.8213152	.5847217	.8112339	13
48	.5562956	.8309845	.5707136	.8211492	.5849577	.8110638	12
49	.5565373	.8308226	.5709524	.8209832	.5851936	.8108936	11
50	0.5567790	0.8306607	0.5711912	0.8208170	0.5854294	0.8107234	10
51	.5570206	.8304987	.5714299	.8206509	.5856652	.8105530	9
52	.5572621	.8303366	.5716686	.8204846	.5859010	.8103826	8
53	.5575036	.8301745	.5719073	.8203183	.5861367	.8102122	7
54	.5577451	.8300123	.5721459	.8201519	.5863724	.8100416	6
55	.5579865	.8298500	.5723844	.8199854	.5866080	.8098710	5
56	.5582279	.8296877	.5726229	.8198189	.5868435	.8097004	4
57	.5584692	.8295252	.5728614	.8196523	.5870790	.8095296	3
58	.5587105	.8293628	.5730998	.8194856	.5873145	.8093588	2
59	.5589517	.8292002	.5733381	.8193189	.5875499	.8091879	1
60	0.5591929	0.8290376	0.5735764	0.8191520	0.5877853	0.8090170	0
	Cosine	Sine	Cosine	Sine	Cosine	Sine	
′	56°		55°		54°		′

TABLE 39. NATURAL SINES AND COSINES (*Continued*)

′	36°		37°		38°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	0.5877853	0.8090170	0.6018150	0.7986355	0.6156615	0.7880108	60
1	.5880206	.8088460	.6020473	.7984604	.6158907	.7878316	59
2	.5882558	.8086749	.6022795	.7982853	.6161198	.7876524	58
3	.5884910	.8085037	.6025117	.7981100	.6163489	.7874732	57
4	.5887262	.8083325	.6027439	.7979347	.6165780	.7872939	56
5	.5889613	.8081612	.6029760	.7977594	.6168069	.7871145	55
6	.5891964	.8079899	.6032080	.7975839	.6170359	.7869350	54
7	.5894314	.8078185	.6034400	.7974084	.6172648	.7867555	53
8	.5896663	.8076470	.6036719	.7972329	.6174936	.7865759	52
9	.5899012	.8074754	.6039038	.7970572	.6177224	.7863963	51
10	0.5901361	0.8073038	0.6041356	0.7968815	0.6179511	0.7862165	50
11	.5903709	.8071321	.6043674	.7967058	.6181798	.7860367	49
12	.5906057	.8069603	.6045991	.7965299	.6184084	.7858569	48
13	.5908404	.8067885	.6048303	.7963540	.6186370	.7856770	47
14	.5910750	.8066166	.6050624	.7961780	.6188655	.7854970	46
15	.5913096	.8064446	.6052940	.7960020	.6190939	.7853169	45
16	.5915442	.8062726	.6055255	.7958259	.6193224	.7851368	44
17	.5917787	.8061005	.6057570	.7956497	.6195507	.7849566	43
18	.5920132	.8059283	.6059884	.7954735	.6197790	.7847764	42
19	.5922476	.8057560	.6062198	.7952972	.6200073	.7845961	41
20	0.5924819	0.8055837	0.6064511	0.7951208	0.6202355	0.7844157	40
21	.5927163	.8054113	.6066824	.7949444	.6204636	.7842352	39
22	.5929505	.8052389	.6069136	.7947678	.6206917	.7840547	38
23	.5931847	.8050664	.6071447	.7945913	.6209198	.7838741	37
24	.5934189	.8048938	.6073758	.7944146	.6211478	.7836935	36
25	.5936530	.8047211	.6076069	.7942379	.6213757	.7835127	35
26	.5938871	.8045484	.6078379	.7940611	.6216036	.7833320	34
27	.5941211	.8043756	.6080689	.7938843	.6218314	.7831511	33
28	.5943550	.8042028	.6082998	.7937074	.6220592	.7829702	32
29	.5945889	.8040299	.6085306	.7935304	.6222870	.7827892	31
30	0.5948228	0.8038569	0.6087614	0.7933533	0.6225146	0.7826082	30
31	.5950566	.8036838	.6089922	.7931762	.6227423	.7824270	29
32	.5952904	.8035107	.6092229	.7929990	.6229698	.7822459	28
33	.5955241	.8033375	.6094535	.7928218	.6231974	.7820646	27
34	.5957577	.8031642	.6096841	.7926445	.6234248	.7818833	26
35	.5959913	.8029909	.6099147	.7924671	.6236522	.7817019	25
36	.5962249	.8028175	.6101452	.7922896	.6238796	.7815205	24
37	.5964584	.8026440	.6103756	.7921121	.6241069	.7813390	23
38	.5966918	.8024705	.6106060	.7919345	.6243342	.7811574	22
39	.5969252	.8022969	.6108363	.7917569	.6245614	.7809757	21
40	0.5971586	0.8021232	0.6110666	0.7915792	0.6247885	0.7807940	20
41	.5973919	.8019495	.6112969	.7914014	.6250156	.7806123	19
42	.5976251	.8017756	.6115270	.7912235	.6252427	.7804304	18
43	.5978583	.8016018	.6117572	.7910456	.6254696	.7802485	17
44	.5980915	.8014278	.6119873	.7908676	.6256966	.7800665	16
45	.5983246	.8012538	.6122173	.7906896	.6259235	.7798845	15
46	.5985577	.8010797	.6124473	.7905115	.6261503	.7797024	14
47	.5987906	.8009056	.6126772	.7903333	.6263771	.7795202	13
48	.5990236	.8007314	.6129071	.7901550	.6266038	.7793380	12
49	.5992565	.8005571	.6131369	.7899767	.6268305	.7791557	11
50	0.5994893	0.8003827	0.6133666	0.7897983	0.6270571	0.7789733	10
51	.5997221	.8002083	.6135964	.7896198	.6272837	.7787909	9
52	.5999549	.8000338	.6138260	.7894413	.6275102	.7786084	8
53	.6001876	.7998593	.6140556	.7892627	.6277366	.7784258	7
54	.6004202	.7996847	.6142852	.7890841	.6279631	.7782431	6
55	.6006528	.7995100	.6145147	.7889054	.6281894	.7780604	5
56	.6008854	.7993352	.6147442	.7887266	.6284157	.7778777	4
57	.6011179	.7991604	.6149736	.7885477	.6286420	.7776949	3
58	.6013503	.7989855	.6152029	.7883688	.6288682	.7775120	2
59	.6015827	.7988105	.6154322	.7881898	.6290943	.7773290	1
60	0.6018150	0.7986355	0.6156615	0.7880108	0.6293204	0.7771460	0
	Cosine	Sine	Cosine	Sine	Cosine	Sine	
′	53°		52°		51°		′

TABLE 39. NATURAL SINES AND COSINES (*Continued*)

′	39°		40°		41°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	0.6293204	0.7771460	0.6427876	0.7660444	0.6560590	0.7547096	60
1	.6295464	.7769629	.6430104	.7658574	.6562785	.7545187	59
2	.6297724	.7767797	.6432332	.7656704	.6564980	.7543278	58
3	.6299983	.7765965	.6434559	.7654832	.6567174	.7541368	57
4	.6302242	.7764132	.6436785	.7652960	.6569367	.7539457	56
5	.6304500	.7762298	.6439011	.7651087	.6571560	.7537546	55
6	.6306758	.7760464	.6441236	.7649214	.6573752	.7535634	54
7	.6309015	.7758629	.6443461	.7647340	.6575944	.7533721	53
8	.6311272	.7756794	.6445685	.7645465	.6578135	.7531808	52
9	.6313528	.7754957	.6447909	.7643590	.6580326	.7529894	51
10	0.6315784	0.7753121	0.6450132	0.7641714	0.6582516	0.7527980	50
11	.6318039	.7751283	.6452355	.7639838	.6584706	.7526065	49
12	.6320293	.7749445	.6454577	.7637960	.6586895	.7524149	48
13	.6322547	.7747606	.6456798	.7636082	.6589083	.7522233	47
14	.6324800	.7745767	.6459019	.7634204	.6591271	.7520316	46
15	.6327053	.7743926	.6461240	.7632325	.6593458	.7518398	45
16	.6329306	.7742086	.6463460	.7630445	.6595645	.7516480	44
17	.6331557	.7740244	.6465679	.7628564	.6597831	.7514561	43
18	.6333809	.7738402	.6467898	.7626683	.6600017	.7512641	42
19	.6336059	.7736559	.6470116	.7624802	.6602202	.7510721	41
20	0.6338310	0.7734716	0.6472334	0.7622919	0.6604386	0.7508800	40
21	.6340559	.7732872	.6474551	.7621036	.6606570	.7506879	39
22	.6342808	.7731027	.6476767	.7619152	.6608754	.7504957	38
23	.6345057	.7729182	.6478984	.7617268	.6610936	.7503034	37
24	.6347305	.7727336	.6481199	.7615383	.6613119	.7501111	36
25	.6349553	.7725489	.6483414	.7613497	.6615300	.7499187	35
26	.6351800	.7723642	.6485628	.7611611	.6617482	.7497262	34
27	.6354046	.7721794	.6487842	.7609724	.6619662	.7495337	33
28	.6356292	.7719945	.6490056	.7607837	.6621842	.7493411	32
29	.6358537	.7718096	.6492268	.7605949	.6624022	.7491484	31
30	0.6360782	0.7716246	0.6494480	0.7604060	0.6626200	0.7489557	30
31	.6363026	.7714395	.6496692	.7602170	.6628379	.7487629	29
32	.6365270	.7712544	.6498903	.7600280	.6630557	.7485701	28
33	.6367513	.7710692	.6501114	.7598389	.6632734	.7483772	27
34	.6369756	.7708840	.6503324	.7596498	.6634910	.7481842	26
35	.6371998	.7706986	.6505533	.7594606	.6637087	.7479912	25
36	.6374240	.7705132	.6507742	.7592713	.6639262	.7477981	24
37	.6376481	.7703278	.6509951	.7590820	.6641437	.7476049	23
38	.6378721	.7701423	.6512158	.7588926	.6643612	.7474117	22
39	.6380961	.7699567	.6514366	.7587031	.6645785	.7472184	21
40	0.6383201	0.7697710	0.6516572	0.7585136	0.6647959	0.7470251	20
41	.6385440	.7695853	.6518778	.7583240	.6650131	.7468317	19
42	.6387678	.7693996	.6520984	.7581343	.6652304	.7466382	18
43	.6389916	.7692137	.6523189	.7579446	.6654475	.7464446	17
44	.6392153	.7690278	.6525394	.7577548	.6656646	.7462510	16
45	.6394390	.7688418	.6527598	.7575650	.6658817	.7460574	15
46	.6396626	.7686558	.6529801	.7573751	.6660987	.7458636	14
47	.6398862	.7684697	.6532004	.7571851	.6663156	.7456699	13
48	.6401097	.7682835	.6534206	.7569951	.6665325	.7454760	12
49	.6403332	.7680973	.6536408	.7568050	.6667493	.7452821	11
50	0.6405566	0.7679110	0.6538609	0.7566148	0.6669661	0.7450881	10
51	.6407799	.7677246	.6540810	.7564246	.6671828	.7448941	9
52	.6410032	.7675382	.6543010	.7562343	.6673994	.7446999	8
53	.6412264	.7673517	.6545209	.7560439	.6676160	.7445058	7
54	.6414496	.7671652	.6547408	.7558535	.6678326	.7443115	6
55	.6416728	.7669785	.6549607	.7556630	.6680490	.7441173	5
56	.6418958	.7667918	.6551804	.7554724	.6682655	.7439229	4
57	.6421189	.7666051	.6554002	.7552818	.6684818	.7437285	3
58	.6423418	.7664183	.6556198	.7550911	.6686981	.7435340	2
59	.6425647	.7662314	.6558395	.7549004	.6689144	.7433394	1
60	0.6427876	0.7660444	0.6560590	0.7547096	0.6691306	0.7431448	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	50°		49°		48°		

TABLE 39. NATURAL SINES AND COSINES (*Continued*)

′	42°		43°		44°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	0.6691306	0.7431448	0.6819984	0.7313537	0.6946584	0.7193398	60
1	.6693468	.7429502	.6822111	.7311553	.6948676	.7191377	59
2	.6695628	.7427554	.6824237	.7309568	.6950767	.7189355	58
3	.6697789	.7425606	.6826363	.7307583	.6952858	.7187333	57
4	.6699948	.7423658	.6828489	.7305597	.6954949	.7185310	56
5	.6702108	.7421708	.6830613	.7303610	.6957039	.7183287	55
6	.6704266	.7419758	.6832738	.7301623	.6959128	.7181263	54
7	.6706424	.7417808	.6834861	.7299635	.6961217	.7179238	53
8	.6708582	.7415857	.6836984	.7297646	.6963305	.7177213	52
9	.6710739	.7413905	.6839107	.7295657	.6965392	.7175187	51
10	0.6712895	0.7411953	0.6841229	0.7293668	0.6967479	0.7173161	50
11	.6715051	.7410000	.6843350	.7291677	.6969565	.7171134	49
12	.6717206	.7408046	.6845471	.7289686	.6971651	.7169106	48
13	.6719361	.7406092	.6847591	.7287695	.6973736	.7167078	47
14	.6721515	.7404137	.6849711	.7285703	.6975821	.7165049	46
15	.6723668	.7402181	.6851830	.7283710	.6977905	.7163019	45
16	.6725821	.7400225	.6853948	.7281716	.6979988	.7160989	44
17	.6727973	.7398268	.6856066	.7279722	.6982071	.7158959	43
18	.6730125	.7396311	.6858184	.7277728	.6984153	.7156927	42
19	.6732276	.7394353	.6860300	.7275732	.6986234	.7154895	41
20	0.6734427	0.7392394	0.6862416	0.7273736	0.6988315	0.7152863	40
21	.6736577	.7390435	.6864532	.7271740	.6990396	.7150830	39
22	.6738727	.7388475	.6866647	.7269743	.6992476	.7148796	38
23	.6740876	.7386515	.6868761	.7267745	.6994555	.7146762	37
24	.6743024	.7384553	.6870875	.7265747	.6996633	.7144727	36
25	.6745172	.7382592	.6872988	.7263748	.6998711	.7142691	35
26	.6747319	.7380629	.6875101	.7261748	.7000789	.7140655	34
27	.6749466	.7378666	.6877213	.7259748	.7002866	.7138618	33
28	.6751612	.7376703	.6879325	.7257747	.7004942	.7136581	32
29	.6753757	.7374738	.6881435	.7255746	.7007018	.7134543	31
30	0.6755902	0.7372773	0.6883546	0.7253744	0.7009093	0.7132504	30
31	.6758046	.7370808	.6885655	.7251741	.7011167	.7130465	29
32	.6760190	.7368842	.6887765	.7249738	.7013241	.7128426	28
33	.6762333	.7366875	.6889873	.7247734	.7015314	.7126385	27
34	.6764476	.7364908	.6891981	.7245729	.7017387	.7124344	26
35	.6766618	.7362940	.6894089	.7243724	.7019459	.7122303	25
36	.6768760	.7360971	.6896195	.7241719	.7021531	.7120260	24
37	.6770901	.7359002	.6898302	.7239712	.7023601	.7118218	23
38	.6773041	.7357032	.6900407	.7237705	.7025672	.7116174	22
39	.6775181	.7355061	.6902512	.7235698	.7027741	.7114130	21
40	0.6777320	0.7353090	0.6904617	0.7233690	0.7029811	0.7112086	20
41	.6779459	.7351118	.6906721	.7231681	.7031879	.7110041	19
42	6781597	.7349146	.6908824	.7229671	.7033947	.7107995	18
43	.6783734	.7347173	.6910927	.7227661	.7036014	.7105948	17
44	.6785871	.7345199	.6913029	.7225651	.7038081	.7103901	16
45	.6788007	.7343225	.6915131	.7223640	.7040147	.7101854	15
46	.6790143	.7341250	.6917232	.7221628	.7042213	.7099806	14
47	.6792278	.7339275	.6919332	.7219615	.7044278	.7097757	13
48	.6794413	.7337299	.6921432	.7217602	.7046342	.7095707	12
49	.6796547	.7335322	.6923531	.7215589	.7048406	.7093657	11
50	0.6798681	0.7333345	0.6925630	0.7213574	0.7050469	0.7091607	10
51	.6800813	.7331367	.6927728	.7211559	.7052532	.7089556	9
52	.6802946	.7329388	.6929825	.7209544	.7054594	.7087504	8
53	.6805078	.7327409	.6931922	.7207528	.7056655	.7085451	7
54	.6807209	.7325429	.6934018	.7205511	.7058716	.7083398	6
55	.6809339	.7323449	.6936114	.7203494	.7060776	.7081345	5
56	.6811469	.7321467	.6938209	.7201476	.7062835	.7079291	4
57	.6813599	.7319486	.6940304	.7199457	.7064894	.7077236	3
58	.6815728	.7317503	.6942398	.7197438	.7066953	.7075180	2
59	.6817856	.7315521	.6944491	.7195418	.7069011	.7073124	1
60	0.6819984	0.7313537	0.6946584	0.7193398	0.7071068	0.7071068	0
	Cosine	Sine	Cosine	Sine	Cosine	Sine	
′	47°		46°		45°		′

TABLE 40. NATURAL TANGENTS AND COTANGENTS

′	0°		1°		2°		′
	Tangent	Cotangent	Tangent	Cotangent	Tangent	Cotangent	
0	0.0000000	Infinite	0.0174551	57.289962	0.0349208	28.636253	60
1	.0002909	3437.7467	.0177460	56.350590	.0352120	28.399397	59
2	.0005818	1718.8732	.0180370	55.441517	.0355033	28.166422	58
3	.0008727	1145.9153	.0183280	54.561300	.0357945	27.937233	57
4	.0011636	859.43630	.0186190	53.708588	.0360858	27.711740	56
5	.0014544	687.54887	.0189100	52.882109	.0363771	27.489853	55
6	.0017453	572.95721	.0192010	52.080673	.0366683	27.271486	54
7	.0020362	491.10600	.0194920	51.303157	.0369596	27.056557	53
8	.0023271	429.71757	.0197830	50.548506	.0372509	26.844984	52
9	.0026180	381.97099	.0200740	49.815726	.0375422	26.636690	51
10	0.0029089	343.77371	0.0203650	49.103881	0.0378335	26.431600	50
11	.0031998	312.52137	.0206560	48.412084	.0381248	26.229638	49
12	.0034907	286.47773	.0209470	47.739501	.0384161	26.030736	48
13	.0037816	264.44080	.0212380	47.085343	.0387074	25.834823	47
14	.0040725	245.55198	.0215291	46.448862	.0389988	25.641832	46
15	.0043634	229.18166	.0218201	45.829351	.0392901	25.451700	45
16	.0046542	214.85762	.0221111	45.226141	.0395814	25.264361	44
17	.0049451	202.21875	.0224021	44.638596	.0398728	25.079757	43
18	.0052360	190.98419	.0226932	44.066113	.0401641	24.897826	42
19	.0055269	180.93220	.0229842	43.508122	.0404555	24.718512	41
20	0.0058178	171.88540	0.0232753	42.964077	0.0407469	24.541758	40
21	.0061087	163.70019	.0235663	42.433464	.0410383	24.367509	39
22	.0063996	156.25908	.0238574	41.915790	.0413296	24.195714	38
23	.0066905	149.46502	.0241484	41.410588	.0416210	24.026320	37
24	.0069814	143.23712	.0244395	40.917412	.0419124	23.859277	36
25	.0072723	137.50745	.0247305	40.435837	.0422038	23.694537	35
26	.0075632	132.21851	.0250216	39.965461	.0424952	23.532052	34
27	.0078541	127.32134	.0253127	39.505895	.0427866	23.371777	33
28	.0081450	122.77396	.0256038	39.056771	.0430781	23.213666	32
29	.0084360	118.54018	.0258948	38.617738	.0433695	23.057677	31
30	0.0087269	114.58865	0.0261859	38.188459	0.0436609	22.903766	30
31	.0090178	110.89205	.0264770	37.768613	.0439524	22.751892	29
32	.0093087	107.42648	.0267681	37.357892	.0442438	22.602015	28
33	.0095996	104.17094	.0270592	36.956001	.0445353	22.454096	27
34	.0098905	101.10690	.0273503	36.562659	.0448268	22.308097	26
35	.0101814	98.217943	.0276414	36.177596	.0451183	22.163980	25
36	.0104724	95.489475	.0279325	35.800553	.0454097	22.021710	24
37	.0107633	92.908487	.0282236	35.431282	.0457012	21.881251	23
38	.0110542	90.463336	.0285148	35.069546	.0459927	21.742569	22
39	.0113451	88.143572	.0288059	34.715115	.0462842	21.605630	21
40	0.0116361	85.939791	0.0290970	34.367771	0.0465757	21.470401	20
41	.0119270	83.843507	.0293882	34.027303	.0468673	21.336851	19
42	.0122179	81.847041	.0296793	33.693509	.0471588	21.204949	18
43	.0125088	79.943430	.0299705	33.366194	.0474503	21.074664	17
44	.0127998	78.126342	.0302616	33.045173	.0477419	20.945966	16
45	.0130907	76.390009	.0305528	32.730264	.0480334	20.818828	15
46	.0133817	74.729165	.0308439	32.421295	.0483250	20.693220	14
47	.0136726	73.138991	.0311351	32.118099	.0486166	20.569115	13
48	.0139635	71.615070	.0314263	31.820516	.0489082	20.446486	12
49	.0142545	70.153346	.0317174	31.528392	.0491997	20.325308	11
50	0.0145454	68.750087	0.0320086	31.241577	0.0494913	20.205553	10
51	.0148364	67.401854	.0322998	30.959928	.0497829	20.087199	9
52	.0151273	66.105473	.0325910	30.683307	.0500746	19.970219	8
53	.0154183	64.858008	.0328822	30.411580	.0503662	19.854591	7
54	.0157093	63.656741	.0331734	30.144619	.0506578	19.740291	6
55	.0160002	62.499154	.0334646	29.882299	.0509495	19.627296	5
56	.0162912	61.382905	.0337558	29.624499	.0512411	19.515584	4
57	.0165821	60.305820	.0340471	29.371106	.0515328	19.405133	3
58	.0168731	59.265872	.0343383	29.122005	.0518244	19.295922	2
59	.0171641	58.261174	.0346295	28.877089	.0521161	19.187930	1
60	0.0174551	57.289962	0.0349208	28.636253	0.0524078	19.081137	0
	Cotangent	Tangent	Cotangent	Tangent	Cotangent	Tangent	
′	89°		88°		87°		′

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TABLE 40. NATURAL TANGENTS AND COTANGENTS (*Continued*)

′	3°		4°		5°		′
	Tangent	Cotangent	Tangent	Cotangent	Tangent	Cotangent	
0	0.0524078	19.081137	0.0699268	14.300666	0.0874887	11.430052	60
1	.0526995	18.975523	.0702191	14.241134	.0877818	11.391885	59
2	.0529912	18.871068	.0705115	14.182092	.0880749	11.353970	58
3	.0532829	18.767754	.0708038	14.123536	.0883681	11.316304	57
4	.0535746	18.665562	.0710961	14.065459	.0886612	11.278885	56
5	.0538663	18.564473	.0713885	14.007856	.0889544	11.241712	55
6	.0541581	18.464471	.0716809	13.950719	.0892476	11.204780	54
7	.0544498	18.365537	.0719733	13.894045	.0895408	11.168089	53
8	.0547416	18.267654	.0722657	13.837827	.0898341	11.131635	52
9	.0550333	18.170807	.0725581	13.782060	.0901273	11.095416	51
10	0.0553251	18.074977	0.0728505	13.726738	0.0904206	11.059431	50
11	.0556169	17.980150	.0731430	13.671856	.0907138	11.023676	49
12	.0559087	17.886310	.0734354	13.617409	.0910071	10.988150	48
13	.0562005	17.793442	.0737279	13.563391	.0913004	10.952850	47
14	.0564923	17.701529	.0740203	13.509799	.0915938	10.917775	46
15	.0567841	17.610559	.0743128	13.456625	.0918871	10.882921	45
16	.0570759	17.520516	.0746053	13.403867	.0921804	10.848288	44
17	.0573678	17.431385	.0748979	13.351518	.0924738	10.813872	43
18	.0576596	17.343155	.0751904	13.299574	.0927672	10.779673	42
19	.0579515	17.255810	.0754829	13.248031	.0930606	10.745687	41
20	0.0582434	17.169337	0.0757755	13.196883	0.0933540	10.711913	40
21	.0585352	17.083724	.0760680	13.146127	.0936474	10.678348	39
22	.0588271	16.998957	.0763606	13.095757	.0939409	10.644992	38
23	.0591190	16.915025	.0766532	13.045769	.0942344	10.611841	37
24	.0594109	16.831915	.0769458	12.906160	.0945278	10.578895	36
25	.0597029	16.749614	.0772384	12.946924	.0948213	10.546151	35
26	.0599948	16.668112	.0775311	12.898058	.0951148	10.513607	34
27	.0602867	16.587396	.0778237	12.849557	.0954084	10.481261	33
28	.0605787	16.507456	.0781164	12.801417	.0957019	10.449112	32
29	.0608706	16.428279	.0784090	12.753634	.0959955	10.417158	31
30	0.0611626	16.349855	0.0787017	12.706205	0.0962890	10.385397	30
31	.0614546	16.272174	.0789944	12.659125	.0965826	10.353827	29
32	.0617466	16.195225	.0792871	12.612390	.0968763	10.322447	28
33	.0620386	16.118998	.0795798	12.565997	.0971699	10.291255	27
34	.0623306	16.043482	.0798726	12.519942	.0974635	10.260249	26
35	.0626226	15.968667	.0801653	12.474221	.0977572	10.229428	25
36	.0629147	15.894545	.0804581	12.428831	.0980509	10.198789	24
37	.0632067	15.821105	.0807509	12.383768	.0983446	10.168332	23
38	.0634988	15.748337	.0810437	12.339028	.0986383	10.138054	22
39	.0637908	15.676233	.0813365	12.294609	.0989320	10.107954	21
40	0.0640829	15.604784	0.0816293	12.250505	0.0992257	10.078031	20
41	.0643750	15.533981	.0819221	12.206716	.0995195	10.048283	19
42	.0646671	15.463814	.0822150	12.163236	.0998133	10.018708	18
43	.0649592	15.394276	.0825078	12.120062	.1001071	9.9893050	17
44	.0652513	15.325358	.0828007	12.077192	.1004009	9.9600724	16
45	.0655435	15.257052	.0830936	12.034622	.1006947	9.9310088	15
46	.0658356	15.189349	.0833865	11.992349	.1009886	9.9021125	14
47	.0661278	15.122242	.0836794	11.950371	.1012824	9.8733823	13
48	.0664199	15.055723	.0839723	11.908682	.1015763	9.8448166	12
49	.0667121	14.989784	.0842653	11.867282	.1018702	9.8164140	11
50	0.0670043	14.924417	0.0845583	11.826167	0.1021641	9.7881732	10
51	.0672965	14.859616	.0848512	11.785333	.1024580	9.7600927	9
52	.0675887	14.795372	.0851442	11.744779	.1027520	9.7321713	8
53	.0678809	14.731679	.0854372	11.704500	.1030460	9.7044075	7
54	.0681732	14.668529	.0857302	11.664495	.1033400	9.6768000	6
55	.0684654	14.605916	.0860233	11.624761	.1036340	9.6493475	5
56	.0687577	14.543833	.0863163	11.585294	.1039280	9.6220486	4
57	.0690499	14.482273	.0866094	11.546093	.1042220	9.5949022	3
58	.0693422	14.421230	.0869025	11.507154	.1045161	9.5679068	2
59	.0696345	14.360696	.0871956	11.468474	.1048101	9.5410613	1
60	0.0699268	14.300666	0.0874887	11.430052	0.1051042	9.5143645	0
	Cotangent	Tangent	Cotangent	Tangent	Cotangent	Tangent	
′	86°		85°		84°		′

TABLE 40. NATURAL TANGENTS AND COTANGENTS (*Continued*)

′	6°		7°		8°		′
	Tangent	Cotangent	Tangent	Cotangent	Tangent	Cotangent	
0	0.1051042	9.5143645	0.1227846	8.1443464	0.1405408	7.1153697	60
1	.1053983	9.4878149	.1230798	8.1248071	.1408375	7.1003826	59
2	.1056925	9.4614116	.1233752	8.1053599	.1411342	7.0854573	58
3	.1059866	9.4351531	.1236705	8.0860042	.1414308	7.0705934	57
4	.1062808	9.4090384	.1239658	8.0667394	.1417276	7.0557905	56
5	.1065750	9.3830663	.1242612	8.0475647	.1420243	7.0410482	55
6	.1068692	9.3572355	.1245566	8.0284796	.1423211	7.0263662	54
7	.1071634	9.3315450	.1248520	8.0094835	.1426179	7.0117441	53
8	.1074576	9.3059936	.1251474	7.9905756	.1429147	6.9971816	52
9	.1077519	9.2805802	.1254429	7.9717555	.1432115	6.9826781	51
10	0.1080462	9.2553035	0.1257384	7.9530224	0.1435084	6.9682335	50
11	.1083405	9.2301627	.1260339	7.9343758	.1438053	6.9538473	49
12	.1086348	9.2051564	.1263294	7.9158151	.1441022	6.9395192	48
13	.1089291	9.1802838	.1266249	7.8973396	.1443991	6.9252489	47
14	.1092234	9.1555436	.1269205	7.8789489	.1446961	6.9110359	46
15	.1095178	9.1309348	.1272161	7.8606423	.1449931	6.8968799	45
16	.1098122	9.1064564	.1275117	7.8424191	.1452901	6.8827807	44
17	.1101066	9.0821074	.1278073	7.8242790	.1455872	6.8687378	43
18	.1104010	9.0578867	.1281030	7.8062212	.1458842	6.8547508	42
19	.1106955	9.0337933	.1283986	7.7882453	.1461813	6.8408196	41
20	0.1109899	9.0098261	0.1286943	7.7703506	0.1464784	6.8269437	40
21	.1112844	8.9859843	.1289900	7.7525366	.1467756	6.8131227	39
22	.1115789	8.9622668	.1292858	7.7348028	.1470727	6.7993565	38
23	.1118734	8.9386726	.1295815	7.7171486	.1473699	6.7856446	37
24	.1121680	8.9152009	.1298773	7.6995735	.1476672	6.7719867	36
25	.1124625	8.8918505	.1301731	7.6820769	.1479644	6.7583826	35
26	.1127571	8.8686206	.1304690	7.6646584	.1482617	6.7448318	34
27	.1130517	8.8455103	.1307648	7.6473174	.1485590	6.7313341	33
28	.1133463	8.8225186	.1310607	7.6300533	.1488563	6.7178891	32
29	.1136410	8.7996446	.1313566	7.6128657	.1491536	6.7044966	31
30	0.1139356	8.7768874	0.1316525	7.5957541	0.1494510	6.6911562	30
31	.1142303	8.7542461	.1319484	7.5787179	.1497484	6.6778677	29
32	.1145250	8.7317198	.1322444	7.5617567	.1500458	6.6646307	28
33	.1148197	8.7093077	.1325404	7.5448699	.1503433	6.6514449	27
34	.1151144	8.6870088	.1328364	7.5280571	.1506408	6.6383100	26
35	.1154092	8.6648223	.1331324	7.5113178	.1509383	6.6252258	25
36	.1157039	8.6427475	.1334285	7.4946514	.1512358	6.6121919	24
37	.1159987	8.6207833	.1337246	7.4780576	.1515333	6.5992080	23
38	.1162936	8.5989290	.1340207	7.4615357	.1518309	6.5862739	22
39	.1165884	8.5771838	.1343168	7.4450855	.1521285	6.5733892	21
40	0.1168832	8.5555468	0.1346129	7.4287064	0.1524262	6.5605538	20
41	.1171781	8.5340172	.1349091	7.4123978	.1527238	6.5477672	19
42	.1174730	8.5125943	.1352053	7.3961595	.1530215	6.5350293	18
43	.1177679	8.4912772	.1355015	7.3799909	.1533192	6.5223396	17
44	.1180628	8.4700651	.1357978	7.3638916	.1536170	6.5096981	16
45	.1183578	8.4489573	.1360940	7.3478610	.1539147	6.4971043	15
46	.1186528	8.4279531	.1363903	7.3318989	.1542125	6.4845581	14
47	.1189478	8.4070511	.1366866	7.3160047	.1545103	6.4720591	13
48	.1192428	8.3862519	.1369830	7.3001780	.1548082	6.4596070	12
49	.1195378	8.3655536	.1372793	7.2844184	.1551061	6.4472017	11
50	0.1198329	8.3449558	0.1375757	7.2687255	0.1554040	6.4348428	10
51	.1201279	8.3244577	.1378721	7.2530987	.1557019	6.4225301	9
52	.1204230	8.3040586	.1381685	7.2375378	.1559998	6.4102633	8
53	.1207182	8.2837579	.1384650	7.2220422	.1562978	6.3980422	7
54	.1210133	8.2635547	.1387615	7.2066116	.1565958	6.3858665	6
55	.1213085	8.2434485	.1390580	7.1912456	.1568939	6.3737359	5
56	.1216036	8.2234384	.1393545	7.1759437	.1571919	6.3616502	4
57	.1218988	8.2035239	.1396510	7.1607056	.1574900	6.3496092	3
58	.1221941	8.1837041	.1399476	7.1455308	.1577881	6.3376126	2
59	.1224893	8.1639786	.1402442	7.1304190	.1580863	6.3256601	1
60	0.1227846	8.1443464	0.1405408	7.1153697	0.1583844	6.3137515	0
	Cotangent	Tangent	Cotangent	Tangent	Cotangent	Tangent	
′	83°		82°		81°		′

TABLE 40. NATURAL TANGENTS AND COTANGENTS (*Continued*)

′	9°		10°		11°		′
	Tangent	Cotangent	Tangent	Cotangent	Tangent	Cotangent	
0	0.1583844	6.3137515	0.1763270	5.6712818	0.1943803	5.1445540	60
1	.1586826	6.3018866	.1766269	5.6616509	.1946822	5.1365763	59
2	.1589809	6.2900651	.1769269	5.6520516	.1949841	5.1286224	58
3	.1592791	6.2782868	.1772269	5.6424838	.1952861	5.1206921	57
4	.1595774	6.2665515	.1775270	5.6329474	.1955881	5.1127855	56
5	.1598757	6.2548588	.1778270	5.6234421	.1958901	5.1049024	55
6	.1601740	6.2432086	.1781271	5.6139680	.1961922	5.0970426	54
7	.1604724	6.2316007	.1784273	5.6045247	.1964943	5.0892061	53
8	.1607708	6.2200347	.1787274	5.5951121	.1967964	5.0813928	52
9	.1610692	6.2085106	.1790276	5.5857302	.1970986	5.0736025	51
10	0.1613677	6.1970279	0.1793279	5.5763786	0.1974008	5.0658352	50
11	.1616662	6.1855867	.1796281	5.5670574	.1977031	5.0580907	49
12	.1619647	6.1741865	.1799284	5.5577663	.1980053	5.0503690	48
13	.1622632	6.1628272	.1802287	5.5485052	.1983076	5.0426700	47
14	.1625618	6.1515085	.1805291	5.5392740	.1986100	5.0349935	46
15	.1628603	6.1402303	.1808295	5.5300724	.1989124	5.0273395	45
16	.1631590	6.1289923	.1811299	5.5209005	.1992148	5.0197078	44
17	.1634576	6.1177943	.1814303	5.5117579	.1995172	5.0120984	43
18	.1637563	6.1066360	.1817308	5.5026446	.1998197	5.0045111	42
19	.1640550	6.0955174	.1820313	5.4935604	.2001222	4.9969459	41
20	0.1643537	6.0844381	0.1823319	5.4845052	0.2004248	4.9894027	40
21	.1646525	6.0733979	.1826324	5.4754788	.2007274	4.9818813	39
22	.1649513	6.0623967	.1829330	5.4664812	.2010300	4.9743817	38
23	.1652501	6.0514343	.1832337	5.4575121	.2013327	4.9669037	37
24	.1655489	6.0405103	.1835343	5.4485715	.2016354	4.9594474	36
25	.1658478	6.0296247	.1838350	5.4396592	.2019381	4.9520125	35
26	.1661467	6.0187772	.1841358	5.4307750	.2022409	4.9445990	34
27	.1664456	6.0079676	.1844365	5.4219188	.2025437	4.9372068	33
28	.1667446	5.9971957	.1847373	5.4130906	.2028465	4.9298358	32
29	.1670436	5.9864614	.1850382	5.4042901	.2031494	4.9224859	31
30	0.1673426	5.9757644	0.1853390	5.3955172	0.2034523	4.9151570	30
31	.1676417	5.9651045	.1856399	5.3867718	.2037552	4.9078491	29
32	.1679407	5.9544815	.1859409	5.3780538	.2040582	4.9005620	28
33	.1682398	5.9438952	.1862418	5.3693630	.2043612	4.8932956	27
34	.1685390	5.9333455	.1865428	5.3606993	.2046643	4.8860499	26
35	.1688381	5.9228322	.1868439	5.3520626	.2049674	4.8788248	25
36	.1691373	5.9123550	.1871449	5.3434527	.2052705	4.8716201	24
37	.1694366	5.9019138	.1874460	5.3348696	.2055737	4.8644359	23
38	.1697358	5.8915084	.1877471	5.3263131	.2058769	4.8572719	22
39	.1700351	5.8811386	.1880483	5.3177830	.2061802	4.8501282	21
40	0.1703344	5.8708042	0.1883495	5.3092793	0.2064834	4.8430045	20
41	.1706338	5.8605051	.1886507	5.3008018	.2067867	4.8359010	19
42	.1709331	5.8502410	.1889520	5.2923505	.2070900	4.8288174	18
43	.1712325	5.8400117	.1892533	5.2839251	.2073934	4.8217536	17
44	.1715320	5.8298172	.1895546	5.2755255	.2076968	4.8147096	16
45	.1718314	5.8196572	.1898559	5.2671517	.2080003	4.8076854	15
46	.1721309	5.8095315	.1901573	5.2588035	.2083038	4.8006808	14
47	.1724304	5.7994400	.1904587	5.2504809	.2086073	4.7936957	13
48	.1727300	5.7893825	.1907602	5.2421836	.2089109	4.7867300	12
49	.1730296	5.7793588	.1910617	5.2339116	.2092145	4.7797837	11
50	0.1733292	5.7693688	0.1913632	5.2256647	0.2095181	4.7728568	10
51	.1736288	5.7594122	.1916648	5.2174428	.2098218	4.7659490	9
52	.1739285	5.7494889	.1919664	5.2092459	.2101255	4.7590603	8
53	.1742282	5.7395988	.1922680	5.2010738	.2104293	4.7521907	7
54	.1745279	5.7297416	.1925696	5.1929264	.2107331	4.7453401	6
55	.1748277	5.7199173	.1928713	5.1848035	.2110369	4.7385083	5
56	.1751275	5.7101256	.1931731	5.1767051	.2113407	4.7316954	4
57	.1754273	5.7003663	.1934748	5.1686311	.2116446	4.7249012	3
58	.1757272	5.6906394	.1937766	5.1605813	.2119486	4.7181256	2
59	.1760271	5.6809446	.1940784	5.1525557	.2122525	4.7113686	1
60	0.1763270	5.6712818	0.1943803	5.1445540	0.2125566	4.7046301	0
	Cotangent	Tangent	Cotangent	Tangent	Cotangent	Tangent	
′	80°		79°		78°		′

TABLE 40. NATURAL TANGENTS AND COTANGENTS (*Continued*)

′	12°		13°		14°		′
	Tangent	Cotangent	Tangent	Cotangent	Tangent	Cotangent	
0	0.2125566	4.7046301	0.2308682	4.3314759	0.2493280	4.0107809	60
1	.2128606	4.6979100	.2311746	4.3257347	.2496370	4.0058164	59
2	.2131647	4.6912083	.2314811	4.3200079	.2499460	4.0008636	58
3	.2134688	4.6845248	.2317876	4.3142955	.2502551	3.9959223	57
4	.2137730	4.6778595	.2320941	4.3085974	.2505642	3.9909924	56
5	.2140772	4.6712124	.2324007	4.3029136	.2508734	3.9860739	55
6	.2143814	4.6645832	.2327073	4.2972440	.2511826	3.9811669	54
7	.2146857	4.6579721	.2330140	4.2915885	.2514919	3.9762712	53
8	.2149900	4.6513788	.2333207	4.2859472	.2518012	3.9713868	52
9	.2152944	4.6448034	.2336274	4.2803199	.2521106	3.9665137	51
10	0.2155988	4.6382457	0.2339342	4.2747066	0.2524200	3.9616518	50
11	.2159032	4.6317056	.2342411	4.2691072	.2527294	3.9568011	49
12	.2162077	4.6251832	.2345479	4.2635218	.2530389	3.9519615	48
13	.2165122	4.6186783	.2348548	4.2579501	.2533484	3.9471331	47
14	.2168167	4.6121908	.2351617	4.2523923	.2536580	3.9423157	46
15	.2171213	4.6057207	.2354687	4.2468482	.2539676	3.9375094	45
16	.2174259	4.5992680	.2357758	4.2413177	.2542773	3.9327141	44
17	.2177306	4.5928325	.2360829	4.2358009	.2545870	3.9279297	43
18	.2180353	4.5864141	.2363900	4.2302977	.2548968	3.9231563	42
19	.2183400	4.5800129	.2366971	4.2248080	.2552066	3.9183937	41
20	0.2186448	4.5736287	0.2370044	4.2193318	0.2555165	3.9136420	40
21	.2189496	4.5672615	.2373116	4.2138690	.2558264	3.9089011	39
22	.2192544	4.5609111	.2376189	4.2084196	.2561363	3.9041710	38
23	.2195593	4.5545776	.2379262	4.2029835	.2564463	3.8994516	37
24	.2198643	4.5482608	.2382336	4.1975606	.2567564	3.8947429	36
25	.2201692	4.5419608	.2385410	4.1921510	.2570664	3.8900448	35
26	.2204742	4.5356773	.2388485	4.1867546	.2573766	3.8853574	34
27	.2207793	4.5294105	.2391560	4.1813713	.2576868	3.8806805	33
28	.2210844	4.5231601	.2394635	4.1760011	.2579970	3.8760142	32
29	.2213895	4.5169261	.2397711	4.1706440	.2583073	3.8713584	31
30	0.2216947	4.5107085	0.2400788	4.1652998	0.2586176	3.8667131	30
31	.2219999	4.5045072	.2403864	4.1599685	.2589280	3.8620782	29
32	.2223051	4.4983221	.2406942	4.1546501	.2592384	3.8574537	28
33	.2226104	4.4921532	.2410019	4.1493446	.2595488	3.8528396	27
34	.2229157	4.4860004	.2413097	4.1440519	.2598593	3.8482358	26
35	.2232211	4.4798636	.2416176	4.1387719	.2601699	3.8436424	25
36	.2235265	4.4737428	.2419255	4.1335046	.2604805	3.8390591	24
37	.2238319	4.4676379	.2422334	4.1282499	.2607911	3.8344861	23
38	.2241374	4.4615489	.2425414	4.1230079	.2611018	3.8299233	22
39	.2244429	4.4554756	.2428494	4.1177784	.2614126	3.8253707	21
40	0.2247485	4.4494181	0.2431575	4.1125614	0.2617234	3.8208281	20
41	.2250541	4.4433762	.2434656	4.1073569	.2620342	3.8162957	19
42	.2253597	4.4373500	.2437737	4.1021649	.2623451	3.8117733	18
43	.2256654	4.4313392	.2440819	4.0969852	.2626560	3.8072609	17
44	.2259711	4.4253439	.2443902	4.0918178	.2629670	3.8027585	16
45	.2262769	4.4193641	.2446984	4.0866627	.2632780	3.7982661	15
46	.2265827	4.4133996	.2450068	4.0815199	.2635891	3.7937835	14
47	.2268885	4.4074504	.2453151	4.0763892	.2639002	3.7893109	13
48	.2271944	4.4015164	.2456236	4.0712707	.2642114	3.7848481	12
49	.2275003	4.3955977	.2459320	4.0661643	.2645226	3.7803951	11
50	0.2278063	4.3896940	0.2462405	4.0610700	0.2648339	3.7759519	10
51	.2281123	4.3838054	.2465491	4.0559877	.2651452	3.7715185	9
52	.2284184	4.3779317	.2468577	4.0509174	.2654566	3.7670947	8
53	.2287244	4.3720731	.2471663	4.0458590	.2657680	3.7626807	7
54	.2290306	4.3662293	.2474750	4.0408125	.2660794	3.7582763	6
55	.2293367	4.3604003	.2477837	4.0357779	.2663909	3.7538815	5
56	.2296429	4.3545861	.2480925	4.0307550	.2667025	3.7494963	4
57	.2299492	4.3487866	.2484013	4.0257440	.2670141	3.7451207	3
58	.2302555	4.3430018	.2487102	4.0207446	.2673257	3.7407546	2
59	.2305618	4.3372316	.2490191	4.0157570	.2676374	3.7363980	1
60	0.2308682	4.3314759	0.2493280	4.0107809	0.2679492	3.7320508	0
	Cotangent	Tangent	Cotangent	Tangent	Cotangent	Tangent	
′	77°		76°		75°		′

TABLE 40. NATURAL TANGENTS AND COTANGENTS (*Continued*)

′	15°		16°		17°		′
	Tangent	Cotangent	Tangent	Cotangent	Tangent	Cotangent	
0	0.2679492	3.7320508	0.2867454	3.4874144	0.3057307	3.2708526	60
1	.2682610	3.7277131	.2870602	3.4835896	.3060488	3.2674529	59
2	.2685728	3.7233847	.2873751	3.4797726	.3063670	3.2640596	58
3	.2688847	3.7190658	.2876900	3.4759632	.3066852	3.2606728	57
4	.2691967	3.7147561	.2880050	3.4721616	.3070034	3.2572924	56
5	.2695087	3.7104558	.2883201	3.4683676	.3073218	3.2539184	55
6	.2698207	3.7061648	.2886352	3.4645813	.3076402	3.2505508	54
7	.2701328	3.7018830	.2889503	3.4608026	.3079586	3.2471895	53
8	.2704449	3.6976104	.2892655	3.4570315	.3082771	3.2438346	52
9	.2707571	3.6933469	.2895808	3.4532679	.3085957	3.2404861	51
10	0.2710694	3.6890927	0.2898961	3.4495120	0.3089143	3.2371438	50
11	.2713817	3.6848475	.2902114	3.4457635	.3092330	3.2338078	49
12	.2716940	3.6806115	.2905269	3.4420226	.3095517	3.2304780	48
13	.2720064	3.6763845	.2908423	3.4382891	.3098705	3.2271546	47
14	.2723188	3.6721665	.2911578	3.4345631	.3101893	3.2238373	46
15	.2726313	3.6679575	.2914734	3.4308446	.3105083	3.2205263	45
16	.2729438	3.6637575	.2917890	3.4271334	.3108272	3.2172215	44
17	.2732564	3.6595665	.2921047	3.4234297	.3111462	3.2139228	43
18	.2735690	3.6553844	.2924205	3.4197333	.3114653	3.2106304	42
19	.2738817	3.6512111	.2927363	3.4160443	.3117845	3.2073440	41
20	0.2741945	3.6470467	0.2930521	3.4123626	0.3121036	3.2040638	40
21	.2745072	3.6428911	.2933680	3.4086882	.3124229	3.2007897	39
22	.2748201	3.6387444	.2936839	3.4050210	.3127422	3.1975217	38
23	.2751330	3.6346064	.2939999	3.4013612	.3130616	3.1942597	37
24	.2754459	3.6304771	.2943160	3.3977085	.3133810	3.1910039	36
25	.2757589	3.6263566	.2946321	3.3940631	.3137005	3.1877540	35
26	.2760719	3.6222447	.2949483	3.3904249	.3140200	3.1845102	34
27	.2763850	3.6181415	.2952645	3.3867938	.3143396	3.1812724	33
28	.2766981	3.6140469	.2955808	3.3831699	.3146593	3.1780406	32
29	.2770113	3.6099609	.2958971	3.3795531	.3149790	3.1748147	31
30	0.2773245	3.6058835	0.2962135	3.3759434	0.3152988	3.1715948	30
31	.2776378	3.6018146	.2965299	3.3723408	.3156186	3.1683808	29
32	.2779512	3.5977543	.2968464	3.3687453	.3159385	3.1651728	28
33	.2782646	3.5937024	.2971630	3.3651568	.3162585	3.1619706	27
34	.2785780	3.5896590	.2974796	3.3615753	.3165785	3.1587744	26
35	.2788915	3.5856241	.2977962	3.3580008	.3168986	3.1555840	25
36	.2792050	3.5815975	.2981129	3.3544333	.3172187	3.1523994	24
37	.2795186	3.5775794	.2984297	3.3508728	.3175389	3.1492207	23
38	.2798322	3.5735696	.2987465	3.3473191	.3178591	3.1460478	22
39	.2801459	3.5695681	.2990634	3.3437724	.3181794	3.1428807	21
40	0.2804597	3.5655749	0.2993803	3.3402326	0.3184998	3.1397194	20
41	.2807735	3.5615900	.2996973	3.3366997	.3188202	3.1365639	19
42	.2810873	3.5576133	.3000144	3.3331736	.3191407	3.1334141	18
43	.2814012	3.5536449	.3003315	3.3296543	.3194613	3.1302701	17
44	.2817152	3.5496846	.3006486	3.3261419	.3197819	3.1271317	16
45	.2820292	3.5457325	.3009658	3.3226362	.3201025	3.1239991	15
46	.2823432	3.5417886	.3012831	3.3191373	.3204232	3.1208722	14
47	.2826573	3.5378528	.3016004	3.3156452	.3207440	3.1177509	13
48	.2829715	3.5339251	.3019178	3.3121598	.3210649	3.1146353	12
49	.2832857	3.5300054	.3022352	3.3086811	.3213858	3.1115254	11
50	0.2835999	3.5260938	0.3025527	3.3052091	0.3217067	3.1084210	10
51	.2839143	3.5221902	.3028703	3.3017438	.3220278	3.1053223	9
52	.2842286	3.5182946	.3031879	3.2982851	.3223489	3.1022291	8
53	.2845430	3.5144070	.3035055	3.2948330	.3226700	3.0991416	7
54	.2848575	3.5105273	.3038232	3.2913876	.3229912	3.0960596	6
55	.2851720	3.5066555	.3041410	3.2879487	.3233125	3.0929831	5
56	.2854866	3.5027916	.3044588	3.2845164	.3236338	3.0899122	4
57	.2858012	3.4989356	.3047767	3.2810907	.3239552	3.0868468	3
58	.2861159	3.4950874	.3050946	3.2776715	.3242766	3.0837869	2
59	.2864306	3.4912470	.3054126	3.2742588	.3245981	3.0807325	1
60	0.2867454	3.4874144	0.3057307	3.2708526	0.3249197	3.0776835	0
′	Cotangent	Tangent	Cotangent	Tangent	Cotangent	Tangent	′
	74°		73°		72°		

TABLE 40. NATURAL TANGENTS AND COTANGENTS (*Continued*)

′	18°		19°		20°		′
	Tangent	Cotangent	Tangent	Cotangent	Tangent	Cotangent	
0	0.3249197	3.0776835	0.3443276	2.9042109	0.3639702	2.7474774	60
1	.3252413	3.0746400	.3446530	2.9014688	.3642997	2.7449927	59
2	.3255630	3.0716020	.3449785	2.8987314	.3646292	2.7425120	58
3	.3258848	3.0685694	.3453040	2.8959986	.3649588	2.7400352	57
4	.3262066	3.0655421	.3456296	2.8932704	.3652885	2.7375623	56
5	.3265284	3.0625203	.3459553	2.8905467	.3656182	2.7350934	55
6	.3268504	3.0595038	.3462810	2.8878277	.3659480	2.7326284	54
7	.3271724	3.0564928	.3466068	2.8851132	.3662779	2.7301674	53
8	.3274944	3.0534870	.3469327	2.8824033	.3666079	2.7277102	52
9	.3278165	3.0504866	.3472586	2.8796979	.3669379	2.7252569	51
10	0.3281387	3.0474915	0.3475846	2.8769970	0.3672680	2.7228076	50
11	.3284610	3.0445018	.3479107	2.8743007	.3675981	2.7203620	49
12	.3287833	3.0415173	.3482368	2.8716088	.3679284	2.7179204	48
13	.3291056	3.0385381	.3485630	2.8689215	.3682587	2.7154826	47
14	.3294281	3.0355641	.3488893	2.8662386	.3685890	2.7130487	46
15	.3297505	3.0325954	.3492156	2.8635602	.3689195	2.7106186	45
16	.3300731	3.0296320	.3495420	2.8608863	.3692500	2.7081923	44
17	.3303957	3.0266737	.3498685	2.8582168	.3695806	2.7057699	43
18	.3307184	3.0237207	.3501950	2.8555517	.3699112	2.7033513	42
19	.3310411	3.0207728	.3505216	2.8528911	.3702420	2.7009364	41
20	0.3313639	3.0178301	0.3508483	2.8502349	0.3705728	2.6985254	40
21	.3316868	3.0148926	.3511750	2.8475831	.3709036	2.6961181	39
22	.3320097	3.0119603	.3515018	2.8449356	.3712346	2.6937147	38
23	.3323327	3.0090330	.3518287	2.8422926	.3715656	2.6913149	37
24	.3326557	3.0061109	.3521556	2.8396539	.3718967	2.6889190	36
25	.3329788	3.0031939	.3524826	2.8370196	.3722278	2.6865267	35
26	.3333020	3.0002820	.3528096	2.8343896	.3725590	2.6841383	34
27	.3336252	2.9973751	.3531368	2.8317640	.3728903	2.6817535	33
28	.3339485	2.9944734	.3534640	2.8291426	.3732217	2.6793725	32
29	.3342719	2.9915766	.3537912	2.8265256	.3735532	2.6769951	31
30	0.3345953	2.9886850	0.3541186	2.8239129	0.3738847	2.6746215	30
31	.3349188	2.9857983	.3544460	2.8213045	.3742163	2.6722516	29
32	.3352424	2.9829167	.3547734	2.8187003	.3745479	2.6698853	28
33	.3355660	2.9800400	.3551010	2.8161004	.3748797	2.6675227	27
34	.3358896	2.9771683	.3554286	2.8135048	.3752115	2.6651638	26
35	.3362134	2.9743016	.3557562	2.8109134	.3755433	2.6628085	25
36	.3365372	2.9714399	.3560840	2.8083263	.3758753	2.6604569	24
37	.3368610	2.9685831	.3564118	2.8057433	.3762073	2.6581089	23
38	.3371850	2.9657312	.3567397	2.8031646	.3765394	2.6557645	22
39	.3375090	2.9628842	.3570676	2.8005901	.3768716	2.6534238	21
40	0.3378330	2.9600422	0.3573956	2.7980198	0.3772038	2.6510867	20
41	.3381571	2.9572050	.3577237	2.7954537	.3775361	2.6487531	19
42	.3384813	2.9543727	.3580518	2.7928917	.3778685	2.6464232	18
43	.3388056	2.9515453	.3583801	2.7903339	.3782010	2.6440969	17
44	.3391299	2.9487227	.3587083	2.7877802	.3785335	2.6417741	16
45	.3394543	2.9459050	.3590367	2.7852307	.3788661	2.6394549	15
46	.3397787	2.9430921	.3593651	2.7826853	.3791988	2.6371392	14
47	.3401032	2.9402840	.3596936	2.7801440	.3795315	2.6348271	13
48	.3404278	2.9374807	.3600222	2.7776069	.3798644	2.6325186	12
49	.3407524	2.9346822	.3603508	2.7750738	.3801973	2.6302136	11
50	0.3410771	2.9318885	0.3606795	2.7725448	0.3805302	2.6279121	10
51	.3414019	2.9290995	.3610082	2.7700199	.3808633	2.6256141	9
52	.3417267	2.9263152	.3613371	2.7674991	.3811964	2.6233196	8
53	.3420516	2.9235358	.3616660	2.7649822	.3815296	2.6210286	7
54	.3423765	2.9207610	.3619949	2.7624695	.3818629	2.6187411	6
55	.3427015	2.9179909	.3623240	2.7599608	.3821962	2.6164571	5
56	.3430266	2.9152256	.3626531	2.7574561	.3825296	2.6141766	4
57	.3433518	2.9124649	.3629823	2.7549554	.3828631	2.6118995	3
58	.3436770	2.9097089	.3633115	2.7524588	.3831967	2.6096259	2
59	.3440023	2.9069576	.3636408	2.7499661	.3835303	2.6073558	1
60	0.3443276	2.9042109	0.3639702	2.7474774	0.3838640	2.6050891	0
′	Cotangent	Tangent	Cotangent	Tangent	Cotangent	Tangent	′
	71°		70°		69°		

TABLE 40. NATURAL TANGENTS AND COTANGENTS (*Continued*)

′	21°		22°		23°		′
	Tangent	Cotangent	Tangent	Cotangent	Tangent	Cotangent	
0	0.3838640	2.6050891	0.4040262	2.4750869	0.4244748	2.3558524	60
1	.3841978	2.6028258	.4043646	2.4730155	.4248182	2.3539483	59
2	.3845317	2.6005659	.4047031	2.4709470	.4251616	2.3520469	58
3	.3848656	2.5983095	.4050417	2.4688816	.4255051	2.3501481	57
4	.3851996	2.5960564	.4053804	2.4668191	.4258487	2.3482519	56
5	.3855337	2.5938068	.4057191	2.4647596	.4261924	2.3463582	55
6	.3858679	2.5915606	.4060579	2.4627030	.4265361	2.3444672	54
7	.3862021	2.5893177	.4063968	2.4606494	.4268800	2.3425787	53
8	.3865364	2.5870782	.4067358	2.4585987	.4272239	2.3406928	52
9	.3868708	2.5848421	.4070748	2.4565510	.4275680	2.3388095	51
10	0.3872053	2.5826094	0.4074139	2.4545061	0.4279121	2.3369287	50
11	.3875398	2.5803800	.4077531	2.4524642	.4282563	2.3350505	49
12	.3878744	2.5781539	.4080924	2.4504252	.4286005	2.3331748	48
13	.3882091	2.5759312	.4084318	2.4483891	.4289449	2.3313017	47
14	.3885439	2.5737118	.4087713	2.4463559	.4292894	2.3294311	46
15	.3888787	2.5714957	.4091108	2.4443256	.4296339	2.3275630	45
16	.3892136	2.5692830	.4094504	2.4422982	.4299785	2.3256975	44
17	.3895486	2.5670735	.4097901	2.4402736	.4303232	2.3238345	43
18	.3898837	2.5648674	.4101299	2.4382519	.4306680	2.3219740	42
19	.3902189	2.5626645	.4104697	2.4362331	.4310129	2.3201160	41
20	0.3905541	2.5604649	0.4108097	2.4342172	0.4313579	2.3182606	40
21	.3908894	2.5582686	.4111497	2.4322041	.4317030	2.3164076	39
22	.3912247	2.5560756	.4114898	2.4301938	.4320481	2.3145571	38
23	.3915602	2.5538858	.4118300	2.4281864	.4323933	2.3127092	37
24	.3918957	2.5516992	.4121703	2.4261819	.4327386	2.3108637	36
25	.3922313	2.5495160	.4125106	2.4241801	.4330840	2.3090206	35
26	.3925670	2.5473359	.4128510	2.4221812	.4334295	2.3071801	34
27	.3929027	2.5451591	.4131915	2.4201851	.4337751	2.3053420	33
28	.3932386	2.5429855	.4135321	2.4181918	.4341208	2.3035064	32
29	.3935745	2.5408151	.4138728	2.4162013	.4344665	2.3016732	31
30	0.3939105	2.5386479	0.4142136	2.4142136	0.4348124	2.2998425	30
31	.3942465	2.5364839	.4145544	2.4122286	.4351583	2.2980143	29
32	.3945827	2.5343231	.4148953	2.4102465	.4355043	2.2961885	28
33	.3949189	2.5321655	.4152363	2.4082672	.4358504	2.2943651	27
34	.3952552	2.5300111	.4155774	2.4062906	.4361966	2.2925442	26
35	.3955916	2.5278598	.4159186	2.4043168	.4365429	2.2907257	25
36	.3959280	2.5257117	.4162598	2.4023457	.4368893	2.2889096	24
37	.3962645	2.5235667	.4166012	2.4003774	.4372357	2.2870959	23
38	.3966011	2.5214249	.4169426	2.3984118	.4375823	2.2852846	22
39	.3969378	2.5192863	.4172841	2.3964490	.4379289	2.2834758	21
40	0.3972746	2.5171507	0.4176257	2.3944889	0.4382756	2.2816693	20
41	.3976114	2.5150183	.4179673	2.3925316	.4386224	2.2798653	19
42	.3979483	2.5128890	.4183091	2.3905769	.4389693	2.2780636	18
43	.3982853	2.5107629	.4186509	2.3886250	.4393163	2.2762643	17
44	.3986224	2.5086398	.4189928	2.3866758	.4396634	2.2744674	16
45	.3989595	2.5065198	.4193348	2.3847293	.4400105	2.2726729	15
46	.3992968	2.5044029	.4196769	2.3827855	.4403578	2.2708807	14
47	.3996341	2.5022891	.4200190	2.3808444	.4407051	2.2690909	13
48	.3999715	2.5001784	.4203613	2.3789060	.4410526	2.2673035	12
49	.4003089	2.4980707	.4207036	2.3769703	.4414001	2.2655184	11
50	0.4006465	2.4959661	0.4210460	2.3750372	0.4417477	2.2637357	10
51	.4009841	2.4938645	.4213885	2.3731068	.4420954	2.2619554	9
52	.4013218	2.4917660	.4217311	2.3711791	.4424432	2.2601773	8
53	.4016596	2.4896706	.4220738	2.3692540	.4427910	2.2584016	7
54	.4019974	2.4875781	.4224165	2.3673316	.4431390	2.2566283	6
55	.4023354	2.4854887	.4227594	2.3654118	.4434871	2.2548572	5
56	.4026734	2.4834023	.4231023	2.3634946	.4438352	2.2530885	4
57	.4030115	2.4813190	.4234453	2.3615801	.4441834	2.2513221	3
58	.4033496	2.4792386	.4237884	2.3596683	.4445318	2.2495580	2
59	.4036879	2.4771612	.4241316	2.3577590	.4448802	2.2477962	1
60	0.4040262	2.4750869	0.4244748	2.3558524	0.4452287	2.2460368	0
′	Cotangent	Tangent	Cotangent	Tangent	Cotangent	Tangent	′
	68°		67°		66°		

TABLE 40. NATURAL TANGENTS AND COTANGENTS (*Continued*)

′	24°		25°		26°		′
	Tangent	Cotangent	Tangent	Cotangent	Tangent	Cotangent	
0	0.4452287	2.2460368	0.4663077	2.1445069	0.4877326	2.0503038	60
1	.4455773	2.2442796	.4666618	2.1428793	.4880927	2.0487910	59
2	.4459260	2.2425247	.4670161	2.1412537	.4884530	2.0472800	58
3	.4462747	2.2407721	.4673705	2.1396301	.4888133	2.0457708	57
4	.4466236	2.2390218	.4677250	2.1380085	.4891737	2.0442634	56
5	.4469726	2.2372738	.4680796	2.1363890	.4895343	2.0427578	55
6	.4473216	2.2355280	.4684342	2.1347714	.4898949	2.0412540	54
7	.4476708	2.2337845	.4687890	2.1331559	.4902557	2.0397519	53
8	.4480200	2.2320433	.4691439	2.1315423	.4906166	2.0382517	52
9	.4483693	2.2303043	.4694988	2.1299308	.4909775	2.0367532	51
10	0.4487187	2.2285676	0.4698539	2.1283213	0.4913386	2.0352565	50
11	.4490682	2.2268331	.4702090	2.1267137	.4916997	2.0337615	49
12	.4494178	2.2251009	.4705643	2.1251082	.4920610	2.0322683	48
13	.4497675	2.2233709	.4709196	2.1235046	.4924224	2.0307769	47
14	.4501173	2.2216432	.4712751	2.1219030	.4927838	2.0292873	46
15	.4504672	2.2199177	.4716306	2.1203034	.4931454	2.0277994	45
16	.4508171	2.2181944	.4719863	2.1187057	.4935071	2.0263133	44
17	.4511672	2.2164733	.4723420	2.1171101	.4938689	2.0248289	43
18	.4515173	2.2147545	.4726978	2.1155164	.4942308	2.0233462	42
19	.4518676	2.2130379	.4730538	2.1139246	.4945928	2.0218654	41
20	0.4522179	2.2113234	0.4734098	2.1123348	0.4949549	2.0203862	40
21	.4525683	2.2096112	.4737659	2.1107470	.4953171	2.0189088	39
22	.4529188	2.2079012	.4741222	2.1091611	.4956794	2.0174331	38
23	.4532694	2.2061934	.4744785	2.1075771	.4960418	2.0159592	37
24	.4536201	2.2044878	.4748349	2.1059951	.4964043	2.0144869	36
25	.4539709	2.2027843	.4751914	2.1044150	.4967669	2.0130164	35
26	.4543218	2.2010831	.4755481	2.1028369	.4971297	2.0115477	34
27	.4546728	2.1993840	.4759048	2.1012607	.4974925	2.0100806	33
28	.4550238	2.1976871	.4762616	2.0996864	.4978554	2.0086153	32
29	.4553750	2.1959923	.4766185	2.0981140	.4982185	2.0071516	31
30	0.4557263	2.1942997	0.4769755	2.0965436	0.4985816	2.0056897	30
31	.4560776	2.1926093	.4773326	2.0949751	.4989449	2.0042295	29
32	.4564290	2.1909210	.4776899	2.0934085	.4993082	2.0027710	28
33	.4567806	2.1892349	.4780472	2.0918437	.4996717	2.0013142	27
34	.4571322	2.1875510	.4784046	2.0902809	.5000352	1.9998590	26
35	.4574839	2.1858691	.4787621	2.0887200	.5003989	1.9984056	25
36	.4578357	2.1841894	.4791197	2.0871610	.5007627	1.9969539	24
37	.4581877	2.1825119	.4794774	2.0856039	.5011266	1.9955038	23
38	.4585397	2.1808364	.4798352	2.0840487	.5014906	1.9940554	22
39	.4588918	2.1791631	.4801932	2.0824953	.5018547	1.9926087	21
40	0.4592439	2.1774920	0.4805512	2.0809438	0.5022189	1.9911637	20
41	.4595962	2.1758229	.4809093	2.0793942	.5025832	1.9897204	19
42	.4599486	2.1741559	.4812675	2.0778465	.5029476	1.9882787	18
43	.4603011	2.1724911	.4816258	2.0763007	.5033121	1.9868387	17
44	.4606537	2.1708283	.4819842	2.0747567	.5036768	1.9854003	16
45	.4610063	2.1691677	.4823427	2.0732146	.5040415	1.9839636	15
46	.4613591	2.1675091	.4827014	2.0716743	.5044063	1.9825286	14
47	.4617119	2.1658527	.4830601	2.0701359	.5047713	1.9810952	13
48	.4620649	2.1641983	.4834189	2.0685994	.5051363	1.9796635	12
49	.4624179	2.1625460	.4837778	2.0670646	.5055015	1.9782334	11
50	0.4627710	2.1608958	0.4841368	2.0655318	0.5058668	1.9768050	10
51	.4631243	2.1592476	.4844959	2.0640008	.5062322	1.9753782	9
52	.4634776	2.1576015	.4848552	2.0624716	.5065977	1.9739531	8
53	.4638310	2.1559575	.4852145	2.0609442	.5069633	1.9725296	7
54	.4641845	2.1543156	.4855739	2.0594187	.5073290	1.9711077	6
55	.4645382	2.1526757	.4859334	2.0578950	.5076948	1.9696874	5
56	.4648919	2.1510378	.4862931	2.0563732	.5080607	1.9682688	4
57	.4652457	2.1494021	.4866528	2.0548531	.5084267	1.9668518	3
58	.4655996	2.1477683	.4870126	2.0533349	.5087929	1.9654364	2
59	.4659536	2.1461366	.4873726	2.0518185	.5091591	1.9640227	1
60	0.4663077	2.1445069	0.4877326	2.0503038	0.5095254	1.9626105	0
′	Cotangent	Tangent	Cotangent	Tangent	Cotangent	Tangent	′
	65°		64°		63°		

Table 40. Natural Tangents and Cotangents (*Continued*)

′	27°		28°		29°		′
	Tangent	Cotangent	Tangent	Cotangent	Tangent	Cotangent	
0	0.5095254	1.9626105	0.5317094	1.8807265	0.5543091	1.8040478	60
1	.5098919	1.9612000	.5320826	1.8794074	.5546894	1.8028108	59
2	.5102585	1.9597910	.5324559	1.8780898	.5550698	1.8015751	58
3	.5106252	1.9583837	.5328293	1.8767736	.5554504	1.8003408	57
4	.5109919	1.9569780	.5332029	1.8754588	.5558311	1.7991077	56
5	.5113588	1.9555739	.5335765	1.8741455	.5562119	1.7978759	55
6	.5117259	1.9541713	.5339503	1.8728336	.5565929	1.7966454	54
7	.5120930	1.9527704	.5343242	1.8715231	.5569739	1.7954162	53
8	.5124602	1.9513711	.5346981	1.8702141	.5573551	1.7941883	52
9	.5128275	1.9499733	.5350723	1.8689065	.5577364	1.7929616	51
10	0.5131950	1.9485772	0.5354465	1.8676003	0.5581179	1.7917362	50
11	.5135625	1.9471826	.5358208	1.8662955	.5584994	1.7905121	49
12	.5139302	1.9457896	.5361953	1.8549921	.5588811	1.7892893	48
13	.5142980	1.9443981	.5365699	1.8636902	.5592629	1.7880678	47
14	.5146658	1.9430083	.5369446	1.8623896	.5596449	1.7868475	46
15	.5150338	1.9416200	.5373194	1.8610905	.5600269	1.7856285	45
16	.5154019	1.9402333	.5376943	1.8597928	.5604091	1.7844107	44
17	.5157702	1.9388481	.5380694	1.8584965	.5607914	1.7831943	43
18	.5161385	1.9374645	.5384445	1.8572015	.5611738	1.7819790	42
19	.5165069	1.9360825	.5388198	1.8559080	.5615564	1.7807651	41
20	0.5168755	1.9347020	0.5391952	1.8546159	0.5619391	1.7795524	40
21	.5172441	1.9333231	.5395707	1.8533252	.5623219	1.7783409	39
22	.5176129	1.9319457	.5399464	1.8520358	.5627048	1.7771307	38
23	.5179818	1.9305699	.5403221	1.8507479	.5630879	1.7759218	37
24	.5183508	1.9291956	.5406980	1.8494613	.5634711	1.7747141	36
25	.5187199	1.9278228	.5410740	1.8481761	.5638543	1.7735076	35
26	.5190891	1.9264516	.5414501	1.8468923	.5642378	1.7723024	34
27	.5194584	1.9250819	.5418263	1.8456099	.5646213	1.7710985	33
28	.5198278	1.9237138	.5422027	1.8443289	.5650050	1.7698958	32
29	.5201974	1.9223472	.5425791	1.8430492	.5653888	1.7686943	31
30	0.5205671	1.9209821	0.5429557	1.8417709	0.5657728	1.7674940	30
31	.5209368	1.9196185	.5433324	1.8404940	.5661568	1.7662950	29
32	.5213067	1.9182565	.5437092	1.8392184	.5665410	1.7650972	28
33	.5216767	1.9168960	.5440862	1.8379442	.5669254	1.7639007	27
34	.5220468	1.9155370	.5444632	1.8366713	.5673098	1.7627053	26
35	.5224170	1.9141795	.5448404	1.8353999	.5676944	1.7615112	25
36	.5227874	1.9128236	.5452177	1.8341297	.5680791	1.7603183	24
37	.5231578	1.9114691	.5455951	1.8328610	.5684639	1.7591267	23
38	.5235284	1.9101162	.5459727	1.8315936	.5688488	1.7579362	22
39	.5238990	1.9087647	.5463503	1.8303275	.5692339	1.7567470	21
40	0.5242698	1.9074147	0.5467281	1.8290628	0.5696191	1.7555590	20
41	.5246407	1.9060663	.5471060	1.8277994	.5700045	1.7543722	19
42	.5250117	1.9047193	.5474840	1.8265374	.5703899	1.7531866	18
43	.5253829	1.9033738	.5478621	1.8252767	.5707755	1.7520023	17
44	.5257541	1.9020299	.5482404	1.8240173	.5711612	1.7508191	16
45	.5261255	1.9006874	.5486188	1.8227593	.5715471	1.7496371	15
46	.5264969	1.8993464	.5489973	1.8215026	.5719331	1.7484564	14
47	.5268685	1.8980068	.5493759	1.8202473	.5723192	1.7472768	13
48	.5272402	1.8966688	.5497547	1.8189932	.5727054	1.7460984	12
49	.5276120	1.8953322	.5501335	1.8177405	.5730918	1.7449213	11
50	0.5279839	1.8939971	0.5505125	1.8164892	0.5734783	1.7437453	10
51	.5283560	1.8926635	.5508916	1.8152391	.5738649	1.7425705	9
52	.5287281	1.8913313	.5512708	1.8139904	.5742516	1.7413969	8
53	.5291004	1.8900006	.5516502	1.8127430	.5746385	1.7402245	7
54	.5294727	1.8886713	.5520297	1.8114969	.5750255	1.7390533	6
55	.5298452	1.8873436	.5524093	1.8102521	.5754126	1.7378833	5
56	.5302178	1.8860172	.5527890	1.8090086	.5757999	1.7367144	4
57	.5305906	1.8846924	.5531688	1.8077664	.5761873	1.7355468	3
58	.5309634	1.8833690	.5535488	1.8065256	.5765748	1.7343803	2
59	.5313364	1.8820470	.5539288	1.8052860	.5769625	1.7332149	1
60	0.5317094	1.8807265	0.5543091	1.8040478	0.5773503	1.7320508	0
′	Cotangent	Tangent	Cotangent	Tangent	Cotangent	Tangent	′
	62°		61°		60°		

TABLE 40. NATURAL TANGENTS AND COTANGENTS (*Continued*)

′	30°		31°		32°		′
	Tangent	Cotangent	Tangent	Cotangent	Tangent	Cotangent	
0	0.5773503	1.7320508	0.6008606	1.6642795	0.6248694	1.6003345	60
1	.5777382	1.7308878	.6012536	1.6631834	.6252739	1.5992991	59
2	.5781262	1.7297260	.6016527	1.6620884	.6256786	1.5982647	58
3	.5785144	1.7285654	.6020490	1.6609945	.6260834	1.5972312	57
4	.5789027	1.7274060	.6024454	1.6599016	.6264884	1.5961987	56
5	.5792912	1.7262477	.6028419	1.6588097	.6268935	1.5951672	55
6	.5796797	1.7250905	.6032386	1.6577189	.6272988	1.5941366	54
7	.5800684	1.7239346	.6036354	1.6566292	.6277042	1.5931070	53
8	.5804573	1.7227797	.6040323	1.6555405	.6281098	1.5920783	52
9	.5808462	1.7216261	.6044294	1.6544529	.6285155	1.5910505	51
10	0.5812353	1.7204736	0.6048266	1.6533663	0.6289215	1.5900238	50
11	.5816245	1.7193222	.6052240	1.6522808	.6293274	1.5889979	49
12	.5820139	1.7181720	.6056215	1.6511963	.6297336	1.5879731	48
13	.5824034	1.7170230	.6060192	1.6501128	.6301399	1.5869491	47
14	.5827930	1.7158751	.6064170	1.6490304	.6305464	1.5859261	46
15	.5831828	1.7147283	.6068149	1.6479490	.6309530	1.5849041	45
16	.5835726	1.7135827	.6072130	1.6468687	.6313598	1.5838830	44
17	.5839627	1.7124382	.6076112	1.6457893	.6317667	1.5828628	43
18	.5843528	1.7112949	.6080095	1.6447111	.6321738	1.5818436	42
19	.5847431	1.7101527	.6084080	1.6436338	.6325810	1.5808253	41
20	0.5851335	1.7090116	0.6088067	1.6425576	0.6329883	1.5798079	40
21	.5855241	1.7078717	.6092054	1.6414824	.6333959	1.5787915	39
22	.5859148	1.7067329	.6096043	1.6404082	.6338035	1.5777760	38
23	.5863056	1.7055953	.6100034	1.6393351	.6342113	1.5767615	37
24	.5866965	1.7044587	.6104026	1.6382630	.6346193	1.5757479	36
25	.5870876	1.7033233	.6108019	1.6371919	.6350274	1.5747352	35
26	.5874788	1.7021890	.6112014	1.6361218	.6354357	1.5737234	34
27	.5878702	1.7010599	.6116011	1.6350528	.6358441	1.5727126	33
28	.5882616	1.6999238	.6120008	1.6339847	.6362527	1.5717026	32
29	.5886533	1.6987929	.6124007	1.6329177	.6366614	1.5706936	31
30	0.5890450	1.6976631	0.6128008	1.6318517	0.6370703	1.5696856	30
31	.5894369	1.6965344	.6132010	1.6307867	.6374793	1.5686784	29
32	.5898289	1.6954069	.6136014	1.6297227	.6378885	1.5676722	28
33	.5902211	1.6942804	.6140018	1.6286597	.6382978	1.5666669	27
34	.5906134	1.6931550	.6144024	1.6275977	.6387073	1.5656625	26
35	.5910058	1.6920308	.6148032	1.6265368	.6391169	1.5646590	25
36	.5913984	1.6909077	.6152041	1.6254768	.6395267	1.5636564	24
37	.5917910	1.6897856	.6156052	1.6244178	.6399366	1.5626548	23
38	.5921839	1.6886647	.6160064	1.6233599	.6403467	1.5616540	22
39	.5925768	1.6875449	.6164077	1.6223029	.6407569	1.5606542	21
40	0.5929699	1.6864261	0.6168092	1.6212469	0.6411673	1.5596552	20
41	.5933632	1.6853085	.6172108	1.6201920	.6415779	1.5586572	19
42	.5937565	1.6841919	.6176126	1.6191380	.6419886	1.5576601	18
43	.5941501	1.6830765	.6180145	1.6180850	.6423994	1.5566639	17
44	.5945437	1.6819621	.6184166	1.6170330	.6428105	1.5556685	16
45	.5949375	1.6808489	.6188188	1.6159820	.6432216	1.5546741	15
46	.5953314	1.6797367	.6192211	1.6149320	.6436329	1.5536806	14
47	.5957255	1.6786256	.6196236	1.6138829	.6440444	1.5526880	13
48	.5961196	1.6775156	.6200263	1.6128349	.6444560	1.5516963	12
49	.5965140	1.6764067	.6204291	1.6117878	.6448678	1.5507054	11
50	0.5969084	1.6752988	0.6208320	1.6107417	0.6452797	1.5497155	10
51	.5973030	1.6741921	.6212351	1.6096966	.6456918	1.5487264	9
52	.5976978	1.6730864	.6216383	1.6086525	.6461041	1.5477383	8
53	.5980926	1.6719818	.6220417	1.6076094	.6465165	1.5467510	7
54	.5984877	1.6708782	.6224452	1.6065672	.6469290	1.5457647	6
55	.5988828	1.6697758	.6228488	1.6055260	.6473417	1.5447792	5
56	.5992781	1.6686744	.6232527	1.6044858	.6477546	1.5437946	4
57	.5996735	1.6675741	.6236566	1.6034465	.6481676	1.5428108	3
58	.6000691	1.6664748	.6240607	1.6024082	.6485808	1.5418280	2
59	.6004648	1.6653766	.6244650	1.6013709	.6489941	1.5408460	1
60	0.6008606	1.6642795	0.6248694	1.6003345	0.6494076	1.5398650	0
′	Cotangent	Tangent	Cotangent	Tangent	Cotangent	Tangent	′
	59°		58°		57°		

TABLE 40. NATURAL TANGENTS AND COTANGENTS (*Continued*)

′	33°		34°		35°		′
	Tangent	Cotangent	Tangent	Cotangent	Tangent	Cotangent	
0	0.6494076	1.5398650	0.6745085	1.4825610	0.7002075	1.4281480	60
1	.6498212	1.5388848	.6749318	1.4816311	.7006411	1.4272642	59
2	.6502350	1.5379054	.6753553	1.4807021	.7010749	1.4263811	58
3	.6506490	1.5369270	.6757790	1.4797738	.7015089	1.4254988	57
4	.6510631	1.5359494	.6762028	1.4788463	.7019430	1.4246171	56
5	.6514774	1.5349727	.6766268	1.4779197	.7023773	1.4237362	55
6	.6518918	1.5339969	.6770509	1.4769938	.7028118	1.4228561	54
7	.6523064	1.5330219	.6774752	1.4760688	.7032464	1.4219766	53
8	.6527211	1.5320479	.6778997	1.4751445	.7036813	1.4210979	52
9	.6531360	1.5310746	.6783243	1.4742210	.7041163	1.4202200	51
10	0.6535511	1.5301023	0.6787492	1.4732983	0.7045515	1.4193427	50
11	.6539663	1.5291308	.6791741	1.4723764	.7049869	1.4184662	49
12	.6543817	1.5281602	.6795993	1.4714553	.7054224	1.4175904	48
13	.6547972	1.5271904	.6800246	1.4705350	.7058581	1.4167153	47
14	.6552129	1.5262215	.6804501	1.4696155	.7062940	1.4158409	46
15	.6556287	1.5252535	.6808758	1.4686967	.7067301	1.4149673	45
16	.6560447	1.5242863	.6813016	1.4677788	.7071664	1.4140943	44
17	.6564609	1.5233200	.6817276	1.4668616	.7076028	1.4132221	43
18	.6568772	1.5223545	.6821537	1.4659452	.7080395	1.4123506	42
19	.6572937	1.5213899	.6825801	1.4650296	.7084763	1.4114799	41
20	0.6577103	1.5204261	0.6830066	1.4641147	0.7089133	1.4106098	40
21	.6581271	1.5194632	.6834333	1.4632007	.7093504	1.4097405	39
22	.6585441	1.5185012	.6838601	1.4622874	.7097878	1.4088718	38
23	.6589612	1.5175400	.6842871	1.4613749	.7102253	1.4080039	37
24	.6593785	1.5165796	.6847143	1.4604632	.7106630	1.4071367	36
25	.6597960	1.5156201	.6851416	1.4595522	.7111009	1.4062702	35
26	.6602136	1.5146614	.6855692	1.4586420	.7115390	1.4054044	34
27	.6606313	1.5137036	.6859969	1.4577326	.7119772	1.4045393	33
28	.6610492	1.5127466	.6864247	1.4568240	.7124157	1.4036749	32
29	.6614673	1.5117905	.6868528	1.4559161	.7128543	1.4028113	31
30	0.6618856	1.5108352	0.6872810	1.4550090	0.7132931	1.4019483	30
31	.6623040	1.5098807	.6877093	1.4541027	.7137320	1.4010860	29
32	.6627225	1.5089271	.6881379	1.4531971	.7141712	1.4002245	28
33	.6631413	1.5079743	.6885666	1.4522923	.7146106	1.3993636	27
34	.6635601	1.5070224	.6889955	1.4513883	.7150501	1.3985034	26
35	.6639792	1.5060713	.6894246	1.4504850	.7154898	1.3976440	25
36	.6643984	1.5051210	.6898538	1.4495825	.7159297	1.3967852	24
37	.6648178	1.5041716	.6902832	1.4486808	.7163698	1.3959272	23
38	.6652373	1.5032229	.6907128	1.4477798	.7168100	1.3950698	22
39	.6656570	1.5022751	.6911425	1.4468796	.7172505	1.3942131	21
40	0.6660769	1.5013282	0.6915725	1.4459801	0.7176911	1.3933571	20
41	.6664969	1.5003821	.6920026	1.4450814	.7181319	1.3925019	19
42	.6669171	1.4994367	.6924328	1.4441834	.7185729	1.3916473	18
43	.6673374	1.4984923	.6928633	1.4432862	.7190140	1.3907934	17
44	.6677580	1.4975486	.6932939	1.4423897	.7194554	1.3899401	16
45	.6681786	1.4966058	.6937247	1.4414940	.7198970	1.3890876	15
46	.6685995	1.4956637	.6941557	1.4405991	.7203387	1.3882358	14
47	.6690205	1.4947225	.6945868	1.4397049	.7207806	1.3873847	13
48	.6694417	1.4937822	.6950181	1.4388114	.7212227	1.3865342	12
49	.6698630	1.4928426	.6954496	1.4379187	.7216650	1.3856844	11
50	0.6702845	1.4919039	0.6958813	1.4370268	0.7221075	1.3848353	10
51	.6707061	1.4909659	.6963131	1.4361356	.7225502	1.3839869	9
52	.6711280	1.4900288	.6967451	1.4352451	.7229930	1.3831392	8
53	.6715500	1.4890925	.6971773	1.4343554	.7234361	1.3822922	7
54	.6719721	1.4881570	.6976097	1.4334664	.7238793	1.3814458	6
55	.6723944	1.4872223	.6980422	1.4325781	.7243227	1.3806001	5
56	.6728169	1.4862884	.6984749	1.4316906	.7247663	1.3797551	4
57	.6732396	1.4853554	.6989078	1.4308039	.7252101	1.3789108	3
58	.6736624	1.4844231	.6993409	1.4299178	.7256540	1.3780672	2
59	.6740854	1.4834916	.6997741	1.4290326	.7260982	1.3772242	1
60	0.6745085	1.4825610	0.7002075	1.4281480	0.7265425	1.3763819	0
	Cotangent	Tangent	Cotangent	Tangent	Cotangent	Tangent	
′	56°		55°		54°		′

TABLE 40. NATURAL TANGENTS AND COTANGENTS (*Continued*)

′	36°		37°		38°		′
	Tangent	Cotangent	Tangent	Cotangent	Tangent	Cotangent	
0	0.7265425	1.3763819	0.7535541	1.3270448	0.7812856	1.2799416	60
1	.7269871	1.3755403	.7540102	1.3262420	.7817542	1.2791745	59
2	.7274318	1.3746994	.7544666	1.3254397	.7822229	1.2784079	58
3	.7278767	1.3738591	.7549232	1.3246381	.7826919	1.2776419	57
4	.7283218	1.3730195	.7553799	1.3238371	.7831611	1.2768765	56
5	.7287671	1.3721806	.7558369	1.3230368	.7836305	1.2761116	55
6	.7292125	1.3713423	.7562941	1.3222370	.7841002	1.2753473	54
7	.7296582	1.3705047	.7567514	1.3214379	.7845700	1.2745835	53
8	.7301041	1.3696678	.7572090	1.3206393	.7850400	1.2738204	52
9	.7305501	1.3688315	.7576668	1.3198414	.7855103	1.2730578	51
10	0.7309963	1.3679959	0.7581248	1.3190441	0.7859808	1.2722957	50
11	.7314428	1.3671610	.7585829	1.3182474	.7864515	1.2715342	49
12	.7318894	1.3663267	.7590413	1.3174513	.7869224	1.2707733	48
13	.7323362	1.3654931	.7594999	1.3166559	.7873935	1.2700130	47
14	.7327832	1.3646602	.7599587	1.3158610	.7878649	1.2692532	46
15	.7332303	1.3638279	.7604177	1.3150668	.7883364	1.2684940	45
16	.7336777	1.3629963	.7608769	1.3142731	.7888082	1.2677353	44
17	.7341253	1.3621653	.7613363	1.3134801	.7892802	1.2669772	43
18	.7345730	1.3613350	.7617959	1.3126876	.7897524	1.2662196	42
19	.7350210	1.3605054	.7622557	1.3118958	.7902248	1.2654626	41
20	0.7354691	1.3596764	0.7627157	1.3111046	0.7906975	1.2647062	40
21	.7359174	1.3588481	.7631759	1.3103140	.7911703	1.2639503	39
22	.7363660	1.3580204	.7636363	1.3095239	.7916434	1.2631950	38
23	.7368147	1.3571934	.7640969	1.3087345	.7921167	1.2624402	37
24	.7372636	1.3563670	.7645577	1.3079457	.7925902	1.2616860	36
25	.7377127	1.3555413	.7650188	1.3071575	.7930640	1.2609323	35
26	.7381620	1.3547162	.7654800	1.3063699	.7935379	1.2601792	34
27	.7386115	1.3538918	.7659414	1.3055828	.7940121	1.2594267	33
28	.7390611	1.3530680	.7664031	1.3047964	.7944865	1.2586747	32
29	.7395110	1.3522449	.7668649	1.3040106	.7949611	1.2579232	31
30	0.7399611	1.3514224	0.7673270	1.3032254	0.7954369	1.2571723	30
31	.7404113	1.3506006	.7677893	1.3024408	.7959110	1.2564219	29
32	.7408618	1.3497794	.7682517	1.3016567	.7963862	1.2556721	28
33	.7413124	1.3489589	.7687144	1.3008733	.7968617	1.2549229	27
34	.7417633	1.3481390	.7691773	1.3000904	.7973374	1.2541742	26
35	.7422143	1.3473198	.7696404	1.2993081	.7978134	1.2534260	25
36	.7426655	1.3465011	.7701037	1.2985265	.7982895	1.2526784	24
37	.7431170	1.3456832	.7705672	1.2977454	.7987659	1.2519313	23
38	.7435686	1.3448658	.7710309	1.2969649	.7992425	1.2511848	22
39	.7440204	1.3440492	.7714948	1.2961850	.7997193	1.2504388	21
40	0.7444724	1.3432331	0.7719589	1.2954057	0.8001963	1.2496933	20
41	.7449246	1.3424177	.7724233	1.2946270	.8006736	1.2489484	19
42	.7453770	1.3416029	.7728878	1.2938488	.8011511	1.2482040	18
43	.7458296	1.3407888	.7733526	1.2930713	.8016288	1.2474602	17
44	.7462824	1.3399753	.7738176	1.2922943	.8021067	1.2467169	16
45	.7467354	1.3391624	.7742827	1.2915179	.8025849	1.2459742	15
46	.7471886	1.3383502	.7747481	1.2907421	.8030632	1.2452320	14
47	.7476420	1.3375386	.7752137	1.2899669	.8035418	1.2444903	13
48	.7480956	1.3367276	.7756795	1.2891922	.8040206	1.2437492	12
49	.7485494	1.3359172	.7761455	1.2884182	.8044997	1.2430086	11
50	0.7490033	1.3351075	0.7766118	1.2876447	0.8049790	1.2422685	10
51	.7494575	1.3342984	.7770782	1.2868718	.8054584	1.2415290	9
52	.7499119	1.3334900	.7775448	1.2860995	.8059382	1.2407900	8
53	.7503665	1.3326822	.7780117	1.2853277	.8064181	1.2400515	7
54	.7508212	1.3318750	.7784788	1.2845566	.8068983	1.2393136	6
55	.7512762	1.3310684	.7789460	1.2837860	.8073787	1.2385762	5
56	.7517314	1.3302624	.7794135	1.2830160	.8078593	1.2378393	4
57	.7521867	1.3294571	.7798812	1.2822466	.8083401	1.2371030	3
58	.7526423	1.3286524	.7803492	1.2814776	.8088212	1.2363672	2
59	.7530981	1.3278483	.7808173	1.2807094	.8093025	1.2356319	1
60	0.7535541	1.3270448	0.7812856	1.2799416	0.8097840	1.2348972	0
′	Cotangent	Tangent	Cotangent	Tangent	Cotangent	Tangent	′
	53°		52°		51°		

TABLE 40. NATURAL TANGENTS AND COTANGENTS (*Continued*)

′	39°		40°		41°		′
	Tangent	Cotangent	Tangent	Cotangent	Tangent	Cotangent	
0	0.8097840	1.2348972	0.8390996	1.1917536	0.8692867	1.1503684	60
1	.8102658	1.2341629	.8395955	1.1910498	.8697976	1.1496928	59
2	.8107478	1.2334292	.8400915	1.1903465	.8703087	1.1490176	58
3	.8112300	1.2326961	.8405878	1.1896437	.8708200	1.1483429	57
4	.8117124	1.2319634	.8410844	1.1889414	.8713316	1.1476687	56
5	.8121951	1.2312313	.8415812	1.1882395	.8718435	1.1469949	55
6	.8126780	1.2304997	.8420782	1.1875382	.8723556	1.1463215	54
7	.8131611	1.2297687	.8425755	1.1868373	.8728680	1.1456486	53
8	.8136444	1.2290381	.8430730	1.1831369	.8733806	1.1449762	52
9	.8141280	1.2283081	.8435708	1.1854370	.8738935	1.1443041	51
10	0.8146118	1.2275786	0.8440688	1.1847376	0.8744067	1.1436326	50
11	.8150958	1.2268496	.8445670	1.1840387	.8749201	1.1429615	49
12	.8155801	1.2261211	.8450655	1.1833402	.8754338	1.1422908	48
13	.8160646	1.2253932	.8455643	1.1826422	.8759478	1.1416206	47
14	.8165493	1.2246658	.8460633	1.1819447	.8764620	1.1409508	46
15	.8170343	1.2239389	.8465625	1.1812477	.8769765	1.1402815	45
16	.8175195	1.2232125	.8470620	1.1805512	.8774912	1.1396126	44
17	.8180049	1.2224866	.8475617	1.1798551	.8780062	1.1389441	43
18	.8184905	1.2217613	.8480617	1.1791595	.8785215	1.1382761	42
19	.8189764	1.2210364	.8485619	1.1784644	.8790370	1.1376086	41
20	0.8194625	1.2203121	0.8490624	1.1777698	0.8795528	1.1369414	40
21	.8199488	1.2195883	.8495631	1.1770756	.8800688	1.1362747	39
22	.8204354	1.2188650	.8500640	1.1763820	.8805852	1.1356085	38
23	.8209222	1.2181422	.8505653	1.1756888	.8811017	1.1349427	37
24	.8214093	1.2174199	.8510667	1.1749960	.8816186	1.1342773	36
25	.8218965	1.2166982	.8515684	1.1743038	.8821357	1.1336124	35
26	.8223840	1.2159769	.8520704	1.1736120	.8826531	1.1329479	34
27	.8228718	1.2152562	.8525726	1.1729207	.8831707	1.1322839	33
28	.8233597	1.2145359	.8530750	1.1722298	.8836886	1.1316203	32
29	.8238479	1.2138162	.8535777	1.1715395	.8842068	1.1309571	31
30	0.8243364	1.2130970	0.8540807	1.1708496	0.8847253	1.1302944	30
31	.8248251	1.2123783	.8545839	1.1701601	.8852440	1.1296321	29
32	.8253140	1.2116601	.8550873	1.1694712	.8857630	1.1289702	28
33	.8258031	1.2109424	.8555910	1.1687827	.8862822	1.1283088	27
34	.8262925	1.2102252	.8560950	1.1680947	.8868017	1.1276478	26
35	.8267821	1.2095085	.8565992	1.1674071	.8873215	1.1269872	25
36	.8272719	1.2087924	.8571037	1.1667200	.8878415	1.1263271	24
37	.8277620	1.2080767	.8576084	1.1660334	.8883619	1.1256674	23
38	.8282523	1.2073615	.8581133	1.1653472	.8888825	1.1250081	22
39	.8287429	1.2066468	.8586185	1.1646615	.8894033	1.1243493	21
40	0.8292337	1.2059327	0.8591240	1.1639763	0.8899244	1.1236909	20
41	.8297247	1.2052190	.8596297	1.1632916	.8904458	1.1230329	19
42	.8302160	1.2045058	.8601357	1.1626073	.8309675	1.1223754	18
43	.8307075	1.2037932	.8606419	1.1619234	.8914894	1.1217183	17
44	.8311992	1.2030810	.8611484	1.1612400	.8920116	1.1210616	16
45	.8316912	1.2023693	.8616551	1.1605571	.8925341	1.1204053	15
46	.8321834	1.2016581	.8321621	1.1598747	.8930569	1.1197495	14
47	.8326759	1.2009475	.8626694	1.1591927	.8935799	1.1190941	13
48	.8331686	1.2002373	.8631768	1.1585112	.8941032	1.1184391	12
49	.8336615	1.1995276	.8636846	1.1578301	.8946268	1.1177846	11
50	0.8341547	1.1988184	0.8641926	1.1571495	0.8951506	1.1171305	10
51	.8346481	1.1981097	.8647009	1.1564693	.8956747	1.1164768	9
52	.8351418	1.1974015	.8652094	1.1557896	.8961991	1.1158235	8
53	.8356357	1.1966938	.8657181	1.1551104	.8967238	1.1151706	7
54	.8361298	1.1959866	.8662272	1.1544316	.8972487	1.1145182	6
55	.8366242	1.1952799	.8667365	1.1537532	.8977739	1.1138662	5
56	.8371188	1.1945736	.8672460	1.1530754	.8982994	1.1132146	4
57	.8376136	1.1938679	.8677558	1.1523979	.8988251	1.1125635	3
58	.8381087	1.1931626	.8682659	1.1517210	.8993512	1.1119127	2
59	.8386041	1.1924579	.8687762	1.1510445	.8998775	1.1112624	1
60	0.8390996	1.1917536	0.8692867	1.1503684	0.9004040	1.1106125	0
′	Cotangent	Tangent	Cotangent	Tangent	Cotangent	Tangent	′
	50°		49°		48°		

TABLE 40. NATURAL TANGENTS AND COTANGENTS (*Continued*)

′	42°		43°		44°		′
	Tangent	Cotangent	Tangent	Cotangent	Tangent	Cotangent	
0	0.9004040	1.1106125	0.9325151	1.0723687	0.9656888	1.0355303	60
1	.9009309	1.1099630	.9330591	1.0717435	.9662511	1.0349277	59
2	.9014580	1.1093140	.9336034	1.0711187	.9668137	1.0343254	58
3	.9019854	1.1086653	.9341479	1.0704943	.9673767	1.0337235	57
4	.9025131	1.1080171	.9346928	1.0698702	.9679399	1.0331220	56
5	.9030411	1.1073693	.9352380	1.0692466	.9685035	1.0325208	55
6	.9035693	1.1067219	.9357834	1.0686233	.9690374	1.0319199	54
7	.9040979	1.1060750	.9363292	1.0680004	.9396316	1.0313195	53
8	.9046267	1.1054284	.9368753	1.0673779	.9701962	1.0307194	52
9	.9051557	1.1047823	.9374216	1.0667558	.9707610	1.0301196	51
10	0.9056851	1.1041365	0.9379683	1.0661341	0.9713262	1.0295203	50
11	.9062147	1.1034913	.9385153	1.0655128	.9718917	1.0289212	49
12	.9067446	1.1028463	.9390625	1.0648918	.9724575	1.0283226	48
13	.9072748	1.1022019	.9396101	1.0642713	.9730236	1.0277243	47
14	.9078053	1.1015578	.9401579	1.0636511	.9735901	1.0271263	46
15	.9083360	1.1009141	.9407061	1.0630313	.9741569	1.0265287	45
16	.9088671	1.1002709	.9412545	1.0624119	.9747240	1.0259315	44
17	.9093984	1.0996281	.9418033	1.0617929	.9752914	1.0253346	43
18	.9099300	1.0989857	.9423523	1.0611742	.9758591	1.0247381	42
19	.9104619	1.0983436	.9429017	1.0605560	.9764272	1.0241419	41
20	0.9109940	1.0977020	0.9434513	1.0599381	0.9769956	1.0235461	40
21	.9115265	1.0970609	.9440013	1.0593206	.9775643	1.0229506	39
22	.9120592	1.0964201	.9445516	1.0587035	.9781333	1.0223555	38
23	.9125922	1.0957797	.9451021	1.0580867	.9787027	1.0217608	37
24	.9131255	1.0951397	.9456530	1.0574704	.9792724	1.0211664	36
25	.9136591	1.0945002	.9462042	1.0568544	.9798424	1.0205723	35
26	.9141929	1.0938610	.9467556	1.0562388	.9804127	1.0199786	34
27	.9147270	1.0932223	.9473074	1.0556235	.9809833	1.0193853	33
28	.9152615	1.0925840	.9478595	1.0550087	.9815543	1.0187923	32
29	.9157962	1.0919460	.9484119	1.0543942	.9821256	1.0181997	31
30	0.9163312	1.0913085	0.9489646	1.0537801	0.9826973	1.0176074	30
31	.9168665	1.0906714	.9495176	1.0531664	.9832692	1.0170155	29
32	.9174020	1.0900347	.9500709	1.0525531	.9838415	1.0164239	28
33	.9179379	1.0893984	.9506245	1.0519401	.9844141	1.0158326	27
34	.9184740	1.0887624	.9511784	1.0513275	.9849871	1.0152418	26
35	.9190104	1.0881269	.9517326	1.0507153	.9855603	1.0146512	25
36	.9195471	1.0874918	.9522871	1.0501034	.9861339	1.0140610	24
37	.9200841	1.0868571	.9528420	1.0494920	.9867079	1.0134712	23
38	.9206214	1.0862228	.9533971	1.0488809	.9872821	1.0128817	22
39	.9211590	1.0855889	.9539526	1.0482702	.9878567	1.0122925	21
40	0.9216969	1.0849554	0.9545083	1.0476598	0.9884316	1.0117038	20
41	.9222350	1.0843223	.9550644	1.0470498	.9890069	1.0111153	19
42	.9227734	1.0836896	.9556208	1.0464402	.9895825	1.0105272	18
43	.9233122	1.0830573	.9561774	1.0458310	.9901584	1.0099394	17
44	.9238512	1.0824254	.9567344	1.0452221	.9907346	1.0093520	16
45	.9243905	1.0817939	.9572917	1.0446136	.9913112	1.0087649	15
46	.9249301	1.0811628	.9578494	1.0440055	.9918881	1.0081782	14
47	.9254700	1.0805321	.9584073	1.0433977	.9924654	1.0075918	13
48	.9260102	1.0799018	.9589655	1.0427904	.9930429	1.0070058	12
49	.9265506	1.0792718	.9595241	1.0421833	.9936208	1.0064201	11
50	0.9270914	1.0786423	0.9600829	1.0415767	0.9941991	1.0058348	10
51	.9276324	1.0780132	.9606421	1.0409704	.9947777	1.0052497	9
52	.9281738	1.0773845	.9612016	1.0403645	.9953566	1.0046651	8
53	.9287154	1.0767561	.9617614	1.0397589	.9959358	1.0040808	7
54	.9292573	1.0761282	.9623215	1.0391538	.9965154	1.0034968	6
55	.9297996	1.0755006	.9628819	1.0385489	.9970953	1.0029131	5
56	.9303421	1.0748734	.9634427	1.0379445	.9976756	1.0023298	4
57	.9308849	1.0742467	.9640037	1.0373404	.9982562	1.0017469	3
58	.9314280	1.0736203	.9645651	1.0367367	.9988371	1.0011642	2
59	.9319714	1.0729943	.9651268	1.0361333	.9994184	1.0005819	1
60	0.9325151	1.0723687	0.9656888	1.0355303	1.0000000	1.0000000	0
′	Cotangent	Tangent	Cotangent	Tangent	Cotangent	Tangent	′
	47°		46°		45°		

TABLE 41. NATURAL VERSINES AND EXTERNAL SECANTS

′	0°		1°		2°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	0.0000000	0.0000000	0.0001523	0.0001523	0.0006092	0.0006095	0
1	.0000000	.0000000	.0001574	.0001574	.0006194	.0006198	1
2	.0000002	.0000002	.0001626	.0001627	.0006296	.0006300	2
3	.0000004	.0000004	.0001679	.0001679	.0006400	.0006404	3
4	.0000007	.0000007	.0001733	.0001733	.0006505	.0006509	4
5	.0000011	.0000011	.0001787	.0001788	.0006610	.0006614	5
6	.0000015	.0000015	.0001843	.0001843	.0006716	.0006721	6
7	.0000021	.0000021	.0001899	.0001900	.0006823	.0006828	7
8	.0000027	.0000027	.0001956	.0001957	.0006931	.0006936	8
9	.0000034	.0000034	.0002014	.0002015	.0007040	.0007045	9
10	0.0000042	0.0000042	0.0002073	0.0002073	0.0007149	0.0007154	10
11	.0000051	.0000051	.0002133	.0002133	.0007260	.0007265	11
12	.0000061	.0000061	.0002193	.0002194	.0007371	.0007376	12
13	.0000072	.0000072	.0002255	.0002255	.0007483	.0007489	13
14	.0000083	.0000083	.0002317	.0002317	.0007596	.0007602	14
15	.0000095	.0000095	.0002380	.0002380	.0007710	.0007716	15
16	.0000108	.0000108	.0002444	.0002444	.0007824	.0007830	16
17	.0000122	.0000122	.0002508	.0002509	.0007940	.0007946	17
18	.0000137	.0000137	.0002574	.0002575	.0008056	.0008063	18
19	.0000153	.0000153	.0002640	.0002641	.0008173	.0008180	19
20	0.0000169	0.0000169	0.0002708	0.0002708	0.0008291	0.0008298	20
21	.0000187	.0000187	.0002776	.0002776	.0008410	.0008417	21
22	.0000205	.0000205	.0002844	.0002845	.0008530	.0008537	22
23	.0000224	.0000224	.0002914	.0002915	.0008650	.0008658	23
24	.0000244	.0000244	.0002985	.0002986	.0008772	.0008779	24
25	.0000264	.0000264	.0003057	.0003058	.0008894	.0008902	25
26	.0000286	.0000286	.0003129	.0003130	.0009017	.0009025	26
27	.0000308	.0000308	.0003202	.0003203	.0009141	.0009149	27
28	.0000332	.0000332	.0003276	.0003277	.0009266	.0009274	28
29	.0000356	.0000356	.0003351	.0003352	.0009391	.0009400	29
30	0.0000381	0.0000381	0.0003427	0.0003428	0.0009518	0.0009527	30
31	.0000407	.0000407	.0003503	.0003505	.0009645	.0009654	31
32	.0000433	.0000433	.0003581	.0003582	.0009773	.0009783	32
33	.0000461	.0000461	.0003659	.0003660	.0009902	.0009912	33
34	.0000489	.0000489	.0003738	.0003739	.0010032	.0010042	34
35	.0000518	.0000518	.0003818	.0003820	.0010163	.0010173	35
36	.0000548	.0000548	.0003899	.0003900	.0010294	.0010305	36
37	.0000579	.0000579	.0003980	.0003982	.0010427	.0010438	37
38	.0000611	.0000611	.0004063	.0004065	.0010560	.0010571	38
39	.0000643	.0000644	.0004146	.0004148	.0010694	.0010705	39
40	0.0000677	0.0000677	0.0004230	0.0004232	0.0010829	0.0010841	40
41	.0000711	.0000711	.0004316	.0004317	.0010965	.0010977	41
42	.0000746	.0000746	.0004401	.0004403	.0011101	.0011114	42
43	.0000782	.0000782	.0004488	0004490	.0011239	.0011251	43
44	.0000819	.0000819	.0004576	.0004578	.0011377	.0011390	44
45	.0000857	.0000857	.0004664	.0004666	.0011516	.0011529	45
46	.0000895	.0000895	.0004753	.0004756	.0011656	.0011670	46
47	.0000935	.0000935	.0004843	.0004846	.0011797	.0011811	47
48	.0000975	.0000975	.0004934	.0004937	.0011939	.0011953	48
49	.0001016	.0001016	.0005026	.0005029	.0012081	.0012096	49
50	0.0001058	0.0001058	0.0005119	0.0005121	0.0012225	0.0012239	50
51	.0001100	.0001101	.0005212	.0005215	.0012369	.0012384	51
52	.0001144	.0001144	.0005307	.0005309	.0012514	.0012529	52
53	.0001188	.0001189	.0005402	.0005405	.0012660	.0012676	53
54	.0001234	.0001234	.0005498	.0005501	.0012806	.0012823	54
55	.0001280	.0001280	.0005595	.0005598	.0012954	.0012971	55
56	.0001327	.0001327	.0005692	.0005696	.0013102	.0013120	56
57	.0001375	.0001375	.0005791	.0005794	.0013252	.0013269	57
58	.0001423	.0001423	.0005890	.0005894	.0013402	.0013420	58
59	.0001473	.0001473	.0005991	.0005994	.0013553	.0013571	59
60	0.0001523	0.0001523	0.0006092	0.0006095	0.0013705	0.0013723	60

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TABLE 41. NATURAL VERSINES AND EXTERNAL SECANTS (*Continued*)

′	3°		4°		5°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	0.0013705	0.0013723	0.0024359	0.0024419	0.0038053	0.0088198	0
1	.0013857	.0013877	.0024563	.0024623	.0038307	.0038454	1
2	.0014011	.0014030	.0024767	.0024829	.0038562	.0038711	2
3	.0014165	.0014185	.0024972	.0025035	.0038817	.0038969	3
4	.0014320	.0014341	.0025178	.0025241	.0039074	.0039227	4
5	.0014476	.0014497	.0025385	.0025449	.0039331	.0039486	5
6	.0014633	.0014655	.0025592	.0025658	.0039589	.0039747	6
7	.0014791	.0014813	.0025801	.0025867	.0039848	.0040008	7
8	.0014950	.0014972	.0026010	.0026078	.0040108	.0040270	8
9	.0015109	.0015132	.0026220	.0026289	.0040369	.0040533	9
10	0.0015269	0.0015293	0.0026431	0.0026501	0.0040630	0.0040796	10
11	.0015430	.0015454	.0026643	.0026714	.0040893	.0041061	11
12	.0015592	.0015617	.0026855	.0026928	.0041156	.0041326	12
13	.0015755	.0015780	.0027069	.0027142	.0041420	.0041592	13
14	.0015919	.0015944	.0027283	.0027358	.0041685	.0041859	14
15	.0016083	.0016109	.0027498	.0027574	.0041951	.0042127	15
16	.0016249	.0016275	.0027714	.0027791	.0042217	.0042396	16
17	.0016415	.0016442	.0027931	.0028009	.0042485	.0042666	17
18	.0016582	.0016609	.0028149	.0028228	.0042753	.0042937	18
19	.0016750	.0016778	.0028367	.0028448	.0043022	.0043208	19
20	0.0016918	0.0016947	0.0028587	0.0028669	0.0043292	0.0043480	20
21	.0017088	.0017117	.0028807	.0028890	.0043563	.0043753	21
22	.0017258	.0017288	.0029028	.0029112	.0043835	.0044028	22
23	.0017430	.0017460	.0029250	.0029336	.0044107	.0044302	23
24	.0017602	.0017633	.0029472	.0029560	.0044380	.0044578	24
25	.0017775	.0017806	.0029696	.0029785	.0044655	.0044855	25
26	.0017948	.0017981	.0029920	.0030010	.0044930	.0045132	26
27	.0018123	.0018156	.0030146	.0030237	.0045205	.0045411	27
28	.0018299	.0018332	.0030372	.0030464	.0045482	.0045690	28
29	.0018475	.0018509	.0030599	.0030693	.0045760	.0045970	29
30	0.0018652	0.0018687	0.0030827	0.0030922	0.0046038	0.0046251	30
31	.0018830	.0018866	.0031055	.0031152	.0046317	.0046533	31
32	.0019009	.0019045	.0031285	.0031383	.0046597	.0046815	32
33	.0019189	.0019225	.0031515	.0031615	.0046878	.0047099	33
34	.0019369	.0019407	.0031746	.0031847	.0047160	.0047383	34
35	.0019550	.0019589	.0031978	.0032081	.0047443	.0047669	35
36	.0019733	.0019772	.0032211	.0032315	.0047726	.0047955	36
37	.0019916	.0019956	.0032445	.0032551	.0048010	.0048242	37
38	.0020100	.0020140	.0032679	.0032787	.0048295	.0048530	38
39	.0020284	.0020326	.0032915	.0033024	.0048581	.0048819	39
40	0.0020470	0.0020512	0.0033151	0.0033261	0.0048868	0.0049108	40
41	.0020657	.0020699	.0033388	.0033500	.0049156	.0049399	41
42	.0020844	.0020887	.0033626	.0033740	.0049444	.0049690	42
43	.0021032	.0021076	.0033865	.0033980	.0049734	.0049982	43
44	.0021221	.0021266	.0034105	.0034221	.0050024	.0050275	44
45	.0021411	.0021457	.0034345	.0034463	.0050315	.0050569	45
46	.0021601	.0021648	.0034586	.0034706	.0050607	.0050864	46
47	.0021793	.0021841	.0034828	.0034950	.0050899	.0051160	47
48	.0021985	.0022034	.0035071	.0035195	.0051193	.0051456	48
49	.0022179	.0022228	.0035315	.0035440	.0051487	.0051754	49
50	0.0022373	0.0022423	0.0035560	0.0035687	0.0051783	0.0052052	50
51	.0022567	.0022619	.0035805	.0035934	.0052079	.0052351	51
52	.0022763	.0022815	.0036052	.0036182	.0052375	.0052651	52
53	.0022960	.0023013	.0036299	.0036431	.0052673	.0052952	53
54	.0023157	.0023211	.0036547	.0036681	.0052972	.0053254	54
55	.0023355	.0023410	.0036796	.0036932	.0053271	.0053557	55
56	.0023555	.0023610	.0037046	.0037183	.0053572	.0053860	56
57	.0023755	.0023811	.0037296	.0037436	.0053873	.0054164	57
58	.0023955	.0024013	.0037548	.0037689	.0054175	.0054470	58
59	.0024157	.0024216	.0037800	.0037943	.0054477	.0054776	59
60	0.0024359	0.0024419	0.0038053	0.0038198	0.0054781	0.0055083	60

TABLE 41. NATURAL VERSINES AND EXTERNAL SECANTS (*Continued*)

′	6°		7°		8°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	0.0054781	0.0055083	0.0074538	0.0075098	0.0097319	0.0098276	0
1	.0055086	.0055391	.0074893	.0075459	.0097725	.0098689	1
2	.0055391	.0055699	.0075249	.0075820	.0098131	.0099103	2
3	.0055697	.0056009	.0075606	.0076182	.0098538	.0099518	3
4	.0056004	.0056319	.0075963	.0076545	.0098945	.0099934	4
5	.0056312	.0056631	.0076321	.0076908	.0099354	.0100351	5
6	.0056621	.0056943	.0076681	.0077273	.0099763	.0100769	6
7	.0056930	.0057256	.0077041	.0077639	.0100174	.0101187	7
8	.0057240	.0057570	.0077401	.0078005	.0100585	.0101607	8
9	.0057552	.0057885	.0077763	.0078372	.0100997	.0102027	9
10	0.0057864	0.0058200	0.0078126	0.0078741	0.0101410	0.0102449	10
11	.0058177	.0058517	.0078489	.0079110	.0101823	.0102871	11
12	.0058490	.0058834	.0078853	.0079480	.0102238	.0103294	12
13	.0058805	.0059153	.0079218	.0079851	.0102653	.0103718	13
14	.0059120	.0059472	.0079584	.0080222	.0103069	.0104143	14
15	.0059437	.0059792	.0079951	.0080595	.0103486	.0104568	15
16	.0059754	.0060113	.0080318	.0080968	.0103904	.0104995	16
17	.0060072	.0060435	.0080686	.0081343	.0104323	.0105422	17
18	.0060390	.0060757	.0081056	.0081718	.0104742	.0105851	18
19	.0060710	.0061081	.0081426	.0082094	.0105162	.0106280	19
20	0.0061031	0.0061405	0.0081796	0.0082471	0.0105584	0.0106710	20
21	.0061352	.0061731	.0082168	.0082849	.0106006	.0107141	21
22	.0061674	.0062057	.0082541	.0083228	.0106428	.0107573	22
23	.0061997	.0062384	.0082914	.0083607	.0106852	.0108006	23
24	.0062321	.0062712	.0083283	.0083988	.0107277	.0108440	24
25	.0062645	.0063040	.0083663	.0084369	.0107702	.0108875	25
26	.0062971	.0063370	.0084039	.0084752	.0108128	.0109310	26
27	.0063297	.0063701	.0084416	.0085135	.0108555	.0109747	27
28	.0063625	.0064032	.0084794	.0085519	.0108983	.0110184	28
29	.0063953	.0064364	.0085172	.0085904	.0109412	.0110622	29
30	0.0064281	0.0064697	0.0085551	0.0086290	0.0109841	0.0111061	30
31	.0064611	.0065031	.0085931	.0086676	.0110272	.0111501	31
32	.0064942	.0065366	.0086312	.0087064	.0110703	.0111942	32
33	.0065273	.0065702	.0086694	.0087452	.0111135	.0112384	33
34	.0065605	.0066039	.0087077	.0087842	.0111568	.0112827	34
35	.0065938	.0066376	.0087460	.0088232	.0112002	.0113270	35
36	.0066272	.0066714	.0087845	.0088623	.0112436	.0113715	36
37	.0066607	.0067054	.0088230	.0089015	.0112872	.0114160	37
38	.0066943	.0067394	.0088616	.0089408	.0113308	.0114606	38
39	.0067279	.0067735	.0089003	.0089802	.0113745	.0115054	39
40	0.0067616	0.0068077	0.0089390	0.0090196	0.0114183	0.0115502	40
41	.0067955	.0068419	.0089779	.0090592	.0114622	.0115951	41
42	.0068294	.0068763	.0090168	.0090988	.0115061	.0116400	42
43	.0068633	.0069108	.0090558	.0091386	.0115502	.0116851	43
44	.0068974	.0069453	.0090949	.0091784	.0115943	.0117303	44
45	.0069315	.0069799	.0091341	.0092183	.0116385	.0117755	45
46	.0069658	.0070146	.0091734	.0092583	.0116828	.0118209	46
47	.0070001	.0070494	.0092127	.0092984	.0117272	.0118663	47
48	.0070345	.0070843	.0092522	.0093386	.0117716	.0119118	48
49	.0070690	.0071193	.0092917	.0093788	.0118162	.0119575	49
50	0.0071035	0.0071544	0.0093313	0.0094192	0.0118608	0.0120032	50
51	.0071382	.0071895	.0093710	.0094596	.0119055	.0120489	51
52	.0071729	.0072248	.0094107	.0095001	.0119503	.0120948	52
53	.0072078	.0072601	.0094506	.0095408	.0119952	.0121408	53
54	.0072427	.0072955	.0094905	.0095815	.0120401	.0121869	54
55	.0072776	.0073310	.0095306	.0096223	.0120852	.0122330	55
56	.0073127	.0073666	.0095707	.0096631	.0121303	.0122793	56
57	.0073479	.0074023	.0096109	.0097041	.0121755	.0123256	57
58	.0073831	.0074380	.0096511	.0097452	.0122208	.0123720	58
59	.0074184	.0074739	.0096915	.0097863	.0122662	.0124185	59
60	0.0074538	0.0075098	0.0097319	0.0098276	0.0123117	0.0124651	60

TABLE 41. NATURAL VERSINES AND EXTERNAL SECANTS (*Continued*)

′	9°		10°		11°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	0.0123117	0.0124651	0.0151922	0.0154266	0.0183728	0.0187167	0
1	.0123572	.0125118	.0152428	.0154787	.0184284	.0187743	1
2	.0124028	.0125586	.0152934	.0155310	.0184840	.0188321	2
3	.0124486	.0126055	.0153442	.0155833	.0185397	.0188899	3
4	.0124943	.0126524	.0153950	.0156357	.0185955	.0189478	4
5	.0125402	.0126995	.0154458	.0156882	.0186514	.0190059	5
6	.0125862	.0127466	.0154968	.0157408	.0187073	.0190640	6
7	.0126322	.0127939	.0155479	.0157934	.0187634	.0191222	7
8	.0126784	.0128412	.0155990	.0158462	.0188195	.0191805	8
9	.0127246	.0128886	.0156502	.0158991	.0188757	.0192389	9
10	0.0127709	0.0129361	0.0157015	0.0159520	0.0189320	0.0192973	10
11	.0128173	.0129837	.0157529	.0160050	.0189884	.0193559	11
12	.0128637	.0130314	.0158044	.0160582	.0190448	.0194146	12
13	.0129103	.0130791	.0158559	.0161114	.0191014	.0194734	13
14	.0129569	.0131270	.0159076	.0161647	.0191580	.0195322	14
15	.0130036	.0131750	.0159593	.0162181	.0192147	.0195912	15
16	.0130504	.0132230	.0160111	.0162716	.0192715	.0196502	16
17	.0130973	.0132711	.0160630	.0163252	.0193284	.0197093	17
18	.0131443	.0133194	.0161150	.0163789	.0193853	.0197686	18
19	.0131913	.0133677	.0161670	.0164327	.0194424	.0198279	19
20	0.0132385	0.0134161	0.0162192	0.0164865	0.0194995	0.0198873	20
21	.0132857	.0134646	.0162714	.0165405	.0195567	.0199468	21
22	.0133330	.0135132	.0163237	.0165946	.0196140	.0200064	22
23	.0133804	.0135618	.0163761	.0166487	.0196714	.0200661	23
24	.0134278	.0136106	.0164285	.0167029	.0197288	.0201259	24
25	.0134751	.0136595	.0164811	.0167573	.0197864	.0201858	25
26	.0135230	.0137084	.0165337	.0168117	.0198440	.0202457	26
27	.0135707	.0137574	.0165864	.0168662	.0199017	.0203058	27
28	.0136185	.0138066	.0166392	.0169208	.0199595	.0203660	28
29	.0136664	.0138558	.0166921	.0169755	.0200173	.0204262	29
30	0.0137144	0.0139051	0.0167451	0.0170303	0.0200753	0.0204866	30
31	.0137625	.0139545	.0167981	.0170851	.0201333	.0205470	31
32	.0138106	.0140040	.0168513	.0171401	.0201914	.0206075	32
33	.0138588	.0140536	.0169045	.0171952	.0202496	.0206682	33
34	.0139071	.0141032	.0169578	.0172503	.0203079	.0207289	34
35	.0139555	.0141530	.0170112	.0173056	.0203663	.0207897	35
36	.0140010	.0142029	.0170647	.0173609	.0204248	.0208506	36
37	.0140525	.0142528	.0171182	.0174163	.0204833	.0209116	37
38	.0141012	.0143028	.0171718	.0174719	.0205419	.0209727	38
39	.0141499	.0143530	.0172256	.0175275	.0206006	.0210339	39
40	0.0141987	0.0144032	0.0172794	0.0175832	0.0206594	0.0210952	40
41	.0142476	.0144535	.0173332	.0176390	.0207182	.0211566	41
42	.0142965	.0145039	.0173872	.0176949	.0207772	.0212180	42
43	.0143456	.0145544	.0174413	.0177509	.0208362	.0212796	43
44	.0143947	.0146050	.0174954	.0178069	.0208953	.0213413	44
45	.0144439	.0146556	.0175496	.0178631	.0209545	.0214030	45
46	.0144932	.0147064	.0176039	.0179194	.0210138	.0214649	46
47	.0145420	.0147572	.0176583	.0179757	.0210732	.0215268	47
48	.0145921	.0148082	.0177127	.0180321	.0211326	.0215888	48
49	.0146417	.0148592	.0177673	.0180887	.0211921	.0216510	49
50	0.0146913	0.0149103	0.0178219	0.0181453	0.0212517	0.0217132	50
51	.0147410	.0149616	.0178766	.0182020	.0213114	.0217755	51
52	.0147908	.0150129	.0179314	.0182588	.0213712	.0218379	52
53	.0148407	.0150643	.0179863	.0183158	.0214311	.0219004	53
54	.0148907	.0151158	.0180413	.0183728	.0214910	.0219630	54
55	.0149407	.0151673	.0180963	.0184298	.0215510	.0220257	55
56	.0149909	.0152190	.0181515	.0184870	.0216111	.0220885	56
57	.0150411	.0152708	.0182067	.0185443	.0216713	.0221514	57
58	.0150914	.0153226	.0182620	.0186017	.0217316	.0222144	58
59	.0151418	.0153746	.0183174	.0186591	.0217920	.0222774	59
60	0.0151922	0.0154266	0.0183728	0.0187167	0.0218524	0.0223406	60

Table 41. Natural Versines and External Secants (*Continued*)

′	12°		13°		14°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	0.0218524	0.0223406	0.0256299	0.0263041	0.0297043	0.0306136	0
1	.0219129	.0224039	.0256954	.0263731	.0297747	.0306884	1
2	.0219735	.0224672	.0257610	.0264421	.0298452	.0307633	2
3	.0220342	.0225307	.0258266	.0265113	.0299158	.0308383	3
4	.0220950	.0225942	.0258923	.0265806	.0299864	.0309134	4
5	.0221558	.0226578	.0259581	.0266499	.0300572	.0309886	5
6	.0222168	.0227216	.0260240	.0267194	.0301280	.0310639	6
7	.0222778	.0227854	.0260900	.0267889	.0301989	.0311393	7
8	.0223389	.0228493	.0261561	.0268586	.0302699	.0312147	8
9	.0224001	.0229133	.0262222	.0269283	.0303409	.0312903	9
10	0.0224613	0.0229774	0.0262884	0.0269982	0.0304121	0.0313660	10
11	.0225227	.0230416	.0263547	.0270681	.0304833	.0314418	11
12	.0225841	.0231059	.0264211	.0271381	.0305547	.0315177	12
13	.0226456	.0231703	.0264876	.0272082	.0306260	.0315936	13
14	.0227072	.0232348	.0265541	.0272785	.0306975	.0316697	14
15	.0227689	.0232994	.0266207	.0273488	.0307691	.0317459	15
16	.0228307	.0233641	.0266875	.0274192	.0308407	.0318222	16
17	.0228925	.0234288	.0267542	.0274897	.0309125	.0318985	17
18	.0229544	.0234937	.0268211	.0275603	.0309843	.0319750	18
19	.0230164	.0235587	.0268881	.0276310	.0310562	.0320516	19
20	0.0230785	0.0236237	0.0269551	0.0277018	0.0311281	0.0321282	20
21	.0231407	.0236889	.0270223	.0277727	.0312002	.0322050	21
22	.0232030	.0237541	.0270895	.0278437	.0312723	.0322818	22
23	.0232653	.0238195	.0271568	.0279148	.0313445	.0323588	23
24	.0233277	.0238849	.0272241	.0279860	.0314168	.0324359	24
25	.0233902	.0239504	.0272916	.0280573	.0314892	.0325130	25
26	.0234528	.0240161	.0273591	.0281287	.0315617	.0325903	26
27	.0235155	.0240818	.0274267	.0282002	.0316342	.0326676	27
28	.0235782	.0241476	.0274944	.0282717	.0317069	.0327451	28
29	.0236411	.0242135	.0275622	.0283434	.0317796	.0328227	29
30	0.0237040	0.0242795	0.0276301	0.0284152	0.0318524	0.0329003	30
31	.0237670	.0243456	.0276980	.0284871	.0319252	.0329781	31
32	.0238301	.0244118	.0277661	.0285590	.0319982	.0330559	32
33	.0238932	.0244781	.0278342	.0286311	.0320712	.0331339	33
34	.0239565	.0245445	.0279024	.0287033	.0321443	.0332119	34
35	.0240198	.0246110	.0279706	.0287755	.0322175	.0332901	35
36	.0240832	.0246776	.0280390	.0288479	.0322908	.0333683	36
37	.0241467	.0247442	.0281074	.0289203	.0323642	.0334467	37
38	.0242103	.0248110	.0281760	.0289929	.0324376	.0335251	38
39	.0242740	.0248779	.0282446	.0290655	.0325112	.0336037	39
40	0.0243377	0.0249448	0.0283133	0.0291383	0.0325848	0.0336823	40
41	.0244015	.0250119	.0283820	.0292111	.0326585	.0337611	41
42	.0244655	.0250790	.0284509	.0292840	.0327322	.0338399	42
43	.0245294	.0251463	.0285198	.0293571	.0328061	.0339188	43
44	.0245935	.0252136	.0285888	.0294302	.0328800	.0339979	44
45	.0246577	.0252811	.0286579	.0295034	.0329541	.0340770	45
46	.0247219	.0253486	.0287271	.0295768	.0330282	.0341563	46
47	.0247862	.0254162	.0287964	.0296502	.0331023	.0342356	47
48	.0248506	.0254839	.0288657	.0297237	.0331766	.0343151	48
49	.0249151	.0255518	.0289351	.0297973	.0332510	.0343946	49
50	0.0249797	0.0256197	0.0290047	0.0298711	0.0333254	0.0344743	50
51	.0250444	.0256877	.0290742	.0299449	.0333999	.0345540	51
52	.0251091	.0257558	.0291439	.0300188	.0334745	.0346338	52
53	.0251739	.0258240	.0292137	.0300928	.0335492	.0347138	53
54	.0252388	.0258923	.0292835	.0301669	.0336239	.0347938	54
55	.0253038	.0259607	.0293534	.0302411	.0336988	.0348740	55
56	.0253689	.0260292	.0294234	.0303154	.0337737	.0349542	56
57	.0254340	.0260978	.0294935	.0303898	.0338487	.0350346	57
58	.0254992	.0261665	.0295637	.0304643	.0339238	.0351150	58
59	.0255645	.0262352	.0296339	.0305389	.0339989	.0351955	59
60	0.0256299	0.0263041	0.0297043	0.0306136	0.0340742	0.0352762	60

TABLE 41. NATURAL VERSINES AND EXTERNAL SECANTS (*Continued*)

′	15°		16°		17°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	0.0340742	0.0352762	0.0387383	0.0402994	0.0436952	0.0456918	0
1	.0341495	.0353569	.0388185	.0403863	.0437803	.0457848	1
2	.0342249	.0354378	.0388988	.0404732	.0438655	.0458780	2
3	.0343004	.0355187	.0389792	.0405602	.0439508	.0459712	3
4	.0343760	.0355998	.0390597	.0406473	.0440361	.0460646	4
5	.0344516	.0356809	.0391402	.0407346	.0441215	.0461581	5
6	.0345274	.0357621	.0392208	.0408219	.0442070	.0462516	6
7	.0346032	.0358435	.0393016	.0409094	.0442926	.0463453	7
8	.0346791	.0359249	.0393823	.0409969	.0443782	.0464391	8
9	.0347551	.0360065	.0394632	.0410845	.0444639	.0465330	9
10	0.0348311	0.0360881	0.0395442	0.0411723	0.0445498	0.0466270	10
11	.0349073	.0361699	.0396252	.0412601	.0446357	.0467211	11
12	.0349835	.0362517	.0397063	.0413481	.0447216	.0468153	12
13	.0350598	.0363337	.0397875	.0414362	.0448077	.0469096	13
14	.0351362	.0364157	.0398688	.0415243	.0448938	.0470040	14
15	.0352127	.0364979	.0399501	.0416126	.0449801	.0470986	15
16	.0352892	.0365801	.0400316	.0417009	.0450664	.0471932	16
17	.0353659	.0366625	.0401131	.0417894	.0451527	.0472879	17
18	.0354426	.0367449	.0401947	.0418780	.0452392	.0473828	18
19	.0355194	.0368275	.0402764	.0419667	.0453257	.0474777	19
20	0.0355963	0.0369101	0.0403582	0.0420554	0.0454124	0.0475728	20
21	.0356732	.0369929	.0404400	.0421443	.0454991	.0476679	21
22	.0357503	.0370757	.0405219	.0422333	.0455859	.0477632	22
23	.0358274	.0371587	.0406039	.0423224	.0456727	.0478586	23
24	.0359046	.0372417	.0406860	.0424116	.0457597	.0479540	24
25	.0359819	.0373249	.0407682	.0425009	.0458467	.0480496	25
26	.0360593	.0374082	.0408504	.0425903	.0459338	.0481453	26
27	.0361367	.0374915	.0409328	.0426798	.0460210	.0482411	27
28	.0362142	.0375750	.0410152	.0427694	.0461083	.0483370	28
29	.0362919	.0376585	.0410977	.0428591	.0461956	.0484330	29
30	0.0363695	0.0377422	0.0411803	0.0429489	0.0462830	0.0485291	30
31	.0364473	.0378260	.0412629	.0430388	.0463706	.0486253	31
32	.0365252	.0379098	.0413457	.0431289	.0464582	.0487217	32
33	.0366031	.0379938	.0414285	.0432190	.0465458	.0488181	33
34	.0366811	.0380779	.0415114	.0433092	.0466336	.0489146	34
35	.0367592	.0381621	.0415944	.0433995	.0467214	.0490113	35
36	.0368374	.0382463	.0416774	.0434900	.0468093	.0491080	36
37	.0369157	.0383307	.0417606	.0435805	.0468973	.0492049	37
38	.0369940	.0384152	.0418438	.0436712	.0469854	.0493019	38
39	.0370725	.0384998	.0419271	.0437619	.0470736	.0493989	39
40	0.0371510	0.0385844	0.0420105	0.0438528	0.0471618	0.0494961	40
41	.0372296	.0386692	.0420940	.0439437	.0472501	.0495934	41
42	.0373083	.0387541	.0421775	.0440348	.0473385	.0496908	42
43	.0373870	.0388391	.0422611	.0441259	.0474270	.0497883	43
44	.0374658	.0389242	.0423448	.0442172	.0475156	.0498859	44
45	.0375448	.0390094	.0424286	.0443086	.0476042	.0499836	45
46	.0376238	.0390947	.0425125	.0444001	.0476929	.0500815	46
47	.0377028	.0391800	.0425965	.0444917	.0477817	.0501794	47
48	.0377820	.0392655	.0426805	.0445833	.0478706	.0502774	48
49	.0378613	.0393511	.0427646	.0446751	.0479596	.0503756	49
50	0.0379406	0.0394368	0.0428488	0.0447670	0.0480486	0.0504738	50
51	.0380200	.0395226	.0429331	.0448590	.0481377	.0505722	51
52	.0380995	.0396085	.0430175	.0449511	.0482269	.0506706	52
53	.0381790	.0396945	.0431019	.0450433	.0483162	.0507692	53
54	.0382587	.0397806	.0431864	.0451357	.0484056	.0508679	54
55	.0383384	.0398669	.0432710	.0452281	.0484950	.0509667	55
56	.0384182	.0399532	.0433557	.0453206	.0485846	.0510656	56
57	.0384981	.0400396	.0434405	.0454132	.0486742	.0511646	57
58	.0385781	.0401261	.0435253	.0455060	.0487639	.0512637	58
59	.0386582	.0402127	.0436102	.0455988	.0488536	.0513629	59
60	0.0387383	0.0402994	0.0436952	0.0456918	0.0489435	0.0514622	60

TABLE 41. NATURAL VERSINES AND EXTERNAL SECANTS (*Continued*)

′	18°		19°		20°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	0.0489435	0.0514622	0.0544814	0.0576207	0.0603074	0.0641778	0
1	.0490334	.0515617	.0545762	.0577267	.0604069	.0642905	1
2	.0491234	.0516612	.0546710	.0578328	.0605065	.0644033	2
3	.0492135	.0517608	.0547659	.0579390	.0606062	.0645163	3
4	.0493037	.0518606	.0548609	.0580453	.0607060	.0646294	4
5	.0493939	.0519605	.0549559	.0581517	.0608058	.0647425	5
6	.0494843	.0520604	.0550511	.0582583	.0609057	.0648558	6
7	.0495747	.0521605	.0551463	.0583649	.0610058	.0649693	7
8	.0496652	.0522607	.0552416	.0584717	.0611058	.0650828	8
9	.0497557	.0523610	.0553370	.0585786	.0612060	.0651964	9
10	0.0498464	0.0524614	0.0554325	0.0586855	0.0613062	0.0653102	10
11	.0499371	.0525619	.0555280	.0587926	.0614066	.0654240	11
12	.0500279	.0526625	.0556236	.0588999	.0615070	.0655380	12
13	.0501188	.0527633	.0557193	.0590072	.0616075	.0656521	13
14	.0502098	.0528641	.0558151	.0591146	.0617080	.0657663	14
15	.0503009	.0529651	.0559110	.0592221	.0618087	.0658807	15
16	.0503920	.0530661	.0560069	.0593298	.0619094	.0659951	16
17	.0504832	.0531673	.0561029	.0594376	.0620102	.0661097	17
18	.0505745	.0532686	.0561990	.0595454	.0621111	.0662243	18
19	.0506659	.0533699	.0562952	.0596534	.0622120	.0663391	19
20	0.0507574	0.0534714	0.0563915	0.0597615	0.0623131	0.0664540	20
21	.0508489	.0535730	.0564878	.0598697	.0624142	.0665690	21
22	.0509405	.0536747	.0565843	.0599781	.0625154	.0666842	22
23	.0510322	.0537765	.0566808	.0600865	.0626167	.0667994	23
24	.0511240	.0538785	.0567773	.0601951	.0627180	.0669148	24
25	.0512158	.0539805	.0568740	.0603037	.0628194	.0670302	25
26	.0513078	.0540826	.0569707	.0604125	.0629210	.0671458	26
27	.0513998	.0541849	.0570676	.0605214	.0630226	.0672615	27
28	.0514919	.0542873	.0571645	.0606304	.0631242	.0673774	28
29	.0515841	.0543897	.0572614	.0607395	.0632260	.0674933	29
30	0.0516763	0.0544923	0.0573585	0.0608487	0.0633278	0.0676094	30
31	.0517687	.0545950	.0574556	.0609580	.0634297	.0677255	31
32	.0518611	.0546978	.0575529	.0610675	.0635317	.0678418	32
33	.0519536	.0548007	.0576502	.0611770	.0636338	.0679582	33
34	.0520462	.0549037	.0577475	.0612867	.0637359	.0680747	34
35	.0521388	.0550068	.0578450	.0613965	.0638382	.0681914	35
36	.0522316	.0551101	.0579425	.0615064	.0639405	.0683081	36
37	.0523244	.0552134	.0580402	.0616164	.0640429	.0684250	37
38	.0524173	.0553169	.0581379	.0617265	.0641453	.0685420	38
39	.0525103	.0554204	.0582356	.0618367	.0642479	.0686591	39
40	0.0526034	0.0555241	0.0583335	0.0619471	0.0643505	0.0687763	40
41	.0526965	.0556279	.0584314	.0620575	.0644532	.0688936	41
42	.0527897	.0557318	.0585295	.0621681	.0645560	.0690110	42
43	.0528830	.0558358	.0586276	.0622788	.0646588	.0691286	43
44	.0529764	.0559399	.0587257	.0623896	.0647618	.0692463	44
45	.0530699	.0560441	.0588240	.0625005	.0648648	.0693641	45
46	.0531634	.0561485	.0589223	.0626115	.0649679	.0694820	46
47	.0532570	.0562529	.0590207	.0627227	.0650711	.0696000	47
48	.0533507	.0563575	.0591192	.0628339	.0651743	.0697182	48
49	.0534445	.0564621	.0592178	.0629453	.0652777	.0698364	49
50	0.0535384	0.0565669	0.0593165	0.0630568	0.0653811	0.0699548	50
51	.0536323	.0566718	.0594152	.0631684	.0654846	.0700733	51
52	.0537264	.0567768	.0595140	.0632801	.0655881	.0701919	52
53	.0538205	.0568819	.0596129	.0633919	.0656918	.0703106	53
54	.0539146	.0569871	.0597119	.0635038	.0657955	.0704295	54
55	.0540089	.0570924	.0598109	.0636158	.0658993	.0705484	55
56	.0541032	.0571978	.0599101	.0637280	.0660032	.0706675	56
57	.0541977	.0573034	.0600093	.0638403	.0661072	.0707867	57
58	.0542922	.0574090	.0601086	.0639527	.0662112	.0709060	58
59	.0543868	.0575148	.0602079	.0640652	.0663154	.0710254	59
60	0.0544814	0.0576207	0.0603074	0.0641778	0.0664196	0.0711450	60

TABLE 41. NATURAL VERSINES AND EXTERNAL SECANTS (*Continued*)

′	21°		22°		23°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	0.0664196	0.0711450	0.0728161	0.0785347	0.0794951	0.0863604	0
1	.0665239	.0712647	.0729252	.0786616	.0796088	.0864946	1
2	.0666282	.0713844	.0730342	.0787885	.0797226	.0866289	2
3	.0667327	.0715043	.0731434	.0789156	.0798365	.0867634	3
4	.0668372	.0716244	.0732526	.0790427	.0799504	.0868979	4
5	.0669418	.0717445	.0733620	.0791700	.0800644	.0870326	5
6	.0670465	.0718647	.0734714	.0792975	.0801785	.0871675	6
7	.0671512	.0719851	.0735808	.0794250	.0802927	.0873024	7
8	.0672561	.0721056	.0736904	.0795527	.0804069	.0874375	8
9	.0673610	.0722262	.0738000	.0796805	.0805212	.0875727	9
10	0.0674660	0.0723469	0.0739098	0.0798084	0.0806356	0.0877080	10
11	.0675710	.0724678	.0740195	.0799364	.0807501	.0878435	11
12	.0676762	.0725887	.0741294	.0800646	.0808647	.0879791	12
13	.0677814	.0727098	.0742394	.0801928	.0809793	.0881148	13
14	.0678867	.0728310	.0743494	.0803212	.0810940	.0882506	14
15	.0679921	.0729523	.0744595	.0804497	.0812088	.0883866	15
16	.0680976	.0730737	.0745697	.0805784	.0813237	.0885226	16
17	.0682031	.0731953	.0746799	.0807071	.0814386	.0886589	17
18	.0683088	.0733170	.0747903	.0808360	.0815536	.0887952	18
19	.0684145	.0734388	.0749007	.0809650	.0816687	.0889317	19
20	0.0685203	0.0735607	0.0750112	0.0810942	0.0817839	0.0890682	20
21	.0686261	.0736827	.0751218	.0812234	.0818991	.0892050	21
22	.0687321	.0738048	.0752324	.0813528	.0820145	.0893418	22
23	.0688381	.0739271	.0753432	.0814823	.0821299	.0894788	23
24	.0689442	.0740495	.0754540	.0816119	.0822454	.0896159	24
25	.0690504	.0741720	.0755649	.0817417	.0823609	.0897531	25
26	.0691566	.0742946	.0756758	.0818715	.0824766	.0898904	26
27	.0692630	.0744173	.0757869	.0820015	.0825923	.0900279	27
28	.0693694	.0745402	.0758980	.0821316	.0827081	.0901655	28
29	.0694759	.0746631	.0760092	.0822618	.0828240	.0903032	29
30	0.0695824	0.0747862	0.0761205	0.0823922	0.0829399	0.0904411	30
31	.0696891	.0749095	.0762318	.0825227	.0830560	.0905791	31
32	.0697958	.0750328	.0763433	.0826533	.0831721	.0907172	32
33	.0699026	.0751562	.0764548	.0827840	.0832882	.0908554	33
34	.0700095	.0752798	.0765664	.0829149	.0834045	.0909938	34
35	.0701165	.0754035	.0766780	.0830458	.0835209	.0911323	35
36	.0702235	.0755273	.0767898	.0831769	.0836373	.0912709	36
37	.0703306	.0756512	.0769016	.0833081	.0837538	.0914097	37
38	.0704378	.0757753	.0770135	.0834395	.0838703	.0915485	38
39	.0705451	.0758995	.0771255	.0835709	.0839870	.0916876	39
40	0.0706525	0.0760237	0.0772376	0.0837025	0.0841037	0.0918267	40
41	.0707599	.0761481	.0773497	.0838342	.0842205	.0919659	41
42	.0708674	.0762727	.0774619	.0839661	.0843374	.0921053	42
43	.0709750	.0763973	.0775742	.0840980	.0844544	.0922448	43
44	.0710827	.0765221	.0776866	.0842301	.0845714	.0923845	44
45	.0711904	.0766470	.0777990	.0843623	.0846885	.0925243	45
46	.0712983	.0767720	.0779116	.0844947	.0848057	.0926642	46
47	.0714062	.0768971	.0780242	.0846271	.0849230	.0928042	47
48	.0715142	.0770224	.0781368	.0847597	.0850403	.0929444	48
49	.0716222	.0771477	.0782496	.0848924	.0851578	.0930846	49
50	0.0717304	0.0772732	0.0783625	0.0850252	0.0852753	0.0932251	50
51	.0718386	.0773988	.0784754	.0851582	.0853928	.0933656	51
52	.0719469	.0775246	.0785884	.0852913	.0855105	.0935063	52
53	.0720553	.0776504	.0787014	.0854245	.0856282	.0936471	53
54	.0721637	.0777764	.0788146	.0855578	.0857460	.0937880	54
55	.0722723	.0779025	.0789278	.0856912	.0858639	.0939291	55
56	.0723809	.0780287	.0790411	.0858248	.0859819	.0940702	56
57	.0724896	.0781550	.0791545	.0859585	.0860999	.0942116	57
58	.0725984	.0782815	.0792680	.0860924	.0862181	.0943530	58
59	.0727072	.0784080	.0793815	.0862263	.0863363	.0944946	59
60	0.0728161	0.0785347	0.0794951	0.0863604	0.0864545	0.0946363	60

TABLE 41. NATURAL VERSINES AND EXTERNAL SECANTS (*Continued*)

′	24°		25°		26°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	0.0864545	0.0946363	0.0936922	0.1033779	0.1012060	0.1126019	0
1	.0865729	.0947781	.0938152	.1035277	.1013335	.1127599	1
2	.0866913	.0949201	.0939382	.1036775	.1014611	.1129179	2
3	.0868098	.0950622	.0940614	.1038275	.1015888	.1130761	3
4	.0869284	.0952044	.0941846	.1039777	.1017166	.1132345	4
5	.0870471	.0953467	.0943078	.1041279	.1018445	.1133929	5
6	.0871658	.0954892	.0944312	.1042783	.1019724	.1135516	6
7	.0872846	.0956318	.0945546	.1044289	.1021004	.1137103	7
8	.0874035	.0957746	.0946781	.1045795	.1022285	.1138692	8
9	.0875225	.0959174	.0948017	.1047303	.1023567	.1140282	9
10	0.0876416	0.0960604	0.0949254	0.1048813	0.1024849	0.1141874	10
11	.0877607	.0962036	.0950491	.1050324	.1026132	.1143467	11
12	.0878799	.0963468	.0951729	.1051836	.1027416	.1145062	12
13	.0879992	.0964902	.0952968	.1053349	.1028701	.1146658	13
14	.0881185	.0966337	.0954208	.1054864	.1029986	.1148255	14
15	.0882380	.0967774	.0955449	.1056380	.1031273	.1149854	15
16	.0883575	.0969212	.0956690	.1057898	.1032560	.1151454	16
17	.0884771	.0970651	.0957932	.1059417	.1033847	.1153056	17
18	.0885967	.0972091	.0959175	.1060937	.1035136	.1154659	18
19	.0887165	.0973533	.0960418	.1062458	.1036425	.1156263	19
20	0.0888363	0.0974976	0.0961662	0.1063981	0.1037715	0.1157869	20
21	.0889562	.0976420	.0962907	.1065506	.1039006	.1159476	21
22	.0890762	.0977866	.0964153	.1067031	.1040297	.1161084	22
23	.0891962	.0979313	.0965400	.1068558	.1041589	.1162694	23
24	.0893163	.0980761	.0966647	.1070087	.1042882	.1164306	24
25	.0894365	.0982211	.0967895	.1071616	.1044176	.1165919	25
26	.0895568	.0983662	.0969144	.1073147	.1045471	.1167533	26
27	.0896772	.0985114	.0970394	.1074680	.1046766	.1169148	27
28	.0897976	.0986568	.0971644	.1076214	.1048062	.1170766	28
29	.0899181	.0988023	.0972895	.1077749	.1049359	.1172384	29
30	0.0900387	0.0989479	0.0974147	0.1079285	0.1050656	0.1174004	30
31	.0901594	.0990936	.0975400	.1080823	.1051955	.1175625	31
32	.0902801	.0992395	.0976653	.1082363	.1053254	.1177248	32
33	.0904010	.0993855	.0977908	.1083903	.1054554	.1178872	33
34	.0905219	.0995317	.0979162	.1085445	.1055854	.1180498	34
35	.0906428	.0996779	.0980418	.1086989	.1057156	.1182124	35
36	.0907639	.0998243	.0981675	.1088533	.1058458	.1183753	36
37	.0908850	.0999709	.0982932	.1090079	.1059760	.1185383	37
38	.0910062	.1001175	.0984190	.1091627	.1061064	.1187014	38
39	.0911275	.1002644	.0985449	.1093176	.1062368	.1188647	39
40	0.0912489	0.1004113	0.0986708	0.1094726	0.1063674	0.1190281	40
41	.0913703	.1005584	.0987969	.1096277	.1064979	.1191916	41
42	.0914918	.1007056	.0989230	.1097830	.1066286	.1193553	42
43	.0916134	.1008529	.0990492	.1099385	.1067594	.1195191	43
44	.0917351	.1010004	.0991754	.1100940	.1068902	.1196831	44
45	.0918568	.1011480	.0993018	.1102498	.1070211	.1198472	45
46	.0919786	.1012957	.0994282	.1104056	.1071520	.1200115	46
47	.0921005	.1014436	.0995547	.1105616	.1072831	.1201759	47
48	.0922225	.1015916	.0996812	.1107177	.1074142	.1203405	48
49	.0923446	.1017397	.0998079	.1108740	.1075454	.1205051	49
50	0.0924667	0.1018879	0.0999346	0.1110304	0.1076766	0.1206700	50
51	.0925889	.1020363	.1000614	.1111869	.1078080	.1208350	51
52	.0927112	.1021849	.1001883	.1113436	.1079394	.1210001	52
53	.0928335	.1023335	.1003152	.1115004	.1080709	.1211653	53
54	.0929560	.1024823	.1004422	.1116573	.1082025	.1213308	54
55	.0930785	.1026313	.1005693	.1118144	.1083341	.1214963	55
56	.0932011	.1027803	.1006965	.1119716	.1084658	.1216620	56
57	.0933238	.1029295	.1008237	.1121290	.1085976	.1218278	57
58	.0934465	.1030789	.1009511	.1122865	.1087295	.1219938	58
59	.0935693	.1032283	.1010785	.1124442	.1088615	.1221600	59
60	0.0936922	0.1033779	0.1012060	0.1126019	0.1089935	0.1223262	60

TABLE 41. NATURAL VERSINES AND EXTERNAL SECANTS (*Continued*)

′	27°		28°		29°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	0.1089935	0.1223262	0.1170524	0.1325701	0.1253803	0.1433541	0
1	.1091256	.1224927	.1171890	.1327453	.1255214	.1435385	1
2	.1092577	.1226592	.1173257	.1329207	.1256625	.1437231	2
3	.1093900	.1228259	.1174624	.1330962	.1258037	.1439078	3
4	.1095223	.1229928	.1175993	.1332719	.1259450	.1440927	4
5	.1096547	.1231598	.1177362	.1334478	.1260863	.1442778	5
6	.1097872	.1233269	.1178731	.1336238	.1262278	.1444630	6
7	.1099197	.1234942	.1180102	.1337999	.1263693	.1446484	7
8	.1100524	.1236616	.1181473	.1339762	.1265109	.1448339	8
9	.1101851	.1238292	.1182845	.1341527	.1266525	.1450196	9
10	0.1103178	0.1239969	0.1184218	0.1343293	0.1267942	0.1452055	10
11	.1104507	.1241648	.1185591	.1345060	.1269360	.1453915	11
12	.1105836	.1243328	.1186965	.1346829	.1270779	.1455776	12
13	.1107166	.1245010	.1188340	.1348600	.1272199	.1457639	13
14	.1108497	.1246693	.1189716	.1350372	.1273619	.1459504	14
15	.1109829	.1248377	.1191093	.1352146	.1275040	.1461371	15
16	.1111161	.1250063	.1192470	.1353921	.1276462	.1463238	16
17	.1112491	.1251750	.1193848	.1355697	.1277884	.1465108	17
18	.1113828	.1253439	.1195226	.1357476	.1279307	.1466979	18
19	.1115162	.1255130	.1196606	.1359255	.1280731	.1468852	19
20	0.1116497	0.1256821	0.1197986	0.1361036	0.1282156	0.1470726	20
21	.1117831	.1258514	.1199367	.1362819	.1283581	.1472602	21
22	.1119170	.1260209	.1200749	.1364603	.1285007	.1474479	22
23	.1120508	.1261905	.1202131	.1366389	.1286434	.1476358	23
24	.1121846	.1263603	.1203514	.1368176	.1287862	.1478239	24
25	.1123185	.1265302	.1204898	.1369965	.1289290	.1480121	25
26	.1124525	.1267003	.1206283	.1371755	.1290719	.1482005	26
27	.1125866	.1268705	.1207668	.1373547	.1292149	.1483890	27
28	.1127207	.1270408	.1209054	.1375341	.1293580	.1485777	28
29	.1128549	.1272113	.1210441	.1377135	.1295011	.1487665	29
30	0.1129892	0.1273819	0.1211829	0.1378932	0.1296443	0.1489555	30
31	.1131235	.1275527	.1213217	.1380730	.1297876	.1491447	31
32	.1132580	.1277237	.1214606	.1382529	.1299309	.1493340	32
33	.1133925	.1278948	.1215996	.1384330	.1300744	.1495235	33
34	.1135270	.1280660	.1217387	.1386133	.1302179	.1497132	34
35	.1136617	.1282374	.1218778	.1387937	.1303614	.1499030	35
36	.1137964	.1284089	.1220170	.1389742	.1305051	.1500930	36
37	.1139312	.1285806	.1221563	.1391550	.1306488	.1502831	37
38	.1140661	.1287524	.1222957	.1393358	.1307926	.1504734	38
39	.1142011	.1289244	.1224351	.1395169	.1309364	.1506638	39
40	0.1143361	0.1290965	0.1225746	0.1396980	0.1310804	0.1508544	40
41	.1144712	.1292687	.1227142	.1398794	.1312244	.1510452	41
42	.1146064	.1294412	.1228538	.1400608	.1313685	.1512361	42
43	.1147416	.1296137	.1229936	.1402425	.1315126	.1514272	43
44	.1148770	.1297864	.1231334	.1404243	.1316569	.1516185	44
45	.1150124	.1299593	.1232732	.1406062	.1318012	.1518099	45
46	.1151478	.1301323	.1234132	.1407883	.1319456	.1520015	46
47	.1152834	.1303055	.1235532	.1409706	.1320900	.1521932	47
48	.1154190	.1304788	.1236933	.1411530	.1322345	.1523851	48
49	.1155547	.1306522	.1238335	.1413356	.1323791	.1525772	49
50	0.1156905	0.1308258	0.1239737	0.1415183	0.1325238	0.1527694	50
51	.1158264	.1309996	.1241141	.1417012	.1326686	.1529618	51
52	.1159623	.1311735	.1242545	.1418842	.1328134	.1531543	52
53	.1160983	.1313475	.1243949	.1420674	.1329583	.1533470	53
54	.1162344	.1315217	.1245355	.1422507	.1331033	.1535399	54
55	.1163705	.1316961	.1246761	.1424342	.1332483	.1537329	55
56	.1165067	.1318706	.1248168	.1426179	.1333934	.1539261	56
57	.1166431	.1320452	.1249575	.1428017	.1335386	.1541195	57
58	.1167794	.1322200	.1250984	.1429857	.1336839	.1543130	58
59	.1169159	.1323950	.1252393	.1431698	.1338292	.1545067	59
60	0.1170524	0.1325701	0.1253803	0.1433541	0.1339746	0.1547005	60

TABLE 41. NATURAL VERSINES AND EXTERNAL SECANTS (*Continued*)

′	30°		31°		32°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	0.1339746	0.1547005	0.1428327	0.1666334	0.1519519	0.1791784	0
1	.1341201	.1548945	.1429826	.1668374	.1521061	.1793928	1
2	.1342656	.1550887	.1431325	.1670416	.1522603	.1796074	2
3	.1344113	.1552830	.1432825	.1672459	.1524147	.1798222	3
4	.1345570	.1554775	.1434326	.1674504	.1525691	.1800372	4
5	.1347027	.1556722	.1435827	.1676551	.1527235	.1802523	5
6	.1348486	.1558670	.1437329	.1678599	.1528781	.1804676	6
7	.1349945	.1560620	.1438832	.1680649	.1530327	.1806831	7
8	.1351405	.1562572	.1440336	.1682701	.1531874	.1808988	8
9	.1352866	.1564525	.1441840	.1684755	.1533421	.1811146	9
10	0.1354327	0.1566480	0.1443345	0.1686810	0.1534970	0.1813307	10
11	.1355789	.1568436	.1444851	.1688867	.1536519	.1815469	11
12	.1357252	.1570394	.1446357	.1690926	.1538068	.1817633	12
13	.1358716	.1572354	.1447865	.1692986	.1539619	.1819798	13
14	.1360180	.1574315	.1449373	.1695048	.1541170	.1821966	14
15	.1361645	.1576278	.1450881	.1697112	.1542722	.1824135	15
16	.1363111	.1578243	.1452391	.1699178	.1544274	.1826306	16
17	.1364577	.1580209	.1453901	.1701245	.1545828	.1828479	17
18	.1366044	.1582177	.1455412	.1703314	.1547382	.1830654	18
19	.1367512	.1584146	.1456923	.1705385	.1548936	.1832830	19
20	0.1368981	0.1586118	0.1458436	0.1707457	0.1550492	0.1835008	20
21	.1370451	.1588091	.1459949	.1709531	.1552048	.1837188	21
22	.1371921	.1590065	.1461462	.1711607	.1553605	.1839370	22
23	.1373392	.1592041	.1462977	.1713685	.1555162	.1841554	23
24	.1374863	.1594019	.1464492	.1715764	.1556721	.1843739	24
25	.1376336	.1595999	.1466008	.1717845	.1558280	.1845927	25
26	.1377809	.1597980	.1467525	.1719928	.1559839	.1848116	26
27	.1379283	.1599963	.1469042	.1722013	.1561400	.1850307	27
28	.1380757	.1601947	.1470560	.1724099	.1562961	.1852500	28
29	.1382232	.1603933	.1472079	.1726187	.1564523	.1854694	29
30	0.1383708	0.1605921	0.1473598	0.1728277	0.1566086	0.1856890	30
31	.1385185	.1607911	.1475119	.1730368	.1567649	.1859089	31
32	.1386663	.1609902	.1476640	.1732462	.1569213	.1861289	32
33	.1388141	.1611894	.1478161	.1734557	.1570778	.1863490	33
34	.1389620	.1613889	.1479684	.1736653	.1572343	.1865694	34
35	.1391099	.1615885	.1481207	.1738752	.1573909	.1867900	35
36	.1392580	.1617883	.1482731	.1740852	.1575476	.1870107	36
37	.1394061	.1619882	.1484255	.1742954	.1577044	.1872316	37
38	.1395543	.1621883	.1485781	.1745058	.1578612	.1874527	38
39	.1397025	.1623886	.1487307	.1747163	.1580181	.1876740	39
40	0.1398509	0.1625891	0.1488833	0.1749270	0.1581751	0.1878954	40
41	.1399993	.1627897	.1490361	.1751379	.1583321	.1881171	41
42	.1401477	.1629905	.1491889	.1753490	.1584892	.1883389	42
43	.1402963	.1631914	.1493418	.1755603	.1586464	.1885609	43
44	.1404449	.1633925	.1494947	.1757717	.1588037	.1887831	44
45	.1405936	.1635938	.1496478	.1759833	.1589610	.1890055	45
46	.1407424	.1637953	.1498009	.1761951	.1591184	.1892280	46
47	.1408912	.1639969	.1499541	.1764070	.1592759	.1894508	47
48	.1410401	.1641987	.1501073	.1766191	.1594334	.1896737	48
49	.1411891	.1644007	.1502606	.1768314	.1595910	.1898968	49
50	0.1413381	0.1646028	0.1504140	0.1770439	0.1597487	0.1901201	50
51	.1414873	.1648051	.1505675	.1772566	.1599064	.1903436	51
52	.1416365	.1650076	.1507210	.1774694	.1600643	.1905673	52
53	.1417857	.1652102	.1508746	.1776824	.1602222	.1907911	53
54	.1419351	.1654130	.1510283	.1778956	.1603801	.1910152	54
55	.1420845	.1656160	.1511821	.1781089	.1605382	.1912394	55
56	.1422340	.1658191	.1513359	.1783225	.1606963	.1914638	56
57	.1423836	.1660224	.1514898	.1785362	.1608545	.1916884	57
58	.1425332	.1662259	.1516438	.1787501	.1610127	.1919132	58
59	.1426829	.1664296	.1517978	.1789642	.1611710	.1921381	59
60	0.1428327	0.1666334	0.1519519	0.1791784	0.1613294	0.1923633	60

TABLE 41. NATURAL VERSINES AND EXTERNAL SECANTS (*Continued*)

′	33°		34°		35°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	0.1613294	0.1923633	0.1709624	0.2062179	0.1808480	0.2207746	0
1	.1614879	.1925886	.1711251	.2064547	.1810148	.2210233	1
2	.1616464	.1928142	.1712879	.2066917	.1811818	.2212723	2
3	.1618050	.1930399	.1714507	.2069288	.1813488	.2215215	3
4	.1619687	.1932658	.1716136	.2071662	.1815159	.2217708	4
5	.1621225	.1934918	.1717766	.2074037	.1816831	.2220204	5
6	.1622813	.1937181	.1719397	.2076415	.1818503	.2222702	6
7	.1624402	.1939446	.1721028	.2078794	.1820176	.2225202	7
8	.1625991	.1941712	.1722660	.2081175	.1821849	.2227703	8
9	.1627582	.1943980	.1724292	.2083559	.1823524	.2230207	9
10	0.1629173	0.1946251	0.1725926	0.2085944	0.1825199	0.2232713	10
11	.1630764	.1948523	.1727560	.2088331	.1826875	.2235222	11
12	.1632357	.1950796	.1729194	.2090720	.1828551	.2237732	12
13	.1633950	.1953072	.1730830	.2093112	.1830228	.2240244	13
14	.1635544	.1955350	.1732466	.2095505	.1831906	.2242758	14
15	.1637138	.1957629	.1734103	.2097900	.1833584	.2245274	15
16	.1638734	.1959911	.1735740	.2100297	.1835264	.2247793	16
17	.1640330	.1962194	.1737378	.2102696	.1836944	.2250313	17
18	.1641926	.1964479	.1739017	.2105097	.1838624	.2252836	18
19	.1643524	.1966767	.1740657	.2107500	.1840305	.2255361	19
20	0.1645122	0.1969056	0.1742297	0.2109905	0.1841987	0.2257887	20
21	.1646721	.1971346	.1743938	.2112312	.1843670	.2260416	21
22	.1648320	.1973639	.1745580	.2114721	.1845353	.2262947	22
23	.1649920	.1975934	.1747222	.2117132	.1847037	.2265480	23
24	.1651521	.1978230	.1748865	.2119545	.1848722	.2268015	24
25	.1653123	.1980529	.1750509	.2121960	.1850407	.2270552	25
26	.1654725	.1982829	.1752153	.2124377	.1852094	.2273091	26
27	.1656328	.1985131	.1753798	.2126795	.1853780	.2275633	27
28	.1657932	.1987435	.1755444	.2129216	.1855468	.2278176	28
29	.1659537	.1989741	.1757091	.2131639	.1857156	.2280722	29
30	0.1661142	0.1992049	0.1758738	0.2134064	0.1858845	0.2283269	30
31	.1662748	.1994359	.1760386	.2136491	.1860534	.2285819	31
32	.1664354	.1996671	.1762035	.2138920	.1862225	.2288371	32
33	.1665962	.1998985	.1763684	.2141351	.1863916	.2290924	33
34	.1667570	.2001300	.1765334	.2143784	.1865607	.2293480	34
35	.1669178	.2003618	.1766985	.2146218	.1867299	.2296039	35
36	.1670788	.2005937	.1768636	.2148655	.1868992	.2298599	36
37	.1672398	.2008258	.1770288	.2151094	.1870686	.2301161	37
38	.1674009	.2010582	.1771941	.2153535	.1872380	.2303725	38
39	.1675620	.2012907	.1773595	.2155978	.1874075	.2306292	39
40	0.1677232	0.2015234	0.1775249	0.2158423	0.1875771	0.2308861	40
41	.1678845	.2017563	.1776904	.2160870	.1877468	.2311432	41
42	.1680459	.2019894	.1778560	.2163319	.1879165	.2314004	42
43	.1682073	.2022226	.1780216	.2165770	.1880863	.2316579	43
44	.1683688	.2024561	.1781873	.2168223	.1882561	.2319156	44
45	.1685304	.2026898	.1783531	.2170678	.1884260	.2321736	45
46	.1686920	.2029236	.1785189	.2173135	.1885960	.2324317	46
47	.1688537	.2031577	.1786848	.2175594	.1887661	.2326900	47
48	.1690155	.2033919	.1788508	.2178055	.1889362	.2329486	48
49	.1691774	.2036264	.1790168	.2180518	.1891064	.2332074	49
50	0.1693393	0.2038610	0.1791830	0.2182983	0.1892766	0.2334664	50
51	.1695013	.2040958	.1793491	.2185450	.1894470	.2337256	51
52	.1696634	.2043308	.1795154	.2187919	.1896174	.2339850	52
53	.1698255	.2045660	.1796817	.2190390	.1897878	.2342446	53
54	.1699877	.2048014	.1798481	.2192864	.1899584	.2345044	54
55	.1701500	.2050370	.1800146	.2195339	.1901290	.2347645	55
56	.1703123	.2052728	.1801811	.2197816	.1902996	.2350248	56
57	.1704748	.2055088	.1803477	.2200296	.1904704	.2352852	57
58	.1706372	.2057450	.1805144	.2202777	.1906412	.2355459	58
59	.1707998	.2059814	.1806811	.2205260	.1908121	.2358069	59
60	0.1709624	0.2062179	0.1808480	0.2207746	0.1909830	0.2360680	60

Table 41. Natural Versines and External Secants (*Continued*)

′	36°		37°		38°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	0.1909830	0.2360680	0.2013645	0.2521357	0.2119892	0.2690182	0
1	.1911540	.2363293	.2015396	.2524102	.2121684	.2693067	1
2	.1913251	.2365909	.2017147	.2526850	.2123476	.2695955	2
3	.1914963	.2368526	.2018900	.2529601	.2125268	.2698845	3
4	.1916675	.2371146	.2020653	.2532353	.2127061	.2701737	4
5	.1918388	.2373768	.2022406	.2535108	.2128855	.2704632	5
6	.1920101	.2376393	.2024161	.2537865	.2130650	.2707529	6
7	.1921815	.2379019	.2025916	.2540625	.2132445	.2710429	7
8	.1923530	.2381647	.2027671	.2543387	.2134241	.2713331	8
9	.1925246	.2384278	.2029428	.2546151	.2136037	.2716235	9
10	0.1926962	0.2386911	0.2031185	0.2548917	0.2137835	0.2719142	10
11	.1928679	.2389546	.2032942	.2551685	.2139633	.2722052	11
12	.1930397	.2392183	.2034701	.2554456	.2141431	.2724963	12
13	.1932115	.2394823	.2036460	.2557229	.2143230	.2727877	13
14	.1933834	.2397464	.2038220	.2560005	.2145030	.2730794	14
15	.1935554	.2400108	.2039980	.2562782	.2146831	.2733712	15
16	.1937274	.2402754	.2041741	.2565562	.2148632	.2736634	16
17	.1938995	.2405402	.2043503	.2568345	.2150434	.2739557	17
18	.1940717	.2408052	.2045265	.2571129	.2152236	.2742484	18
19	.1942440	.2410704	.2047028	.2573916	.2154039	.2745412	19
20	0.1944163	0.2413359	0.2048792	0.2576705	0.2155843	0.2748343	20
21	.1945887	.2416016	.2050556	.2579497	.2157648	.2751276	21
22	.1947611	.2418675	.2052322	.2582291	.2159453	.2754212	22
23	.1949336	.2421336	.2054087	.2585087	.2161259	.2757151	23
24	.1951062	.2423999	.2055854	.2587885	.2163065	.2760091	24
25	.1952789	.2426665	.2057621	.2590686	.2164873	.2763034	25
26	.1954516	.2429333	.2059389	.2593489	.2166680	.2765980	26
27	.1956244	.2432003	.2061157	.2596294	.2168489	.2768928	27
28	.1957972	.2434675	.2062926	.2599102	.2170298	.2771878	28
29	.1959701	.2437349	.2064696	.2601912	.2172108	.2774831	29
30	0.1961431	0.2440026	0.2066467	0.2604724	0.2173918	0.2777787	30
31	.1963162	.2442704	.2068238	.2607539	.2175730	.2780744	31
32	.1964893	.2445385	.2070010	.2610356	.2177541	.2783705	32
33	.1966625	.2448069	.2071782	.2613175	.2179354	.2786667	33
34	.1968358	.2450754	.2073555	.2615997	.2181167	.2789632	34
35	.1970091	.2453442	.2075329	.2618820	.2182981	.2792600	35
36	.1971825	.2456131	.2077104	.2621647	.2184795	.2795570	36
37	.1973560	.2458823	.2078879	.2624475	.2186610	.2798543	37
38	.1975295	.2461518	.2080655	.2627306	.2188426	.2801518	38
39	.1977031	.2464214	.2082431	.2630140	.2190243	.2804495	39
40	0.1978768	0.2466913	0.2084208	0.2632975	0.2192060	0.2807475	40
41	.1980505	.2469614	.2085986	.2635813	.2193877	.2810457	41
42	.1982244	.2472317	.2087765	.2638653	.2195696	.2813442	42
43	.1983982	.2475022	.2089544	.2641496	.2197515	.2816430	43
44	.1985722	.2477730	.2091324	.2644341	.2199335	.2819419	44
45	.1987462	.2480440	.2093104	.2647188	.2201155	.2822412	45
46	.1989203	.2483152	.2094885	.2650038	.2202976	.2825407	46
47	.1990944	.2485866	.2096667	.2652890	.2204798	.2828404	47
48	.1992686	.2488583	.2098450	.2655745	.2206620	.2831404	48
49	.1994429	.2491302	.2100233	.2658601	.2208443	.2834406	49
50	0.1996173	0.2494023	0.2102017	0.2661460	0.2210267	0.2837411	50
51	.1997917	.2496746	.2103802	.2664322	.2212091	.2840418	51
52	.1999662	.2499471	.2105587	.2667186	.2213916	.2843428	52
53	.2001407	.2502199	.2107373	.2670052	.2215742	.2846440	53
54	.2003153	.2504929	.2109159	.2672921	.2217569	.2849445	54
55	.2004900	.2507661	.2110946	.2675792	.2219396	.2852472	55
56	.2006648	.2510396	.2112734	.2678665	.2221223	.2855492	56
57	.2008396	.2513133	.2114523	.2681541	.2223051	.2858514	57
58	.2010145	.2515872	.2116312	.2684419	.2224880	.2861539	58
59	.2011895	.2518613	.2118102	.2687299	.2226710	.2864566	59
60	0.2013645	0.2521357	0.2119892	0.2690182	0.2228540	0.2867596	60

TABLE 41. NATURAL VERSINES AND EXTERNAL SECANTS (*Continued*)

′	39°		40°		41°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	0.2228540	0.2867596	0.2339556	0.3054073	0.2452904	0.3250130	0
1	.2230371	.2870628	.2341426	.3057261	.2454813	.3253482	1
2	.2232203	.2873663	.2343296	.3060451	.2456722	.3256837	2
3	.2234035	.2876700	.2345168	.3063644	.2458632	.3260194	3
4	.2235868	.2879740	.2347040	.3066839	.2460543	.3263554	4
5	.2237702	.2882782	.2348913	.3070038	.2462454	.3266918	5
6	.2239536	.2885827	.2350786	.3073239	.2464366	.3270284	6
7	.2241371	.2888875	.2352660	.3076442	.2466279	.3273653	7
8	.2243206	.2891925	.2354535	.3079649	.2468192	.3277024	8
9	.2245043	.2894977	.2356410	.3082858	.2470106	.3280399	9
10	0.2246879	0.2898032	0.2358286	0.3086069	0.2472020	0.3283776	10
11	.2248717	.2901090	.2360162	.3089284	.2473935	.3287156	11
12	.2250555	.2904150	.2362040	.3092501	.2475851	.3290539	12
13	.2252394	.2907213	.2363918	.3095720	.2477767	.3293925	13
14	.2254233	.2910278	.2365796	.3098943	.2479684	.3297314	14
15	.2256074	.2913346	.2367675	.3102168	.2481602	.3300706	15
16	.2257914	.2916416	.2369555	.3105396	.2483520	.3304100	16
17	.2259756	.2919489	.2371436	.3108626	.2485439	.3307497	17
18	.2261598	.2922564	.2373317	.3111860	.2487359	.3310897	18
19	.2263441	.2925642	.2375198	.3115095	.2489279	.3314301	19
20	0.2265284	0.2928723	0.2377081	0.3118334	0.2491200	0.3317707	20
21	.2267128	.2931806	.2378964	.3121575	.2493121	.3321115	21
22	.2268973	.2934892	.2380848	.3124820	.2495043	.3324527	22
23	.2270818	.2937980	.2382732	.3128066	.2496966	.3327942	23
24	.2272664	.2941071	.2384617	.3131316	.2498889	.3331359	24
25	.2274511	.2944164	.2386503	.3134568	.2500813	.3334779	25
26	.2276358	.2947260	.2388389	.3137823	.2502738	.3338203	26
27	.2278206	.2950359	.2390276	.3141081	.2504663	.3341629	27
28	.2280055	.2953460	.2392163	.3144341	.2506589	.3345058	28
29	.2281904	.2956564	.2394051	.3147604	.2508516	.3348489	29
30	0.2283754	0.2959670	0.2395940	0.3150870	0.2510443	0.3351924	30
31	.2285605	.2962779	.2397830	.3154139	.2512371	.3355362	31
32	.2287456	.2965890	.2399720	.3157410	.2514299	.3358802	32
33	.2289308	.2969004	.2401611	.3160684	.2516228	.3362246	33
34	.2291160	.2972121	.2403502	.3163961	.2518158	.3365692	34
35	.2293014	.2975240	.2405394	.3167240	.2520088	.3369141	35
36	.2294868	.2978362	.2407287	.3170523	.2522019	.3372594	36
37	.2296722	.2981487	.2409180	.3173808	.2523951	.3376049	37
38	.2298577	.2984614	.2411074	.3177096	.2525883	.3379507	38
39	.2300433	.2987743	.2412969	.3180386	.2527816	.3382968	39
40	0.2302290	0.2990876	0.2414864	0.3183680	0.2529749	0.3386432	40
41	.2304147	.2994011	.2416760	.3186976	.2531683	.3389898	41
42	.2306004	.2997148	.2418657	.3190274	.2533618	.3393368	42
43	.2307863	.3000288	.2420554	.3193576	.2535554	.3396841	43
44	.2309722	.3003431	.2422452	.3196881	.2537490	.3400316	44
45	.2311582	.3006576	.2424350	.3200188	.2539426	.3403795	45
46	.2313442	.3009724	.2426249	.3203498	.2541364	.3407276	46
47	.2315303	.3012875	.2428149	.3206810	.2543301	.3410761	47
48	.2317165	.3016028	.2430049	.3210126	.2545240	.3414248	48
49	.2319027	.3019184	.2431950	.3213444	.2547179	.3417738	49
50	0.2320890	0.3022343	0.2433852	0.3216765	0.2549119	0.3421232	50
51	.2322754	.3025504	.2435754	.3220089	.2551059	.3424728	51
52	.2324618	.3028667	.2437657	.3223416	.2553001	.3423227	52
53	.2326483	.3031834	.2439561	.3226745	.2554942	.3431729	53
54	.2328348	.3035003	.2441465	.3230078	.2556885	.3435234	54
55	.2330215	.3038175	.2443370	.3233413	.2558827	.3438742	55
56	.2332082	.3041349	.2445276	.3236750	.2560771	.3442253	56
57	.2333949	.3044526	.2447182	.3240091	.2562715	.3445767	57
58	.2335817	.3047706	.2449089	.3243435	.2564660	.3449284	58
59	.2337686	.3050888	.2450996	.3246781	.2566606	.3452804	59
60	0.2339556	0.3054073	0.2452904	0.3250130	0.2568552	0.3456327	60

TABLE 41. NATURAL VERSINES AND EXTERNAL SECANTS (*Continued*)

′	42°		43°		44°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	0.2568552	0.3456327	0.2686463	0.3673275	0.2806602	0.3901636	0
1	.2570498	.3459853	.2688447	.3676985	.2808623	.3905543	1
2	.2572446	.3463382	.2690432	.3680699	.2810645	.3909453	2
3	.2574394	.3466914	.2692417	.3684416	.2812667	.3913366	3
4	.2576342	.3470449	.2694403	.3688136	.2814690	.3917283	4
5	.2578292	.3473987	.2696390	.3691859	.2816713	.3921203	5
6	.2580242	.3477528	.2698377	.3695586	.2818737	.3925127	6
7	.2582192	.3481072	.2700365	.3699315	.2820762	.3929054	7
8	.2584143	.3484619	.2702354	.3703048	.2822787	.3932985	8
9	.2586095	.3488168	.2704343	.3706784	.2824813	.3936918	9
10	0.2588047	0.3491721	0.2706332	0.3710523	0.2826839	0.3940856	10
11	.2590000	.3495277	.2708323	.3714266	.2828866	.3944796	11
12	.2591954	.3498836	.2710314	.3718011	.2830894	.3948740	12
13	.2593908	.3502398	.2712305	.3721760	.2832922	.3952688	13
14	.2595863	.3505963	.2714297	.3725512	.2834951	.3956639	14
15	.2597819	.3509531	.2716290	.3729268	.2836981	.3960593	15
16	.2599775	.3513102	.2718284	.3733026	.2839011	.3964551	16
17	.2601732	.3516677	.2720278	.3736788	.2841041	.3968512	17
18	.2603689	.3520254	.2722272	.3740553	.2843073	.3972477	18
19	.2605647	.3523834	.2724268	.3744321	.2845105	.3976445	19
20	0.2607606	0.3527417	0.2726264	0.3748092	0.2847137	0.3980416	20
21	.2609565	.3531003	.2728260	.3751867	.2849170	.3984391	21
22	.2611525	.3534593	.2730257	.3755645	.2851204	.3988369	22
23	.2613485	.3538185	.2732255	.3759426	.2853238	.3992351	23
24	.2615447	.3541780	.2734253	.3763210	.2855273	.3996336	24
25	.2617408	.3545379	.2736252	.3766998	.2857309	.4000325	25
26	.2619371	.3548980	.2738252	.3770789	.2859345	.4004317	26
27	.2621334	.3552585	.2740252	.3774583	.2861382	.4008313	27
28	.2623297	.3556193	.2742253	.3778380	.2863419	.4012312	28
29	.2625262	.3559803	.2744254	.3782181	.2865457	.4016315	29
30	0.2627227	0.3563417	0.2746256	0.3785985	0.2867496	0.4020321	30
31	.2629192	.3567034	.2748259	.3789792	.2869535	.4024330	31
32	.2631158	.3570654	.2750262	.3793602	.2871574	.4028343	32
33	.2633125	.3574277	.2752266	.3797416	.2873615	.4032360	33
34	.2635092	.3577903	.2754271	.3801233	.2875656	.4036380	34
35	.2637060	.3581532	.2756276	.3805053	.2877697	.4040403	35
36	.2639029	.3585164	.2758281	.3808877	.2879740	.4044430	36
37	.2640998	.3588800	.2760288	.3812704	.2881782	.4048461	37
38	.2642968	.3592438	.2762295	.3816534	.2883826	.4052494	38
39	.2644939	.3596080	.2764302	.3820367	.2885870	.4056532	39
40	0.2646910	0.3599725	0.2766310	0.3824204	0.2887914	0.4060573	40
41	.2648882	.3603372	.2768319	.3828044	.2889959	.4064617	41
42	.2650854	.3607023	.2770329	.3831887	.2892005	.4068665	42
43	.2652827	.3610677	.2772339	.3835734	.2894052	.4072717	43
44	.2654801	.3614334	.2774349	.3839584	.2896099	.4076772	44
45	.2656775	.3617995	.2776360	.3843437	.2898146	.4080831	45
46	.2658750	.3621658	.2778372	.3847294	.2900194	.4084893	46
47	.2660725	.3625324	.2780385	.3851153	.2902243	.4088958	47
48	.2662701	.3628994	.2782398	.3855017	.2904293	.4093028	48
49	.2664678	.3632667	.2784411	.3858883	.2906343	.4097100	49
50	0.2666655	0.3636343	0.2786426	0.3862753	0.2908393	0.4101177	50
51	.2668633	.3640022	.2788441	.3866626	.2910444	.4105257	51
52	.2670612	.3643704	.2790456	.3870503	.2912496	.4109340	52
53	.2672591	.3647389	.2792472	.3874383	.2914549	.4113427	53
54	.2674571	.3651078	.2794489	.3878266	.2916602	.4117517	54
55	.2676551	.3654770	.2796506	.3882153	.2918655	.4121612	55
56	.2678533	.3658464	.2798524	.3886043	.2920709	.4125709	56
57	.2680514	.3662162	.2800543	.3889936	.2922764	.4129810	57
58	.2682497	.3665863	.2802562	.3893832	.2924820	.4133915	58
59	.2684479	.3669567	.2804582	.3897733	.2926876	.4138024	59
60	0.2686463	0.3673275	0.2806602	0.3901636	0.2928932	0.4142136	60

TABLE 41. NATURAL VERSINES AND EXTERNAL SECANTS (*Continued*)

′	45°		46°		47°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	0.2928932	0.4142136	0.3053416	0.4395565	0.3180016	0.4662792	0
1	.2930989	.4146251	.3055509	.4399904	.3182144	.4667368	1
2	.2933047	.4150370	.3057602	.4404246	.3184272	.4671948	2
3	.2935106	.4154493	.3059696	.4408592	.3186401	.4676532	3
4	.2937165	.4158619	.3061791	.4412941	.3188531	.4681120	4
5	.2939224	.4162749	.3063886	.4417295	.3190661	.4685713	5
6	.2941284	.4166883	.3065982	.4421652	.3192791	.4690309	6
7	.2943345	.4171020	.3068078	.4426013	.3194922	.4694910	7
8	.2945406	.4175161	.3070175	.4430379	.3197054	.4699514	8
9	.2947468	.4179306	.3072272	.4434748	.3199187	.4704123	9
10	0.2949531	0.4183454	0.3074370	0.4439120	0.3201319	0.4708736	10
11	.2951594	.4187605	.3076469	.4443497	.3203453	.4713354	11
12	.2953658	.4191761	.3078568	.4447878	.3205587	.4717975	12
13	.2955722	.4195920	.3080668	.4452262	.3207722	.4722600	13
14	.2957787	.4200082	.3082768	.4456651	.3209857	.4727230	14
15	.2959853	.4204248	.3084869	.4461043	.3211993	.4731864	15
16	.2961919	.4208418	.3086971	.4465439	.3214129	.4736502	16
17	.2963986	.4212592	.3089073	.4469839	.3216266	.4741144	17
18	.2966053	.4216769	.3091176	.4474243	.3218403	.4745790	18
19	.2968121	.4220950	.3093279	.4478651	.3220541	.4750440	19
20	0.2970189	0.4225134	0.3095383	0.4483063	0.3222680	0.4755095	20
21	.2972259	.4229323	.3097488	.4487478	.3224819	.4759754	21
22	.2974328	.4233514	.3099593	.4491898	.3226959	.4764417	22
23	.2976399	.4237710	.3101698	.4496322	.3229099	.4769084	23
24	.2978469	.4241909	.3103805	.4500749	.3231240	.4773755	24
25	.2980541	.4246112	.3105911	.4505181	.3233382	.4778431	25
26	.2982613	.4250319	.3108019	.4509616	.3235524	.4783111	26
27	.2984686	.4254529	.3110127	.4514055	.3237667	.4787795	27
28	.2986759	.4258743	.3112235	.4518498	.3239810	.4792483	28
29	.2988833	.4262961	.3114345	.4522946	.3241954	.4797176	29
30	0.2990907	0.4267182	0.3116454	0.4527397	0.3244098	0.4801872	30
31	.2992982	.4271407	.3118565	.4531852	.3246243	.4806573	31
32	.2995058	.4275636	.3120675	.4536311	.3248388	.4811278	32
33	.2997134	.4279868	.3122787	.4540774	.3250534	.4815988	33
34	.2999211	.4284105	.3124899	.4545241	.3252681	.4820702	34
35	.3001289	.4288345	.3127012	.4549712	.3254828	.4825420	35
36	.3003367	.4292588	.3129125	.4554187	.3256976	.4830142	36
37	.3005445	.4296836	.3131239	.4558666	.3259124	.4834868	37
38	.3007524	.4301087	.3133353	.4563149	.3261273	.4839599	38
39	.3009604	.4305342	.3135468	.4567636	.3263423	.4844334	39
40	0.3011685	0.4309600	0.3137584	0.4572127	0.3265573	0.4849073	40
41	.3013766	.4313863	.3139700	.4576621	.3267724	.4853817	41
42	.3015847	.4318129	.3141816	.4581120	.3269875	.4858565	42
43	.3017929	.4322399	.3143934	.4585623	.3272027	.4863317	43
44	.3020012	.4326672	.3146052	.4590130	.3274179	.4868073	44
45	.3022095	.4330950	.3148170	.4594641	.3276332	.4872834	45
46	.3024179	.4335231	.3150289	.4599156	.3278485	.4877599	46
47	.3026264	.4339516	.3152409	.4603675	.3280639	.4882369	47
48	.3028349	.4343805	.3154529	.4608198	.3282794	.4887142	48
49	.3030435	.4348097	.3156650	.4612726	.3284949	.4891920	49
50	0.3032521	0.4352393	0.3158771	0.4617257	0.3287105	0.4896703	50
51	.3034608	.4356693	.3160893	.4621792	.3289261	.4901489	51
52	.3036695	.4360997	.3163016	.4626331	.3291418	.4906280	52
53	.3038783	.4365305	.3165139	.4630875	.3293576	.4911076	53
54	.3040872	.4369616	.3167262	.4635422	.3295734	.4915876	54
55	.3042961	.4373932	.3169387	.4639973	.3297892	.4920680	55
56	.3045051	.4378251	.3171511	.4644529	.3300052	.4925488	56
57	.3047142	.4382574	.3173637	.4649089	.3302211	.4930301	58
58	.3049233	.4386900	.3175763	.4653652	.3304372	.4935118	58
59	.3051324	.4391231	.3177889	.4658220	.3306532	.4939940	59
60	0.3053416	0.4395565	0.3180016	0.4662792	0.3308694	0.4944765	60

Table 41. Natural Versines and External Secants (*Continued*)

′	48°		49°		50°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	0.3308694	0.4944765	0.3439410	0.5242531	0.3572124	0.5557238	0
1	.3310856	.4949596	.3441605	.5247634	.3574353	.5562634	1
2	.3313019	.4954431	.3443802	.5252741	.3576582	.5568035	2
3	.3315182	.4959270	.3445998	.5257854	.3578811	.5573441	3
4	.3317345	.4964113	.3448196	.5262971	.3581042	.5578852	4
5	.3319510	.4968961	.3450393	.5268093	.3583272	.5584268	5
6	.3321674	.4973813	.3452592	.5273219	.3585504	.5589689	6
7	.3323840	.4978670	.3454791	.5278351	.3587736	.5595115	7
8	.3326006	.4983531	.3456990	.5283487	.3589968	.5600546	8
9	.3328172	.4988397	.3459190	.5288627	.3592201	.5605982	9
10	0.3330339	0.4993267	0.3461391	0.5293773	0.3594434	0.5611424	10
11	.3332507	.4998141	.3463592	.5298923	.3596668	.5616871	11
12	.3334675	.5003020	.3465794	.5304078	.3598903	.5622322	12
13	.3336844	.5007903	.3467996	.5309238	.3601138	.5627779	13
14	.3339013	.5012791	.3470199	.5314403	.3603374	.5633241	14
15	.3341183	.5017683	.3472402	.5319572	.3605610	.5638708	15
16	.3343354	.5022580	.3474606	.5324746	.3607847	.5644181	16
17	.3345525	.5027481	.3476811	.5329925	.3610084	.5649658	17
18	.3347696	.5032387	.3479016	.5335109	.3612322	.5655141	18
19	.3349869	.5037297	.3481222	.5340297	.3614560	.5660628	19
20	0.3352041	0.5042211	0.3483428	0.5345491	0.3616799	0.5666121	20
21	.3354215	.5047131	.3485634	.5350689	.3619039	.5671619	21
22	.3356388	.5052054	.3487842	.5355892	.3621279	.5677123	22
23	.3358563	.5056982	.3490049	.5361100	.3623519	.5682631	23
24	.3360738	.5061915	.3492258	.5366313	.3625760	.5688145	24
25	.3362913	.5066852	.3494467	.5371530	.3628002	.5693664	25
26	.3365090	.5071793	.3496676	.5376752	.3630244	.5699188	26
27	.3367266	.5076739	.3498886	.5381980	.3632487	.5704717	27
28	.3369443	.5081690	.3501097	.5387212	.3634730	.5710252	28
29	.3371621	.5086645	.3503308	.5392449	.3636974	.5715792	29
30	0.3373800	0.5091605	0.3505520	0.5397690	0.3639218	0.5721337	30
31	.3375978	.5096569	.3507732	.5402937	.3641463	.5726887	31
32	.3378158	.5101538	.3509944	.5408189	.3643708	.5732443	32
33	.3380338	.5106511	.3512158	.5413445	.3645954	.5738004	33
34	.3382518	.5111489	.3514372	.5418706	.3648200	.5743570	34
35	.3384700	.5116472	.3516586	.5423973	.3650447	.5749141	35
36	.3386881	.5121459	.3518801	.5429244	.3652695	.5754718	36
37	.3389064	.5126450	.3521016	.5434520	.3654943	.5760300	37
38	.3391246	.5131446	.3523233	.5439801	.3657192	.5765887	38
39	.3393430	.5136447	.3525449	.5445087	.3659441	.5771479	39
40	0.3395614	0.5141452	0.3527666	0.5450378	0.3661690	0.5777077	40
41	.3397798	.5146462	.3529884	.5455673	.3663941	.5782680	41
42	.3399983	.5151477	.3532102	.5460974	.3666191	.5788289	42
43	.3402169	.5156496	.3534321	.5466280	.3668443	.5793902	43
44	.3404355	.5161520	.3536540	.5471590	.3670694	.5799521	44
45	.3406542	.5166548	.3538760	.5476906	.3672947	.5805146	45
46	.3408729	.5171581	.3540981	.5482226	.3675200	.5810776	46
47	.3410917	.5176619	.3543202	.5487552	.3677453	.5816411	47
48	.3413105	.5181661	.3545423	.5492882	.3679707	.5822051	48
49	.3415294	.5186708	.3547645	.5498218	.3681961	.5827697	49
50	0.3417484	0.5191759	0.3549868	0.5503558	0.3684216	0.5833348	50
51	.3419674	.5196815	.3552091	.5508904	.3686472	.5839005	51
52	.3421865	.5201876	.3554315	.5514254	.3688728	.5844667	52
53	.3424056	.5206942	.3556539	.5519610	.3690985	.5850334	53
54	.3426248	.5212012	.3558764	.5524970	.3693242	.5856007	54
55	.3428440	.5217087	.3560989	.5530335	.3695500	.5861685	55
56	.3430633	.5222166	.3563215	.5535706	.3697758	.5867369	56
57	.3432826	.5227250	.3565441	.5541081	.3700017	.5873058	57
58	.3435020	.5232339	.3567668	.5546462	.3702276	.5878752	58
59	.3437215	.5237433	.3569896	.5551848	.3704536	.5884452	59
60	0.3439410	0.5242531	0.3572124	0.5557238	0.3706796	0.5890157	60

TABLE 41. NATURAL VERSINES AND EXTERNAL SECANTS (*Continued*)

′	51°		52°		53°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	0.3706796	0.5890157	0.3843385	0.6242692	0.3981850	0.6616401	0
1	.3709057	.5895868	.3845678	.6248743	.3984173	.6622819	1
2	.3711318	.5901584	.3847971	.6254799	.3986497	.6629243	2
3	.3713580	.5907306	.3850264	.6260861	.3988821	.6635673	3
4	.3715843	.5913033	.3852558	.6266929	.3991146	.6642110	4
5	.3718106	.5918766	.3854853	.6273003	.3993472	.6648553	5
6	.3720369	.5924504	.3857148	.6279083	.3995798	.6655002	6
7	.3722634	.5930247	.3859444	.6285169	.3998124	.6661458	7
8	.3724898	.5935996	.3861740	.6291261	.4000451	.6667920	8
9	.3727163	.5941751	.3864036	.6297359	.4002779	.6674389	9
10	0.3729429	0.5947511	0.3866334	0.6303462	0.4005107	0.6680864	10
11	.3731695	.5953276	.3868631	.6309572	.4007435	.6687345	11
12	.3733962	.5959048	.3870929	.6315688	.4009764	.6693833	12
13	.3736229	.5964824	.3873228	.6321809	.4012094	.6700328	13
14	.3738497	.5970606	.3875527	.6327937	.4014423	.6706828	14
15	.3740765	.5976394	.3877827	.6334070	.4016754	.6713336	15
16	.3743034	.5982187	.3880127	.6340210	.4019085	.6719850	16
17	.3745304	.5987983	.3882428	.6346355	.4021417	.6726370	17
18	.3747573	.5993790	.3884730	.6352507	.4023749	.6732897	18
19	.3749844	.5999600	.3887031	.6358664	.4026081	.6739430	19
20	0.3752115	0.6005416	0.3889334	0.6364828	0.4028414	0.6745970	20
21	.3754386	.6011237	.3891637	.6370997	.4030748	.6752517	21
22	.3756658	.6017064	.3893940	.6377173	.4033082	.6759070	22
23	.3758931	.6022896	.3896244	.6383355	.4035416	.6765629	23
24	.3761204	.6028734	.3898548	.6389542	.4037751	.6772195	24
25	.3763478	.6034577	.3900853	.6395736	.4040087	.6778768	25
26	.3765752	.6040426	.3903159	.6401936	.4042423	.6785347	26
27	.3768026	.6046281	.3905465	.6408142	.4044759	.6791933	27
28	.3770302	.6052142	.3907771	.6414354	.4047096	.6798525	28
29	.3772577	.6058008	.3910078	.6420572	.4049434	.6805124	29
30	0.3774854	0.6063879	0.3912386	0.6426796	0.4051772	0.6811730	30
31	.3777130	.6069757	.3914694	.6433027	.4054111	.6818342	31
32	.3779408	.6075640	.3917002	.6439263	.4056450	.6824961	32
33	.3781686	.6081528	.3919311	.6445506	.4058789	.6831586	33
34	.3783964	.6087423	.3921621	.6451754	.4061129	.6838219	34
35	.3786243	.6093323	.3923931	.6458009	.4063470	.6844857	35
36	.3788522	.6099228	.3926242	.6464270	.4065811	.6851503	36
37	.3790802	.6105140	.3928553	.6470537	.4068153	.6858155	37
38	.3793083	.6111057	.3930864	.6476811	.4070495	.6864814	38
39	.3795364	.6116980	.3933176	.6483090	.4072837	.6871479	39
40	0.3797645	0.6122908	0.3935489	0.6489376	0.4075181	0.6878151	40
41	.3799927	.6128843	.3937802	.6495668	.4077524	.6884830	41
42	.3802210	.6134783	.3940116	.6501966	.4079868	.6891516	42
43	.3804493	.6140728	.3942430	.6508270	.4082213	.6898208	43
44	.3806776	.6146680	.3944745	.6514581	.4084558	.6904907	44
45	.3809061	.6152637	.3947060	.6520898	.4086904	.6911613	45
46	.3811345	.6158600	.3949376	.6527221	.4089250	.6918326	46
47	.3813630	.6164569	.3951692	.6533550	.4091596	.6925045	47
48	.3815916	.6170544	.3954009	.6539885	.4093943	.6931771	48
49	.3818202	.6176524	.3956326	.6546227	.4096291	.6938504	49
50	0.3820489	0.6182510	0.3958644	0.6552575	0.4098639	0.6945244	50
51	.3822776	.6188502	.3960962	.6558929	.4100988	.6951990	51
52	.3825064	.6194500	.3963281	.6565290	.4103337	.6958744	52
53	.3827352	.6200504	.3965600	.6571657	.4105686	.6965504	53
54	.3829641	.6206513	.3967920	.6578030	.4108036	.6972271	54
55	.3831931	.6212528	.3970240	.6584409	.4110387	.6979044	55
56	.3834220	.6218549	.3972561	.6590795	.4112738	.6985825	56
57	.3836511	.6224576	.3974883	.6597187	.4115090	.6992612	57
58	.3838802	.6230609	.3977205	.6603586	.4117442	.6999407	58
59	.3841093	.6236648	.3979527	.6609990	.4119794	.7006208	59
60	0.3843385	0.6242692	0.3981850	0.6616401	0.4122147	0.7013016	60

TABLE 41. NATURAL VERSINES AND EXTERNAL SECANTS (*Continued*)

′	54°		55°		56°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	0.4122147	0.7013016	0.4264236	0.7434468	0.4408071	0.7882916	0
1	.4124501	.7019831	.4266619	.7441715	.4410483	.7890633	1
2	.4126855	.7026653	.4269002	.7448969	.4412895	.7898357	2
3	.4129210	.7033482	.4271386	.7456230	.4415308	.7906090	3
4	.4131565	.7040318	.4273771	.7463499	.4417721	.7913831	4
5	.4133920	.7047160	.4276156	.7470776	.4420135	.7921580	5
6	.4136276	.7054010	.4278541	.7478060	.4422549	.7929337	6
7	.4138633	.7060867	.4280927	.7485352	.4424964	.7937102	7
8	.4140990	.7067730	.4283314	.7492651	.4427379	.7944876	8
9	.4143348	.7074601	.4285701	.7499958	.4429794	.7952658	9
10	0.4145706	0.7081478	0.4288088	0.7507273	0.4432210	0.7960449	10
11	.4148064	.7088362	.4290476	.7514595	.4434627	.7968247	11
12	.4150423	.7095254	.4292864	.7521924	.4437044	.7976054	12
13	.4152783	.7102152	.4295253	.7529262	.4439461	.7983869	13
14	.4155143	.7109058	.4297643	.7536607	.4441879	.7991693	14
15	.4157503	.7115970	.4300032	.7543959	.4444298	.7999524	15
16	.4159864	.7122890	.4302423	.7551320	.4446717	.8007365	16
17	.4162226	.7129817	.4304813	.7558687	.4449136	.8015213	17
18	.4164588	.7136750	.4307205	.7566063	.4451556	.8023070	18
19	.4166950	.7143691	.4309597	.7573446	.4453976	.8030935	19
20	0.4169313	0.7150639	0.4311989	0.7580837	0.4456397	0.8038809	20
21	.4171677	.7157594	.4314381	.7588236	.4458818	.8046691	21
22	.4174041	.7164556	.4316775	.7595642	.4461240	.8054582	22
23	.4176405	.7171525	.4319168	.7603057	.4463662	.8062481	23
24	.4178770	.7178501	.4321563	.7610478	.4466085	.8070388	24
25	.4181136	.7185484	.4323957	.7617908	.4468508	.8078304	25
26	.4183502	.7192475	.4326352	.7625345	.4470931	.8086228	26
27	.4185868	.7199472	.4328748	.7632791	.4473355	.8094161	27
28	.4188235	.7206477	.4331144	.7640244	.4475780	.8102102	28
29	.4190603	.7213489	.4333541	.7647704	.4478205	.8110052	29
30	0.4192970	0.7220508	0.4335938	0.7655173	0.4480630	0.8118010	30
31	.4195339	.7227534	.4338335	.7662649	.4483056	.8125977	31
32	.4197708	.7234568	.4340733	.7670133	.4485482	.8133953	32
33	.4200077	.7241609	.4343132	.7677625	.4487909	.8141937	33
34	.4202447	.7248657	.4345531	.7685125	.4490337	.8149929	34
35	.4204817	.7255712	.4347930	.7692633	.4492764	.8157930	35
36	.4207188	.7262774	.4350330	.7700149	.4495193	.8165940	36
37	.4209560	.7269844	.4352730	.7707672	.4497621	.8173958	37
38	.4211931	.7276921	.4355131	.7715204	.4500050	.8181985	38
39	.4214304	.7284005	.4357533	.7722743	.4502480	.8190021	39
40	0.4216677	0.7291096	0.4359934	0.7730290	0.4504910	0.8198065	40
41	.4219050	.7298195	.4362337	.7737845	.4507341	.8206118	41
42	.4221424	.7305301	.4364740	.7745409	.4509772	.8214179	42
43	.4223798	.7312414	.4367143	.7752980	.4512203	.8222249	43
44	.4226173	.7319535	.4369547	.7760559	.4514635	.8230328	44
45	.4228548	.7326663	.4371951	.7768146	.4517068	.8238416	45
46	.4230924	.7333798	.4374355	.7775741	.4519501	.8246512	46
47	.4233300	.7340941	.4376761	.7783344	.4521934	.8254617	47
48	.4235677	.7348091	.4379166	.7790955	.4524368	.8262731	48
49	.4238054	.7355248	.4381572	.7798574	.4526802	.8270854	49
50	0.4240432	0.7362413	0.4383979	0.7806201	0.4529237	0.8278985	50
51	.4242810	.7369585	.4386386	.7813836	.4531672	.8287125	51
52	.4245189	.7376764	.4388794	.7821479	.4534108	.8295274	52
53	.4247568	.7383951	.4391202	.7829131	.4536544	.8303432	53
54	.4249947	.7391145	.4393610	.7836790	.4538980	.8311599	54
55	.4252328	.7398347	.4396019	.7844457	.4541417	.8319774	55
56	.4254708	.7405556	.4398428	.7852133	.4543855	.8327959	56
57	.4257089	.7412773	.4400838	.7859817	.4546293	.8336152	57
58	.4259471	.7419997	.4403249	.7867508	.4548731	.8344354	58
59	.4261853	.7427229	.4405660	.7875208	.4551170	.8352565	59
60	0.4264236	0.7434468	0.4408071	0.7882916	0.4553610	0.8360785	60

TABLE 41. NATURAL VERSINES AND EXTERNAL SECANTS (*Continued*)

′	57°		58°		59°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	0.4553610	0.8360785	0.4700807	0.8870799	0.4849619	0.9416040	0
1	.4556049	.8369013	.4703274	.8879589	.4852113	.9425445	1
2	.4558490	.8377251	.4705742	.8888388	.4854607	.9434861	2
3	.4560931	.8385498	.4708210	.8897197	.4857101	.9444288	3
4	.4563372	.8393753	.4710678	.8906016	.4859596	.9453725	4
5	.4565813	.8402018	.4713147	.8914845	.4862092	.9463173	5
6	.4568256	.8410292	.4715617	.8923684	.4864587	.9472632	6
7	.4570698	.8418574	.4718086	.8932532	.4867084	.9482102	7
8	.4573141	.8426866	.4720557	.8941391	.4869580	.9491583	8
9	.4575585	.8435166	.4723027	.8950259	.4872077	.9501075	9
10	0.4578029	0.8443476	0.4725498	0.8959138	0.4874575	0.9510577	10
11	.4580473	.8451795	.4727970	.8968026	.4877073	.9520091	11
12	.4582918	.8460123	.4730442	.8976924	.4879571	.9529615	12
13	.4585363	.8468460	.4732915	.8985832	.4882070	.9539150	13
14	.4587809	.8476806	.4735387	.8994750	.4884569	.9548697	14
15	.4590255	.8485161	.4737861	.9003678	.4887069	.9558254	15
16	.4592702	.8493525	.4740335	.9012616	.4889569	.9567822	16
17	.4595149	.8501898	.4742809	.9021564	.4892070	.9577402	17
18	.4597597	.8510281	.4745283	.9030522	.4894571	.9586992	18
19	.4600045	.8518672	.4747759	.9039491	.4897072	.9596593	19
20	0.4602493	0.8527073	0.4750234	0.9048469	0.4899574	0.9606206	20
21	.4604942	.8535483	.4752710	.9057457	.4902076	.9615829	21
22	.4607392	.8543903	.4755187	.9066456	.4904579	.9625464	22
23	.4609842	.8552331	.4757664	.9075464	.4907082	.9635110	23
24	.4612292	.8560769	.4760141	.9084483	.4909586	.9644767	24
25	.4614743	.8569216	.4762619	.9093512	.4912090	.9654435	25
26	.4617194	.8577672	.4765097	.9102551	.4914594	.9664114	26
27	.4619646	.8586138	.4767576	.9111600	.4917099	.9673805	27
28	.4622098	.8594612	.4770055	.9120659	.4919604	.9683507	28
29	.4624551	.8603097	.4772534	.9129729	.4922110	.9693220	29
30	0.4627004	0.8611590	0.4775014	0.9138809	0.4924616	0.9702944	30
31	.4629457	.8620093	.4777495	.9147899	.4927123	.9712680	31
32	.4631911	.8628605	.4779976	.9156999	.4929630	.9722427	32
33	.4634366	.8637126	.4782457	.9166110	.4932137	.9732185	33
34	.4636821	.8645657	.4784939	.9175230	.4934645	.9741954	34
35	.4639276	.8654197	.4787421	.9184362	.4937154	.9751735	35
36	.4641732	.8662747	.4789904	.9193503	.4939662	.9761527	36
37	.4644188	.8671306	.4792387	.9202655	.4942172	.9771331	37
38	.4646645	.8679875	.4794870	.9211817	.4944681	.9781146	38
39	.4649102	.8688453	.4797354	.9220990	.4947191	.9790972	39
40	0.4651560	0.8697040	0.4799839	0.9230173	0.4949702	0.9800810	40
41	.4654018	.8705637	.4802324	.9239366	.4952212	.9810659	41
42	.4656477	.8714244	.4804809	.9248570	.4954724	.9820520	42
43	.4658935	.8722859	.4807295	.9257784	.4957235	.9830393	43
44	.4661395	.8731485	.4809781	.9267009	.4959748	.9840276	44
45	.4663855	.8740120	.4812267	.9276244	.4962260	.9850172	45
46	.4666315	.8748764	.4814754	.9285490	.4964773	.9860080	46
47	.4668776	.8757419	.4817242	.9294746	.4967287	.9869997	47
48	.4671237	.8766082	.4819730	.9304013	.4969801	.9879927	48
49	.4673699	.8774755	.4822218	.9313290	.4972315	.9889869	49
50	0.4676161	0.8783438	0.4824707	0.9322578	0.4974830	0.9899822	50
51	.4678624	.8792131	.4827196	.9331876	.4977345	.9909787	51
52	.4681087	.8800833	.4829686	.9341185	.4979860	.9919764	52
53	.4683550	.8809545	.4832176	.9350505	.4982376	.9929752	53
54	.4686014	.8818266	.4834667	.9359835	.4984893	.9939753	54
55	.4688479	.8826998	.4837158	.9369176	.4987409	.9949764	55
56	.4690943	.8835738	.4839649	.9378527	.4989927	.9959788	56
57	.4693409	.8844489	.4842141	.9387889	.4992444	.9969823	57
58	.4695875	.8853249	.4844633	.9397262	.4994963	.9979870	58
59	.4698341	.8862019	.4847126	.9406646	.4997481	.9989929	59
60	0.4700807	0.8870799	0.4849619	0.9416040	0.5000000	1.0000000	60

TABLE 41. NATURAL VERSINES AND EXTERNAL SECANTS (*Continued*)

′	60°		61°		62°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	0.5000000	1.0000000	0.5151904	1.0626653	0.5305284	1.1300545	0
1	.5002519	1.0010083	.5154448	1.0637484	.5307853	1.1312205	1
2	.5005039	1.0020177	.5156993	1.0648328	.5310422	1.1323880	2
3	.5007559	1.0030283	.5159538	1.0659183	.5312991	1.1335570	3
4	.5010080	1.0040402	.5162084	1.0670056	.5315561	1.1347274	4
5	.5012601	1.0050532	.5164630	1.0680940	.5318131	1.1358993	5
6	.5015123	1.0060674	.5167176	1.0691836	.5320702	1.1370726	6
7	.5017645	1.0070828	.5169723	1.0702746	.5323273	1.1382475	7
8	.5020167	1.0080994	.5172270	1.0713670	.5325844	1.1394238	8
9	.5022690	1.0091172	.5174818	1.0724606	.5328416	1.1406015	9
10	0.5025213	1.0101362	0.5177366	1.0735556	0.5330988	1.1417808	10
11	.5027736	1.0111564	.5179914	1.0746519	.5333561	1.1429615	11
12	.5030260	1.0121779	.5182463	1.0757496	.5336134	1.1441438	12
13	.5032785	1.0132005	.5185013	1.0768483	.5338707	1.1453275	13
14	.5035310	1.0142243	.5187562	1.0779489	.5341281	1.1465127	14
15	.5037835	1.0152494	.5190112	1.0790506	.5343855	1.1476993	15
16	.5040361	1.0162756	.5192663	1.0801536	.5346429	1.1488875	16
17	.5042887	1.0173031	.5195214	1.0812580	.5349004	1.1500772	17
18	.5045413	1.0183318	.5197765	1.0823637	.5351580	1.1512684	18
19	.5047940	1.0193618	.5200317	1.0834708	.5354155	1.1524611	19
20	0.5050468	1.0203929	0.5202869	1.0845792	0.5356731	1.1536553	20
21	.5052995	1.0214253	.5205421	1.0856890	.5359308	1.1548510	21
22	.5055524	1.0224589	.5207974	1.0868002	.5361885	1.1560482	22
23	.5058052	1.0234937	.5210528	1.0879127	.5364462	1.1572469	23
24	.5060581	1.0245297	.5213081	1.0890265	.5367040	1.1584471	24
25	.5063111	1.0255670	.5215636	1.0901418	.5369618	1.1596489	25
26	.5065641	1.0266056	.5218190	1.0912584	.5372196	1.1608522	26
27	.5068171	1.0276453	.5220745	1.0923764	.5374775	1.1620570	27
28	.5070702	1.0286863	.5223300	1.0934957	.5377354	1.1632633	28
29	.5073233	1.0297286	.5225856	1.0946164	.5379934	1.1644712	29
30	0.5075764	1.0307720	0.5228412	1.0957385	0.5382514	1.1656806	30
31	.5078296	1.0318168	.5230969	1.0968620	.5385094	1.1668915	31
32	.5080829	1.0328628	.5233526	1.0979869	.5387675	1.1681040	32
33	.5083362	1.0339100	.5236083	1.0991131	.5390256	1.1693180	33
34	.5085895	1.0349585	.5238641	1.1002408	.5392833	1.1705335	34
35	.5088428	1.0360082	.5241199	1.1013698	.5395420	1.1717506	35
36	.5090962	1.0370592	.5243758	1.1025002	.5398002	1.1729693	36
37	.5093497	1.0381114	.5246317	1.1036320	.5400585	1.1741895	37
38	.5096032	1.0391649	.5248876	1.1047652	.5403168	1.1754113	38
39	.5098567	1.0402197	.5251436	1.1058998	.5405752	1.1766346	39
40	0.5101103	1.0412757	0.5253996	1.1070359	0.5408335	1.1778595	40
41	.5103639	1.0423330	.5256557	1.1081733	.5410920	1.1790859	41
42	.5106175	1.0433916	.5259118	1.1093121	.5413504	1.1803139	42
43	.5108712	1.0444515	.5261679	1.1104523	.5416090	1.1815435	43
44	.5111250	1.0455126	.5264241	1.1115940	.5418675	1.1827746	44
45	.5113788	1.0465750	.5266803	1.1127371	.5421261	1.1840074	45
46	.5116326	1.0476386	.5269366	1.1138815	.5423847	1.1852417	46
47	.5118864	1.0487036	.5271929	1.1150274	.5426434	1.1864775	47
48	.5121403	1.0497698	.5274492	1.1161748	.5429021	1.1877150	48
49	.5123943	1.0508373	.5277056	1.1173235	.5431608	1.1889541	49
50	0.5126483	1.0519061	0.5279620	1.1184737	0.5434196	1.1901947	50
51	.5129023	1.0529762	.5282185	1.1196253	.5436784	1.1914370	51
52	.5131564	1.0540476	.5284750	1.1207783	.5439373	1.1926808	52
53	.5134105	1.0551203	.5287315	1.1219328	.5441962	1.1939262	53
54	.5136646	1.0561942	.5289881	1.1230887	.5444551	1.1951733	54
55	.5139188	1.0572695	.5292447	1.1242460	.5447141	1.1964219	55
56	.5141730	1.0583460	.5295014	1.1254048	.5449731	1.1976721	56
57	.5144273	1.0594239	.5297581	1.1265651	.5452321	1.1989240	57
58	.5146816	1.0605031	.5300148	1.1277267	.5454912	1.2001775	58
59	.5149360	1.0615836	.5302716	1.1288899	.5457503	1.2014326	59
60	0.5151904	1.0626653	0.5305284	1.1300545	0.5460095	1.2026893	60

TABLE 41. NATURAL VERSINES AND EXTERNAL SECANTS (*Continued*)

′	63°		64°		65°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	0.5460095	1.2026893	0.5616289	1.2811720	0.5773817	1.3662016	0
1	.5462687	1.2039476	.5618903	1.2825335	.5776454	1.3676787	1
2	.5465279	1.2052075	.5621518	1.2838967	.5779091	1.3691578	2
3	.5467872	1.2064691	.5624134	1.2852618	.5781728	1.3706390	3
4	.5470465	1.2077323	.5626749	1.2866286	.5784366	1.3721222	4
5	.5473059	1.2089972	.5629366	1.2879974	.5787004	1.3736075	5
6	.5475653	1.2102637	.5631982	1.2893679	.5789642	1.3750949	6
7	.5478247	1.2115318	.5634599	1.2907403	.5792281	1.3765843	7
8	.5480842	1.2128016	.5637216	1.2921145	.5794920	1.3780758	8
9	.5483437	1.2140730	.5639834	1.2934906	.5797559	1.3795694	9
10	0.5486033	1.2153460	0.5642452	1.2948685	0.5800199	1.3810650	10
11	.5488628	1.2166208	.5645070	1.2962483	.5802839	1.3825627	11
12	.5491225	1.2178971	.5647689	1.2976299	.5805479	1.3840625	12
13	.5493821	1.2191752	.5650308	1.2990134	.5808120	1.3855645	13
14	.5496418	1.2204548	.5652928	1.3003988	.5810761	1.3870685	14
15	.5499016	1.2217362	.5655547	1.3017860	.5813403	1.3885746	15
16	.5501613	1.2230192	.5658168	1.3031751	.5816044	1.3900828	16
17	.5504211	1.2243039	.5660788	1.3045660	.5818687	1.3915931	17
18	.5506810	1.2255903	.5663409	1.3059588	.5821329	1.3931055	18
19	.5509409	1.2268783	.5666030	1.3073536	.5823972	1.3946201	19
20	0.5512008	1.2281681	0.5668652	1.3087501	0.5826615	1.3961367	20
21	.5514608	1.2294595	.5671274	1.3101486	.5829259	1.3976555	21
22	.5517208	1.2307526	.5673897	1.3115490	.5831903	1.3991764	22
23	.5519808	1.2320474	.5676519	1.3129513	.5834547	1.4006995	23
24	.5522409	1.2333438	.5679143	1.3143554	.5837192	1.4022247	24
25	.5525010	1.2346420	.5681766	1.3157615	.5839837	1.4037520	25
26	.5527612	1.2359419	.5684390	1.3171695	.5842483	1.4052815	26
27	.5530214	1.2372435	.5687014	1.3185794	.5845128	1.4068132	27
28	.5532816	1.2385468	.5689639	1.3199912	.5847774	1.4083469	28
29	.5535419	1.2398517	.5692264	1.3214049	.5850421	1.4098829	29
30	0.5538022	1.2411585	0.5694889	1.3228205	0.5853068	1.4114210	30
31	.5540625	1.2424669	.5697515	1.3242381	.5855715	1.4129613	31
32	.5543229	1.2437770	.5700141	1.3256575	.5858362	1.4145038	32
33	.5545833	1.2450889	.5702767	1.3270790	.5861010	1.4160484	33
34	.5548438	1.2464025	.5705394	1.3285023	.5863658	1.4175952	34
35	.5551043	1.2477178	.5708021	1.3299276	.5866307	1.4191442	35
36	.5553648	1.2490348	.5710649	1.3313548	.5868956	1.4206954	36
37	.5556254	1.2503536	.5713277	1.3327840	.5871605	1.4222488	37
38	.5558860	1.2516741	.5715905	1.3342152	.5874255	1.4238044	38
39	.5561466	1.2529964	.5718533	1.3356482	.5876904	1.4253622	39
40	0.5564073	1.2543204	0.5721162	1.3370833	0.5879555	1.4269222	40
41	.5566681	1.2556461	.5723792	1.3385203	.5882205	1.4284844	41
42	.5569288	1.2569736	.5726421	1.3399593	.5884856	1.4300489	42
43	.5571896	1.2583029	.5729051	1.3414002	.5887508	1.4316155	43
44	.5574504	1.2596339	.5731682	1.3428432	.5890159	1.4331844	44
45	.5577113	1.2609667	.5734313	1.3442881	.5892811	1.4347555	45
46	.5579722	1.2623012	.5736944	1.3457349	.5895464	1.4363289	46
47	.5582332	1.2636376	.5739575	1.3471838	.5898117	1.4379045	47
48	.5584941	1.2649756	.5742207	1.3486347	.5900770	1.4394823	48
49	.5587552	1.2663155	.5744839	1.3500875	.5903423	1.4410624	49
50	0.5590162	1.2676571	0.5747472	1.3515424	0.5906077	1.4426448	50
51	.5592773	1.2690005	.5750105	1.3529992	.5908731	1.4442294	51
52	.5595385	1.2703457	.5752738	1.3544581	.5911385	1.4458163	52
53	.5597996	1.2716927	.5755372	1.3559189	.5914040	1.4474054	53
54	.5600608	1.2730415	.5758006	1.3573818	.5916695	1.4489968	54
55	.5603221	1.2743921	.5760640	1.3588467	.5919351	1.4505905	55
56	.5605834	1.2757445	.5763275	1.3603136	.5922007	1.4521865	56
57	.5608447	1.2770987	.5765910	1.3617826	.5924663	1.4537848	57
58	.5611060	1.2784546	.5768545	1.3632535	.5927319	1.4553853	58
59	.5613674	1.2798124	.5771181	1.3647265	.5929976	1.4569882	59
60	0.5616289	1.2811720	0.5773817	1.3662016	0.5932634	1.4585933	60

TABLE 41. NATURAL VERSINES AND EXTERNAL SECANTS (*Continued*)

′	66°		67°		68°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	0.5932634	1.4585933	0.6092689	1.5593047	0.6253934	1.6694672	0
1	.5935291	1.4602008	.6095367	1.5610599	.6256631	1.6713906	1
2	.5937949	1.4618106	.6098045	1.5628176	.6259329	1.6733171	2
3	.5940607	1.4634227	.6100723	1.5645781	.6262027	1.6752465	3
4	.5943266	1.4650371	.6103402	1.5663412	.6264725	1.6771790	4
5	.5945925	1.4666538	.6106081	1.5681069	.6267423	1.6791145	5
6	.5948584	1.4682729	.6108760	1.5698752	.6270122	1.6810530	6
7	.5951244	1.4698943	.6111440	1.5716462	.6272821	1.6829945	7
8	.5953904	1.4715181	.6114120	1.5734199	.6275521	1.6849391	8
9	.5956564	1.4731442	.6116801	1.5751963	.6278220	1.6868867	9
10	0.5959225	1.4747726	0.6119482	1.5769753	0.6280921	1.6888374	10
11	.5961886	1.4764034	.6122163	1.5787570	.6283621	1.6907912	11
12	.5964547	1.4780366	.6124844	1.5805414	.6286332	1.6927480	12
13	.5967209	1.4796721	.6127526	1.5823284	.6289023	1.6947079	13
14	.5969871	1.4813100	.6130208	1.5841182	.6291724	1.6966709	14
15	.5972533	1.4829503	.6132890	1.5859107	.6294426	1.6986370	15
16	.5975196	1.4845929	.6135573	1.5877058	.6297128	1.7006061	16
17	.5977859	1.4862380	.6138256	1.5895037	.6299830	1.7025784	17
18	.5980522	1.4878854	.6140940	1.5913043	.6302532	1.7045538	18
19	.5983186	1.4895352	.6143623	1.5931077	.6305235	1.7065323	19
20	0.5985850	1.4911874	0.6146307	1.5949137	0.6307939	1.7085139	20
21	.5988514	1.4928421	.6148992	1.5967225	.6310642	1.7104987	21
22	.5991179	1.4944991	.6151676	1.5985341	.6313346	1.7124866	22
23	.5993844	1.4961586	.6154361	1.6003484	.6316050	1.7144777	23
24	.5996510	1.4978204	.6157047	1.6021654	.6318754	1.7164719	24
25	.5999175	1.4994848	.6159732	1.6039852	.6321459	1.7184693	25
26	.6001842	1.5011515	.6162418	1.6058078	.6324164	1.7204698	26
27	.6004508	1.5028207	.6165105	1.6076332	.6326870	1.7224735	27
28	.6007175	1.5044923	.6167791	1.6094613	.6329575	1.7244804	28
29	.6009842	1.5061663	.6170478	1.6112922	.6332281	1.7264905	29
30	0.6012509	1.5078428	0.6173166	1.6131259	0.6334988	1.7285038	30
31	.6015177	1.5095218	.6175853	1.6149624	.6337694	1.7305203	31
32	.6017845	1.5112032	.6178541	1.6168018	.6340401	1.7325400	32
33	.6020514	1.5128871	.6181230	1.6186439	.6343109	1.7345630	33
34	.6023182	1.5145735	.6183918	1.6204888	.6345816	1.7365892	34
35	.6025852	1.5162624	.6186607	1.6223366	.6348524	1.7386186	35
36	.6028521	1.5179537	.6189296	1.6241872	.6351232	1.7406512	36
37	.6031191	1.5196475	.6191986	1.6260406	.6353941	1.7426871	37
38	.6033861	1.5213438	.6194676	1.6278969	.6356649	1.7447263	38
39	.6036532	1.5230426	.6197366	1.6297560	.6359359	1.7467687	39
40	0.6039202	1.5247440	0.6200056	1.6316180	0.6362068	1.7488144	40
41	.6041873	1.5264478	.6202747	1.6334828	.6364778	1.7508634	41
42	.6044545	1.5281541	.6205438	1.6353506	.6367488	1.7529157	42
43	.6047217	1.5298630	.6208130	1.6372211	.6370198	1.7549712	43
44	.6049889	1.5315744	.6210822	1.6390946	.6372909	1.7570301	44
45	.6052561	1.5332883	.6213514	1.6409710	.6375620	1.7590923	45
46	.6055234	1.5350048	.6216206	1.6428502	.6378331	1.7611578	46
47	.6057907	1.5367238	.6218899	1.6447323	.6381042	1.7632267	47
48	.6060581	1.5384453	.6221592	1.6466174	.6383754	1.7652988	48
49	.6063255	1.5401694	.6224286	1.6485054	.6386466	1.7673744	49
50	0.6065929	1.5418961	0.6226979	1.6503962	0.6389179	1.7694532	50
51	.6068603	1.5436253	.6229673	1.6522901	.6391892	1.7715355	51
52	.6071278	1.5453571	.6232368	1.6541868	.6394605	1.7736211	52
53	.6073953	1.5470915	.6235062	1.6560865	.6397318	1.7757100	53
54	.6076629	1.5488284	.6237757	1.6579891	.6400032	1.7778024	54
55	.6079305	1.5505680	.6240453	1.6598947	.6402746	1.7798982	55
56	.6081981	1.5523101	.6243148	1.6618033	.6405460	1.7819973	56
57	.6084657	1.5540548	.6245844	1.6637148	.6408175	1.7840999	57
58	.6087334	1.5558022	.6248541	1.6656292	.6410890	1.7862059	58
59	.6090011	1.5575521	.6251237	1.6675467	.6413605	1.7883153	59
60	0.6092689	1.5593047	0.6253934	1.6694672	0.6416321	1.7904281	60

TABLE 41. NATURAL VERSINES AND EXTERNAL SECANTS (*Continued*)

′	69°		70°		71°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	0.6416321	1.7904281	0.6579799	1.9238044	0.6744318	2.0715535	0
1	.6419036	1.7925444	.6582532	1.9261431	.6747069	2.0741507	1
2	.6421752	1.7946641	.6585266	1.9284858	.6749820	2.0767525	2
3	.6424469	1.7967873	.6588000	1.9308326	.6752571	2.0793590	3
4	.6427186	1.7989140	.6590735	1.9331833	.6755322	2.0819702	4
5	.6429903	1.8010441	.6593469	1.9355380	.6758074	2.0845860	5
6	.6432620	1.8031777	.6596204	1.9378968	.6760826	2.0872066	6
7	.6435338	1.8053148	.6598940	1.9402597	.6763578	2.0898319	7
8	.6438056	1.8074554	.6601675	1.9426265	.6766330	2.0924620	8
9	.6440774	1.8095995	.6604411	1.9449975	.6769083	2.0950967	9
10	0.6443492	1.8117471	0.6607148	1.9473725	0.6771836	2.0977363	10
11	.6446211	1.8138982	.6609884	1.9497516	.6774589	2.1003805	11
12	.6448930	1.8160529	.6612621	1.9521348	.6777343	2.1030296	12
13	.6451650	1.8182111	.6615358	1.9545221	.6780097	2.1056835	13
14	.6454370	1.8203729	.6618095	1.9569135	.6782851	2.1083422	14
15	.6457090	1.8225382	.6620833	1.9593090	.6785605	2.1110057	15
16	.6459810	1.8247071	.6623571	1.9617087	.6788360	2.1136740	16
17	.6462531	1.8268796	.6626309	1.9641125	.6791115	2.1163472	17
18	.6465252	1.8290556	.6629047	1.9665205	.6793870	2.1190252	18
19	.6467973	1.8312353	.6631786	1.9689327	.6796626	2.1217081	19
20	0.6470694	1.8334185	0.6634525	1.9713490	0.6799381	2.1243959	20
21	.6473416	1.8356054	.6637265	1.9737695	.6802137	2.1270886	21
22	.6476138	1.8377958	.6640004	1.9761942	.6804894	2.1297862	22
23	.6478861	1.8399899	.6642744	1.9786231	.6807650	2.1324887	23
24	.6481584	1.8421877	.6645484	1.9810563	.6810407	2.1351962	24
25	.6484307	1.8443891	.6648225	1.9834936	.6813164	2.1379086	25
26	.6487030	1.8465941	.6650966	1.9859352	.6815921	2.1406259	26
27	.6489753	1.8488028	.6653707	1.9883811	.6818679	2.1433483	27
28	.6492477	1.8510152	.6656448	1.9908312	.6821437	2.1460756	28
29	.6495202	1.8532312	.6659190	1.9932856	.6824195	2.1488079	29
30	0.6497926	1.8554510	0.6661931	1.9957443	0.6826953	2.1515453	30
31	.6500651	1.8576744	.6664674	1.9982073	.6829712	2.1542877	31
32	.6503376	1.8599015	.6667416	2.0006746	.6832471	2.1570351	32
33	.6506102	1.8621324	.6670159	2.0031462	.6835230	2.1597876	33
34	.6508827	1.8643670	.6672902	2.0056221	.6837990	2.1625452	34
35	.6511553	1.8666053	.6675645	2.0081024	.6840750	2.1653078	35
36	.6514280	1.8688474	.6678389	2.0105870	.6843510	2.1680756	36
37	.6517006	1.8710932	.6681133	2.0130760	.6846270	2.1708484	37
38	.6519733	1.8733428	.6683877	2.0155694	.6849031	2.1736264	38
39	.6522460	1.8755961	.6686621	2.0180672	.6851791	2.1764095	39
40	0.6525188	1.8778532	0.6689366	2.0205693	0.6854552	2.1791978	40
41	.6527915	1.8801142	.6692111	2.0230759	.6857314	2.1819913	41
42	.6530643	1.8823789	.6694856	2.0255868	.6860075	2.1847899	42
43	.6533372	1.8846474	.6697602	2.0281023	.6862837	2.1875937	43
44	.6536100	1.8869198	.6700347	2.0306221	.6865600	2.1904028	44
45	.6538829	1.8891960	.6703094	2.0331464	.6868362	2.1932170	45
46	.6541559	1.8914760	.6705840	2.0356752	.6871125	2.1960365	46
47	.6544288	1.8937598	.6708587	2.0382084	.6873888	2.1988613	47
48	.6547018	1.8960475	.6711334	2.0407462	.6876651	2.2016913	48
49	.6549748	1.8983391	.6714081	2.0432884	.6879414	2.2045266	49
50	0.6552479	1.9006346	0.6716828	2.0458352	0.6882178	2.2073673	50
51	.6555209	1.9029339	.6719576	2.0483864	.6884942	2.2102132	51
52	.6557940	1.9052372	.6722324	2.0509423	.6887706	2.2130644	52
53	.6560671	1.9075443	.6725072	2.0535026	.6890471	2.2159210	53
54	.6563403	1.9098553	.6727821	2.0560675	.6893236	2.2187830	54
55	.6566135	1.9121703	.6730570	2.0586370	.6896001	2.2216503	55
56	.6568867	1.9144892	.6733319	2.0612111	.6898766	2.2245230	56
57	.6571600	1.9168121	.6736068	2.0637898	.6901532	2.2274011	57
58	.6574332	1.9191389	.6738818	2.0663731	.6904298	2.2302846	58
59	.6577065	1.9214697	.6741568	2.0689610	.6907064	2.2331736	59
60	0.6579799	1.9238044	0.6744318	2.0715535	0.6909830	2.2360680	60

TABLE 41. NATURAL VERSINES AND EXTERNAL SECANTS (*Continued*)

′	72°		73°		74°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	0.6909830	2.2360680	0.7076283	2.4203036	0.7243626	2.6279553	0
1	.6912597	2.2389678	.7079065	2.4235611	.7246423	2.6316395	1
2	.6915364	2.2418732	.7081847	2.4268251	.7249219	2.6353316	2
3	.6918131	2.2447840	.7084629	2.4300956	.7252016	2.6390315	3
4	.6920898	2.2477003	.7087412	2.4333727	.7254813	2.6427392	4
5	.6923666	2.2506222	.7090195	2.4366563	.7257610	2.6464548	5
6	.6926434	2.2535496	.7092978	2.4399465	.7260408	2.6501783	6
7	.6929202	2.2564825	.7095761	2.4432433	.7263206	2.6539097	7
8	.6931970	2.2594211	.7098545	2.4465467	.7266003	2.6576491	8
9	.6934739	2.2623652	.7101329	2.4498568	.7268802	2.6613964	9
10	0.6937508	2.2653149	0.7104113	2.4531735	0.7271600	2.6651518	10
11	.6940277	2.2682702	.7106897	2.4564969	.7274399	2.6689151	11
12	.6943047	2.2712311	.7109682	2.4598269	.7277198	2.6726865	12
13	.6945817	2.2741977	.7112467	2.4631637	.7279997	2.6764660	13
14	.6948587	2.2771700	.7115252	2.4665073	.7282796	2.6802536	14
15	.6951357	2.2801479	.7118037	2.4698576	.7285596	2.6840493	15
16	.6954128	2.2831316	.7120823	2.4732146	.7288395	2.6878532	16
17	.6956898	2.2861209	.7123609	2.4765785	.7291195	2.6916652	17
18	.6959669	2.2891160	.7126395	2.4799492	.7293996	2.6954854	18
19	.6962441	2.2921168	.7129181	2.4833267	.7296796	2.6993139	19
20	0.6965212	2.2951234	0.7131968	2.4867110	0.7299597	2.7031506	20
21	.6967984	2.2981357	.7134754	2.4901023	.7302398	2.7069956	21
22	.6970756	2.3011539	.7137542	2.4935004	.7305199	2.7108489	22
23	.6973529	2.3041778	.7140329	2.4969055	.7308000	2.7147105	23
24	.6976301	2.3072076	.7143116	2.5003175	.7310802	2.7185805	24
25	.6979074	2.3102432	.7145904	2.5037365	.7313604	2.7224589	25
26	.6981847	2.3132847	.7148692	2.5071625	.7316406	2.7263457	26
27	.6984620	2.3163320	.7151480	2.5105954	.7319208	2.7302409	27
28	.6987394	2.3193853	.7154269	2.5140354	.7322011	2.7341446	28
29	.6990168	2.3224444	.7157058	2.5174824	.7324813	2.7380568	29
30	0.6992942	2.3255095	0.7159847	2.5209365	0.7327616	2.7419775	30
31	.6995716	2.3285805	.7162636	2.5243977	.7330419	2.7459068	31
32	.6998491	2.3316575	.7165425	2.5278660	.7333223	2.7498447	32
33	.7001266	2.3347405	.7168215	2.5313414	.7336027	2.7537911	33
34	.7004041	2.3378294	.7171005	2.5348240	.7338830	2.7577462	34
35	.7006816	2.3409244	.7173795	2.5383138	.7341634	2.7617100	35
36	.7009592	2.3440254	.7176585	2.5418107	.7344439	2.7656824	36
37	.7012368	2.3471324	.7179376	2.5453149	.7347243	2.7696636	37
38	.7015144	2.3502455	.7182167	2.5488263	.7350048	2.7736535	38
39	.7017921	2.3533647	.7184958	2.5523450	.7352853	2.7776522	39
40	0.7020697	2.3564900	0.7187749	2.5558710	0.7355658	2.7816596	40
41	.7023474	2.3596214	.7190541	2.5594042	.7358464	2.7856760	41
42	.7026251	2.3627589	.7193333	2.5629448	.7361270	2.7897011	42
43	.7029029	2.3659026	.7196125	2.5664928	.7364075	2.7937352	43
44	.7031806	2.3690524	.7198917	2.5700481	.7366882	2.7977782	44
45	.7034584	2.3722084	.7201710	2.5736108	.7369688	2.8018301	45
46	.7037362	2.3753707	.7204503	2.5771810	.7372494	2.8058911	46
47	.7040141	2.3785391	.7207296	2.5807586	.7375301	2.8099610	47
48	.7042919	2.3817138	.7210089	2.5843437	.7378108	2.8140399	48
49	.7045698	2.3848948	.7212882	2.5879362	.7380915	2.8181280	49
50	0.7048478	2.3880820	0.7215676	2.5915363	0.7383723	2.8222251	50
51	.7051257	2.3912755	.7218470	2.5951439	.7386531	2.8263313	51
52	.7054037	2.3944754	.7221264	2.5987590	.7389338	2.8304467	52
53	.7056817	2.3976816	.7224059	2.6023818	.7392147	2.8345713	53
54	.7059597	2.4008941	.7226853	2.6060121	.7394955	2.8387052	54
55	.7062377	2.4041130	.7229648	2.6096501	.7397763	2.8428482	55
56	.7065158	2.4073382	.7232444	2.6132957	.7400572	2.8470006	56
57	.7067939	2.4105699	.7235239	2.6169490	.7403381	2.8511622	57
58	.7070720	2.4138080	.7238035	2.6206101	.7406190	2.8553332	58
59	.7073501	2.4170526	.7240830	2.6242788	.7409000	2.8595135	59
60	0.7076283	2.4203036	0.7243626	2.6279553	0.7411810	2.8637033	60

TABLE 41. NATURAL VERSINES AND EXTERNAL SECANTS (*Continued*)

′	75°		76°		77°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	0.7411810	2.8637033	0.7580781	3.1335655	0.7750489	3.4454115	0
1	.7414619	2.8679025	.7583604	3.1383939	.7753324	3.4510198	1
2	.7417430	2.8721112	.7586426	3.1432339	.7756158	3.4566428	2
3	.7420240	2.8763293	.7589249	3.1480856	.7758993	3.4622803	3
4	.7423050	2.8805570	.7592073	3.1529491	.7761828	3.4679324	4
5	.7425861	2.8847943	.7594896	3.1578243	.7764663	3.4735993	5
6	.7428672	2.8890411	.7597720	3.1627114	.7767499	3.4792810	6
7	.7431483	2.8932976	.7600543	3.1676102	.7770334	3.4849775	7
8	.7434295	2.8975637	.7603367	3.1725210	.7773170	3.4906889	8
9	.7437106	2.9018395	.7606192	3.1774438	.7776006	3.4964152	9
10	0.7439918	2.9061250	0.7609016	3.1823785	0.7778842	3.5021565	10
11	.7442730	2.9104203	.7611841	3.1873258	.7781679	3.5079129	11
12	.7445542	2.9147254	.7614665	3.1922840	.7784515	3.5136844	12
13	.7448355	2.9190403	.7617490	3.1972549	.7787352	3.5194711	13
14	.7451168	2.9233651	.7620316	3.2022380	.7790189	3.5252730	14
15	.7453981	2.9276997	.7623141	3.2072333	.7793026	3.5310903	15
16	.7456794	2.9320443	.7625967	3.2122408	.7795863	3.5369229	16
17	.7459607	2.9363988	.7628793	3.2172606	.7798700	3.5427709	17
18	.7462421	2.9407633	.7631619	3.2222928	.7801538	3.5486344	18
19	.7465234	2.9451379	.7634445	3.2273373	.7804376	3.5545134	19
20	0.7468048	2.9495224	0.7637271	3.2323943	0.7807214	3.5604080	20
21	.7470863	2.9539171	.7640098	3.2374637	.7810052	3.5663183	21
22	.7473677	2.9583219	.7642925	3.2425457	.7812890	3.5722444	22
23	.7476492	2.9627369	.7645752	3.2476402	.7815729	3.5781862	23
24	.7479306	2.9671621	.7648579	3.2527474	.7818568	3.5841439	24
25	.7482121	2.9715975	.7651406	3.2578671	.7821407	3.5901174	25
26	.7484937	2.9760431	.7654234	3.2629996	.7824246	3.5961070	26
27	.7487752	2.9804991	.7657062	3.2681449	.7827085	3.6021126	27
28	.7490568	2.9849654	.7659890	3.2733029	.7829924	3.6081343	28
29	.7493384	2.9894421	.7662718	3.2784738	.7832764	3.6141722	29
30	0.7496200	2.9939292	0.7665546	3.2836576	0.7835604	3.6202263	30
31	.7499016	2.9984267	.7668375	3.2888543	.7838444	3.6262967	31
32	.7501833	3.0029347	.7671204	3.2940640	.7841284	3.6323835	32
33	.7504650	3.0074532	.7674033	3.2992867	.7844124	3.6384867	33
34	.7507467	3.0119823	.7676862	3.3045225	.7846965	3.6446064	34
35	.7510284	3.0165219	.7679691	3.3097715	.7849806	3.6507427	35
36	.7513101	3.0210722	.7682521	3.3150336	.7852647	3.6568956	36
37	.7515919	3.0256332	.7685351	3.3203090	.7855488	3.6630652	37
38	.7518737	3.0302048	.7688181	3.3255977	.7858329	3.6692516	38
39	.7521555	3.0347872	.7691011	3.3308996	.7861171	3.6754548	39
40	0.7524373	3.0393804	0.7693841	3.3362150	0.7864012	3.6816748	40
41	.7527191	3.0439844	.7696672	3.3415438	.7866854	3.6879119	41
42	.7530010	3.0485992	.7699503	3.3468861	.7869696	3.6941660	42
43	.7532829	3.0532249	.7702334	3.3522419	.7872538	3.7004372	43
44	.7535648	3.0578615	.7705165	3.3576113	.7875381	3.7067256	44
45	.7538467	3.0625091	.7707996	3.3629943	.7878223	3.7130313	45
46	.7541287	3.0671677	.7710828	3.3683910	.7881066	3.7193542	46
47	.7544106	3.0718374	.7713659	3.3738015	.7883909	3.7256945	47
48	.7546926	3.0765181	.7716491	3.3792257	.7886752	3.7320524	48
49	.7549746	3.0812100	.7719323	3.3846638	.7889595	3.7384277	49
50	0.7552567	3.0859130	0.7722156	2.3901158	0.7892439	3.7448206	50
51	.7555387	3.0906272	.7724988	3.3955817	.7895282	3.7512312	51
52	.7558208	3.0953526	.7727821	3.4010616	.7898126	3.7576596	52
53	.7561029	3.1000893	.7730654	3.4065556	.7900970	3.7641058	53
54	.7563850	3.1048374	.7733487	3.4120637	.7903814	3.7705699	54
55	.7566671	3.1095967	.7736320	3.4175859	.7906659	3.7770519	55
56	.7569493	3.1143675	.7739154	3.4231224	.7909503	3.7835520	56
57	.7572315	3.1191498	.7741987	3.4286731	.7912348	3.7900702	57
58	.7575137	3.1239435	.7744821	3.4342382	.7915193	3.7966066	58
59	.7577959	3.1287487	.7747655	3.4398176	.7918038	3.8031613	59
60	0 7580781	3.1335655	0.7750489	3.4454115	0.7920883	3.8097343	60

TABLE 41. NATURAL VERSINES AND EXTERNAL SECANTS (*Continued*)

′	78°		79°		80°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	0.7920883	3.8097343	0.8091910	4.2408431	0.8263518	4.7587705	0
1	.7923728	3.8163258	.8094766	4.2486979	.8266383	4.7682867	1
2	.7926574	3.8229357	.8097621	4.2565768	.8269248	4.7778350	2
3	.7929420	3.8295643	.8100477	4.2644798	.8272113	4.7874153	3
4	.7932266	3.8362114	.8103333	4.2724070	.8274978	4.7970280	4
5	.7935112	3.8428774	.8106189	4.2803587	.8277844	4.8066732	5
6	.7937958	3.8495621	.8109046	4.2883347	.8280709	4.8163510	6
7	.7940805	3.8562657	.8111902	4.2963354	.8283575	4.8260617	7
8	.7943651	3.8629883	.8114759	4.3043608	.8286440	4.8358053	8
9	.7946498	3.8697299	.8117615	4.3124109	.8289306	4.8455820	9
10	0.7949345	3.8764907	0.8120472	4.3204860	0.8292172	4.8553921	10
11	.7952192	3.8832707	.8123330	4.3285861	.8295039	4.8652356	11
12	.7955039	3.8900700	.8126187	4.3367114	.8297905	4.8751128	12
13	.7957887	3.8968886	.8129044	4.3448620	.8300772	4.8850238	13
14	.7960735	3.9037267	.8131902	4.3530379	.8303638	4.8949688	14
15	.7963582	3.9105844	.8134760	4.3612393	.8306505	4.9049479	15
16	.7966431	3.9174616	.8137618	4.3694664	.8309372	4.9149614	16
17	.7969279	3.9243586	.8140476	4.3777192	.8312239	4.9250095	17
18	.7972127	3.9312754	.8143334	4.3859979	.8315106	4.9350922	18
19	.7974976	3.9382120	.8146192	4.3943026	.8317974	4.9452098	19
20	0.7977824	3.9451687	0.8149051	4.4026333	0.8320841	4.9553625	20
21	.7980673	3.9521453	.8151909	4.4109903	.8323709	4.9655504	21
22	.7983522	3.9591421	.8154768	4.4193737	.8326577	4.9757737	22
23	.7986371	3.9661591	.8157627	4.4277835	.8329444	4.9860326	23
24	.7989221	3.9731964	.8160486	4.4362199	.8332313	4.9963274	24
25	.7992070	3.9802541	.8163346	4.4446831	.8335181	5.0066581	25
26	.7994920	3.9873323	.8166205	4.4531731	.8338049	5.0170250	26
27	.7997770	3.9944311	.8169065	4.4616901	.8340918	5.0274282	27
28	.8000620	4.0015505	.8171925	4.4702342	.8343786	5.0378680	28
29	.8003470	4.0086907	.8174785	4.4788056	.8346655	5.0483445	29
30	0.8006321	4.0158517	0.8177645	4.4874043	0.8349524	5.0588580	30
31	.8009171	4.0230337	.8180505	4.4960305	.8352393	5.0694085	31
32	.8012022	4.0302367	.8183365	4.5046843	.8355262	5.0799964	32
33	.8014873	4.0374607	.8186226	4.5133659	.8358132	5.0906219	33
34	.8017724	4.0447060	.8189087	4.5220754	.8361001	5.1012850	34
35	.8020575	4.0519726	.8191948	4.5308129	.8363871	5.1119861	35
36	.8023427	4.0592606	.8194809	4.5395786	.8366740	5.1227253	36
37	.8026278	4.0665701	.8197670	4.5483726	.8369610	5.1335028	37
38	.8029130	4.0739012	.8200531	4.5571951	.8372480	5.1443189	38
39	.8031982	4.0812539	.8203393	4.5660460	.8375350	5.1551736	39
40	0.8034834	4.0886284	0.8206254	4.5749258	0.8378221	5.1660674	40
41	.8037686	4.0960248	.8209116	4.5838343	.8381091	5.1770003	41
42	.8040539	4.1034431	.8211978	4.5927719	.8383962	5.1879725	42
43	.8043391	4.1108835	.8214840	4.6017386	.8386833	5.1989843	43
44	.8046244	4.1183461	.8217702	4.6107345	.8389703	5.2100359	44
45	.8049097	4.1258309	.8220565	4.6197599	.8392574	5.2211275	45
46	.8051950	4.1333381	.8223427	4.6288148	.8395445	5.2322594	46
47	.8054803	4.1408677	.8226290	4.6378995	.8398317	5.2434316	47
48	.8057656	4.1484199	.8229153	4.6470140	.8401188	5.2546446	48
49	.8060510	4.1559948	.8232016	4.6561584	.8404060	5.2658984	49
50	0.8063364	4.1635924	0.8234879	4.6653331	0.8406931	5.2771933	50
51	.8066218	4.1712128	.8237742	4.6745380	.8409803	5.2885295	51
52	.8069072	4.1788563	.8240605	4.6837734	.8412675	5.2999073	52
53	.8071926	4.1865228	.8243469	4.6930393	.8415547	5.3113269	53
54	.8074780	4.1942125	.8246333	4.7023360	.8418419	5.3227884	54
55	.8077635	4.2019254	.8249197	4.7116636	.8421292	5.3342923	55
56	.8080490	4.2096618	.8252061	4.7210223	.8424164	5.3458386	56
57	.8083344	4.2174216	.8254925	4.7304121	.8427037	5.3574276	57
58	.8086199	4.2252050	.8257789	4.7398333	.8429909	5.3690595	58
59	.8089055	4.2330121	.8260654	4.7492861	.8432782	5.3807347	59
60	0.8091910	4.2408431	0.8263518	4.7587705	0.8435655	5.3924532	60

TABLE 41. NATURAL VERSINES AND EXTERNAL SECANTS (*Continued*)

′	81°		82°		83°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	0.8435655	5.3924532	0.8608269	6.1852965	0.8781307	7.2055090	0
1	.8438528	5.4042154	.8611150	6.2001996	.8784194	7.2249952	1
2	.8441402	5.4160216	.8614030	6.2151653	.8787081	7.2445748	2
3	.8444275	5.4278719	.8616911	6.2301940	.8789969	7.2642485	3
4	.8447149	5.4397666	.8619792	6.2452859	.8792856	7.2840171	4
5	.8450022	5.4517059	.8622673	6.2604417	.8795744	7.3038812	5
6	.8452896	5.4636901	.8625555	6.2756616	.8798632	7.3238415	6
7	.8455770	5.4757195	.8628436	6.2909460	.8801519	7.3438986	7
8	.8458644	5.4877944	.8631317	6.3062954	.8804407	7.3640534	8
9	.8461518	5.4999148	.8634199	6.3217102	.8807296	7.3843065	9
10	0.8464393	5.5120812	0.8637081	6.3371909	0.8810184	7.4046586	10
11	.8467267	5.5242938	.8639962	6.3527377	.8813072	7.4251105	11
12	.8470142	5.5365528	.8642844	6.3683512	.8815960	7.4456629	12
13	.8473016	5.5488586	.8645726	6.3840318	.8818849	7.4663165	13
14	.8475891	5.5612113	.8648608	6.3997798	.8821737	7.4870721	14
15	.8478766	5.5736112	.8651491	6.4155959	.8824626	7.5079304	15
16	.8481641	5.5860587	.8654373	6.4314803	.8827515	7.5288923	16
17	.8484516	5.5985540	.8657256	6.4474335	.8830404	7.5499584	17
18	.8487392	5.6110973	.8660138	6.4634560	.8833293	7.5711295	18
19	.8490267	5.6236890	.8663021	6.4795482	.8836182	7.5924065	19
20	0.8493143	5.6363293	0.8665904	6.4957106	0.8839071	7.6137901	20
21	.8496019	5.6490184	.8668787	6.5119437	.8841960	7.6352812	21
22	.8498894	5.6617568	.8671670	6.5282478	.8844849	7.6568805	22
23	.8501770	5.6745446	.8674553	6.5446236	.8847739	7.6785889	23
24	.8504647	5.6873822	.8677436	6.5610713	.8850628	7.7004071	24
25	.8507523	5.7002699	.8680319	6.5775916	.8853518	7.7223361	25
26	.8510399	5.7132079	.8683203	6.5941849	.8856408	7.7443766	26
27	.8513276	5.7261965	.8686087	6.6108516	.8859298	7.7665295	27
28	.8516152	5.7392360	.8688970	6.6275923	.8862188	7.7887957	28
29	.8519029	5.7523268	.8691854	6.6444075	.8865078	7.8111761	29
30	0.8521906	5.7654691	0.8694738	6.6612076	0.8867968	7.8336715	30
31	.8524783	5.7786632	.8697622	6.6782631	.8870858	7.8562828	31
32	.8527660	5.7919005	.8700506	6.6953047	.8873748	7.8790109	32
33	.8530537	5.8052082	.8703391	6.7124227	.8876639	7.9018567	33
34	.8533415	5.8185597	.8706275	6.7296176	.8879529	7.9248211	34
35	.8536292	5.8319642	.8709159	6.7468901	.8882420	7.9479051	35
36	.8539170	5.8454222	.8712044	6.7642406	.8885311	7.9711095	36
37	.8542047	5.8589328	.8714929	6.7816697	.8888201	7.9944354	37
38	.8544925	5.8724995	.8717814	6.7991778	.8891092	8.0178837	38
39	.8547803	5.8861195	.8720698	6.8167656	.8893983	8.0414553	39
40	0.8550681	5.8997942	0.8723584	6.8344335	0.8896874	8.0651512	40
41	.8553560	5.9135239	.8726469	6.8521821	.8899766	8.0889725	41
42	.8556438	5.9273089	.8729354	6.8700120	.8902657	8.1129200	42
43	.8559316	5.9411496	.8732239	6.8879238	.8905548	8.1369949	43
44	.8562195	5.9550464	.8735125	6.9059179	.8908440	8.1611980	44
45	.8565074	5.9689994	.8738010	6.9239950	.8911331	8.1855305	45
46	.8567953	5.9830092	.8740896	6.9421556	.8914223	8.2099934	46
47	.8570832	5.9970760	.8743782	6.9604003	.8917115	8.2345877	47
48	.8573711	6.0112001	.8746668	6.9787298	.8920006	8.2593145	48
49	.8576590	6.0253820	.8749554	6.9971445	.8922898	8.2841749	49
50	0.8579469	6.0396220	0.8752440	7.0156450	0.8925790	8.3091699	50
51	.8582349	6.0539205	.8755326	7.0342321	.8928682	8.3343006	51
52	.8585228	6.0682777	.8758212	7.0529062	.8931575	8.3595682	52
53	.8588108	6.0826941	.8761099	7.0716681	.8934467	8.3849738	53
54	.8590988	6.0971700	.8763985	7.0905182	.8937359	8.4105184	54
55	.8593868	6.1117059	.8766872	7.1094573	.8940252	8.4362033	55
56	.8596748	6.1263019	.8769759	7.1284860	.8943144	8.4620296	56
57	.8599628	6.1409587	.8772645	7.1476048	.8946037	8.4879984	57
58	.8602508	6.1556764	.8775532	7.1668145	.8948930	8.5141110	58
59	.8605388	6.1704556	.8778419	7.1861157	.8951822	8.5403686	59
60	0.8608269	6.1852965	0.8781307	7.2055090	0.8954715	8.5667722	60

TABLE 41. NATURAL VERSINES AND EXTERNAL SECANTS (*Continued*)

′	84°		85°		86°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	0.8954715	8.5667722	0.9128443	10.473713	0.9302435	13.335587	0
1	.8957608	8.5933233	.9131340	10.511990	.9305337	13.395471	1
2	.8960501	8.6200229	.9134238	10.550523	.9308239	13.455859	2
3	.8963395	8.6468724	.9137136	10.589316	.9311141	13.516757	3
4	.8966288	8.6738730	.9140034	10.628372	.9314043	13.578172	4
5	.8969181	8.7010260	.9142933	10.667693	.9316945	13.640109	5
6	.8972075	8.7283327	.9145831	10.707282	.9319847	13.702576	6
7	.8974968	8.7557944	.9148729	10.747141	.9322749	13.765580	7
8	.8977862	8.7834124	.9151627	10.787274	.9325651	13.829128	8
9	.8980755	8.8111880	.9154526	10.827683	.9328554	13.893226	9
10	0.8983649	8.8391227	0.9157424	10.868370	0.9331456	13.957882	10
11	.8986543	8.8672176	.9160323	10.909340	.9334359	14.023103	11
12	.8989437	8.8954744	.9163222	10.950595	.9337261	14.088896	12
13	.8992331	8.9238943	.9166120	10.992137	.9340164	14.155270	13
14	.8995225	8.9524787	.9169019	11.033970	.9343066	14.222231	14
15	.8998119	8.9812291	.9171918	11.076098	.9345969	14.289788	15
16	.9001014	9.010147	.9174817	11.118522	.9348871	14.357949	16
17	.9003908	9.039234	.9177716	11.161246	.9351774	14.426721	17
18	.9006803	9.068491	.9180615	11.204274	.9354677	14.496114	18
19	.9009697	9.097920	.9183514	11.247608	.9357580	14.566135	19
20	0.9012592	9.127522	0.9186413	11.291252	0.9360483	14.636793	20
21	.9015486	9.157300	.9189313	11.335210	.9363386	14.708096	21
22	.9018381	9.187254	.9192212	11.379484	.9366289	14.780054	22
23	.9021276	9.217386	.9195111	11.424078	.9369192	14.852676	23
24	.9024171	9.247697	.9198011	11.468995	.9372095	14.925971	24
25	.9027066	9.278190	.9200910	11.514240	.9374998	14.999948	25
26	.9029961	9.308866	.9203810	11.559815	.9377901	15.074617	26
27	.9032856	9.339726	.9206710	11.605724	.9380804	15.149987	27
28	.9035752	9.370772	.9209609	11.651971	.9383708	15.226069	28
29	.9038647	9.402007	.9212509	11.698560	.9386611	15.302873	29
30	0.9041542	9.433431	0.9215409	11.745495	0.9389515	15.380408	30
31	.9044438	9.465046	.9218309	11.792779	.9392418	15.458686	31
32	.9047334	9.496854	.9221209	11.840416	.9395322	15.537717	32
33	.9050229	9.528857	.9224109	11.888410	.9398225	15.617512	33
34	.9053125	9.561057	.9227009	11.936765	.9401129	15.698082	34
35	.9056021	9.593455	.9229909	11.985486	.9404033	15.779439	35
36	.9058917	9.626054	.0232810	12.034576	.9406936	15.861594	36
37	.9061813	9.658854	.9235710	12.084040	.9409840	15.944559	37
38	.9064709	9.691859	.9238610	12.133882	.9412744	16.028346	38
39	.9067605	9.725070	.9241511	12.184106	.9415648	16.112966	39
40	0.9070501	9.758488	0.9244411	12.234717	0.9418552	16.198434	40
41	.9073398	9.792117	.9247312	12.285719	.9421456	16.284761	41
42	.9076294	9.825957	.9250213	12.337116	.9424360	16.371960	42
43	.9079191	9.860011	.9253113	12.388914	.9427264	16.460046	43
44	.9082087	9.894281	.9256014	12.441118	.9430168	16.549030	44
45	.9084984	9.928768	.9258915	12.493731	.9433072	16.638928	45
46	.9087881	9.963476	.9261816	12.546758	.9435976	16.729753	46
47	.9090777	9.998406	.9264717	12.600205	.9438881	16.821520	47
48	.9093674	10.033560	.9267618	12.654077	.9441785	16.914243	48
49	.9096571	10.068940	.9270519	12.708379	.9444689	17.007937	49
50	0.9099468	10.104549	0.9273420	12.763115	0.9447594	17.102619	50
51	.9102365	10.140389	.9276322	12.818291	.9450498	17.198303	51
52	.9105262	10.176462	.9279223	12.873913	.9453403	17.295005	52
53	.9108160	10.212770	.9282124	12.929985	.9456307	17.392742	53
54	.9111057	10.249316	.9285026	12.986514	.9459212	17.491530	54
55	.9113954	10.286101	.9287927	13.043504	.9462117	17.591387	55
56	.9116852	10.323129	.9290829	13.100963	.9465021	17.692330	56
57	.9119749	10.360402	.9293730	13.158894	.9467926	17.794377	57
58	.9122647	10.397922	.9296632	13.217304	.9470831	17.897545	58
59	.9125545	10.435692	.9299533	13.276200	.9473736	18.001854	59
60	0.9128443	10.473713	0.9302435	13.335587	0.9476640	18.107323	60

TABLE 41. NATURAL VERSINES AND EXTERNAL SECANTS (*Continued*)

′	87°		88°		89°		′
	Vers.	Ex. sec.	Vers.	Ex. sec.	Vers.	Ex. sec.	
0	0.9476640	18.107323	0.9651005	27.653708	0.9825476	56.298688	0
1	.9479545	18.213970	.9653912	27.894398	.9828384	57.269755	1
2	.9482450	18.321816	.9656819	28.139169	.9831293	58.274308	2
3	.9485355	18.430882	.9659726	28.388124	.9834201	59.314110	3
4	.9488260	18.541187	.9662634	28.641373	.9837110	60.391050	4
5	.9491165	18.652754	.9665541	28.899026	.9840018	61.507153	5
6	.9494071	18.765604	.9668448	29.161201	.9842927	62.664595	6
7	.9496976	18.879758	.9671356	29.428017	.9845835	63.865716	7
8	.9499881	18.995241	.9674263	29.699598	.9848744	65.113036	8
9	.9502786	19.112075	.9677170	29.976074	.9851652	66.409272	9
10	0.9505692	19.230284	0.9680078	30.257577	0.9854561	67.757360	10
11	.9508597	19.349893	.9682985	30.544246	.9857470	69.160474	11
12	.9511502	19.470926	.9685892	30.836225	.9860378	70.622052	12
13	.9514408	19.593409	.9688800	31.133663	.9863287	72.145827	13
14	.9517313	19.717368	.9691707	31.436713	.9866195	73.735856	14
15	.9520219	19.842830	.9694615	31.745537	.9869104	75.396554	15
16	.9523124	19.969824	.9697522	32.060300	.9872013	77.132742	16
17	.9526030	20.098376	.9700430	32.381176	.9874921	78.949684	17
18	.9528935	20.228515	.9703338	32.708345	.9877830	80.853150	18
19	.9531841	20.360272	.9706245	33.041994	.9880739	82.849470	19
20	0.9534747	20.493676	0.9709153	33.382316	0.9883647	84.945609	20
21	.9537653	20.628759	.9712060	33.729515	.9886556	87.149244	21
22	.9540558	20.765553	.9714968	34.083800	.9889465	89.468863	22
23	.9543464	20.904090	.9717876	34.445391	.9892373	91.913869	23
24	.9546370	21.044403	.9720784	34.814517	.9895282	94.494711	24
25	.9549276	21.186528	.9723691	35.191414	.9898191	97.223033	25
26	.9552182	21.330499	.9726599	35.576332	.9901100	100.11185	26
27	.9555088	21.476353	.9729507	35.969528	.9904008	103.17574	27
28	.9557994	21.624126	.9732415	36.371273	.9906917	106.43114	28
29	.9560900	21.773857	.9735323	36.781849	.9909826	109.89656	29
30	0.9563806	21.925586	0.9738231	37.201550	0.9912735	113.59301	30
31	.9566712	22.079351	.9741138	37.630683	.9915643	117.54440	31
32	.9569618	22.235196	.9744046	38.069571	.9918552	121.77803	32
33	.9572525	22.393161	.9746954	38.518549	.9921461	126.32526	33
34	.9575431	22.553291	.9749862	38.977969	.9924370	131.22229	34
35	.9578337	22.715630	.9752770	39.448201	.9927279	136.51108	35
26	.9581243	22.880224	.9755678	39.929630	.9930187	142.24061	36
37	.9584150	23.047121	.9758586	40.422660	.9933096	148.46837	37
38	.9587056	23.216370	.9761494	40.927717	.9936005	155.26228	38
39	.9589963	23.388020	.9764402	41.445245	.9938914	162.70325	39
40	0.9592869	23.562123	0.9767310	41.975713	0.9941823	170.88831	40
41	.9595776	23.738731	.9770219	42.519612	.9944732	179.93496	41
42	.9598682	23.917900	.9773127	43.077458	.9947640	189.98680	42
43	.9601589	24.099685	.9776035	43.649795	.9950549	201.22122	43
44	.9604495	24.284144	.9778943	44.237195	.9953458	213.85995	44
45	.9607402	24.471337	.9781851	44.840260	.9956367	228.18385	45
46	.9610308	24.661324	.9784759	45.459625	.9959276	244.55402	46
47	.9613215	24.854169	.9787668	46.095961	.9962185	263.44269	47
48	.9616122	25.049937	.9790576	46.749974	.9965093	285.47948	48
49	.9619029	25.248694	.9793484	47.422411	.9968002	311.52297	49
50	0.9621935	25.450510	0.9796392	48.114062	0.9970911	342.77516	50
51	.9624842	25.655455	.9799301	48.825762	.9973820	380.97230	51
52	.9627749	25.863603	.9802209	49.558396	.9976729	428.71873	52
53	.9630656	26.075030	.9805117	50.312902	.9979638	490.10702	53
54	.9633563	26.289814	.9808026	51.090272	.9982547	571.95809	54
55	.9636470	26.508035	.9810934	51.891564	.9985456	686.54960	55
56	.9639377	26.729777	.9813842	52.717896	.9988364	858.43689	56
57	.9642284	26.955125	.9816751	53.570464	.9991273	1144.9157	57
58	.9645191	27.184168	.9819659	54.450534	.9994182	1717.8735	58
59	.9648098	27.416997	.9822568	55.359462	.9997091	3436.7468	59
60	0.9651005	27.653708	0.9825476	56.298688	1.0000000	Infinite	60

TABLE 42. VALUES OF A AND B, WHERE A = (LOG COSECANT) × 10,000, B = (LOG SECANT) × 10,000

	0°		1°			2°			3°			4°			
′	A	B	A	d	B	A	d	B	A	d	B	A	d	B	′
0		0.0	175814	717	6.6	145718	360	26.5	128120	240	59.6	115642	181	106	60
1	353627	0.0	175097	706	6.8	145358	357	26.9	127880	239	60.2	115461	179	107	59
2	323524	0.0	174391	695	7.1	145001	355	27.4	127641	238	60.9	115282	179	108	58
3	305915	0.0	173696	684	7.3	144646	351	27.8	127403	237	61.6	115103	178	109	57
4	293421	0.0	173012	673	7.5	144295	349	28.3	127166	235	62.2	114925	177	109	56
5	283730	0.0	172339	663	7.8	143946	346	28.7	126931	234	62.9	114748	177	110	55
6	275812	0.1	171676	653	8.0	143600	343	29.2	126697	232	63.6	114571	176	111	54
7	269118	0.1	171023	644	8.3	143257	340	29.6	126465	232	64.3	114395	175	112	53
8	263318	0.1	170379	634	8.5	142916	337	30.1	126233	230	65.0	114220	175	113	52
9	258203	0.1	169745	624	8.8	142579	335	30.6	126003	229	65.7	114045	173	114	51
10	253627	0.2	169121	616	9.0	142243	332	31.1	125774	228	66.4	113872	173	115	50
11	249488	0.2	168505	608	9.3	141911	330	31.5	125546	226	67.1	113699	173	116	49
12	245709	0.3	167897	599	9.5	141581	328	32.0	125320	225	67.8	113526	171	117	48
13	242233	0.3	167298	590	9.8	141253	325	32.5	125095	225	68.5	113355	171	118	47
14	239015	0.4	166708	583	10.1	140928	323	33.0	124870	223	69.2	113184	171	119	46
15	236018	0.4	166125	575	10.3	140605	320	33.5	124647	222	69.9	113013	170	120	45
16	233216	0.5	165550	568	10.6	140285	318	34.0	124425	220	70.6	112843	168	121	44
17	230583	0.5	164982	560	10.9	139967	316	34.5	124205	220	71.3	112675	169	121	43
18	228100	0.6	164422	553	11.2	139651	313	35.0	123985	219	72.1	112506	168	122	42
19	225752	0.7	163869	547	11.5	139338	311	35.5	123766	217	72.8	112338	167	123	41
20	223525	0.7	163322	539	11.8	139027	309	36.0	123549	216	73.5	112171	166	124	40
21	221406	0.8	162783	533	12.1	138718	307	36.5	123333	216	74.3	112005	166	125	39
22	219385	0.9	162250	526	12.4	138411	305	37.1	123117	214	75.0	111839	165	126	38
23	217455	1.0	161724	520	12.7	138106	302	37.6	122903	213	75.8	111674	164	127	37
24	215607	1.1	161204	514	13.0	137804	300	38.1	122690	212	76.5	111510	164	128	36
25	213834	1.1	160690	508	13.3	137504	299	38.6	122478	211	77.3	111346	163	129	35
26	212130	1.2	160182	502	13.6	137205	296	39.2	122267	210	78.0	111183	163	130	34
27	210491	1.3	159680	496	13.9	136909	294	39.7	122057	209	78.8	111020	162	131	33
28	208912	1.4	159184	491	14.2	136615	293	40.3	121848	208	79.5	110858	161	132	32
29	207388	1.5	158693	485	14.6	136322	290	40.8	121640	208	80.3	110697	161	133	31
30	205916	1.7	158208	480	14.9	136032	288	41.4	121432	206	81.1	110536	161	134	30
31	204492	1.8	157728	474	15.2	135744	287	41.9	121226	205	81.9	110375	159	135	29
32	203113	1.9	157254	470	15.6	135457	284	42.5	121021	204	82.6	110216	159	136	28
33	201777	2.0	156784	464	15.9	135173	283	43.0	120817	203	83.4	110057	159	137	27
34	200480	2.1	156320	459	16.2	134890	281	43.6	120614	202	84.2	109898	158	138	26
35	199221	2.3	155861	455	16.6	134609	279	44.2	120412	201	85.0	109740	157	139	25
36	197998	2.4	155406	450	16.9	134330	277	44.7	120211	201	85.8	109583	157	140	24
37	196808	2.5	154956	445	17.3	134053	276	45.3	120010	199	86.6	109426	156	141	23
38	195650	2.7	154511	441	17.7	133777	274	45.9	119811	199	87.4	109270	155	142	22
39	194522	2.8	154070	436	18.0	133503	272	46.5	119612	197	88.2	109115	155	143	21
40	193422	2.9	153634	432	18.4	133231	270	47.1	119415	197	89.0	108960	155	144	20
41	192350	3.1	153201	428	18.8	132961	269	47.6	119218	196	89.8	108805	154	145	19
42	191304	3.2	152774	424	19.1	132692	267	48.2	119022	195	90.6	108651	153	146	18
43	190282	3.4	152350	419	19.5	132425	265	48.8	118827	194	91.4	108498	153	147	17
44	189283	3.6	151931	415	19.9	132160	264	49.4	118633	193	92.3	108345	152	148	16
45	188307	3.7	151515	411	20.3	131896	262	50.0	118440	192	93.1	108193	152	149	15
46	187353	3.9	151104	408	20.7	131634	261	50.7	118248	192	93.9	108041	151	150	14
47	186419	4.1	150696	404	21.0	131373	259	51.3	118056	190	94.8	107890	151	152	13
48	185505	4.2	150292	400	21.4	131114	258	51.9	117866	190	95.6	107739	150	153	12
49	184609	4.4	149892	396	21.8	130856	256	52.5	117676	189	96.4	107589	150	154	11
50	183732	4.6	149496	393	22.2	130600	254	53.1	117487	188	97.3	107439	149	155	10
51	182872	4.8	149103	389	22.6	130346	253	53.8	117299	187	98.1	107290	149	156	9
52	182029	5.0	148713	386	23.1	130093	252	54.4	117112	187	99.0	107141	148	157	8
53	181202	5.2	148327	382	23.5	129841	250	55.0	116925	186	99.8	106993	147	158	7
54	180390	5.4	147945	379	23.9	129591	249	55.7	116739	185	100.7	106846	147	159	6
55	179593	5.6	147566	376	24.3	129342	247	56.3	116554	184	101.6	106699	147	160	5
56	178811	5.8	147190	373	24.7	129095	246	56.9	116370	183	102.4	106552	146	161	4
57	178042	6.0	146817	370	25.2	128849	244	57.6	116187	183	103.3	106406	146	162	3
58	177287	6.2	146448	367	25.6	128605	243	58.2	116004	181	104.2	106260	145	163	2
59	176544	6.4	146081	363	26.0	128362	242	58.9	115823	181	105.0	106115	145	164	1
60	175814	6.6	145718		26.5	128120		59.6	115642		105.9	105970		166	0
′	B	A	B	d	A	B	d	A	B	d	A	B	d	A	′
	89°		88°			87°			86°			85°			

TABLE 42. VALUES OF A AND B, WHERE A = (LOG COSECANT) × 10,000, B = (LOG SECANT) × 10,000 (*Continued*)

	5°				6°				7°				8°				
′	A	d	B	d	A	d	B	d	A	d	B	d	A	d	B	d	′
0	105970	144	166	1	98077	120	239	1	91411	103	325	1	85644	89	425	1	60
1	105826	143	167	1	97957	120	240	1	91308	103	326	2	85555	90	426	2	59
2	105683	144	168	1	97837	120	241	2	91205	102	328	2	85465	89	428	2	58
3	105539	142	169	1	97717	119	243	1	91103	102	330	1	85376	90	430	2	57
4	105397	143	170	1	97598	118	244	1	91001	102	331	2	85286	89	432	2	56
5	105254	141	171	1	97480	119	245	2	90899	101	333	1	85197	88	434	1	55
6	105113	142	172	1	97361	118	247	1	90798	102	334	2	85109	89	435	2	54
7	104971	141	173	2	97243	117	248	1	90696	101	336	1	85020	89	437	2	53
8	104830	140	175	1	97126	118	249	2	90595	101	337	2	84931	88	439	2	52
9	104690	140	176	1	97008	117	251	1	90494	100	339	2	84843	88	441	2	51
10	104550	139	177	1	96891	117	252	1	90394	101	341	1	84755	88	443	1	50
11	104411	139	178	1	96774	116	253	2	90293	100	342	2	84667	88	444	2	49
12	104272	139	179	1	96658	116	255	1	90193	100	344	1	84579	87	446	2	48
13	104133	138	180	1	96542	116	256	2	90093	99	345	2	84492	88	448	2	47
14	103995	138	181	2	96426	116	258	1	89994	100	347	2	84404	87	450	2	46
15	103857	137	183	1	96310	115	259	1	89894	99	349	1	84317	87	452	2	45
16	103720	137	184	1	96195	115	260	2	89795	99	350	2	84230	87	454	1	44
17	103583	136	185	1	96080	114	262	1	89696	98	352	1	84143	87	455	2	43
18	103447	136	186	1	95966	115	263	1	89598	99	353	2	84056	86	457	2	42
19	103311	136	187	1	95851	113	264	2	89499	98	355	2	83970	86	459	2	41
20	103175	135	188	2	95738	114	266	1	89401	98	357	1	83884	87	461	2	40
21	103040	135	190	1	95624	114	267	2	89303	98	358	2	83797	86	463	2	39
22	102905	134	191	1	95510	113	269	1	89205	98	360	2	83711	85	465	2	38
23	102771	134	192	1	95397	112	270	2	89107	97	362	1	83626	86	467	1	37
24	102637	133	193	1	95285	113	272	1	89010	97	363	2	83540	85	468	2	36
25	102504	133	194	2	95172	112	273	1	88913	97	365	2	83455	86	470	2	35
26	102371	133	196	1	95060	112	274	2	88816	97	367	1	83369	85	472	2	34
27	102238	132	197	1	94948	112	276	1	88719	96	368	2	83284	85	474	2	33
28	102106	132	198	1	94836	111	277	2	88623	97	370	1	83199	85	476	2	32
29	101974	131	199	1	94725	111	279	1	88526	96	371	2	83114	84	478	2	31
30	101843	131	200	2	94614	111	280	2	88430	96	373	2	83030	85	480	2	30
31	101712	131	202	1	94503	110	282	1	88334	95	375	1	82945	84	482	1	29
32	101581	130	203	1	94393	110	283	1	88239	96	376	2	82861	84	483	2	28
33	101451	130	204	1	94283	110	284	2	88143	95	378	2	82777	84	485	2	27
34	101321	129	205	2	94173	110	286	1	88048	95	380	2	82693	84	487	2	26
35	101192	129	207	1	94063	109	287	2	87953	95	382	1	82609	83	489	2	25
36	101063	129	208	1	93954	109	289	1	87858	95	383	2	82526	84	491	2	24
37	100934	128	209	1	93845	109	290	2	87764	95	385	2	82442	83	493	2	23
38	100806	128	210	2	93736	108	292	1	87669	94	387	1	82359	83	495	2	22
39	100678	128	212	1	93628	109	293	2	87575	94	388	2	82276	83	497	2	21
40	100550	127	213	1	93519	108	295	1	87481	93	390	2	82193	83	499	2	20
41	100423	127	214	1	93411	107	296	2	87388	94	392	1	82110	83	501	2	19
42	100296	126	215	2	93304	108	298	1	87294	93	393	2	82027	82	503	2	18
43	100170	126	217	1	93196	107	299	2	87201	93	395	2	81945	82	505	1	17
44	100044	126	218	1	93089	107	301	1	87108	93	397	2	81863	83	506	2	16
45	99918	125	219	1	92982	106	302	2	87015	93	399	1	81780	82	508	2	15
46	99793	125	220	2	92876	107	304	1	86922	93	400	2	81698	81	510	2	14
47	99668	124	222	1	92769	106	305	2	86829	92	402	2	81617	82	512	2	13
48	99544	125	223	1	92663	105	307	1	86737	92	404	1	81535	82	514	2	12
49	99419	123	224	1	92558	106	308	2	86645	92	405	2	81453	81	516	2	11
50	99296	124	225	2	92452	105	310	1	86553	92	407	2	81372	81	518	2	10
51	99172	123	227	1	92347	105	311	2	86461	91	409	2	81291	81	520	2	9
52	99049	123	228	1	92242	105	313	1	86370	92	411	1	81210	81	522	2	8
53	98926	122	229	2	92137	105	314	2	86278	91	412	2	81129	81	524	2	7
54	98804	122	231	1	92032	104	316	1	86187	91	414	2	81048	81	526	2	6
55	98682	122	232	1	91928	104	317	2	86096	90	416	2	80967	80	528	2	5
56	98560	121	233	2	91824	104	319	1	86006	91	418	1	80887	80	530	2	4
57	98439	121	235	1	91720	103	320	1	85915	90	419	2	80807	80	532	2	3
58	98318	121	236	1	91617	103	322	1	85825	91	421	2	80727	80	534	2	2
59	98197	120	237	2	91514	103	323	2	85734	90	423	2	80647	80	536	2	1
60	98077		239		91411		325		85644		425		80567		538		0
′	B	d	A	d	B	d	A	d	B	d	A	d	B	d	A	d	
	84°				83°				82°				81°				

TABLE 42. VALUES OF A AND B, WHERE A = (LOG COSECANT) × 10,000, B = (LOG SECANT) × 10,000 (*Continued*)

	9°				10°				11°				12°				
′	A	d	B	d	A	d	B	d	A	d	B	d	A	d	B	d	′
0	80567	80	538	2	76033	72	665	2	71940	65	805	3	68212	59	960	2	60
1	80487	79	540	2	75961	71	667	2	71875	65	808	2	68153	60	962	3	59
2	80408	80	542	2	75890	71	669	3	71810	64	810	3	68093	59	965	3	58
3	80328	79	544	2	75819	72	672	2	71746	65	813	2	68034	59	968	2	57
4	80249	79	546	2	75747	71	674	2	71681	65	815	3	67975	59	970	3	56
5	80170	79	548	2	75676	71	676	2	71616	64	818	2	67916	59	973	3	55
6	80091	79	550	2	75605	71	678	3	71552	64	820	3	67857	59	976	2	54
7	80012	79	552	2	75534	70	681	2	71488	65	823	2	67798	59	978	3	53
8	79933	78	554	2	75464	71	683	2	71423	64	825	3	67739	58	981	3	52
9	79855	78	556	2	75393	70	685	2	71359	64	828	2	67681	59	984	3	51
10	79777	79	558	2	75323	71	687	3	71295	64	830	3	67622	59	987	2	50
11	79698	78	560	2	75252	70	690	2	71231	64	833	2	67563	58	989	3	49
12	79620	78	562	2	75182	70	692	2	71167	63	835	3	67505	58	992	3	48
13	79542	77	564	2	75112	70	694	2	71104	64	838	2	67447	59	995	3	47
14	79465	78	566	2	75042	70	696	3	71040	64	840	3	67388	58	998	2	46
15	79387	78	568	3	74972	70	699	2	70976	63	843	2	67330	58	1000	3	45
16	79309	77	571	2	74902	70	701	2	70913	63	845	3	67272	58	1003	3	44
17	79232	77	573	2	74832	69	703	3	70850	64	848	2	67214	58	1006	3	43
18	79155	77	575	2	74763	70	706	2	70786	63	850	3	67156	58	1009	2	42
19	79078	77	577	2	74693	69	708	2	70723	63	853	2	67098	58	1011	3	41
20	79001	77	579	2	74624	69	710	2	70660	63	855	3	67040	58	1014	3	40
21	78924	77	581	2	74555	69	712	3	70597	63	858	2	66982	57	1017	3	39
22	78847	76	583	2	74486	69	715	2	70534	63	860	3	66925	58	1020	2	38
23	78771	77	585	2	74417	69	717	2	70471	62	863	2	66867	57	1022	3	37
24	78694	76	587	2	74348	69	719	3	70409	63	865	3	66810	58	1025	3	36
25	78618	76	589	2	74279	69	722	2	70346	62	868	2	66752	57	1028	3	35
26	78542	76	591	2	74210	68	724	2	70284	63	870	3	66695	57	1031	2	34
27	78466	76	593	3	74142	69	726	3	70221	62	873	3	66638	58	1033	3	33
28	78390	75	596	2	74073	68	729	2	70159	62	876	2	66580	57	1036	3	32
29	78315	76	598	2	74005	68	731	2	70097	63	878	3	66523	57	1039	3	31
30	78239	75	600	2	73937	68	733	3	70034	62	881	2	66466	57	1042	3	30
31	78164	76	602	2	73869	68	736	2	69972	62	883	3	66409	56	1045	2	29
32	78088	75	604	2	73801	68	738	2	69910	61	886	2	66353	57	1047	3	28
33	78013	75	606	2	73733	68	740	3	69849	62	888	3	66296	57	1050	3	27
34	77938	75	608	2	73665	68	743	2	69787	62	891	3	66239	57	1053	3	26
35	77863	74	610	2	73597	67	745	3	69725	61	894	2	66182	56	1056	3	25
36	77789	75	612	3	73530	68	748	2	69664	62	896	3	66126	57	1059	3	24
37	77714	75	615	2	73462	67	750	2	69602	61	899	2	66069	56	1062	2	23
38	77639	74	617	2	73395	67	752	3	69541	62	901	3	66013	56	1064	3	22
39	77565	74	619	2	73328	67	755	2	69479	61	904	3	65957	57	1067	3	21
40	77491	74	621	2	73261	67	757	2	69418	61	907	2	65900	56	1070	3	20
41	77417	74	623	2	73194	67	759	3	69357	61	909	3	65844	56	1073	3	19
42	77343	74	625	3	73127	67	762	2	69296	61	912	2	65788	56	1076	3	18
43	77269	74	628	2	73060	67	764	3	69235	61	914	3	65732	56	1079	2	17
44	77195	73	630	2	72993	66	767	2	69174	61	917	3	65676	56	1081	3	16
45	77122	74	632	2	72927	67	769	2	69113	60	920	2	65620	56	1084	3	15
46	77048	73	634	2	72860	66	771	3	69053	61	922	3	65564	55	1087	3	14
47	76975	73	636	2	72794	67	774	2	68992	60	925	3	65509	56	1090	3	13
48	76902	73	638	3	72727	66	776	3	68932	61	928	2	65453	55	1093	3	12
49	76829	73	641	2	72661	66	779	2	68871	60	930	3	65398	56	1096	3	11
50	76756	73	643	2	72595	67	781	2	68811	61	933	3	65342	55	1099	3	10
51	76683	73	645	2	72529	66	783	3	68750	60	936	2	65287	56	1102	2	9
52	76610	72	647	2	72463	65	786	2	68690	60	938	3	65231	55	1104	3	8
53	76538	73	649	3	72398	66	788	3	68630	60	941	3	65176	55	1107	3	7
54	76465	72	652	2	72332	66	791	2	68570	60	944	2	65121	55	1110	3	6
55	76393	72	654	2	72266	65	793	3	68510	59	946	3	65066	55	1113	3	5
56	76321	73	656	2	72201	65	796	2	68451	60	949	3	65011	55	1116	3	4
57	76248	71	658	2	72136	66	798	2	68391	60	952	2	64956	55	1119	3	3
58	76177	72	660	3	72070	65	800	3	68331	59	954	3	64901	55	1122	3	2
59	76105	72	663	2	72005	65	803	2	68272	60	957	3	64846	55	1125	3	1
60	76033		665		71940		805		68212		960		64791		1128		0
′	B	d	A	d	B	d	A	d	B	d	A	d	B	d	A	d	′
	80°				79°				78°				77°				

TABLE 42. VALUES OF *A* AND *B*, WHERE *A* = (LOG COSECANT) × 10,000, *B* = (LOG SECANT) × 10,000 (*Continued*)

	13°				14°				15°				16°				
′	A	d	B	d	A	d	B	d	A	d	B	d	A	d	B	d	′
0	64791	54	1128	3	61632	50	1310	3	58700	47	1506	3	55966	44	1716	3	60
1	64737	55	1131	2	61582	51	1313	3	58653	47	1509	3	55922	44	1719	4	59
2	64682	55	1133	3	61531	50	1316	3	58606	47	1512	4	55878	44	1723	4	58
3	64627	54	1136	3	61481	51	1319	3	58559	47	1516	3	55834	44	1727	3	57
4	64573	54	1139	3	61430	50	1322	3	58512	47	1519	4	55790	43	1730	4	56
5	64519	55	1142	3	61380	50	1325	4	58465	47	1523	3	55747	44	1734	4	55
6	64464	54	1145	3	61330	51	1329	3	58418	46	1526	3	55703	44	1738	3	54
7	64410	54	1148	3	61279	50	1332	3	58372	47	1529	4	55659	44	1741	4	53
8	64356	54	1151	3	61229	50	1335	3	58325	47	1533	3	55615	43	1745	4	52
9	64302	54	1154	3	61179	50	1338	3	58278	46	1536	4	55572	44	1749	3	51
10	64248	54	1157	3	61129	50	1341	3	58232	47	1540	3	55528	44	1752	4	50
11	64194	54	1160	3	61079	50	1344	4	58185	46	1543	4	55484	43	1756	4	49
12	64140	54	1163	3	61029	50	1348	3	58139	47	1547	3	55441	43	1760	3	48
13	64086	54	1166	3	60979	50	1351	3	58092	46	1550	3	55398	44	1763	4	47
14	64032	54	1169	3	60929	50	1354	3	58046	47	1553	4	55354	43	1767	4	46
15	63978	53	1172	3	60879	49	1357	3	57999	46	1557	3	55311	44	1771	3	45
16	63925	54	1175	3	60830	50	1360	4	57953	46	1560	4	55267	43	1774	4	44
17	63871	53	1178	3	60780	50	1364	3	57907	46	1564	3	55224	43	1778	4	43
18	63818	54	1181	3	60730	51	1367	3	57860	46	1567	4	55181	43	1782	3	42
19	63764	53	1184	3	60681	50	1370	3	57814	46	1571	3	55138	43	1785	4	41
20	63711	53	1187	3	60631	49	1373	4	57768	46	1574	4	55095	43	1789	4	40
21	63658	53	1190	3	60582	49	1377	3	57722	46	1578	3	55052	44	1793	3	39
22	63605	54	1193	3	60533	50	1380	3	57676	46	1581	4	55008	43	1796	4	38
23	63551	53	1196	3	60483	49	1383	3	57630	46	1585	3	54965	42	1800	4	37
24	63498	53	1199	3	60434	49	1386	4	57584	45	1588	3	54923	43	1804	4	36
25	63445	53	1202	3	60385	49	1390	3	57539	46	1591	4	54880	43	1808	3	35
26	63392	52	1205	3	60336	49	1393	3	57493	46	1595	3	54837	43	1811	4	34
27	63340	53	1208	3	60287	49	1396	3	57447	46	1598	4	54794	43	1815	4	33
28	63287	53	1211	3	60238	49	1399	4	57401	45	1602	3	54751	43	1819	4	32
29	63234	53	1214	3	60189	49	1403	3	57356	46	1605	4	54708	42	1823	3	31
30	63181	52	1217	3	60140	49	1406	3	57310	45	1609	3	54666	43	1826	4	30
31	63129	53	1220	3	60091	49	1409	3	57265	46	1612	4	54623	42	1830	4	29
32	63076	52	1223	3	60042	48	1412	4	57219	45	1616	3	54581	43	1834	4	28
33	63024	52	1226	3	59994	49	1416	3	57174	46	1619	4	54538	42	1838	3	27
34	62972	53	1229	3	59945	48	1419	3	57128	45	1623	4	54496	43	1841	4	26
35	62919	52	1232	3	59897	49	1422	4	57083	45	1627	3	54453	42	1845	4	25
36	62867	52	1235	3	59848	48	1426	3	57038	46	1630	4	54411	43	1849	4	24
37	62815	52	1238	3	59800	49	1429	3	56992	45	1634	3	54368	42	1853	3	23
38	62763	52	1241	3	59751	48	1432	3	56947	45	1637	4	54326	42	1856	4	22
39	62711	52	1244	3	59703	49	1435	4	56902	45	1641	3	54284	42	1860	4	21
40	62659	52	1247	3	59654	48	1439	3	56857	45	1644	4	54242	43	1864	4	20
41	62607	52	1250	4	59606	48	1442	3	56812	45	1648	3	54199	42	1868	3	19
42	62555	52	1254	3	59558	48	1445	4	56767	45	1651	4	54157	42	1871	4	18
43	62503	52	1257	3	59510	48	1449	3	56722	45	1655	3	54115	42	1875	4	17
44	62451	51	1260	3	59462	48	1452	3	56677	44	1658	4	54073	42	1879	4	16
45	62400	52	1263	3	59414	48	1455	4	56633	45	1662	4	54031	42	1883	4	15
46	62348	51	1266	3	59366	48	1459	3	56588	45	1666	3	53989	42	1887	3	14
47	62297	52	1269	3	59318	48	1462	3	56543	45	1669	4	53947	42	1890	4	13
48	62245	51	1272	3	59270	48	1465	4	56498	44	1673	3	53905	41	1894	4	12
49	62194	52	1275	3	59222	47	1469	3	56454	45	1676	4	53864	42	1898	4	11
50	62142	51	1278	3	59175	48	1472	3	56409	44	1680	3	53822	42	1902	4	10
51	62091	51	1281	4	59127	48	1475	4	56365	45	1683	4	53780	42	1906	4	9
52	62040	51	1285	3	59079	47	1479	3	56320	44	1687	4	53738	41	1910	3	8
53	61989	51	1288	3	59032	48	1482	3	56276	45	1691	3	53697	42	1913	4	7
54	61938	51	1291	3	58984	47	1485	4	56231	44	1694	4	53655	41	1917	4	6
55	61887	51	1294	3	58937	48	1489	3	56187	44	1698	3	53614	42	1921	4	5
56	61836	51	1297	3	58889	47	1492	3	56143	44	1701	4	53572	41	1925	4	4
57	61785	51	1300	3	58842	47	1495	4	56099	45	1705	4	53531	42	1929	4	3
58	61734	51	1303	3	58795	47	1499	3	56054	44	1709	3	53489	41	1933	4	2
59	61683	51	1306	4	58748	48	1502	4	56010	44	1712	4	53448	42	1937	3	1
60	61632		1310		58700		1506		55966		1716		53406		1940		0
′	B	d	A	d	B	d	A	d	B	d	A	d	B	d	A	d	′
	76°				75°				74°				73°				

Table 42. Values of A and B, Where A = (Log Cosecant) × 10,000, B = (Log Secant) × 10,000 (*Continued*)

	17°				18°				19°				20°				
′	A	d	B	d	A	d	B	d	A	d	B	d	A	d	B	d	′
0	53406	41	1940	4	51002	39	2179	4	48736	37	2433	4	46595	35	2701	5	60
1	53365	41	1944	4	50963	39	2183	5	48699	37	2437	5	46560	35	2706	5	59
2	53324	41	1948	4	50924	39	2188	4	48662	36	2442	4	46525	34	2711	4	58
3	53283	41	1952	4	50885	38	2192	4	48626	37	2446	4	46491	35	2715	5	57
4	53242	42	1956	4	50847	39	2196	4	48589	36	2450	5	46456	34	2720	4	56
5	53200	41	1960	4	50808	39	2200	4	48553	37	2455	4	46422	35	2724	5	55
6	53159	41	1964	4	50769	38	2204	4	48516	36	2459	5	46387	34	2729	5	54
7	53118	41	1968	3	50731	39	2208	4	48480	37	2464	4	46353	35	2734	4	53
8	53077	41	1971	4	50692	39	2212	4	48443	36	2468	4	46318	34	2738	5	52
9	53036	41	1975	4	50653	38	2216	5	48407	36	2472	5	46284	35	2743	5	51
10	52995	40	1979	4	50615	39	2221	4	48371	37	2477	4	46249	34	2748	4	50
11	52955	41	1983	4	50576	38	2225	4	48334	36	2481	4	46215	34	2752	5	49
12	52914	41	1987	4	50538	38	2229	4	48298	36	2485	5	46181	35	2757	5	48
13	52873	41	1991	4	50500	39	2233	4	48262	36	2490	4	46146	34	2762	4	47
14	52832	41	1995	4	50461	38	2237	4	48226	37	2494	5	46112	34	2766	5	46
15	52791	40	1999	4	50423	38	2241	5	48189	36	2499	4	46078	35	2771	5	45
16	52751	41	2003	4	50385	39	2246	4	48153	36	2503	5	46043	34	2776	4	44
17	52710	40	2007	4	50346	38	2250	4	48117	36	2508	4	46009	34	2780	5	43
18	52670	41	2011	3	50308	38	2254	4	48081	36	2512	4	45975	34	2785	5	42
19	52629	40	2014	4	50270	38	2258	4	48045	36	2516	5	45941	34	2790	4	41
20	52589	41	2018	4	50232	38	2262	4	48009	36	2521	4	45907	34	2794	5	40
21	52548	40	2022	4	50194	38	2266	5	47973	36	2525	5	45873	34	2799	5	39
22	52508	41	2026	4	50156	38	2271	4	47937	36	2530	4	45839	34	2804	4	38
23	52467	40	2030	4	50118	38	2275	4	47901	36	2534	5	45805	34	2808	5	37
24	52427	40	2034	4	50080	38	2279	4	47865	36	2539	4	45771	34	2813	5	36
25	52387	41	2038	4	50042	38	2283	4	47829	36	2543	4	45737	34	2818	4	35
26	52346	40	2042	4	50004	38	2287	5	47793	35	2547	5	45703	34	2822	5	34
27	52306	40	2046	4	49966	38	2292	4	47758	36	2552	4	45669	34	2827	5	33
28	52266	40	2050	4	49928	38	2296	4	47722	36	2556	5	45635	34	2832	5	32
29	52226	40	2054	4	49890	38	2300	4	47686	36	2561	4	45601	34	2837	4	31
30	52186	40	2058	4	49852	37	2304	5	47650	35	2565	5	45567	33	2841	5	30
31	52146	40	2062	4	49815	38	2309	4	47615	36	2570	4	45534	34	2846	5	29
32	52106	40	2066	4	49777	38	2313	4	47579	35	2574	5	45500	34	2851	4	28
33	52066	40	2070	4	49739	37	2317	4	47544	36	2579	4	45466	33	2855	5	27
34	52026	40	2074	4	49702	38	2321	5	47508	35	2583	5	45433	34	2860	5	26
35	51986	40	2078	4	49664	38	2326	4	47473	36	2588	4	45399	34	2865	5	25
36	51946	40	2082	4	49626	37	2330	4	47437	35	2592	5	45365	33	2870	4	24
37	51906	39	2086	4	49589	38	2334	4	47402	36	2597	4	45332	34	2874	5	23
38	51867	40	2090	4	49551	37	2338	5	47366	35	2601	5	45298	33	2879	5	22
39	51827	40	2094	4	49514	37	2343	4	47331	36	2606	4	45265	34	2884	5	21
40	51787	39	2098	4	49477	38	2347	4	47295	35	2610	5	45231	33	2889	4	20
41	51748	40	2102	4	49439	37	2351	4	47260	35	2615	4	45198	34	2893	5	19
42	51708	40	2106	4	49402	37	2355	5	47225	36	2619	5	45164	33	2898	5	18
43	51668	39	2110	4	49365	38	2360	4	47189	35	2624	4	45131	34	2903	5	17
44	51629	40	2114	4	49327	37	2364	4	47154	35	2628	5	45097	33	2908	5	16
45	51589	39	2118	4	49290	37	2368	4	47119	34	2633	4	45064	33	2913	4	15
46	51550	40	2122	4	49253	37	2372	5	47084	35	2637	5	45031	34	2917	5	14
47	51510	39	2126	4	49216	37	2377	4	47049	35	2642	5	44997	33	2922	5	13
48	51471	39	2130	4	49179	37	2381	4	47014	35	2647	4	44964	33	2927	5	12
49	51432	39	2134	5	49142	38	2385	5	46979	35	2651	5	44931	33	2932	5	11
50	51393	40	2139	4	49104	37	2390	4	46944	36	2656	4	44898	34	2937	4	10
51	51353	39	2143	4	49067	37	2394	4	46908	34	2660	5	44864	33	2941	5	9
52	51314	39	2147	4	49030	37	2398	5	46874	35	2665	4	44831	33	2946	5	8
53	51275	39	2151	4	48993	36	2403	4	46839	35	2669	5	44798	33	2951	5	7
54	51236	39	2155	4	48957	37	2407	4	46804	35	2674	4	44765	33	2956	5	6
55	51197	39	2159	4	48920	37	2411	5	46769	35	2678	5	44732	33	2961	4	5
56	51158	39	2163	4	48883	37	2416	4	46734	35	2683	5	44699	33	2965	5	4
57	51119	39	2167	4	48846	37	2420	4	46699	35	2688	4	44666	33	2970	5	3
58	51080	39	2171	4	48809	36	2424	5	46664	34	2692	5	44633	33	2975	5	2
59	51041	39	2175	4	48773	37	2429	4	46630	35	2697	4	44600	33	2980	5	1
60	51002		2179		48736		2433		46595		2701		44567		2985		0
	B	d	A	d	B	d	A	d	B	d	A	d	B	d	A	d	′
	72°				71°				70°				69°				

TABLE 42. VALUES OF A AND B, WHERE A = (LOG COSECANT) × 10,000, B = (LOG SECANT) × 10,000 (*Continued*)

	21°				22°				23°				24°				
′	A	d	B	d	A	d	B	d	A	d	B	d	A	d	B	d	′
0	44567	33	2985	5	42642	31	3283	6	40812	30	3597	6	39069	29	3927	6	60
1	44534	33	2990	5	42611	31	3289	5	40782	29	3603	5	39040	28	3933	5	59
2	44501	33	2995	4	42580	31	3294	5	40753	30	3608	5	39012	28	3938	6	58
3	44468	32	2999	5	42549	31	3299	5	40723	30	3613	6	38984	29	3944	6	57
4	44436	33	3004	5	42518	32	3304	5	40693	29	3619	5	38955	28	3950	5	56
5	44403	33	3009	5	42486	31	3309	5	40664	30	3624	6	38927	28	3955	6	55
6	44370	33	3014	5	42455	31	3314	5	40634	30	3630	5	38899	28	3961	5	54
7	44337	32	3019	5	42424	31	3319	5	40604	29	3635	5	38871	29	3966	6	53
8	44305	33	3024	5	42393	31	3324	6	40575	30	3640	6	38842	28	3972	6	52
9	44272	33	3029	5	42362	31	3330	5	40545	29	3646	5	38814	28	3978	5	51
10	44239	32	3034	4	42331	31	3335	5	40516	30	3651	6	38786	28	3983	6	50
11	44207	33	3038	5	42300	31	3340	5	40486	29	3657	5	38758	28	3989	6	49
12	44174	32	3043	5	42269	31	3345	5	40457	30	3662	5	38730	28	3995	5	48
13	44142	33	3048	5	42238	31	3350	5	40427	29	3667	6	38702	28	4000	6	47
14	44109	32	3053	5	42207	31	3355	5	40398	30	3673	5	38674	28	4006	6	46
15	44077	33	3058	5	42176	31	3360	6	40368	29	3678	6	38646	28	4012	6	45
16	44044	32	3063	5	42145	30	3366	5	40339	29	3684	5	38618	29	4018	5	44
17	44012	33	3068	5	42115	31	3371	5	40310	30	3689	6	38589	27	4023	6	43
18	43979	32	3073	5	42084	31	3376	5	40280	29	3695	5	38562	28	4029	6	42
19	43947	32	3078	5	42053	31	3381	5	40251	29	3700	6	38534	28	4035	5	41
20	43915	33	3083	5	42022	30	3386	6	40222	30	3706	5	38506	28	4040	6	40
21	43882	32	3088	5	41992	31	3392	5	40192	29	3711	5	38478	28	4046	6	39
22	43850	32	3093	4	41961	31	3397	5	40163	29	3716	6	38450	28	4052	6	38
23	43818	33	3097	5	41930	31	3402	5	40134	29	3722	5	38422	28	4058	5	37
24	43785	32	3102	5	41899	30	3407	5	40105	29	3727	6	38394	28	4063	6	36
25	43753	32	3107	5	41869	31	3412	6	40076	30	3733	5	38366	28	4069	6	35
26	43721	32	3112	5	41838	30	3418	5	40046	29	3738	6	38338	27	4075	5	34
27	43689	32	3117	5	41808	31	3423	5	40017	29	3744	5	38311	28	4080	6	33
28	43657	32	3122	5	41777	30	3428	5	39988	29	3749	6	38283	28	4086	6	32
29	43625	33	3127	5	41747	31	3433	5	39959	29	3755	5	38255	28	4092	6	31
30	43592	32	3132	5	41716	30	3438	6	39930	29	3760	6	38227	27	4098	5	30
31	43560	32	3137	5	41686	31	3444	5	39901	31	3766	5	38200	28	4103	6	29
32	43528	32	3142	5	41655	30	3449	5	39872	29	3771	6	38172	28	4109	6	28
33	43496	32	3147	5	41625	31	3454	5	39843	29	3777	5	38144	27	4115	6	27
34	43464	32	3152	5	41594	30	3459	6	39814	29	3782	6	38117	28	4121	6	26
35	43432	31	3157	5	41564	31	3465	5	39785	29	3788	5	38089	28	4127	5	25
36	43401	32	3162	5	41533	30	3470	5	39756	29	3793	6	38061	27	4132	6	24
37	43369	32	3167	5	41503	30	3475	5	39727	29	3799	5	38034	28	4138	6	23
38	43337	32	3172	5	41473	30	3480	6	39698	29	3804	6	38006	27	4144	6	22
39	43305	32	3177	5	41443	31	3486	5	39669	28	3810	5	37979	28	4150	6	21
40	43273	32	3182	5	41412	30	3491	5	39641	29	3815	6	37951	27	4156	5	20
41	43241	31	3187	5	41382	30	3496	6	39612	29	3821	5	37924	28	4161	6	19
42	43210	32	3192	5	41352	30	3502	5	39583	29	3826	6	37896	27	4167	6	18
43	43178	32	3197	5	41322	31	3507	5	39554	28	3832	6	37869	28	4173	6	17
44	43146	32	3202	5	41291	30	3512	5	39526	29	3838	5	37841	27	4179	6	16
45	43114	31	3207	5	41261	30	3517	6	39497	29	3843	6	37814	28	4185	5	15
46	43083	32	3212	5	41231	30	3523	5	39468	29	3849	5	37786	27	4190	6	14
47	43051	31	3217	5	41201	30	3528	5	39439	28	3854	6	37759	27	4196	6	13
48	43020	32	3222	6	41171	30	3533	6	39411	29	3860	5	37732	28	4202	6	12
49	42938	32	3228	5	41141	30	3539	5	39382	28	3865	6	37704	27	4208	6	11
50	42956	31	3233	5	41111	30	3544	5	39354	29	3871	6	37677	27	4214	6	10
51	42925	32	3238	5	41081	30	3549	6	39325	29	3877	5	37650	27	4220	5	9
52	42893	31	3243	5	41051	30	3555	5	39296	28	3882	6	37623	28	4225	6	8
53	42862	31	3248	5	41021	30	3560	5	39268	29	3888	5	37595	27	4231	6	7
54	42831	32	3253	5	40991	30	3565	6	39239	28	3893	6	37568	27	4237	6	6
55	42799	31	3258	5	40961	30	3571	5	39211	29	3899	6	37541	27	4243	6	5
56	42768	32	3263	5	40931	31	3576	5	39182	28	3905	5	37514	27	4249	6	4
57	42736	31	3268	5	40902	30	3581	6	39154	29	3910	6	37487	28	4255	6	3
58	42705	31	3273	5	40872	30	3587	5	39125	28	3916	5	37459	27	4261	6	2
59	42674	32	3278	5	40842	30	3592	5	39097	28	3921	6	37432	27	4267	5	1
60	42642		3283		40812		3597		39069		3927		37405		4272		0
′	B	d	A	d	B	d	A	d	B	d	A	d	B	d	A	d	′
	68°				67°				66°				65°				

TABLE 42. VALUES OF A AND B, WHERE A = (LOG COSECANT) × 10,000, B = (LOG SECANT) × 10,000 (*Continued*)

	25°				26°				27°				28°				
′	A	d	B	d	A	d	B	d	A	d	B	d	A	d	B	d	′
0	37405	27	4272	6	35816	26	4634	6	34295	24	5012	6	32839	24	5407	6	60
1	37378	27	4278	6	35790	26	4640	6	34271	25	5018	7	32815	23	5413	7	59
2	37351	27	4284	6	35764	26	4646	6	34246	25	5025	6	32792	24	5420	7	58
3	37324	27	4290	6	35738	26	4652	7	34221	25	5031	7	32768	24	5427	6	57
4	37297	27	4296	6	35712	25	4659	6	34196	24	5038	6	32744	24	5433	7	56
5	37270	27	4302	6	35687	26	4665	6	34172	25	5044	7	32720	23	5440	7	55
6	37243	27	4308	6	35661	26	4671	6	34147	25	5051	6	32697	24	5447	7	54
7	37216	27	4314	6	35635	26	4677	6	34122	24	5057	7	32673	23	5454	6	53
8	37189	27	4320	6	35609	26	4683	7	34098	25	5064	6	32650	24	5460	7	52
9	37162	27	4326	6	35583	25	4690	6	34073	25	5070	7	32626	24	5467	7	51
10	37135	27	4332	5	35558	26	4696	6	34048	24	5077	6	32602	23	5474	7	50
11	37108	26	4337	6	35532	26	4702	6	34024	25	5083	6	32579	24	5481	6	49
12	37082	27	4343	6	35506	25	4708	6	33999	24	5089	7	32555	23	5487	7	48
13	37055	27	4349	6	35481	26	4714	7	33975	25	5096	6	32532	24	5494	7	47
14	37028	27	4355	6	35455	26	4721	6	33950	25	5102	7	32508	23	5501	7	46
15	37001	27	4361	6	35429	25	4727	6	33925	24	5109	6	32485	24	5508	7	45
16	36974	26	4367	6	35404	26	4733	6	33901	25	5115	7	32461	23	5515	6	44
17	36948	27	4373	6	35378	25	4739	7	33876	24	5122	7	32438	24	5521	7	43
18	36921	27	4379	6	35353	26	4746	6	33852	25	5129	6	32414	23	5528	7	42
19	36894	27	4385	6	35327	25	4752	6	33827	24	5135	7	32391	24	5535	7	41
20	36867	26	4391	6	35302	26	4758	6	33803	24	5142	6	32367	23	5542	7	40
21	36841	27	4397	6	35276	25	4764	7	33779	25	5148	7	32344	24	5549	6	39
22	36814	27	4403	6	35251	26	4771	6	33754	24	5155	6	32320	23	5555	7	38
23	36787	26	4409	6	35225	25	4777	6	33730	25	5161	7	32297	23	5562	7	37
24	36761	27	4415	6	35200	26	4783	6	33705	24	5168	6	32274	24	5569	7	36
25	36734	26	4421	6	35174	25	4789	7	33681	24	5174	7	32250	23	5576	7	35
26	36708	27	4427	6	35149	26	4796	6	33657	25	5181	6	32227	23	5583	7	34
27	36681	26	4433	6	35123	25	4802	6	33632	24	5187	7	32204	24	5590	6	33
28	36655	27	4439	6	35098	25	4808	7	33608	24	5194	7	32180	23	5596	7	32
29	36628	26	4445	6	35073	26	4815	6	33584	25	5201	6	32157	23	5603	7	31
30	36602	27	4451	6	35047	25	4821	6	33559	24	5207	7	32134	24	5610	7	30
31	36575	26	4457	6	35022	25	4827	6	33535	24	5214	6	32110	23	5617	7	29
32	36549	27	4463	6	34997	26	4833	7	33511	24	5220	7	32087	23	5624	7	28
33	36522	26	4469	6	34971	25	4840	6	33487	24	5227	6	32064	23	5631	7	27
34	36496	27	4475	6	34946	25	4846	6	33463	25	5233	7	32041	23	5638	7	26
35	36469	26	4481	6	34921	25	4852	7	33438	24	5240	7	32018	24	5645	6	25
36	36443	26	4487	6	34896	26	4859	6	33414	24	5247	6	31994	23	5651	7	24
37	36417	27	4493	7	34870	25	4865	6	33390	24	5253	7	31971	23	5658	7	23
38	36390	26	4500	6	34845	25	4871	7	33366	24	5260	6	31948	23	5665	7	22
39	36364	26	4506	6	34820	25	4878	6	33342	24	5266	7	31925	23	5672	7	21
40	36338	27	4512	6	34795	25	4884	6	33318	24	5273	7	31902	23	5679	7	20
41	36311	26	4518	6	34770	25	4890	7	33294	25	5280	6	31879	23	5686	7	19
42	36285	26	4524	6	34745	26	4897	6	33269	24	5286	7	31856	23	5693	7	18
43	36259	26	4530	6	34719	25	4903	7	33245	24	5293	7	31833	23	5700	7	17
44	36233	27	4536	6	34694	25	4910	6	33221	24	5300	6	31810	23	5707	7	16
45	36206	26	4542	6	34669	25	4916	6	33197	24	5306	7	31787	24	5714	7	15
46	36180	26	4548	6	34644	25	4922	7	33173	24	5313	7	31763	23	5721	6	14
47	36154	26	4554	6	34619	25	4929	6	33149	24	5320	6	31740	23	5727	7	13
48	36128	26	4560	6	34594	25	4935	6	33125	24	5326	7	31717	22	5734	7	12
49	36102	26	4566	7	34569	25	4941	7	33101	23	5333	7	31695	23	5741	7	11
50	36076	26	4573	6	34544	25	4948	6	33078	24	5340	6	31672	23	5748	7	10
51	36050	26	4579	6	34519	25	4954	7	33054	24	5346	7	31649	23	5755	7	9
52	36024	26	4585	6	34494	25	4961	6	33030	24	5353	7	31626	23	5762	7	8
53	35998	26	4591	6	34469	25	4967	6	33006	24	5360	6	31603	23	5769	7	7
54	35972	26	4597	6	34444	24	4973	7	32982	24	5366	7	31580	23	5776	7	6
55	35946	26	4603	6	34420	25	4980	6	32958	24	5373	7	31557	23	5783	7	5
56	35920	26	4609	7	34395	25	4986	7	32934	24	5380	6	31534	23	5790	7	4
57	35894	26	4616	6	34370	25	4993	6	32910	23	5386	7	31511	23	5797	7	3
58	35868	26	4622	6	34345	25	4999	6	32887	24	5393	7	31488	22	5804	7	2
59	35842	26	4628	6	34320	25	5005	7	32863	24	5400	7	31466	23	5811	7	1
60	35816		4634		34295		5012		32839		5407		31443		5818		0
′	B	d	A	d	B	d	A	d	B	d	A	d	B	d	A	d	′
	64°				63°				62°				61°				

TABLE 42. VALUES OF *A* AND *B*, WHERE *A* = (LOG COSECANT) × 10,000, *B* = (LOG SECANT) × 10,000 (*Continued*)

	29°				30°				31°				32°				
'	A	d	B	d	A	d	B	d	A	d	B	d	A	d	B	d	'
0	31443	23	5818	7	30103	22	6247	7	28816	21	6693	8	27579	20	7158	8	60
1	31420	23	5825	7	30081	22	6254	8	28795	21	6701	8	27559	20	7166	8	59
2	31397	22	5832	7	30059	22	6262	7	28774	21	6709	7	27539	21	7174	8	58
3	31375	23	5839	7	30037	21	6269	7	28753	21	6716	8	27518	20	7182	8	57
4	31352	23	5846	7	30016	22	6276	7	28732	21	6724	7	27498	20	7190	7	56
5	31329	23	5853	7	29994	22	6283	8	28711	21	6731	8	27478	20	7197	8	55
6	31306	22	5860	7	29972	22	6291	7	28690	21	6739	8	27458	20	7205	8	54
7	31284	23	5867	7	29950	22	6298	7	28669	21	6747	7	27438	20	7213	8	53
8	31261	23	5874	7	29928	21	6305	8	28648	21	6754	8	27418	20	7221	8	52
9	31238	22	5881	7	29907	22	6313	7	28627	20	6762	8	27398	20	7229	8	51
10	31216	23	5888	7	29885	22	6320	7	28607	21	6770	7	27378	21	7237	8	50
11	31193	22	5895	7	29863	22	6327	8	28586	21	6777	8	27357	20	7245	8	49
12	31171	23	5902	8	29841	21	6335	7	28565	21	6785	8	27337	20	7253	8	48
13	31148	23	5910	7	29820	22	6342	8	28544	21	6793	7	27317	20	7261	8	47
14	31125	22	5917	7	29798	22	6350	7	28523	21	6800	8	27297	20	7269	8	46
15	31103	23	5924	7	29776	21	6357	7	28502	21	6808	8	27277	20	7277	8	45
16	31080	22	5931	7	29755	22	6364	8	28481	20	6816	7	27257	20	7285	8	44
17	31058	23	5938	7	29733	21	6372	7	28461	21	6823	8	27237	20	7293	8	43
18	31035	22	5945	7	29712	22	6379	7	28440	21	6831	8	27217	20	7301	8	42
19	31013	23	5952	7	29690	22	6386	8	28419	21	6839	7	27197	20	7309	8	41
20	30990	22	5959	7	29668	21	6394	7	28398	20	6846	8	27177	20	7317	8	40
21	30968	23	5966	7	29647	22	6401	8	28378	21	6854	8	27157	20	7325	8	39
22	30945	22	5973	7	29625	21	6409	7	28357	21	6862	7	27137	20	7333	8	38
23	30923	23	5980	8	29604	22	6416	7	28336	21	6869	8	27117	19	7341	8	37
24	30900	22	5988	7	29582	21	6423	8	28315	20	6877	8	27098	20	7349	8	36
25	30878	22	5995	7	29561	22	6431	7	28295	21	6885	7	27078	20	7357	8	35
26	30856	23	6002	7	29539	21	6438	8	28274	21	6892	8	27058	20	7365	8	34
27	30833	22	6009	7	29518	22	6446	7	28253	20	6900	8	27038	20	7373	8	33
28	30811	23	6016	7	29496	21	6453	8	28233	21	6908	8	27018	20	7381	8	32
29	30788	22	6023	7	29475	22	6461	7	28212	21	6916	7	26998	20	7389	8	31
30	30766	22	6030	7	29453	21	6468	7	28191	20	6923	8	26978	19	7397	8	30
31	30744	23	6037	8	29432	22	6475	8	28171	21	6931	8	26959	20	7405	8	29
32	30721	22	6045	7	29410	21	6483	7	28150	20	6939	8	26939	20	7413	8	28
33	30699	22	6052	7	29389	22	6490	8	28130	21	6947	7	26919	20	7421	8	27
34	30677	22	6059	7	29367	21	6498	7	28109	20	6954	8	26899	20	7429	8	26
35	30655	23	6066	7	29346	21	6505	8	28089	21	6962	8	26879	19	7437	8	25
36	30632	22	6073	7	29325	22	6513	7	28068	20	6970	8	26860	20	7445	9	24
37	30610	22	6080	8	29303	21	6520	8	28048	21	6978	8	26840	20	7454	8	23
38	30588	22	6088	7	29282	21	6528	7	28027	21	6986	7	26820	20	7462	8	22
39	30566	22	6095	7	29261	22	6535	8	28006	20	6993	8	26800	19	7470	8	21
40	30544	23	6102	7	29239	21	6543	7	27986	20	7001	8	26781	20	7478	8	20
41	30521	22	6109	7	29218	21	6550	8	27966	21	7009	8	26761	20	7486	8	19
42	30499	22	6116	8	29197	21	6558	7	27945	20	7017	7	26741	19	7494	8	18
43	30477	22	6124	7	29176	22	6565	8	27925	21	7024	8	26722	20	7502	8	17
44	30455	22	6131	7	29154	21	6573	7	27904	20	7032	8	26702	20	7510	8	16
45	30433	22	6138	7	29133	21	6580	8	27884	21	7040	8	26682	19	7518	9	15
46	30411	22	6145	8	29112	21	6588	7	27863	20	7048	8	26663	20	7527	8	14
47	30389	22	6153	7	29091	22	6595	8	27843	20	7056	8	26643	20	7535	8	13
48	30367	22	6160	7	29069	21	6603	7	27823	21	7064	7	26623	19	7543	8	12
49	30345	22	6167	7	29048	21	6610	8	27802	20	7071	8	26604	20	7551	8	11
50	30323	22	6174	7	29027	21	6618	7	27782	20	7079	8	26584	19	7559	8	10
51	30301	22	6181	8	29006	21	6625	8	27762	21	7087	8	26565	20	7567	8	9
52	30279	22	6189	7	28985	21	6633	7	27741	20	7095	8	26545	19	7575	9	8
53	30257	22	6196	7	28964	22	6640	8	27721	20	7103	8	26526	20	7584	8	7
54	30235	22	6203	8	28942	21	6648	8	27701	21	7111	8	26506	19	7592	8	6
55	30213	22	6211	7	28921	21	6656	7	27680	20	7119	7	26487	20	7600	8	5
56	30191	22	6218	7	28900	21	6663	8	27660	20	7126	8	26467	19	7608	8	4
57	30169	22	6225	7	28879	21	6671	7	27640	21	7134	8	26448	20	7616	8	3
58	30147	22	6232	8	28858	21	6678	8	27619	20	7142	8	26428	19	7624	9	2
59	30125	22	6240	7	28837	21	6686	7	27599	20	7150	8	26409	20	7633	8	1
60	30103		6247		28816		6693		27579		7158		26389		7641		0
'	B	d	A	d	B	d	A	d	B	d	A	d	B	d	A	d	'
	60°				59°				58°				57°				

TABLE 42. VALUES OF A AND B, WHERE A = (LOG COSECANT) × 10,000, B = (LOG SECANT) × 10,000 (*Continued*)

	33°				34°				35°				36°				
′	A	d	B	d	A	d	B	d	A	d	B	d	A	d	B	d	′
0	26389	19	7641	8	25244	19	8143	8	24141	18	8664	8	23078	17	9204	9	60
1	26370	20	7649	8	25225	19	8151	9	24123	18	8672	9	23061	18	9213	10	59
2	26350	19	7657	8	25206	18	8160	8	24105	18	8681	9	23043	17	9223	9	58
3	26331	20	7665	9	25188	19	8168	9	24087	18	8690	9	23026	17	9232	9	57
4	26311	19	7674	8	25169	19	8177	8	24069	18	8699	9	23009	18	9241	9	56
5	26292	19	7682	8	25150	18	8185	9	24051	18	8708	9	22991	17	9250	9	55
6	26273	20	7690	8	25132	19	8194	8	24033	18	8717	9	22974	17	9259	10	54
7	26253	19	7698	9	25113	19	8202	9	24015	18	8726	8	22957	18	9269	9	53
8	26234	19	7707	8	25094	18	8211	8	23997	18	8734	9	22939	17	9278	9	52
9	26215	20	7715	8	25076	19	8219	9	23979	18	8743	9	22922	17	9287	9	51
10	26195	19	7723	8	25057	18	8228	9	23961	18	8752	9	22905	17	9296	10	50
11	26176	19	7731	9	25039	19	8237	8	23943	18	8761	9	22888	18	9306	9	49
12	26157	20	7740	8	25020	19	8245	9	23925	18	8770	9	22870	17	9315	9	48
13	26137	19	7748	8	25001	18	8254	8	23907	18	8779	9	22853	17	9324	9	47
14	26118	19	7756	9	24983	19	8262	9	23889	18	8788	9	22836	17	9333	10	46
15	26099	20	7765	8	24964	18	8271	9	23871	17	8797	9	22819	18	9343	9	45
16	26079	19	7773	8	24946	19	8280	8	23854	18	8806	9	22801	17	9352	9	44
17	26060	19	7781	8	24927	18	8288	9	23836	18	8815	9	22784	17	9361	9	43
18	26041	19	7789	9	24909	19	8297	8	23818	18	8824	9	22767	17	9370	10	42
19	26022	19	7798	8	24890	18	8305	9	23800	18	8833	9	22750	18	9380	9	41
20	26003	20	7806	8	24872	19	8314	9	23782	18	8842	9	22732	17	9389	9	40
21	25983	19	7814	9	24853	18	8323	8	23764	17	8851	8	22715	17	9398	10	39
22	25964	19	7823	8	24835	19	8331	9	23747	18	8859	9	22698	17	9408	9	38
23	25945	19	7831	8	24816	18	8340	9	23729	18	8868	9	22681	17	9417	9	37
24	25926	19	7839	9	24798	19	8349	8	23711	18	8877	9	22664	17	9426	9	36
25	25907	20	7848	8	24779	18	8357	9	23693	17	8886	9	22647	17	9435	10	35
26	25887	19	7856	8	24761	19	8366	9	23676	18	8895	9	22630	17	9445	9	34
27	25868	19	7864	9	24742	18	8375	8	23658	18	8904	9	22613	18	9454	9	33
28	25849	19	7873	8	24724	18	8383	9	23640	18	8913	9	22595	17	9463	10	32
29	25830	19	7881	8	24706	19	8392	9	23622	17	8922	9	22578	17	9473	9	31
30	25811	19	7889	9	24687	18	8401	8	23605	18	8931	9	22561	17	9482	9	30
31	25792	19	7898	8	24669	19	8409	9	23587	18	8940	9	22544	17	9491	10	29
32	25773	19	7906	8	24650	18	8418	9	23569	17	8949	9	22527	17	9501	9	28
33	25754	19	7914	9	24632	18	8427	8	23552	18	8958	9	22510	17	9510	10	27
34	25735	19	7923	8	24614	19	8435	9	23534	18	8967	10	22493	17	9520	9	26
35	25716	19	7931	9	24595	18	8444	9	23516	17	8977	9	22476	17	9529	9	25
36	25697	19	7940	8	24577	18	8453	9	23499	18	8986	9	22459	17	9538	10	24
37	25678	19	7948	8	24559	18	8462	8	23481	18	8995	9	22442	17	9548	9	23
38	25659	19	7956	9	24541	19	8470	9	23463	17	9004	9	22425	17	9557	9	22
39	25640	19	7965	8	24522	18	8479	9	23446	18	9013	9	22408	17	9566	10	21
40	25621	19	7973	9	24504	18	8488	8	23428	18	9022	9	22391	17	9576	9	20
41	25602	19	7982	8	24486	19	8496	9	23410	17	9031	9	22374	17	9585	10	19
42	25583	19	7990	8	24467	18	8505	9	23393	18	9040	9	22357	17	9595	9	18
43	25564	19	7998	9	24449	18	8514	9	23375	17	9049	9	22340	17	9604	10	17
44	25545	19	8007	8	24431	18	8523	8	23358	18	9058	9	22323	17	9614	9	16
45	25526	19	8015	9	24413	18	8531	9	23340	17	9067	9	22306	17	9623	9	15
46	25507	19	8024	8	24395	19	8540	9	23323	18	9076	9	22289	17	9632	10	14
47	25488	19	8032	9	24376	18	8549	9	23305	17	9085	9	22272	16	9642	9	13
48	25469	18	8041	8	24358	18	8558	9	23288	18	9094	10	22256	17	9651	10	12
49	25451	19	8049	9	24340	18	8567	8	23270	17	9104	9	22239	17	9661	9	11
50	25432	19	8058	8	24322	18	8575	9	23253	18	9113	9	22222	17	9670	10	10
51	25413	19	8066	9	24304	18	8584	9	23235	17	9122	9	22205	17	9680	9	9
52	25394	19	8075	8	24286	19	8593	9	23218	18	9131	9	22188	17	9689	10	8
53	25375	19	8083	9	24267	18	8602	9	23200	17	9140	9	22171	17	9699	9	7
54	25356	18	8092	8	24249	18	8611	8	23183	18	9149	9	22154	16	9708	10	6
55	25338	19	8100	9	24231	18	8619	9	23165	17	9158	10	22138	17	9718	9	5
56	25319	19	8109	8	24213	18	8628	9	23148	18	9168	9	22121	17	9727	10	4
57	25300	19	8117	9	24195	18	8637	9	23130	17	9177	9	22104	17	9737	9	3
58	25281	18	8126	8	24177	18	8646	9	23113	17	9186	9	22087	17	9746	10	2
59	25263	19	8134	9	24159	18	8655	9	23096	18	9195	9	22070	16	9756	9	1
60	25244		8143		24141		8664		23078		9204		22054		9765		0
′	B	d	A	d	B	d	A	d	B	d	A	d	B	d	A	d	′
	56°				55°				54°				53°				

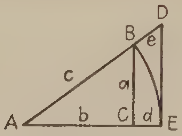
TABLE 42. VALUES OF A AND B, WHERE A = (LOG COSECANT) × 10,000, B = (LOG SECANT) × 10,000 (*Continued*)

	37°				38°				39°				40°				
′	A	d	B	d	A	d	B	d	A	d	B	d	A	d	B	d	′
0	22054	17	9765	10	21066	16	10347	10	20113	16	10950	10	19193	15	11575	10	60
1	22037	17	9775	9	21050	17	10357	10	20097	15	10960	10	19178	15	11585	11	59
2	22020	17	9784	10	21033	16	10367	9	20082	16	10970	10	19163	15	11596	10	58
3	22003	16	9794	9	21017	16	10376	10	20066	16	10980	11	19148	15	11606	11	57
4	21987	17	9803	10	21001	16	10386	10	20050	15	10991	10	19133	15	11617	11	56
5	21970	17	9813	9	20985	16	10396	10	20035	16	11001	10	19118	15	11628	10	55
6	21953	16	9822	10	20969	16	10406	10	20019	15	11011	11	19103	15	11638	11	54
7	21937	17	9832	9	20953	16	10416	10	20004	16	11022	10	19088	15	11649	11	53
8	21920	17	9841	10	20937	16	10426	10	19988	15	11032	10	19073	15	11660	10	52
9	21903	16	9851	10	20921	16	10436	10	19973	16	11042	10	19058	15	11670	11	51
10	21887	17	9861	9	20905	16	10446	10	19957	15	11052	11	19043	15	11681	11	50
11	21870	17	9870	10	20889	17	10456	10	19942	16	11063	10	19028	15	11692	10	49
12	21853	16	9880	9	20872	16	10466	10	19926	15	11073	10	19013	15	11702	11	48
13	21837	17	9889	10	20856	16	10476	10	19911	16	11083	11	18998	15	11713	11	47
14	21820	17	9899	10	20840	16	10486	10	19895	15	11094	10	18983	15	11724	10	46
15	21803	16	9909	9	20824	16	10496	9	19880	16	11104	10	18968	15	11734	11	45
16	21787	17	9918	10	20808	16	10505	10	19864	15	11114	11	18953	14	11745	11	44
17	21770	16	9928	9	20792	16	10515	10	19849	15	11125	10	18939	15	11756	10	43
18	21754	17	9937	10	20776	16	10525	10	19834	16	11135	10	18924	15	11766	11	42
19	21737	17	9947	10	20760	16	10535	10	19818	15	11145	11	18909	15	11777	11	41
20	21720	16	9957	9	20744	16	10545	10	19803	16	11156	10	18894	15	11788	11	40
21	21704	17	9966	10	20728	16	10555	10	19787	15	11166	10	18879	15	11799	10	39
22	21687	16	9976	10	20712	16	10565	10	19772	16	11176	11	18864	15	11809	11	38
23	21671	17	9986	9	20696	15	10575	10	19756	15	11187	10	18849	15	11820	11	37
24	21654	16	9995	10	20681	16	10585	10	19741	15	11197	10	18834	14	11831	11	36
25	21638	17	10005	10	20665	16	10595	10	19726	16	11207	11	18820	15	11842	10	35
26	21621	16	10015	9	20649	16	10605	10	19710	15	11218	10	18805	15	11852	11	34
27	21605	17	10024	10	20633	16	10615	10	19695	15	11228	11	18790	15	11863	11	33
28	21588	16	10034	10	20617	16	10625	11	19680	16	11239	10	18775	15	11874	11	32
29	21572	17	10044	9	20601	16	10636	10	19664	15	11249	10	18760	14	11885	10	31
30	21555	16	10053	10	20585	16	10646	10	19649	15	11259	11	18746	15	11895	11	30
31	21539	17	10063	10	20569	16	10656	10	19634	16	11270	10	18731	15	11906	11	29
32	21522	16	10073	9	20553	16	10666	10	19618	15	11280	11	18716	15	11917	11	28
33	21506	16	10082	10	20537	15	10676	10	19603	15	11291	10	18701	15	11928	11	27
34	21490	17	10092	10	20522	16	10686	10	19588	16	11301	11	18686	14	11939	10	26
35	21473	16	10102	10	20506	16	10696	10	19572	15	11312	10	18672	15	11949	11	25
36	21457	17	10112	9	20490	16	10706	10	19557	15	11322	10	18657	15	11960	11	24
37	21440	16	10121	10	20474	16	10716	10	19542	15	11332	11	18642	14	11971	11	23
38	21424	16	10131	10	20458	16	10726	10	19527	16	11343	10	18628	15	11982	11	22
39	21408	17	10141	10	20442	15	10736	10	19511	15	11353	11	18613	15	11993	11	21
40	21391	16	10151	11	20427	16	10746	10	19496	15	11364	10	18598	15	12004	11	20
41	21375	17	10160	10	20411	16	10756	11	19481	15	11374	11	18583	14	12015	10	19
42	21358	16	10170	10	20395	16	10767	10	19466	16	11385	10	18569	15	12025	11	18
43	21342	16	10180	10	20379	16	10777	10	19450	15	11395	11	18554	15	12036	11	17
44	21326	17	10190	9	20364	16	10787	10	19435	15	11406	10	18539	14	12047	11	16
45	21309	16	10199	10	20348	16	10797	10	19420	15	11416	11	18525	15	12058	11	15
46	21293	16	10209	10	20332	16	10807	10	19405	15	11427	10	18510	15	12069	11	14
47	21277	16	10219	10	20316	15	10817	10	19390	15	11437	11	18495	14	12080	11	13
48	21261	17	10229	10	20301	16	10827	11	19375	16	11448	10	18481	15	12091	11	12
49	21244	16	10239	9	20285	16	10838	10	19359	15	11458	11	18466	15	12102	11	11
50	21228	16	10248	10	20269	15	10848	10	19344	15	11469	10	18451	14	12113	10	10
51	21212	17	10258	10	20254	16	10858	10	19329	15	11479	11	18437	15	12123	11	9
52	21195	16	10268	10	20238	16	10868	10	19314	15	11490	11	18422	14	12134	11	8
53	21179	16	10278	10	20222	15	10878	10	19299	15	11501	10	18408	15	12145	11	7
54	21163	16	10288	10	20207	16	10888	11	19284	15	11511	11	18393	15	12156	11	6
55	21147	16	10298	9	20191	16	10899	10	19269	15	11522	10	18378	14	12167	11	5
56	21131	17	10307	10	20175	15	10909	10	19254	16	11532	11	18364	15	12178	11	4
57	21114	16	10317	10	20160	16	10919	10	19238	15	11543	10	18349	14	12189	11	3
58	21098	16	10327	10	20144	16	10929	11	19223	15	11553	11	18335	15	12200	11	2
59	21082	16	10337	10	20128	15	10940	10	19208	15	11564	11	18320	14	12211	11	1
60	21066		10347		20113		10950		19193		11575		18306		12222		0
′	B	d	A	d	B	d	A	d	B	d	A	d	B	d	A	d	′
	52°				51°				50°				49°				

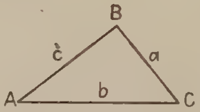
TABLE 42. VALUES OF A AND B, WHERE A = (LOG COSECANT) × 10,000, B = (LOG SECANT) × 10,000 (*Continued*)

	41°				42°				43°				44°				
'	A	d	B	d	A	d	B	d	A	d	B	d	A	d	B	d	'
0	18306	15	12222	11	17449	14	12893	11	16622	14	13587	12	15823	13	14307	12	60
1	18291	14	12233	11	17435	14	12904	11	16608	13	13599	12	15810	13	14319	12	59
2	18277	15	12244	11	17421	14	12915	12	16595	14	13611	12	15797	13	14331	12	58
3	18262	14	12255	11	17407	14	12927	11	16581	13	13623	11	15784	13	14343	12	57
4	18248	15	12266	11	17393	14	12938	12	16568	14	13634	12	15771	13	14355	13	56
5	18233	14	12277	11	17379	14	12950	11	16554	13	13646	12	15758	13	14368	12	55
6	18219	15	12288	11	17365	14	12961	11	16541	14	13658	12	15745	14	14380	12	54
7	18204	14	12299	11	17351	14	12972	12	16527	13	13670	12	15731	13	14392	12	53
8	18190	15	12310	11	17337	14	12984	11	16514	14	13682	12	15718	13	14404	13	52
9	18175	14	12321	11	17323	14	12995	12	16500	13	13694	11	15705	13	14417	12	51
10	18161	15	12332	11	17309	14	13007	11	16487	14	13705	12	15692	13	14429	12	50
11	18146	14	12343	11	17295	14	13018	12	16473	13	13717	12	15679	13	14441	12	49
12	18132	14	12354	11	17281	14	13030	11	16460	14	13729	12	15666	13	14453	13	48
13	18118	15	12365	11	17267	14	13041	12	16446	13	13741	12	15653	13	14466	12	47
14	18103	14	12376	11	17253	14	13053	11	16433	14	13753	12	15640	13	14478	12	46
15	18089	15	12387	12	17239	14	13064	12	16419	13	13765	12	15627	12	14490	13	45
16	18074	14	12399	11	17225	13	13076	11	16406	14	13777	12	15615	13	14503	12	44
17	18060	15	12410	11	17212	14	13087	11	16392	13	13789	11	15602	13	14515	12	43
18	18045	14	12421	11	17198	14	13098	12	16379	13	13800	12	15589	13	14527	13	42
19	18031	14	12432	11	17184	14	13110	11	16366	14	13812	12	15576	13	14540	12	41
20	18017	15	12443	11	17170	14	13121	12	16352	13	13824	12	15563	13	14552	12	40
21	18002	14	12454	11	17156	14	13133	12	16339	13	13836	12	15550	13	14564	13	39
22	17988	14	12465	11	17142	14	13145	11	16326	14	13848	12	15537	13	14577	12	38
23	17974	15	12476	11	17128	13	13156	12	16312	13	13860	12	15524	13	14589	12	37
24	17959	14	12487	12	17115	14	13168	11	16299	14	13872	12	15511	13	14601	13	36
25	17945	14	12499	11	17101	14	13179	12	16285	13	13884	12	15498	13	14614	12	35
26	17931	15	12510	11	17087	14	13191	11	16272	13	13896	12	15485	13	14626	13	34
27	17916	14	12521	11	17073	14	13202	12	16259	14	13908	12	15472	12	14639	12	33
28	17902	14	12532	11	17059	14	13214	11	16245	13	13920	12	15460	13	14651	12	32
29	17888	14	12543	11	17045	13	13225	12	16232	13	13932	12	15447	13	14663	13	31
30	17874	15	12554	12	17032	14	13237	11	16219	14	13944	12	15434	13	14676	12	30
31	17859	14	12566	11	17018	14	13248	12	16205	13	13956	12	15421	13	14688	13	29
32	17845	14	12577	11	17004	14	13260	12	16192	13	13968	12	15408	13	14701	12	28
33	17831	15	12588	11	16990	13	13272	11	16179	13	13980	12	15395	13	14713	13	27
34	17816	14	12599	11	16977	14	13283	12	16166	14	13992	12	15382	12	14726	12	26
35	17802	14	12610	12	16963	14	13295	11	16152	13	14004	12	15370	13	14738	12	25
36	17788	14	12622	11	16949	14	13306	12	16139	13	14016	12	15357	13	14750	13	24
37	17774	14	12633	11	16935	13	13318	12	16126	13	14028	12	15344	13	14763	12	23
38	17760	15	12644	11	16922	14	13330	11	16113	14	14040	12	15331	13	14775	13	22
39	17745	14	12655	11	16908	14	13341	12	16099	13	14052	12	15318	12	14788	12	21
40	17731	14	12666	12	16894	14	13353	12	16086	13	14064	12	15306	13	14800	13	20
41	17717	14	12678	11	16880	13	13365	11	16073	13	14076	12	15293	13	14813	12	19
42	17703	14	12689	11	16867	14	13376	12	16060	14	14088	12	15280	13	14825	13	18
43	17689	15	12700	12	16853	14	13388	12	16046	13	14100	12	15267	12	14838	12	17
44	17674	14	12712	11	16839	13	13400	11	16033	13	14112	12	15255	13	14850	13	16
45	17660	14	12723	11	16826	14	13411	12	16020	13	14124	12	15242	13	14863	12	15
46	17646	14	12734	11	16812	14	13423	12	16007	13	14136	13	15229	13	14875	13	14
47	17632	14	12745	12	16798	13	13435	11	15994	14	14149	12	15216	12	14888	12	13
48	17618	14	12757	11	16785	14	13446	12	15980	13	14161	12	15204	13	14900	13	12
49	17604	14	12768	11	16771	13	13458	12	15967	13	14173	12	15191	13	14913	13	11
50	17590	14	12779	12	16758	14	13470	12	15954	13	14185	12	15178	13	14926	12	10
51	17576	15	12791	11	16744	14	13482	11	15941	13	14197	12	15165	12	14938	13	9
52	17561	14	12802	11	16730	13	13493	12	15928	13	14209	12	15153	13	14951	12	8
53	17547	14	12813	12	16717	14	13505	12	15915	13	14221	13	15140	13	14963	13	7
54	17533	14	12825	11	16703	13	13517	11	15902	14	14234	12	15127	12	14976	12	6
55	17519	14	12836	11	16690	14	13528	12	15888	13	14246	12	15115	13	14988	13	5
56	17505	14	12847	12	16676	14	13540	12	15875	13	14258	12	15102	13	15001	13	4
57	17491	14	12859	11	16662	13	13552	12	15862	13	14270	12	15089	12	15014	12	3
58	17477	14	12870	11	16649	14	13564	11	15849	13	14282	12	15077	13	15026	13	2
59	17463	14	12881	12	16635	13	13575	12	15836	13	14294	13	15064	13	15039	12	1
60	17449		12893		16622		13587		15823		14307		15051		15051		0
'	B	d	A	d	B	d	A	d	B	d	A	d	B	d	A	d	'
	48°				47°				46°				45°				

TABLE 43. TRIGONOMETRIC FORMULAS



Right Triangle



Oblique Triangle

RIGHT TRIANGLES

$$\sin A = \frac{a}{c} = \cos B \qquad \sec A = \frac{c}{b} = \operatorname{cosec} B$$

$$\cos A = \frac{b}{c} = \sin B \qquad \operatorname{cosec} A = \frac{c}{a} = \sec B$$

$$\tan A = \frac{a}{b} = \cot B \qquad \operatorname{vers} A = \frac{c-b}{c} = \frac{d}{c} = 1 - \cos A$$

$$\cot A = \frac{b}{a} = \tan B \qquad \operatorname{exsec} A = \frac{e}{c} = \sec A - 1$$

$$a = c \sin A = c \cos B = b \tan A = b \cot B = \sqrt{c^2 - b^2}$$

$$b = c \cos A = c \sin B = a \cot A = a \tan B = \sqrt{c^2 - a^2}$$

$$c = \sqrt{a^2 + b^2} \qquad d = c \operatorname{vers} A \qquad e = c \operatorname{exsec} A$$

OBLIQUE TRIANGLES: $A + B + C = 180°$; $s = \frac{1}{2}(a + b + c)$

Given	Sought	Formulas
A, B, a	b, c	$b = \frac{a}{\sin A}\cdot \sin B \qquad c = \frac{a}{\sin A}\cdot \sin(A + B)$
A, a, b	B, c	$\sin B = \frac{\sin A}{a}\cdot b \qquad c = \frac{a}{\sin A}\cdot \sin C$
C, a, b	A, c, B	$\tan A = \frac{a \sin C}{b - a \cos C} \qquad c = \frac{a \sin C}{\sin A} \qquad b > a$
C, b, a	B, c, A	$\tan B = \frac{b \sin C}{a - b \cos C} \qquad c = \frac{b \sin C}{\sin B} \qquad a > b$
A, b, c	a	$a^2 = b^2 + c^2 - 2bc \cos A$
B, a, c	b	$b^2 = a^2 + c^2 - 2ac \cos B$
C, a, b	c	$c^2 = a^2 + b^2 - 2ab \cos C$
a, b, c	A, B, C	$\sin^2 \frac{1}{2}A = \frac{s-b}{b}\cdot\frac{s-c}{c}$
	(logs)	$\sin^2 \frac{1}{2}B = \frac{s-a}{a}\cdot\frac{s-c}{c}$
		$\sin^2 \frac{1}{2}C = \frac{s-a}{a}\cdot\frac{s-b}{b}$
a, b, c	A, B, C	$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$
	(Natural functions)	$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$
		$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

TABLE 43. TRIGONOMETRIC FORMULAS (*Continued*)

General Formulas

Given	*Area of Triangle*
a, b, c	$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$
C, a, b	$\text{Area} = \frac{1}{2}ab \sin C$
A, B, C, a	$\text{Area} = \dfrac{a^2 \sin B \sin C}{2 \sin A}$

$$\sin A = \sqrt{1 - \cos^2 A} = 2 \sin \tfrac{1}{2}A \cos \tfrac{1}{2}A = \sqrt{\frac{1 - \cos 2A}{2}}$$

$$\cos A = \sqrt{1 - \sin^2 A} = \cos^2 \tfrac{1}{2}A - \sin^2 \tfrac{1}{2}A = 1 - 2 \sin^2 \tfrac{1}{2}A$$

$$= 2 \cos^2 \tfrac{1}{2}A - 1 = \sqrt{\frac{1 + \cos 2A}{2}}$$

$$\operatorname{vers} A = 1 - \cos A = 2 \sin^2 \tfrac{1}{2}A$$

$$\tan A = \frac{1}{\cot A} = \frac{\sin A}{\cos A} = \sqrt{\sec^2 A - 1} = \frac{1 - \cos 2A}{\sin 2A} = \frac{\sin 2A}{1 + \cos 2A}$$

$$\cot A = \frac{1}{\tan A} = \frac{\cos A}{\sin A} = \sqrt{\operatorname{cosec}^2 A - 1} = \frac{1 + \cos 2A}{\sin 2A}$$

$$\sec A = \frac{1}{\cos A} = \sqrt{1 + \tan^2 A}$$

$$\operatorname{exsec} A = \sec A - 1 = \tan A \tan \tfrac{1}{2}A$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \sqrt{1 + \cot^2 A}$$

$$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin A + \sin B = 2 \sin \tfrac{1}{2}(A + B) \cos \tfrac{1}{2}(A - B)$$

$$\sin A - \sin B = 2 \cos \tfrac{1}{2}(A + B) \sin \tfrac{1}{2}(A - B)$$

$$\cos A + \cos B = 2 \cos \tfrac{1}{2}(A + B) \cos \tfrac{1}{2}(A - B)$$

$$\cos A - \cos B = -2 \sin \tfrac{1}{2}(A + B) \sin \tfrac{1}{2}(A - B)$$

Trig. Function	*Angles*							
	0°	30°	45°	60°	90°	180°	270°	360°
sine	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	−1	0
cosine	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	−1	0	1
tangent	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	−∞	0

TABLE 44. USEFUL NUMBERS AND FORMULAS

Ratio of circumference to diameter $= \pi = 3.14159265$

Circumference of circle $= 2\pi r$

1 radian $= \frac{180}{\pi}$ degrees $= 57.29578$ degrees

1 degree $= \frac{\pi}{180}$ radians $= 0.0174533$ radian $= 60$ min. $= 3600$ sec.

Length of arc $=$ radius $\times$ central angle (in radians)

$$= \frac{\pi r}{180} \times \text{central angles (in degrees)}$$

Acceleration due to gravity $= 32.2$ feet per second^2 (approx)

1 meter $= 39.37$ inches $= 3.2808$ feet

1 foot $= 0.3048$ meter $= 12$ inches

1 square yard $= 9$ square feet $= 144$ square inches

1 kilometer $= 1{,}000$ meters $= 0.62137$ mile

1 mile $= 5{,}280$ feet $= 1{,}760$ yards $= 1.60935$ kilometers

1 acre $= 43{,}560$ square feet $= 4{,}840$ square yards

Area of:

Trapezoid: $\frac{1}{2}$(sum of parallel sides) $\times$ (perpendicular height)

Circle: πr^2, where $r =$ radius

Sector of circle: Arc $\times \frac{1}{2}$ radius $= \frac{1}{2}r^2 \times$ central angle (in radians) $= 0.0087266r^2 \times$ central angle (in degrees)

Segment of circle: $\frac{r^2}{2}\left(\frac{\pi\theta}{180} - \sin\theta\right)$, where $\theta =$ central angle (in degrees)

Ellipse: $\pi ab = 0.78540 \times$ long diameter $\times$ short diameter

Parabola: Base $\times \frac{2}{3}$ perpendicular height

Volume of:

Prism or cylinder: Area base $\times$ perpendicular height

Pyramid or cone: Area base $\times \frac{1}{3}$ perpendicular height

Frustum of pyramid or cone: (Sum of end areas, A_1 and A_2, + square root of their product) $\times \frac{1}{3}$ perpendicular height h

$$= (A_1 + A_2 + \sqrt{A_1A_2})\frac{h}{3}$$

Sphere: $\frac{4}{3}\pi r^3$

Weight of:

Cement = 94 lb per cu ft

Concrete = 150 lb per cu ft

Earth = 100 lb per cu ft

Sand = 110 lb per cu ft

Steel = 490 lb per cu ft

Water = 62.5 lb per cu ft

TABLE 45. SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
1	1	1	1.0000	1.0000	1.000000000	3.1416	0.7854
2	4	8	1.4142	1.2599	.500000000	6.2832	3.1416
3	9	27	1.7321	1.4422	.333333333	9.4248	7.0686
4	16	64	2.0000	1.5874	.250000000	12.5664	12.5664
5	25	125	2.2361	1.7100	.200000000	15.7080	19.635
6	36	216	2.4495	1.8171	.166666667	18.850	28.274
7	49	343	2.6458	1.9129	.142857143	21.991	38.485
8	64	512	2.8284	2.0000	.125000000	25.133	50.266
9	81	729	3.0000	2.0801	.111111111	28.274	63.617
10	100	1,000	3.1623	2.1544	.100000000	31.416	78.540
11	121	1,331	3.3166	2.2240	.090909091	34.558	95.033
12	144	1,728	3.4641	2.2894	.083333333	37.699	113.10
13	169	2,197	3.6056	2.3513	.076923077	40.841	132.73
14	196	2,744	3.7417	2.4101	.071428571	43.982	153.94
15	225	3,375	3.8730	2.4662	.066666667	47.124	176.71
16	256	4,096	4.0000	2.5198	.062500000	50.265	201.06
17	289	4,913	4.1231	2.5713	.058823529	53.407	226.98
18	324	5,832	4.2426	2.6207	.055555556	56.549	254.47
19	361	6,859	4.3589	2.6684	.052631579	59.690	283.53
20	400	8,000	4.4721	2.7144	.050000000	62.832	314.16
21	441	9,261	4.5826	2.7589	.047619048	65.973	346.36
22	484	10,648	4.6904	2.8020	.045454545	69.115	380.13
23	529	12,167	4.7958	2.8439	.043478261	72.257	415.48
24	576	13,824	4.8990	2.8845	.041666667	75.398	452.39
25	625	15,625	5.0000	2.9240	.040000000	78.540	490.87
26	676	17,576	5.0990	2.9625	.038461538	81.681	530.93
27	729	19,683	5.1962	3.0000	.037037037	84.823	572.56
28	784	21,952	5.2915	3.0366	.035714286	87.965	615.75
29	841	24,389	5.3852	3.0723	.034482759	91.106	660.52
30	900	27,000	5.4772	3.1072	.033333333	94.248	706.86
31	961	29,791	5.5678	3.1414	.032258065	97.389	754.77
32	1,024	32,768	5.6569	3.1748	.031250000	100.53	804.25
33	1,089	35,937	5.7446	3.2075	.030303030	103.67	855.30
34	1,156	39,304	5.8310	3.2396	.029411765	106.81	907.92
35	1,225	42,875	5.9161	3.2717	.028571429	109.96	962.11
36	1,296	46,656	6.0000	3.3019	.027777778	113.10	1,017.88
37	1,369	50,653	6.0828	3.3322	.027027027	116.24	1,075.21
38	1,444	54,872	6.1644	3.3620	.026315789	119.38	1,134.11
39	1,521	59,319	6.2450	3.3912	.025641026	122.52	1,194.59
40	1,600	64,000	6.3246	3.4200	.025000000	125.66	1,256.64
41	1,681	68,921	6.4031	3.4482	.024390244	128.81	1,320.25
42	1,764	74,088	6.4807	3.4760	.023809524	131.95	1,385.44
43	1,849	79,507	6.5574	3.5034	.023255814	135.09	1,452.20
44	1,936	85,184	6.6332	3.5303	.022727273	138.23	1,520.53
45	2,025	91,125	6.7082	3.5569	.022222222	141.37	1,590.43
46	2,116	97,336	6 7823	3.5830	.021739130	144.51	1,661.90
47	2,209	103,823	6 8557	3.6088	.021276600	147.65	1,734.94
48	2,304	110,592	6.9282	3.6342	.020833333	150.80	1,809.56
49	2,401	117,649	7.0000	3.6593	.020408163	153.94	1,885.74
50	2,500	125,000	7.0711	3.6840	.020000000	157.08	1,963.50
51	2,601	132,651	7.1414	3.7084	.019607843	160.22	2,042.82
52	2,704	140,608	7.2111	3.7325	.019230769	163.36	2,123.72
53	2,809	148,877	7.2801	3.7563	.018867925	166.50	2,206.18
54	2,916	157,464	7.3485	3.7798	.018518519	169.65	2,290.22
55	3,025	166,375	7.4162	3.8030	.018181818	172.79	2,375.83

From "Coal Miners' Pocketbook."

TABLE 45. SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS (*Continued*)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum	Area.
56	3,136	175,616	7.4833	3.8259	.017857143	175.93	2,463.01
57	3,249	185,193	7.5498	3.8485	.017543860	179.07	2,551.76
58	3,364	195,112	7.6158	3.8709	.017241379	182.21	2,642.08
59	3,481	205,379	7.6811	3.8930	.016949153	185.35	2,733.97
60	3,600	216,000	7.7460	3.9149	.016666667	188.50	2,827.43
61	3,721	226,981	7.8102	3.9365	.016393443	191.64	2,922.47
62	3,844	238,328	7.8740	3.9579	.016129032	194.78	3,019.07
63	3,969	250,047	7.9373	3.9791	.015873016	197.92	3,117.25
64	4,096	262,144	8.0000	4.0000	.015625000	201.06	3,216.99
65	4,225	274,625	8.0623	4.0207	.015384615	204.20	3,318.31
66	4,356	287,496	8.1240	4.0412	.015151515	207.34	3,421.19
67	4,489	300,763	8.1854	4.0615	.014925373	210.49	3,525.65
68	4,624	314,432	8.2462	4.0817	.014705882	213.63	3,631.68
69	4,761	328,509	8.3066	4.1016	.014492754	216.77	3,739.28
70	4,900	343,000	8.3666	4.1213	.014285714	219.91	3,848.45
71	5,041	357,911	8.4261	4.1408	.014084517	223.05	3,959.19
72	5,184	373,248	8.4853	4.1602	.013888889	226.19	4,071.50
73	5,329	389,017	8.5440	4.1793	.013698630	229.34	4,185.39
74	5,476	405,224	8.6023	4.1983	.013513514	232.48	4,300.84
75	5,625	421,875	8.6603	4.2172	.013333333	235.62	4,417.86
76	5,776	438,976	8.7178	4.2358	.013157895	238.76	4,536.46
77	5,929	456,533	8.7750	4.2543	.012987013	241.90	4,656.63
78	6,084	474,552	8.8318	4.2727	.012820513	245.04	4,778.36
79	6,241	493,039	8.8882	4.2908	.012658228	248.19	4,901.67
80	6,400	512,000	8.9443	4.3089	.012500000	251.33	5,026.55
81	6,561	531,441	9.0000	4.3267	.012345679	254.47	5,153.00
82	6,724	551,368	9.0554	4.3445	.012195122	257.61	5,281.02
83	6,889	571,787	9.1104	4.3621	.012048193	260.75	5,410.61
84	7,056	592,704	9.1652	4.3795	.011904762	263.89	5,541.77
85	7,225	614,125	9.2195	4.3968	.011764706	267.04	5,674.50
86	7,396	636,056	9.2736	4.4140	.011627907	270.18	5,808.80
87	7,569	658,503	9.3274	4.4310	.011494253	273.32	5,944.68
88	7,744	681,472	9.3808	4.4480	.011363636	276.46	6,082.12
89	7,921	704,969	9.4340	4.4647	.011235955	279.60	6,221.14
90	8,100	729,000	9.4868	4.4814	.011111111	282.74	6,361.73
91	8,281	753,571	9.5394	4.4979	.010989011	285.88	6,503.88
92	8,464	778,688	9.5917	4.5144	.010869565	289.03	6,647.61
93	8,649	804,357	9.6437	4.5307	.010752688	292.17	6,792.91
94	8,836	830,584	9.6954	4.5468	.010638298	295.31	6,939.78
95	9,025	857,375	9.7468	4.5629	.010526316	298.45	7,088.22
96	9,216	884,736	9.7980	4.5789	.010416667	301.59	7,238.23
97	9,409	912,673	9.8489	4.5947	.010309278	304.73	7,389.81
98	9,604	941,192	9.8995	4.6104	.010204082	307.88	7,542.96
99	9,801	970,299	9.9499	4.6261	.010101010	311.02	7,697.69
100	10,000	1,000,000	10.0000	4.6416	.010000000	314.16	7,853.98
101	10,201	1,030,301	10.0499	4.6570	.009900990	317.30	8,011 85
102	10,404	1,061,208	10.0995	4.6723	.009803922	320.44	8,171.28
103	10,609	1,092,727	10.1489	4.6875	.009708738	323.58	8,332.29
104	10,816	1,124,864	10.1980	4.7027	.009615385	326.73	8,494.87
105	11,025	1,157,625	10.2470	4.7177	.009523810	329.87	8,659.01
106	11,236	1,191,016	10.2956	4.7326	.009433962	333.01	8,824.73
107	11,449	1,225,043	10.3441	4.7475	.009345794	336.15	8,992.02
108	11,664	1,259,712	10.3923	4.7622	.009259259	339.29	9,160.88
109	11,881	1,295,029	10.4403	4.7769	.009174312	342.43	9,331.32
110	12,100	1,331,000	10.4881	4.7914	.009090909	345.58	9,503.32
111	12,321	1,367,631	10.5357	4.8059	.009009009	348.72	9,676.89
112	12,544	1,404,928	10.5830	4.8203	.008928571	351.86	9,852.03
113	12,769	1,442,897	10.6301	4.8346	.008849558	355.00	10,028.75
114	12,996	1,481,544	10.6771	4.8488	.008771930	358.14	10,207.03
115	13,225	1,520,875	10.7238	4.8629	.008695652	361.28	10,386.89
116	13,456	1,560,896	10.7703	4.8770	.008620690	364.42	10,568.32
117	13,689	1,601,613	10.8167	4.8910	.008547009	367.57	10,751.32
118	13,924	1,643,032	10.8628	4.9049	.008474576	370,71	10,935.88

Table 45. Squares, Cubes, Square and Cube Roots, Circumferences, and Areas (*Continued*)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
119	14,161	1,685,159	10.9087	4.9187	.008403361	373.85	11,122.02
120	14,400	1,728,000	10.9545	4.9324	.008333333	376.99	11,309.73
121	14,641	1,771,561	11.0000	4.9461	.008264463	380.13	11,499.01
122	14,884	1,815,848	11.0454	4.9597	.008196721	383.27	11,689.87
123	15,129	1,860,867	11.0905	4.9732	.008130081	386.42	11,882.29
124	15,376	1,906,624	11.1355	4.9866	.008064516	389.56	12,076.28
125	15,625	1,953,125	11.1803	5.0000	.008000000	392.70	12,271.85
126	15,876	2,000,376	11.2250	5.0133	.007936508	395.84	12,468.98
127	16,129	2,048,383	11.2694	5.0265	.007874016	398.98	12,667.69
128	16,384	2,097,152	11.3137	5.0397	.007812500	402.12	12,867.96
129	16,641	2,146,689	11.3578	5.0528	.007751938	405.27	13,069.81
130	16,900	2,197,000	11.4018	5.0658	.007692308	408.41	13,273.23
131	17,161	2,248,091	11.4455	5.0788	.007633588	411.55	13,478.22
132	17,424	2,299,968	11.4891	5.0916	.007575758	414.69	13,684.78
133	17,689	2,352,637	11.5326	5.1045	.007518797	417.83	13,892.91
134	17,956	2,406,104	11.5758	5.1172	.007462687	420.97	14,102.61
135	18,225	2,460,375	11.6190	5.1299	.007407407	424.12	14,313.88
136	18,496	2,515,456	11.6619	5.1426	.007352941	427.26	14,526.72
137	18,769	2,571,353	11.7047	5.1551	.007299270	430.40	14,741.14
138	19,044	2,628,072	11.7473	5.1676	.007246377	433.54	14,957.12
139	19,321	2,685,619	11.7898	5.1801	.007194245	436.68	15,174.68
140	19,600	2,744,000	11.8322	5.1925	.007142857	439.82	15,393.80
141	19,881	2,803,221	11.8743	5.2048	.007092199	442.96	15,614.50
142	20,164	2,863,288	11.9164	5.2171	.007042254	446.11	15,836.77
143	20,449	2,924,207	11.9583	5.2293	.006993007	449.25	16,060.61
144	20,736	2,985,984	12.0000	5.2415	.006944444	452.39	16,286.02
145	21,025	3,048,625	12.0416	5.2536	.006896552	455.53	16,513.00
146	21,316	3,112,136	12.0830	5.2656	.006849315	458.67	16,741.55
147	21,609	3,176,523	12.1244	5.2776	.006802721	461.81	16,971.67
148	21,904	3,241,792	12.1655	5.2896	.006756757	464.96	17,203.36
149	22,201	3,307,949	12.2066	5.3015	.006711409	468.10	17,436.62
150	22,500	3,375,000	12.2474	5.3133	.006666667	471.24	17,671.46
151	22,801	3,442,951	12.2882	5.3251	.006622517	474.38	17,907.86
152	23,104	3,511,008	12.3288	5.3368	.006578947	477.52	18,145.84
153	23,409	3,581,577	12.3693	5.3485	.006535948	480.66	18,385.39
154	23,716	3,652,264	12.4097	5.3601	.006493506	483.81	18,626.50
155	24,025	3,723,875	12.4499	5.3717	.006451613	486.95	18,869.19
156	24,336	3,796,416	12.4900	5.3832	.006410256	490.09	19,113.45
157	24,649	3,869,893	12.5300	5.3947	.006369427	493.23	19,359.28
158	24,964	3,944,312	12.5698	5.4061	.006329114	496.37	19,606.68
159	25,281	4,019,679	12.6095	5.4175	.006289308	499.51	19,855.65
160	25,600	4,096,000	12.6491	5.4288	.006250000	502.65	20,106.19
161	25,921	4,173,281	12.6886	5.4401	.006211180	505.80	20,358.31
162	26,244	4,251,528	12.7279	5.4514	.006172840	508.94	20,611.99
163	26,569	4,330,747	12.7671	5.4626	.006134969	512.08	20,867.24
164	26,896	4,410,944	12.8062	5.4737	.006097561	515.22	21,124.07
165	27,225	4,492,125	12.8452	5.4848	.006060606	518.36	21,382.46
166	27,556	4,574,296	12.8841	5.4959	.006024096	521.50	21,642.43
167	27,889	4,657,463	12.9228	5.5069	.005988024	524.65	21,903.97
168	28,224	4,741,632	12.9615	5.5178	.005952381	527.79	22,167.08
169	28,561	4,826,809	13.0000	5.5288	.005917160	530.93	22,431.76
170	28,900	4,913,000	13.9384	5.5397	.005882353	534.07	22,698.01
171	29,241	5,000,211	13.0767	5.5505	.005847953	537.21	22,965.83
172	29,584	5,088,448	13.1149	5.5613	.005813953	540.35	23,235.22
173	29,929	5,177,717	13.1529	5.5721	.005780347	543.50	23,506.18
174	30,276	5,268,024	13.1909	5.5828	.005747126	546.64	23,778.71
175	30,625	5,359,375	13.2288	5.5934	.005714286	549.78	24,052.82
176	30,976	5,451.776	13.2665	5.6041	.005681818	552.92	24,328.49
177	31,329	5,545,233	13.3041	5.6147	.005649718	556.06	24,605.74
178	31,684	5,639,752	13.3417	5.6252	.005617978	559.20	24,884.56
179	32,041	5,735,339	13.3791	5.6357	.005586592	562.35	25,164.94
180	32,400	5,832,000	13.4164	5.6462	.005555556	565.49	25,446.90
181	32,761	5,929,741	13.4536	5.6567	.005524862	568.63	25,730.48

TABLE 45. SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS (*Continued*)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
182	33,124	6,028,568	13.4907	5.6671	.005494505	571.77	26,015.53
183	33,489	6,128,487	13.5277	5.6774	.005464481	574.91	26,302.20
184	33,856	6,229,504	13.5647	5.6877	.005434783	578.05	26,590.44
185	34,225	6,331,625	13.6015	5.6980	.005405405	581.19	26,880.25
186	34,596	6,434,856	13.6382	5.7083	.005376344	584.34	27,171.63
187	34,969	6,539,203	13.6748	5.7185	.005347594	587.48	27,464.59
188	35,344	6,644,672	13.7113	5.7287	.005319149	590.62	27,759.11
189	35,721	6,751,269	13.7477	5.7388	.005291005	593.76	28,055.21
190	36,100	6,859,000	13.7840	5.7489	.005263158	596.90	28,352.87
191	36,481	6,967,871	13.8203	5.7590	.005235602	600.04	28,652.11
192	36,864	7,077,888	13.8564	5.7690	.005208333	603.19	28,952.92
193	37,249	7,189,017	13.8924	5.7790	.005181347	606.33	29,255.30
194	37,636	7,301,384	13.9284	5.7890	.005154639	609.47	29,559.25
195	38,025	7,414,875	13.9642	5.7989	.005128205	612.61	29,864.77
196	38,416	7,529,536	14.0000	5.8088	.005102041	615.75	30,171.86
197	38,809	7,645,373	14.0357	5.8186	.005076142	618.89	30,480.52
198	39,204	7,762,392	14.0712	5.8285	.005050505	622.04	30,790.75
199	39,601	7,880,599	14.1067	5.8383	.005025126	625.18	31,102.55
200	40,000	8,000,000	14.1421	5.8480	.005000000	628.32	31,415.93
201	40,401	8,120,601	14.1774	5.8578	.004975124	631.46	31,730.87
202	40,804	8,242,408	14.2127	5.8675	.004950495	634.60	32,047.39
203	41,209	8,365,427	14.2478	5.8771	.004926108	637.74	32,365.47
204	41,616	8,489,664	14.2829	5.8868	.004901961	640.88	32,685.13
205	42,025	8,615,125	14.3178	5.8964	.004878049	644.03	33,006.36
206	42,436	8,741,816	14.3527	5.9059	.004854369	647.17	33,329.16
207	42,849	8,869,743	14.3875	5.9155	.004830918	650.31	33,653.53
208	43,264	8,998,912	14.4222	5.9250	.004807692	653.45	33,979.47
209	43,681	9,129,329	14.4568	5.9345	.004784689	656.59	34,306.98
210	44,100	9,261,000	14.4914	5.9439	.004761905	659.73	34,636.06
211	44,521	9,393,931	14.5258	5.9533	.004739336	662.88	34,966.71
212	44,944	9,528.128	14.5602	5.9627	.004716981	666.02	35,298.94
213	45,369	9,663,597	14.5945	5.9721	.004694836	669.16	35,632.73
214	45,796	9,800,344	14.6287	5.9814	.004672897	672.30	35,968.09
215	46,225	9,938,375	14.6629	5.9907	.004651163	675.44	36,305.03
216	46,656	10,077,696	14.6969	6.0000	.004629630	678.58	36,643.54
217	47,089	10,218,313	14.7309	6.0092	.004608295	681.73	36,983.61
218	47,524	10,360,232	14.7648	6.0185	.004587156	684.87	37,325.26
219	47,961	10,503,459	14.7986	6.0277	.004566210	688.01	37,668.48
220	48,400	10,648,000	14.8324	6.0368	.004545455	691.15	38,013.27
221	48,841	10,793,861	14.8661	6.0459	.004524887	694.29	38,359.63
222	49,284	10,941,048	14.8997	6.0550	.004504505	697.43	38,707.56
223	49,729	11,089,567	14.9332	6.0641	.004484305	700.58	39,057.07
224	50,176	11,239,424	14.9666	6.0732	.004464286	703.72	39,408.14
225	50,625	11,390,625	15.0000	6.0822	.004444444	706.86	39,760.78
226	51,076	11,543,176	15.0333	6.0912	.004424779	710.00	40,115.00
227	51,529	11,697,083	15.0665	6.1002	.004405286	713.14	40,470.78
228	51,984	11,852,352	15.0997	6.1091	.004385965	716.28	40,828.14
229	52,441	12,008,989	15.1327	6.1180	.004366812	719.42	41,187.07
230	52,900	12,167,000	15.1658	6.1269	.004347826	722.57	41,547.56
231	53,361	12,326,391	15.1987	6.1358	.004329004	725.71	41,909.63
232	53,824	12,487,168	15.2315	6.1446	.004310345	728.85	42,273.27
233	54,289	12,649,337	15.2643	6.1534	.004291845	731.99	42,638.48
234	54,756	12,812,904	15.2971	6.1622	.004273504	735.13	43,005.26
235	55,225	12,977,875	15.3297	6.1710	.004255319	738.27	43,373.61
236	55,696	13,144,256	15.3623	6.1797	.004237288	741.42	43,743.54
237	56,169	13,312,053	15.3948	6.1885	.004219409	744.56	44,115.03
238	56,644	13,481,272	15.4272	6.1672	.004201681	747.70	44,488.09
239	57,121	13,651,919	15.4596	6.2058	.004184100	750.84	44,862.73
240	57,600	13,824,000	15.4919	6.2145	.004166667	753.98	45,238.93
241	58,081	13,997,521	15.5242	6.2231	.004149378	757.12	45,616.71
242	58,564	14,172,488	15.5563	6.2317	.004132231	760.27	45,996.06
243	59,049	14,348,907	15.5885	6.2403	.004115226	763.41	46,376.98
244	59,536	14,526,784	15.6205	6.2488	.004098361	766.55	46,759.47

Table 45. Squares, Cubes, Square and Cube Roots, Circumferences, and Areas (*Continued*)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
245	60,025	14,706,125	15.6525	6.2573	.004081633	769.69	47,143.52
246	60,516	14,886,936	15.6844	6.2658	.004065041	772.83	47,529.16
247	61,009	15,069,223	15.7162	6.2743	.004048583	775.97	47,916.36
248	61,504	15,252,992	15.7480	6.2828	.004032258	779.11	48,305.13
249	62,001	15,438,249	15.7797	6.2912	.004016064	782.26	48,695.47
250	62,500	15,625,000	15.8114	6.2996	.004000000	785.40	49,087.39
251	63,001	15,813,251	15.8430	6.3080	.003984064	788.54	49,480.87
252	63,504	16,003,008	15.8745	6.3164	.003968254	791.68	49,875.92
253	64,009	16,194,277	15.9060	6.3247	.003952569	794.82	50,272.55
254	64,516	16,387,064	15.9374	6.3330	.003937008	797.96	50,670.75
255	65,025	16,581,375	15.9687	6.3413	.003921569	801.11	51,070.52
256	65,536	16,777,216	16.0000	6.3496	.003906250	804.25	51,471.85
257	66,049	16,974,593	16.0312	6.3579	.003891051	807.39	51,874.76
258	66,564	17,173,512	16.0624	6.3661	.003875969	810.53	52,279.24
259	67,081	17,373,979	16.0935	6.3743	.003861004	813.67	52,685.29
260	67,600	17,576,000	16.1245	6.3825	.003846154	816.81	53,092.92
261	68,121	17,779,581	16.1555	6.3907	.003831418	819.96	53,502.11
262	68,644	17,984,728	16.1864	6.3988	.003816794	823.10	53,912.87
263	69,169	18,191,447	16.2173	6.4070	.003802281	826.24	54,325.21
264	69,696	18,399,744	16.2481	6.4151	.003787879	829.38	54,739.11
265	70,225	18,609,625	16.2788	6.4232	.003773585	832.52	55,154.59
266	70,756	18,821,096	16.3095	6.4312	.003759398	835.66	55,571.63
267	71,289	19,034,163	16.3401	6.4393	.003745318	838.81	55,990.25
268	71,824	19,248,832	16.3707	6.4473	.003731343	841.95	56,410.44
269	72,361	19,465,109	16.4012	6.4553	.003717472	845.09	56,832.20
270	72,900	19,683,000	16.4317	6.4633	.003703704	848.23	57,255.53
271	73,441	19,902,511	16.4621	6.4713	.003690037	851.37	57,680.43
272	73,984	20,123,643	16.4924	6.4792	.003676471	854.51	58,106.90
273	74,529	20,346,417	16.5227	6.4872	.003663004	857.65	58,534.94
274	75,076	20,570,824	16.5529	6.4951	.003649635	860.80	58,964.55
275	75,625	20,796,875	16.5831	6.5030	.003636364	863.94	59,395.74
276	76,176	21,024,576	16.6132	6.5108	.003623188	867.08	59,828.49
277	76,729	21,253,933	16.6433	6.5187	.003610108	870.22	60,262.82
278	77,284	21,484,952	16.6783	6.5265	.003597122	873.36	60,698.71
279	77,841	21,717,639	16.7033	6.5343	.003584229	876.50	61,136.18
280	78,400	21,952,000	16.7332	6.5421	.003571429	879.65	61,575.22
281	78,961	22,188,041	16.7631	6.5499	.003558719	882.79	62,015.82
282	79,524	22,425,768	16.7929	6.5577	.003546099	885.93	62,458.00
283	80,089	22,665,187	16.8226	6.5654	.003533569	889.07	62,901.75
284	80,656	22,906,304	16.8523	6.5731	.003522127	892.21	63,347.07
285	81,225	23,149,125	16.8819	6.5808	.003508772	895.35	63,793.97
286	81,796	23,393,656	16.9115	6.5885	.003496503	898.50	64,242.43
287	82,369	23,639,903	16.9411	6.5962	.003484321	901.64	64,692.46
288	82,944	23,887,872	16.9706	6.6039	.003472222	904.78	65,144.07
289	83,521	24,137,569	17.0000	6.6115	.003460208	907.92	65,597.24
290	84,100	24,389,000	17.0294	6.6191	.003448276	911.06	66,051.99
291	84,681	24,642,171	17.0587	6.6267	.003436426	914.20	66,508.30
292	85,264	24,897,088	17.0880	6.6343	.003424658	917.35	66,966.19
293	85,849	25,153,757	17.1172	6.6419	.003412969	920.49	67,425.65
294	86,436	25,412,184	17.1464	6.6494	.003401361	923.63	67,886.68
295	87,025	25,672,375	17.1756	6.6569	.003389831	926.77	68,349.28
296	87,616	25,934,836	17.2047	6.6644	.003378378	929.91	68,813.45
297	88,209	26,198,073	17.2337	6.6719	.003367003	933.05	69,279.19
298	88,804	26,463,592	17.2627	6.6794	.003355705	936.19	69,746.50
299	89,401	26,730,899	17.2916	6.6869	.003344482	939.34	70,215.38
300	90,000	27,000,000	17.3205	6.6943	.003333333	942.48	70,685.83
301	90,601	27,270,901	17.3494	6.7018	.003322259	945.62	71,157.86
302	91,204	27,543,608	17.3781	6.7092	.003311258	948.76	71,631.45
303	91,809	27,818,127	17.4069	6.7166	.003301330	951.90	72,106.62
304	92,416	28,094,464	17.4356	6.7240	.003289474	955.04	72,583.36
305	93,025	28,372,625	17.4642	6.7313	.003278689	958.19	73,061.66
306	93,636	28,652,616	17.4929	6.7387	.003267974	961.33	73,541.54
307	94,249	28,934,443	17.5214	6.7460	.003257329	964.47	74,022.99

TABLE 45. SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS (*Continued*)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
308	94,864	29,218,112	17.5499	6.7533	.003246753	967.61	74,506.01
309	95,481	29,503,629	17.5784	6.7606	.003236246	970.75	74,990.60
310	96,100	29,791,000	17.6068	6.7679	.003225806	973.89	75,476.76
311	96,721	30,080,231	17.6352	6.7752	.003215434	977.04	75,964.50
312	97,344	30,371,328	17.6635	6.7824	.003205128	980.18	76,453.80
313	97,969	30,664,297	17.6918	6.7897	.003194888	983.32	76,944.67
314	98,596	30,959,144	17.7200	6.7969	.003184713	986.46	77,437.12
315	99,225	31,255,875	17.7482	6.8041	.003174603	989.60	77,931.13
316	99,856	31,554,496	17.7764	6.8113	.003164557	992.74	78,426.72
317	100,489	31,855,013	17.8045	6.8185	.003154574	995.88	78,923.88
318	101,124	32,157,432	17.8326	6.8256	.003144654	999.03	79,422.60
319	101,761	32,461,759	17.8606	6.8328	.003134796	1,002.17	79,922.90
320	102,400	32,768,000	17.8885	6.8399	.003125000	1,005.31	80,424.77
321	103,041	33,076,161	17.9165	6.8470	.003115265	1,008.45	80,928.21
322	103,684	33,386,248	17.9444	6.8541	.003105590	1,011.59	81,433.22
323	104,329	33,698,267	17.9722	6.8612	.003095975	1,014.73	81,939.80
324	104,976	34,012,224	18.0000	6.8683	.003086420	1,017.88	82,447.96
325	105,625	34,328,125	18.0278	6.8753	.003076923	1,021.02	82,957.68
326	106,276	34,645,976	18.0555	6.8824	.003067485	1,024.16	83,468.98
327	106.929	34,965,783	18.0831	6.8894	.003058104	1,027.30	83,981.84
328	107,584	35,287,552	18.1108	6.8964	.003048780	1,030.44	84,496.28
329	108,241	35,611,289	18.1384	6.9034	.003039514	1,033.58	85,012.28
330	108,900	35,937,000	18.1659	6.9104	.003030303	1,036.73	85,529.86
331	109,561	36,264,691	18.1934	6.9174	.003021148	1,039.87	86,049.01
332	110,224	36,594,368	18.2209	6.9244	.003012048	1,043.01	86,569.73
333	110,889	36,926,037	18.2483	6.9313	.003003003	1,046.15	87,092.02
334	111,556	37,259,704	18.2757	6.9382	.002994012	1,049.29	87,615.88
335	112,225	37,595,375	18.3030	6.9451	.002985075	1,052.43	88,141.31
336	112,896	37,933,056	18.3303	6.9521	.002976190	1,055.58	88,668.31
337	113,569	38,272,753	18.3576	6.9589	.002967359	1,058.72	89,196.88
338	114,244	38,614,472	18.3848	6.9658	.002958580	1,061.86	89,727.03
339	114,921	38,958,219	18.4120	6.9727	.002949853	1,065.00	90,258.74
340	115,600	39,304,000	18.4391	6.9795	.002941176	1,068.14	90,792.03
341	116,281	39,651,821	18.4662	6.9864	.002932551	1,071.28	91,326.88
342	116,964	40,001,688	18.4932	6.9932	.002923977	1,074.42	91,863.31
343	117,649	40,353,607	18.5203	7.0000	.002915452	1,077.57	92,401.31
344	118,336	40,707,584	18.5472	7.0068	.002906977	1,080.71	92,940.88
345	119,025	41,063,625	18.5742	7.0136	.002898551	1,083.85	93,482.02
346	119,716	41,421,736	18.6011	7.0203	.002890173	1,086.99	94,024.73
347	120,409	41,781,923	18.6279	7.0271	.002881844	1,090.13	94,569.01
348	121,104	42,144,192	18.6548	7.0338	.002873563	1,093.27	95,114.86
349	121,801	42,508,549	18.6815	7.0406	.002865330	1,096.42	95,662.28
350	122,500	42,875,000	18.7083	7.0473	.002857143	1,099.56	96,211.28
351	123,201	43,243,551	18.7350	7.0540	.002849003	1,102.70	96,761.84
352	123,904	43,614,208	18.7617	7.0607	.002840909	1,105.84	97,313.97
353	124,609	43,986,977	18.7883	7.0674	.002832861	1,108.98	97,867.68
354	125,316	44,361,864	18.8149	7.0740	.002824859	1,112.12	98,422.96
355	126,025	44,738,875	18.8414	7.0807	.002816901	1,115.27	98,979.80
356	126,736	45,118,016	18.8680	7.0873	.002808989	1,118.41	99,538.22
357	127,449	45,499,293	18.8944	7.0940	.002801120	1,121.55	100,098.21
358	128,164	45,882,712	18.9209	7.1006	.002793296	1,124.69	100,659.77
359	128,881	46,268,279	18.9473	7.1072	.002785515	1,127.83	101,222.90
360	129,600	46,656,000	18.9737	7.1138	.002777778	1,130.97	101,787.60
361	130,321	47,045,881	19.0000	7.1204	.002770083	1,134.11	102,353.87
362	131,044	47,437,928	19.0263	7.1269	.002762431	1,137.26	102,921.72
363	131,769	47,832,147	19.0526	7.1335	.002754821	1,140.40	103,491.13
364	132,496	48,228,544	19.0788	7.1400	.002747253	1,143.54	104,062.12
365	133,225	48,627,125	19.1050	7.1466	.002739726	1,146.68	104,634.67
366	133,956	49,027,896	19.1311	7.1531	.002732240	1,149.82	105,208.80
367	134,689	49,430,863	19.1572	7.1596	.002724796	1,152.96	105,784.49
368	135,424	49,836,032	19.1833	7.1661	.002717391	1,156.11	106,361.76
369	136,161	50,243,409	19.2094	7.1726	.002710027	1,159.25	106,940.60
370	136,900	50,653,000	19.2354	7.1791	.002702703	1,162.39	107,521.01

TABLE 45. SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS (*Continued*)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
371	137,641	51,064,811	19.2614	7.1855	.002695418	1,165.53	108,102.99
372	138,384	51,478,848	19.2873	7.1920	.002688172	1,168.67	108,686.54
373	139,129	51,895,117	19.3132	7.1984	.002680965	1,171.81	109,271.66
374	139,876	52,313,624	19.3391	7.2048	.002673797	1,174.96	109,858.35
375	140,625	52,734,375	19.3649	7.2112	.002666667	1,178.10	110,446.62
376	141,376	53,157,376	19.3907	7.2177	.002659574	1,181.24	111,036.45
377	142,129	53,582,633	19.4165	7.2240	.002652520	1,184.38	111,627.86
378	142,884	54,010,152	19.4422	7.2304	.002645503	1,187.52	112,220.83
379	143,641	54,439,939	19.4679	7.2368	.002638521	1,190.66	112,815.38
380	144,400	54,872,000	19.4936	7.2432	.002631579	1,193.81	113,411.49
381	145,161	55,306,341	19.5192	7.2495	.002624672	1,196.95	114,009.18
382	145,924	55,742,968	19.5448	7.2558	.002617801	1,200.09	114,608.44
383	146,689	56,181,887	19.5704	7.2622	.002610966	1,203.23	115,209.27
384	147,456	56,623,104	19.5959	7.2685	.002604167	1,206.37	115,811.67
385	148,225	57,066,625	19.6214	7.2748	.002597403	1,209.51	116,415.64
386	148,996	57,512,456	19.6469	7.2811	.002590674	1,212.65	117,021.18
387	149,769	57,960,603	19.6723	7.2874	.002583979	1,215.80	117,628.30
388	150,544	58,411,072	19.6977	7.2936	.002577320	1,218.94	118,236.98
389	151,321	58,863,869	19.7231	7.2999	.002570694	1,222.08	118,847.24
390	152,100	59,319,000	19.7484	7.3061	.002564103	1,225.22	119,459.06
391	152,881	59,776,471	19.7737	7.3124	.002557545	1,228.36	120,072.46
392	153,664	60,236,288	19.7990	7.3186	.002551020	1,231.50	120,687.42
393	154,449	60,698,457	19.8242	7.3248	.002544529	1,234.65	121,303.96
394	155,236	61,162,984	19.8494	7.3310	.002538071	1,237.79	121,922.07
395	156,025	61,629,875	19.8746	7.3372	.002531646	1,240.93	122,541.75
396	156,816	62,099,136	19.8997	7.3434	.002525253	1,244.07	123,163.00
397	157,609	62,570,773	19.9249	7.3496	.002518892	1,247.21	123,785.82
398	158,404	63,044,792	19.9499	7.3558	.002512563	1,250.35	124,410.21
399	159,201	63,521,199	19.9750	7.3619	.002506266	1,253.50	125,036.17
400	160,000	64,000,000	20.0000	7.3681	.002500000	1,256.64	125,663.71
401	160,801	64,481,201	20.0250	7.3742	.002493766	1,259.78	126,292.81
402	161,604	64,964,808	20.0499	7.3803	.002487562	1,262.92	126,923.48
403	162,409	65,450,827	20.0749	7.3864	.002481390	1,266.06	127,555.73
404	163,216	65,939,264	20.0998	7.3925	.002475248	1,269.20	128,189.55
405	164,025	66,430,125	20.1246	7.3986	.002469136	1,272.35	128,824.93
406	164,836	66,923,416	20.1494	7.4047	.002463054	1,275.49	129,461.89
407	165,649	67,419,143	20.1742	7.4108	.002457002	1,278.63	130,100.42
408	166,464	67,917,312	20.1990	7.4169	.002450980	1,281.77	130,740.52
409	167,281	68,417,929	20.2237	7.4229	.002444988	1,284.91	131,382.19
410	168,100	68,921,000	20.2485	7.4290	.002439024	1,288.05	132,025.43
411	168,921	69,426,531	20.2731	7.4350	.002433090	1,291.19	132,670.24
412	169,744	69,934,528	20.2978	7.4410	.002427184	1,294.34	133,316.63
413	170,569	70,444,997	20.3224	7.4470	.002421308	1,297.48	133,964.58
414	171,396	70,957,944	20.3470	7.4530	.002415459	1,300.62	134,614.10
415	172,225	71,473,375	20.3715	7.4590	.002409639	1,303.76	135,265.20
416	173,056	71,991,296	20.3961	7.4650	.002406846	1,306.90	135,917.86
417	173,889	72,511,713	20.4206	7.4710	.002398082	1,310.04	136,572.10
418	174,724	73,034,632	20.4450	7.4770	.002392344	1,313.19	137,227.91
419	175,561	73,560,059	20.4695	7.4829	.002386635	1,316.33	137,885.29
420	176,400	74,088,000	20.4939	7.4889	.002380952	1,319.47	138,544.24
421	177,241	74,618,461	20.5183	7.4948	.002375297	1,322.61	139,204.76
422	178,084	75,151,448	20.5426	7.5007	.002369668	1,325.75	139,866.85
423	178,929	75,686,967	20.5670	7.5067	.002364066	1,328.89	140,530.51
424	179,776	76,225,024	20.5913	7.5126	.002358491	1,332.04	141,195.74
425	180,625	76,765,625	20.6155	7.5185	.002352941	1,335.18	141,862.54
426	181,476	77,308,776	20.6398	7.5244	.002347418	1,338.32	142,530.92
427	182,329	77,854,483	20.6640	7.5302	.002341920	1,341.46	143,200.86
428	183,184	78,402,752	20.6882	7.5361	.002336449	1,344.60	143,872.38
429	184,041	78,953,589	20.7123	7.5420	.002331002	1,347.74	144,545.46
430	184,900	79,507,000	20.7364	7.5478	.002325581	1,350.88	145,220.12
431	185,761	80,062,991	20.7605	7.5537	.002320186	1,354.03	145,896.35
432	186,624	80,621,568	20.7846	7.5595	.002314815	1,357.17	146,574.15
433	187,489	81,182,737	20.8087	7.5654	.002309469	1,360.31	147,253.52

TABLE 45. SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS (*Continued*)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
434	188,356	81,746,504	20.8327	7.5712	.002304147	1,363.45	147,934.46
435	189,225	82,312,875	20.8567	7.5770	.002298851	1,366.59	148,616.97
436	190,096	82,881,856	20.8806	7.5828	.002293578	1,369.73	149,301.05
437	190,969	83,453,453	20.9045	7.5886	.002288330	1,372.88	149,986.70
438	191,844	84,027,672	20.9284	7.5944	.002283105	1,376.02	150,673.93
439	192,721	84,604,519	20.9523	7.6001	.002277904	1,379.16	151,362.72
440	193,600	85,184,000	20.9762	7.6059	.002272727	1,382.30	152,053.08
441	194.481	85,766,121	21.0000	7.6117	.002267574	1,385.44	152,745.02
442	195,364	86,350,888	21.0238	7.6174	.002262443	1,388.58	153,438.53
443	196,249	86,938,307	21.0476	7.6232	.002257336	1,391.73	154,133.60
444	197,136	87,528,384	21.0713	7.6289	.002252252	1,394.87	154,830.25
445	198,025	88,121,125	21.0950	7.6346	.002247191	1,398.01	155,528.47
446	198,916	88,716,536	21.1187	7.6403	.002242152	1,401.15	156,228.26
447	199,809	89,314,623	21.1424	7.6460	.002237136	1,404.29	156,929.62
448	200,704	89,915,392	21.1660	7.6517	.002232143	1,407.43	157,632.55
449	201,601	90,518,849	21.1896	7.6574	.002227171	1,410.58	158,337.06
450	202,500	91,125,000	21.2132	7.6631	.002222222	1,413.72	159,043.13
451	203,401	91,733,851	21.2368	7.6688	.002217295	1,416.86	159,750.77
452	204,304	92,345,408	21.2603	7.6744	.002212389	1,420.00	160,459.99
453	205,209	92,959,677	21.2838	7.6801	.002207506	1,423.14	161,170.77
454	206,116	93,576,664	21.3073	7.6857	.002202643	1,426.28	161,883.13
455	207,025	94,196,375	21.3307	7.6914	.002197802	1,429.42	162,597.05
456	207,936	94,818,816	21.3542	7.6970	.002192982	1,432.57	163,312.55
457	208,849	95,443,993	21.3776	7.7026	.002188184	1,435.71	164,029.62
458	209,764	96,071,912	21.4009	7.7082	.002183406	1,438.85	164,748.26
459	210,681	96,702,579	21.4243	7.7188	.002178649	1,441.99	165,468.47
460	211,600	97,336,000	21.4476	7.7194	.002173913	1,445.13	166,190.25
461	212,521	97,972,181	21.4709	7.7250	.002169197	1,448.27	166,913.60
462	213,444	98,611,128	21.4942	7.7306	.002164502	1,451.42	167,638.53
463	214,369	99,252,847	21.5174	7.7362	.002159827	1,454.56	168,365.02
464	215,296	99,897,344	21.5407	7.7418	.002155172	1,457.70	169,093.08
465	216,225	100,544,625	21.5639	7.7473	.002150538	1,460.84	169,822.72
466	217,156	101,194,696	21.5870	7.7529	.002145923	1,463.98	170,553.92
467	218,089	101,847,563	21.6102	7.7584	.002141328	1,467.12	171,286.70
468	219,024	102,503,232	21.6333	7.7639	.002136752	1,470.27	172,021.05
469	219,961	103,161,709	21.6564	7.7695	.002132196	1,473.41	172,756.97
470	220,900	103,823,000	21.6795	7.7750	.002127660	1,476.55	173,494.45
471	221,841	104,487,111	21.7025	7.7805	.002123142	1,479.69	174,233.51
472	222,784	105,154,048	21.7256	7.7860	.002118644	1,482.83	174,974.14
473	223,729	105,823,817	21.7486	7.7915	.002114165	1,485.97	175,716.35
474	224,676	106,496,424	21.7715	7.7970	.002109705	1,489.11	176,460.12
475	225,625	107,171,875	21.7945	7.8025	.002105263	1,492.26	177,205.46
476	226,576	107,850,176	21.8174	7.8079	.002100840	1,495.40	177,952.37
477	227,529	108,531,333	21.8403	7.8134	.002096486	1,498.54	178,700.86
478	228,484	109,215,352	21.8632	7.8188	.002092050	1,501.68	179,450.91
479	229,441	109,902,239	21.8861	7.8243	.002087683	1,504.82	180,202.54
480	230,400	110,592,000	21.9089	7.8297	.002083333	1,507.96	180,955.74
481	231,361	111,284,641	21.9317	7.8352	.002079002	1,511.11	181,710.50
482	232,324	111,980,168	21.9545	7.8406	.002074689	1,514.25	182,466.84
483	233,289	112,678,587	21.9775	7.8460	.002070393	1.517.39	183,224.75
484	234,256	113,379,904	22.0000	7.8514	.002066116	1,520.53	183,984.23
485	235,225	114,084,125	22.0227	7.8568	.002061856	1,523.67	184,745.28
486	236,196	114,791,256	22.0454	7.8622	.002057613	1,526.81	185,507.90
487	237,169	115,501,303	22.0681	7.8676	.002053388	1,529.96	186,272.10
488	238,144	116,214,272	22.0907	7.8730	.002049180	1,533.10	187,037.86
489	239,121	116,930,169	22.1133	7.8784	.002044990	1,536.24	187,805.19
490	240,100	117,649,000	22.1359	7.8837	.002040816	1,539.38	188,574.10
491	241,081	118,370,771	22.1585	7.8891	.002036660	1,542.52	189,344.57
492	242,064	119,095,488	22.1811	7.8944	.002032520	1,545.66	190,116.62
493	243,049	119,823,157	22.2036	7.8998	.002028398	1,548.81	190,890.24
494	244,036	120,553,784	22.2261	7.9051	.002024291	1,551.95	191,665.43
495	245,025	121,287,375	22.2486	7.9105	.002020292	1,555.09	192,442.18
496	246,016	122,023,936	22.2711	7.9158	.002016129	1,558.23	193,220.51

TABLE 45. SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS (*Continued*)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
497	247,009	122,763,473	22.2935	7.9211	.002012072	1,561.37	194,000.41
498	248,004	123,505,992	22.3159	7.9264	.002008032	1,564.51	194,781.89
499	249,001	124,251,499	22.3383	7.9317	.002004008	1,567.65	195,564.93
500	250,000	125,000,000	22.3607	7.9370	.002000000	1,570.80	196,349.54
501	251,001	125,751,501	22.3830	7.9423	.001996008	1,573.94	197,135.72
502	252,004	126,506,008	22.4054	7.9476	.001992032	1,577.08	197,923.48
503	253,009	127,263,527	22.4277	7.9528	.001988072	1,580.22	198,712.80
504	254,016	128,024,064	22.4499	7.9581	.001984127	1,583.36	199,503.70
505	255,025	128,787,625	22.4722	7.9634	.001980198	1,586.50	200,296.17
506	256,036	129,554,216	22.4944	7.9686	.001976285	1,589.65	201,090.20
507	257,049	130,323,843	22.5167	7.9739	.001972387	1,592.79	201,885.81
508	258,064	131,096,512	22.5389	7.9791	.001968504	1,595.93	202,682.99
509	259,081	131,872,229	22.5610	7.9843	.001964637	1,599.07	203,481.74
510	260,100	132,651,000	22.5832	7.9895	.001960785	1,602.21	204,282.06
511	261,121	133,432,831	22.6053	7.9948	.001956947	1,605.35	205,083.95
512	262,144	134,217,728	22.6274	8.0000	.001953125	1,608.50	205,887.42
513	263,169	135,005,697	22.6495	8.0052	.001949318	1,611.64	206,692.45
514	264,196	135,796,744	22.6716	8.0104	.001945525	1,614.78	207,499.05
515	265,225	136,590,875	22.6936	8.0156	.001941748	1,617.92	208,307.23
516	266,256	137,388,096	22.7156	8.0208	.001937984	1,621.06	209,116.97
517	267,289	138,188,413	22.7376	8.0260	.001934236	1,624.20	209,928.29
518	268,324	138,991,832	22.7596	8.0311	.001930502	1,627.34	210,741.18
519	269,361	139,798,359	22.7816	8.0363	.001926782	1,630.49	211,555.63
520	270,400	140,608,000	22.8035	8.0415	.001923077	1,633.63	212,371.66
521	271,441	141,420,761	22.8254	8.0466	.001919386	1,636.77	213,189.26
522	272,484	142,236,648	22.8473	8.0517	.001915709	1,639.91	214,008.43
523	273,529	143,055,667	22.8692	8.0569	.001912046	1,643.05	214,829.17
524	274,576	143,877,824	22.8910	8.0620	.001908397	1,646.19	215,651.49
525	275,625	144,703,125	22.9129	8.0671	.001904762	1,649.34	216,475.37
526	276,676	145,531,576	22.9347	8.0723	.001901141	1,652.48	217,300.82
527	277,729	146,363,183	22.9565	8.0774	.001897533	1,655.62	218,127.85
528	278,784	147,197,952	22.9783	8.0825	.001893939	1,658.76	218,956.44
529	279,841	148,035,889	23.0000	8.0876	.001890359	1,661.90	219,786.61
530	280,900	148,877,001	23.0217	8.0927	.001886792	1,665.04	220,618.34
531	281,961	149,721,291	23.0434	8.0978	.001883239	1,668.19	221,451.65
532	283,024	150,568,768	23.0651	8.1028	.001879699	1,671.33	222,286.53
533	284,089	151,419,437	23.0868	8.1079	.001876173	1,674.47	223,122.98
534	285,156	152,273,304	23.1084	8.1130	.001872659	1,677.61	223,961.00
535	286,225	153,130,375	23.1301	8.1180	.001869159	1,680.75	224,800.59
536	287,296	153,990,656	23.1517	8.1231	.001865672	1,683.89	225,641.75
537	288,369	154,854,153	23.1733	8.1281	.001862197	1,687.04	226,484.48
538	289,444	155,720,872	23.1948	8.1332	.001858736	1,690.18	227,328.79
539	290,521	156,590,819	23.2164	8.1382	.001855288	1,693.32	228,174.66
540	291,600	157,464,000	23.2379	8.1433	.001851852	1,696.46	229,022.10
541	292,681	158,340,421	23.2594	8.1483	.001848429	1,699.60	229,871.12
542	293,764	159,220,088	23.2809	8.1533	.001845018	1,702.74	230,721.71
543	294,849	160,103,007	23.3024	8.1583	.001841621	1,705.88	231,573.86
544	295,936	160,989,184	23.3238	8.1633	.001838235	1,709.03	232,427.59
545	297,025	161,878,625	23.3452	8.1683	.001834862	1,712.17	233,282.89
546	298,116	162,771,336	23.3666	8.1733	.001831502	1,715.31	234,139.76
547	299,209	163,667,323	23.3880	8.1783	.001828154	1,718.45	234,998.20
548	300,304	164,566,592	23.4094	8.1833	.001824818	1,721.59	235,858.21
549	301,401	165,469,149	23.4307	8.1882	.001821494	1,724.73	236,719.79
550	302,500	166,375,000	23.4521	8.1932	.001818182	1,727.88	237,582.94
551	303,601	167,284,151	23.4734	8.1982	.001814882	1,731.02	238,447.67
552	304,704	168,196,608	23.4947	8.2031	.001811594	1,734.16	239,313.96
553	305,809	169,112,377	23.5160	8.2081	.001808318	1,737.30	240,181.83
554	306,916	170,031,464	23.5372	8.2130	.001805054	1,740.44	241,051.26
555	308,025	170,953,875	23.5584	8.2180	.001801802	1,743.58	241,922.27
556	309,136	171,879,616	23.5797	8.2229	.001798561	1,746.73	242,794.85
557	310,249	172,808,693	23.6008	8.2278	.001795332	1,749.87	243,668.99
558	311,364	173,741,112	23.6220	8.2327	.001792115	1,753.01	244,544.71
559	812,481	174,676,879	23,6432	8.2377	.001788909	1,756.15	245,422.00

TABLE 45. SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS (*Continued*)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
560	313,600	175,616,000	23.6643	8.2426	.001785714	1,759.29	246,300.86
561	314,721	176,558,481	23.6854	8.2475	.001782531	1,762.43	247,181.30
562	315,844	177,504,328	23.7065	8.2524	.001779359	1,765.58	248,063.30
563	316,969	178,453,547	23.7276	8.2573	.001776199	1,768.72	248,946.87
564	318,096	179,406,144	23.7487	8.2621	.001773050	1,771.86	249,832.01
565	319,225	180,362,125	23.7697	8.2670	.001769912	1,775.00	250,718.73
566	320,356	181,321,496	23.7908	8.2719	.001766784	1,778.14	251,607.01
567	321,489	182,284,263	23.8118	8.2768	.001763668	1,781.28	252,496.87
568	322,624	183,250,432	23.8328	8.2816	.001760563	1,784.42	253,388.30
569	323,761	184,220,009	23.8537	8.2865	.001757469	1,787.57	254,281.29
570	324,900	185,193,000	23.8747	8.2913	.001754386	1,790.71	255,175.86
571	326,041	186,169,411	23.8956	8.2962	.001751313	1,793.85	256,072.00
572	327,184	187,149,248	23.9165	8.3010	.001748252	1,796.99	256,969.71
573	328,329	188,132,517	23.9374	8.3059	.001745201	1,800.13	257,868.99
574	329,476	189,119,224	23.9583	8.3107	.001742164	1,803.27	258,769.85
575	330,625	190,109,375	23.9792	8.3155	.001739130	1,806.42	259,672.27
576	331,776	191,102,976	24.0000	8.3203	.001736111	1,809.56	260,576.26
577	332,929	192,100,033	24.0208	8.3251	.001733102	1,812.70	261,481.83
578	334,084	193,100,552	24.0416	8.3300	.001730104	1,815.84	262,388.96
579	335,241	194,104,539	24.0624	8.3348	.001727116	1,818.98	263,297.67
580	336,400	195,112,000	24.0832	8.3396	.001724138	1,822.12	264,207.94
581	337,561	196,122,941	24.1039	8.3443	.001721170	1,825.27	265,119.79
582	338,724	197,137,368	24.1247	8.3491	.001718213	1,828.41	266,033.21
583	339,889	198,155,287	24.1454	8.3539	.001715266	1,831.55	266,948.20
584	341,056	199,176,704	24.1661	8.3587	.001712329	1,834.69	267,864.76
585	342,225	200,201,625	24.1868	8.3634	.001709402	1,837.83	268,782.89
586	343,396	201,230,056	24.2074	8.3682	.001706485	1,840.97	269,702.59
587	344,569	202,262,003	24.2281	8.3730	.001703578	1,844.11	270,623.86
588	345,744	203,297,472	24.2487	8.3777	.001700680	1,847.26	271,546.70
589	346,921	204,336,469	24.2693	8.3825	.001697793	1,850.40	272,471.12
590	348,100	205,379,000	24.2899	8.3872	.001694915	1,853.54	273,397.10
591	349,281	206,425,071	24.3105	8.3919	.001692047	1,856.68	274,324.66
592	350,464	207,474,688	24.3311	8.3967	.001689189	1,859.82	275,253.78
593	351,649	208,527,857	24.3516	8.4014	.001686341	1,862.96	276,184.48
594	352,836	209,584,584	24.3721	8.4061	.001683502	1,866.11	277,116.75
595	354,025	210,644,875	24.3926	8.4108	.001680672	1,869.25	278,050.58
596	355,216	211,708,736	24.4131	8.4155	.001677852	1,872.39	278,985.99
597	356,409	212,776,173	24.4336	8.4202	.001675042	1,875.53	279,922.97
598	357,604	213,847,192	24.4540	8.4249	.001672241	1,878.67	280,861.52
599	358,801	214,921,799	24.4745	8.4296	.001669449	1,881.81	281,801.65
600	360,000	216,000,000	24.4949	8.4343	.001666667	1,884.96	282,743.34
601	361,201	217,081,801	24.5153	8.4390	.001663894	1,888.10	283,686.60
602	362,404	218,167,208	24.5357	8.4437	.001661130	1,891.24	284,631.44
603	363,609	219,256,227	24.5561	8.4484	.001658375	1,894.38	285,577.84
604	364,816	220,348,864	24.5764	8.4530	.001655629	1,897.52	286,525.82
605	366,025	221,445,125	24.5968	8.4577	.001652893	1,900.66	287,475.36
606	367,236	222,545,016	24.6171	8.4623	.001650165	1,903.81	288,426.48
607	368,449	223,648,543	24.6374	8.4670	.001647446	1,906.95	289,379.17
608	369,664	224,755,712	24.6577	8.4716	.001644737	1,910.09	290,333.43
609	370,881	225,866,529	24.6779	8.4763	.001642036	1,913.23	291,289.26
610	372,100	226,981,000	24.6982	8.4809	.001639344	1,916.37	292,246.66
611	373,321	228,099,131	24.7184	8.4856	.001636661	1,919.51	293,205.63
612	374,544	229,220,928	24.7386	8.4902	.001633987	1,922.65	294,166.17
613	375,769	230,346,397	24.7588	8.4948	.001631321	1,925.80	295,128.28
614	376,996	231,475,544	24.7790	8.4994	.001628664	1,928.94	296,091.97
615	378,225	232,608,375	24.7992	8.5040	.001626016	1,932.08	297,057.22
616	379,456	233,744,896	24.8193	8.5086	.001623377	1,935.22	298,024.05
617	380,689	234,885,113	24.8395	8.5132	.001620746	1,938.36	298,992.44
618	381,924	236,029,032	24.8596	8.5178	.001618123	1,941.50	299,962.41
619	383,161	237,176,659	24.8797	8.5224	.001615509	1,944.65	300,933.95
620	384,400	238,328,000	24.8998	8.5270	.001612903	1,947.79	301,907.05
621	385,641	239,483,061	24.9199	8.5316	.001610306	1,950.93	302,881.73
622	386,884	240,641,848	24.9399	8.5362	.001607717	1,954.07	303,857.98

TABLE 45. SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS (*Continued*)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
623	388,129	241,804,367	24.9600	8.5408	.001605136	1,957.21	304,835.80
624	389,376	242,970,624	24.9800	8.5453	.001602564	1,960.35	305,815.20
625	390,625	244,140,625	25.0000	8.5499	.001600000	1,963.50	306,796.16
626	391,876	245,314,376	25.0200	8.5544	.001597444	1,966.64	307,778.69
627	393,129	246,491,883	25.0400	8.5589	.001594896	1,969.78	308,762.79
628	394,384	247,673,152	25.0599	8.5635	.001592357	1,972.92	309,748.47
629	395,641	248,858,189	25.0799	8.5681	.001589825	1,976.06	310.735.71
630	396,900	250,047,000	25.0998	8.5726	.001587302	1,979.20	311.724.53
631	398,161	251,239,591	25.1197	8.5772	.001584786	1,982.35	312,714.92
632	399,424	252,435,968	25.1396	8.5817	.001582278	1,985.49	313.706.88
633	400,689	253,636,137	25.1595	8.5862	.001579779	1,988.63	314.700.40
634	401,956	254,840,104	25.1794	8.5907	.001577287	1,991.77	315.695.50
635	403,225	256,047,875	25.1992	8.5952	.001574803	1,994.91	316.692.17
636	404,496	257,259,456	25.2190	8.5997	.001572327	1,998.05	317,690.42
637	405,769	258,474,853	25.2389	8.6043	.001569859	2,001.19	318,690.23
638	407,044	259,694,072	25.2587	8.6088	.001567398	2,004.34	319,691.61
639	408,321	260,917,119	25.2784	8.6132	.001564945	2,007.48	320,694.56
640	409,600	262,144,000	25.2982	8.6177	.001562500	2,010.62	321,699.09
641	410,881	263,374,721	25.3180	8.6222	.001560062	2,013.76	322.705.18
642	412,164	264,609,288	25.3377	8.6267	.001557632	2,016.90	323,712.85
643	413,449	265,847,707	25.3574	8.6312	.001555210	2,020.04	324,722.09
644	414,736	267,089,984	25.3772	8.6357	.001552795	2,023.19	325,732.89
645	416,125	268,336,125	25.3969	8.6401	.001550388	2,026.33	326,745.27
646	417,316	269,585,136	25.4165	8.6446	.001547988	2,029.47	327,759.22
647	418,609	270,840,023	25.4362	8.6490	.001545595	2,032.61	328,774.74
648	419,904	272,097,792	25.4558	8.6535	.001543210	2,035.75	329,791.83
649	421,201	273,359,449	25.4755	8.6579	.001540832	2,038.89	330,810.49
650	422,500	274,625,000	25.4951	8.6624	.001538462	2,042.04	331,830.72
651	423,801	275,894,451	25.5147	8.6668	.001536098	2,045.18	332,852.53
652	425,104	277,167,808	25.5343	8.6713	.001533742	2,048.32	333,875.90
653	426,409	278,445,077	25.5539	8.6757	.001531394	2,051.46	334,900.85
654	427,716	279,726,264	25.5734	8.6801	.001529052	2,054.60	335.927.36
655	429,025	281,011,375	25.5930	8.6845	.001526718	2,057.74	336,955.45
656	430,336	282,300,416	25.6125	8.6890	.001524390	2,060.88	337,985.10
657	431,639	283,593,393	25.6320	8.6934	.001522070	2,064.03	339,016.33
658	432,964	284,890,312	25.6515	8.6978	.001519751	2,067.17	340,049.13
659	434,281	286,191,179	25.6710	8.7022	.001517451	2,070.31	341,083.50
660	435,600	287,496,000	25.6905	8.7066	.001515152	2,073.45	342,119.44
661	436,921	288,804,781	25.7099	8.7110	.001512859	2,076.59	343,156.95
662	438,244	290,117,528	25.7294	8.7154	.001510574	2,079.73	344,196.03
663	439,569	291,434,247	25.7488	8.7198	.001508296	2,082.88	345,236.69
664	440,896	292,754,944	25.7682	8.7241	.001506024	2,086.02	346,278.91
665	442,225	294,079,625	25.7876	8.7285	.001503759	2,089.16	347,322.70
666	443,556	295,408,296	25.8070	8.7329	.001501502	2,092.30	348,368.07
667	444,899	296,740,963	25.8263	8.7373	.001499250	2,095.44	349,415.00
668	446,224	298,077,632	25.8457	8.7416	.001497006	2,098.58	350,463.51
669	447,561	299,418,309	25.8650	8.7460	.001494768	2,101.73	351,513.59
670	448,900	300,763,000	25.8844	8.7503	.001492537	2,104.87	352,565.24
671	450,241	302,111,711	25.9037	8.7547	.001490313	2,108.01	353,618.45
672	451,584	303,464,448	25.9230	8.7590	.001488095	2,111.15	354,673.24
673	452,929	304,821,217	25.9422	8.7634	.001485884	2,114.29	355,729.60
674	454,276	306,182,024	25.9615	8.7677	.001483680	2,117.43	356,787.54
675	455,625	307,546,875	25.9808	8.7721	.001481481	2,120.58	357,847.04
676	456,976	308,915,776	26.0000	8.7764	.001479290	2,123.72	358,908.11
677	458,329	310,288,733	26.0192	8.7807	.001477105	2,126.86	359,970.75
678	459,684	311,665,752	26.0384	8.7850	.001474926	2,130.00	361,034.97
679	461,041	313,046,839	26.0576	8.7893	.001472754	2,133.14	362,100.75
680	462,400	314,432,000	26.0768	8.7937	.001470588	2,136.28	363,168.11
681	463,761	315,821,241	26.0960	8.7980	.001468429	2,139.42	364,237.04
682	465,124	317,214,568	26.1151	8.8023	.001466276	2,142.57	365,307.54
683	466,489	318,611,987	26.1343	8.8066	.001464129	2,145.71	366,379.60
684	467,856	320,013,504	26.1534	8.8109	.001461988	2,148.85	367,453.24
685	469,225	321,419,125	26.1725	8.8152	.001459854	2,151.99	368,528.45

TABLE 45. SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS (*Continued*)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
686	470,596	322,828,856	26.1916	8.8194	.001457726	2,155.13	369,605.23
687	471,969	324,242,703	26.2107	8.8237	.001455604	2,158.27	370,683.59
688	473,344	325,660,672	26.2298	8.8280	.001453488	2,161.42	371,763.51
689	474,721	327,082,769	26.2488	8.8323	.001451379	2,164.56	372,845.00
690	476,100	328,509,000	26.2679	8.8366	.001449275	2,167.70	373,928.07
691	477,481	329,939,371	26.2869	8.8408	.001447178	2,170.84	375,012.70
692	478,864	331,373,888	26.3059	8.8451	.001445087	2,173.98	376,098.91
693	480,249	332,812,557	26.3249	8.8493	.001443001	2,177.12	377,186.68
694	481,636	334,255,384	26.3439	8.8536	.001440922	2,180.27	378,276.03
695	483,025	335,702,375	26.3629	8.8578	.001438849	2,183.41	379,366.95
696	484,416	337,153,536	26.3818	8.8621	.001436782	2,186.55	380,459.44
697	485,809	338,608,873	26.4008	8.8663	.001434720	2,189.69	381,553.50
698	487,204	340,068,392	26.4197	8.8706	.001432665	2,192.83	382,649.13
699	488,601	341,532,099	26.4386	8.8748	.001430615	2,195.97	383,746.33
700	490,000	343,000,000	26.4575	8.8790	.001428571	2,199.11	384,845.10
701	491,401	344,472,101	26.4764	8.8833	.001426534	2,202.26	385,945.44
702	492,804	345,948,408	26.4953	8.8875	.001424501	2,205.40	387,047.36
703	494,209	347,428,927	26.5141	8.8917	.001422475	2,208.54	388,150.84
704	495,616	348,913,664	26.5330	8.8959	.001420455	2,211.68	389,255.90
705	497,025	350,402,625	26.5518	8.9001	.001418440	2,214.82	390,362.52
706	498,436	351,895,816	26.5707	8.9043	.001416431	2,217.96	391,470.72
707	499,849	353,393,243	26.5895	8.9085	.001414427	2,221.11	392,580.49
708	501,264	354,894,912	26.6083	8.9127	.001412429	2,224.25	393,691.82
709	502,681	356,400,829	26.6271	8.9169	.001410437	2,227.39	394,804.73
710	504,100	357,911,000	26.6458	8.9211	.001408451	2,230.53	395,919.21
711	505,521	359,425,431	26.6646	8.9253	.001406470	2,233.67	397,035.26
712	506,944	360,944,128	26.6833	8.9295	.001404494	2,236.81	398,152.89
713	508,369	362,467,097	26.7021	8.9337	.001402525	2,239.96	399,272.08
714	509,796	363,994,344	26.7208	8.9378	.001400560	2,243.10	400,392.84
715	511,225	365,525,875	26.7395	8.9420	.001398601	2,246.24	401,515.18
716	512,656	367,061,696	26.7582	8.9462	.001396648	2,249.38	402,639.08
717	514,089	368,601,813	26.7769	8.9503	.001394700	2,252.52	403,764.56
718	515,524	370,146,232	26.7955	8.9545	.001392758	2,255.66	404,891.60
719	516,961	371,694,959	26.8142	8.9587	.001390821	2,258.81	406,020.22
720	518,400	373,248,000	26.8328	8.9628	.001388889	2,261.95	407,150.41
721	519,841	374,805,361	26.8514	8.9670	.001386963	2,265.09	408,282.17
722	521,284	376,367,048	26.8701	8.9711	.001385042	2,268.23	409,415.50
723	522,729	377,933,067	26.8887	8.9752	.001383126	2,271.37	410,550.40
724	524,176	379,503,424	26.9072	8.9794	.001381215	2,274.51	411,686.87
725	525,625	381,078,125	26.9258	8.9835	.001379310	2,277.65	412,824.91
726	527,076	382,657,176	26.9444	8.9876	.001377410	2,280.80	413,964.52
727	528,529	384,240,583	26.9629	8.9918	.001375516	2,283.94	415,105.71
728	529,984	385,828,352	26.9815	8.9959	.001373626	2,287.08	416,248.46
729	531,441	387,420,489	27.0000	9.0000	.001371742	2,290.22	417,392.79
730	532,900	389,017,000	27.0185	9.0041	.001369863	2,293.36	418,538.68
731	534,361	390,617,891	27.0370	9.0082	.001367989	2,296.50	419,686.15
732	535,824	392,223,168	27.0555	9.0123	.001366120	2,299.65	420,835.19
733	537,289	393,832,837	27.0740	9.0164	.001364256	2,302.79	421,985.79
734	538,756	395,446,904	27.0924	9.0205	.001362398	2,305.93	423,137.97
735	540,225	397,065,375	27.1109	9.0246	.001360544	2,309.07	424,291.72
736	541,696	398,688,256	27.1293	9.0287	.001358696	2,312.21	425,447.04
737	543,169	400,315,553	27.1477	9.0328	.001356852	2,315.35	426,603.94
738	544,644	401,947,272	27.1662	9.0369	.001355014	2,318.50	427,762.40
739	546,121	403,583,419	27.1846	9.0410	.001353180	2,321.64	428,922.43
740	547,600	405,224,000	27.2029	9.0450	.001351351	2,324.78	430,084.03
741	549,081	406,869,021	27.2213	9.0491	.001349528	2,327.92	431,247.21
742	550,564	408,518,488	27.2397	9.0532	.001347709	2,331.06	432,411.95
743	552,049	410,172,407	27.2580	9.0572	.001345895	2,334.20	433,578.27
744	553,536	411,830,784	27.2764	9.0613	.001344086	2,337.34	434,746.16
745	555,025	413,493,625	27.2947	9.0654	.001342282	2,340.49	435,915.62
746	556,516	415,160,936	27.3130	9.0694	.001340483	2,343.63	437,086.64
747	558,009	416,832,728	27.3313	9.0735	.001338688	2,246.77	438,259.24
748	559,504	418,508,992	27.3496	9.0775	.001336898	2,349.91	439,433.41

TABLE 45. SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS (*Continued*)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
749	561,001	420,189,749	27.3679	9.0816	.001335113	2,353.05	440,609.16
750	562,500	421,875,000	27.3861	9.0856	.001333333	2,356.19	441,786.47
751	564,001	423,564,751	27.4044	9.0896	.001331558	2,359.34	442,965.35
752	565,504	425,259,008	27.4226	9.0937	.001329787	2,362.48	444,145.80
753	567,009	426,957,777	27.4408	9.0977	.001328021	2,365.62	445,327.83
754	568,516	428,661,064	27.4591	9.1017	.001326260	2,368.76	446,511.42
755	570,025	430,368,875	27.4773	9.1057	.001324503	2,371.90	447,696.59
756	571,536	432,081,216	27.4955	9.1098	.001322751	2,375.04	448,883.32
757	573,049	433,798,093	27.5136	9.1138	.001321004	2,378.19	450,071.63
758	574,564	435,519,512	27.5318	9.1178	.001319261	2.381.33	451,261.51
759	576,081	437,245,479	27.5500	9.1218	.001317523	2,384.47	452,452.96
760	577,600	438,976,000	27.5681	9.1258	.001315789	2,387.61	453,645.98
761	579,121	440,711,081	27.5862	9.1298	.001314060	2,390.75	454,840.57
762	580,644	442,450,728	27.6043	9.1338	.001312336	2,393.89	456,036.73
763	582,169	444,194,947	27.6225	9.1378	.001310616	2,397.04	457,234.46
764	583,696	445,943,744	27.6405	9.1418	.001308901	2,400.18	458,433.77
765	585,225	447,697,125	27.6586	9.1458	.001307190	2,403.32	459,634.64
766	586,756	449,455,096	27.6767	9.1498	.001305483	2,406.46	460,837.08
767	588,289	451,217,663	27.6948	9.1537	.001303781	2,409.60	462,041.10
768	589,824	452,984,832	27.7128	9.1577	.001302083	2,412.74	463,246.69
769	591,361	454,756,609	27.7308	9.1617	.001300390	2,415.88	464,453.84
770	592,900	456,533,000	27.7489	9.1657	.001298701	2,419.03	465,662.57
771	594,441	458,314,011	27.7669	9.1696	.001297017	2,422.17	466,872.87
772	595,984	460,099,648	27.7849	9.1736	.001295337	2,425.31	468,084.74
773	597,529	461,889,917	27.8029	9.1775	.001293661	2,428.45	469,298.18
774	599,076	463,684,824	27.8209	9.1815	.001291990	2,431.59	470,513.19
775	600,625	465,484,375	27.8388	9.1855	.001290323	2,434.73	471,729.77
776	602,176	467,288,576	27.8568	9.1894	.001288660	2,437.88	472,947.92
777	603,729	469,097,433	27.8747	9.1933	.001287001	2,441.02	474,167.65
778	605,284	470,910,952	27.8927	9.1973	.001285347	2,444.16	475,388.94
779	606,841	472,729,139	27.9106	9.2012	.001283697	2,447.30	476,611.81
780	608,400	474,552,000	27.9285	9.2052	.001282051	2,450.44	477,836.24
781	609,961	476,379,541	27.9464	9.2091	.001280410	2,453.58	479,062.25
782	611,524	478,211,768	27.9643	9.2130	.001278772	2,456.73	480,289.83
783	613,089	480,048,687	27.9821	9.2170	.001277139	2,459.87	481,518.97
784	614,656	481,890,304	28.0000	9.2209	.001275510	2,463.01	482,749.69
785	616,225	483,736,625	28.0179	9.2248	.001273885	2,466.15	483,981.98
786	617,796	485,587,656	28.0357	9.2287	.001272265	2,469.29	485,215.84
787	619,369	487,443,403	28.0535	9.2326	.001270648	2,472.43	486,451.28
788	620,944	489,303,872	28.0713	9.2365	.001269036	2,475.58	487,688.28
789	622,521	491,169,069	28.0891	9.2404	.001267427	2,478.72	488,926.85
790	624,100	493,039,000	28.1069	9.2443	.001265823	2,481.86	490,166.99
791	625,681	494,913,671	28.1247	9.2482	.001264223	2,485.00	491,408.71
792	627,264	496,793,088	28.1425	9.2521	.001262626	2,488.14	492,651.99
793	628,849	498,677,257	28.1603	9.2560	.001261034	2,491.28	493,896.85
794	630,436	500,566,184	28.1780	9.2599	.001259446	2,494.42	495,143.28
795	632,025	502,459,875	28.1957	9.2638	.001257862	2,497.57	496,391.27
796	633,616	504,358,336	28.2135	9.2677	.001256281	2,500.71	497,640.84
797	635,209	506,261,573	28.2312	9.2716	.001254705	2,503.85	498,891.98
798	636,804	508,169,592	28.2489	9.2754	.001253133	2,506.99	500,144.69
799	638,401	510,082,399	28.2666	9.2793	.001251364	2,510.13	501,398.97
800	640,000	512,000,000	28.2843	9.2832	.001250000	2,513.27	502,654.82
801	641,601	513,922,401	28.3019	9.2870	.001248439	2,516.42	503,912.25
802	643,204	515,849,608	28.3196	9.2909	.001246883	2,519.56	505,171.24
803	644,809	517,781,627	28.3373	9.2948	.001245330	2,522.70	506,431.80
804	646,416	519,718,464	28.3549	9.2986	.001243781	2,525.84	507,693.94
805	648,025	521,660,125	28.3725	9.3025	.001242236	2,528.98	508,957.64
806	649,636	523,606,616	28.3901	9.3063	.001240695	2,532.12	510,222.92
807	651,249	525,557,943	28.4077	9.3102	.001239157	2,535.27	511,489.77
808	652,864	527,514,112	28.4253	9.3140	.001237624	2,538.41	512,758.19
809	654,481	529,475,129	28.4429	9.3179	.001236094	2,541.55	514,028.18
810	656,100	531,441,000	28.4605	9.3217	.001234568	2,544.69	515,299.74
811	657,721	533,411,731	28.4781	9.3255	.001233046	2,547.83	516,572.87

TABLE 45. SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS (*Continued*)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
812	659,344	535,387,328	28.4956	9.3294	.001231527	2,550.97	517,847.57
813	660,969	537,367,797	28.5132	9.3332	.001230012	2,554.11	519,123.84
814	662,596	539,353,144	28.5307	9.3370	.001228501	2,557.26	520,401.68
815	664,225	541,343,375	28.5482	9.3408	.001226994	2,560.40	521,681.10
816	665,856	543,338,496	28.5657	9.3447	.001225490	2,563.54	522,962.08
817	667,489	545,338,513	28.5832	9.3485	.001223990	2,566.68	524,244.63
818	669,124	547,343,432	28.6007	9.3523	.001222494	2,569.82	525,528.76
819	670,761	549,353,259	28.6182	9.3561	.001221001	2,572.96	526,814.46
820	672,400	551,368,000	28.6356	9.3599	.001219512	2,576.11	528,101.73
821	674,041	553,387,661	28.6531	9.3637	.001218027	2,579.25	529,390.56
822	675,584	555,412,248	28.6705	9.3675	.001216545	2,582.39	530,680.97
823	677,329	557,441,767	28.6880	9.3713	.001215067	2,585.53	531,972.95
824	678,976	559,476,224	28.7054	9.3751	.001213592	2,588.67	533,266.50
825	680,625	561,515,625	28.7228	9.3789	.001212121	2,591.81	534,561.62
826	682,276	563,559,976	28.7402	9.3827	.001210654	2,594.96	535,858.32
827	683,929	565,609,283	28.7576	9.3865	.001209190	2,598.10	537,156.58
828	685,584	567,663,552	28.7750	9.3902	.001207729	2,601.24	538,456.41
829	687,241	569,722,789	28.7924	9.3940	.001206273	2,604.38	539,757.82
830	688,900	571,787,000	28.8097	9.3978	.001204819	2,607.52	541,060.79
831	690,561	573,856,191	28.8271	9.4016	.001203369	2,610.66	542,365.34
832	692,224	575,930,368	28.8444	9.4053	.001201923	2,613.81	543,671.46
833	693,889	578,009,537	28.8617	9.4091	.001200480	2,616.95	544,979.15
834	695,556	580,093,704	28.8791	9.4129	.001199041	2,620.09	546,288.40
835	697,225	582,182,875	28.8964	9.4166	.001197605	2,623.23	547,599.23
836	698,896	584,277,056	28.9137	9.4204	.001196172	2,626.37	548,911.63
837	700,569	586,376,253	28.9310	9.4241	.001194743	2,629.51	550,225.61
838	702,244	588,480,472	28.9482	9.4279	.001193317	2,632.65	551,541.15
839	703,921	590,589,719	28.9655	9.4316	.001191895	2,635.80	552,858.26
840	705,600	592,704,000	28.9828	9.4354	.001190476	2,638.94	554,176.94
841	707,281	594,823,321	29.0000	9.4391	.001189061	2,642.08	555,497.20
842	708,964	596,947,688	29.0172	9.4429	.001187648	2,645.22	556,819.02
843	710,649	599,077,107	29.0345	9.4466	.001186240	2,648.36	558,142.42
844	712,336	601,211,584	29.0517	9.4503	.001184834	2,651.50	559,467.39
845	714,025	603,351,125	29.0689	9.4541	.001183432	2,654.65	560,793.92
846	715,716	605,495,736	29.0861	9.4578	.001182033	2,657.79	562,122.03
847	717,409	607,645,423	29.1033	9.4615	.001180638	2,660.93	563,451.71
848	719,104	609,800,192	29.1204	9.4652	.001179245	2,664.07	564,782.96
849	720,801	611,960,049	29.1376	9.4690	.001177856	2,667.21	566,115.78
850	722,500	614,125,000	29.1548	9.4727	.001176471	2,670.35	567,450.17
851	724,201	616,295,051	29.1719	9.4764	.001175088	2,673.50	568,786.14
852	725,904	618,470,208	29.1890	9.4801	.001173709	2,676.64	570,123.67
853	727,609	620,650,477	29.2062	9.4838	.001172333	2,679.78	571,462.77
854	729,316	622,835,864	29.2233	9.4875	.001170960	2,682.92	572,803.45
855	731,025	625,026,375	29.2404	9.4912	.001169591	2,686.06	574,145.69
856	732,736	627,222,016	29.2575	9.4949	.001168224	2,689.20	575,489.51
857	734,449	629,422,793	29.2746	9.4986	.001166861	2,692.34	576,834.90
858	736,164	631,628,712	29.2916	9.5023	.001165501	2,695.49	578,181.85
859	737,881	633,839,779	29.3087	9.5060	.001164144	2,698.63	579,530.38
860	739,600	636,056,000	29.3258	9.5097	.001162791	2,701.77	580,880.48
861	741,321	638,277,381	29.3428	9.5135	.001161440	2,704.91	582,232.15
862	743,044	640,503,928	29.3598	9.5171	.001160093	2,708.05	583,585.39
863	744,769	642,735,647	29.3769	9.5207	.001158749	2,711.19	584,940.20
864	746,496	644,972,544	29.3939	9.5244	.001157407	2,714.34	586,296.59
865	748,225	647,214,625	29.4109	9.5281	.001156069	2,717.48	587,654.54
866	749,956	649,461,896	29.4279	9.5317	.001154734	2,720.62	589,014.07
867	751,689	651,714,363	29.4449	9.5354	.001153403	2,723.76	590,375.16
868	753,424	653,972,032	29.4618	9.5391	.001152074	2,726.90	591,737.83
869	755,161	656,234,909	29.4788	9.5427	.001150748	2,730.04	593,102.06
870	756,900	658,503,000	29.4958	9.5464	.001149425	2,733.19	594,467.87
871	758,641	660,776,311	29.5127	9.5501	.001148106	2,736.33	595,835.25
872	760,384	663,054,848	29.5296	9.5537	.001146789	2,739.47	597,204.20
873	762,129	665,338,617	29.5466	9.5574	.001145475	2,742.61	598,574.72
874	763,876	667,627,624	29.5635	9.5610	.001144165	2,745.75	599,946.81

TABLE 45. SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS (*Continued*)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
875	765,625	669,921,875	29.5804	9.5647	.001142857	2,748.89	601,320.47
876	767,376	672,221,376	29.5973	9.5683	.001141553	2,752.04	602,695.70
877	769,129	674,526,133	29.6142	9.5719	.001140251	2,755.18	604,072.50
878	770,884	676,836,152	29.6311	9.5756	.001138952	2,758.32	605,450.88
879	772,641	679,151.439	29.6479	9.5792	.001137656	2,761.46	606,830.82
880	774,400	681,472,000	29.6648	9.5828	.001136364	2,764.60	608,212.34
881	776,161	683,797.841	29.6816	9.5865	.001135074	2,767.74	609,595.42
882	777,924	686,128,968	29.6985	9.5901	.001133787	2,770.88	610,980.08
883	779,689	688,465,387	29.7153	9.5937	.001132503	2,774.03	612,366.31
884	781,456	690,807,104	29.7321	9.5973	.001131222	2,777.17	613,754.11
885	783,225	693,154,125	29.7489	9.6010	.001129944	2,780.31	615,143.48
886	784,996	695,506,456	29.7658	9.6046	.001128668	2,783.45	616,534.42
887	786,769	697,864,103	29.7825	9.6082	.001127396	2,786.59	617,926.93
888	788,544	700,227,072	29.7993	9.6118	.001126126	2,789.73	619,321.01
889	790,321	702,595,369	29.8161	9.6154	.001124859	2,792.88	620,716.66
890	792,100	704,969,000	29.8329	9.6190	.001123596	2,796.02	622,113.89
891	793,881	707,347,971	29.8496	9.6226	.001122334	2,799.16	623,512.68
892	795,664	707,932,288	29.8664	9.6262	.001121076	2,802.30	624,913.04
893	797,449	712,121,957	29.8831	9.6298	.001119821	2,805.44	626,314.98
894	799,236	714,516,984	29.8998	9.6334	.001118568	2,808.58	627,718.49
895	801,025	716,917,375	29.9166	9.6370	.001117818	2,811.73	629,123.56
896	802,816	719,323,136	29.9333	9.6406	.001116071	2,814.87	630,530.21
897	804,609	721,734,273	29.9500	9.6442	.001114827	2,818.01	631,938.43
898	806,404	724,150,792	29.9666	9.6477	.001113586	2,821.15	633,348.22
899	808,201	726,572,699	29.9833	9.6513	.001112347	2,824.29	634,759.58
900	810,000	729,000,000	30.0000	9.6549	.001111111	2,827.43	636,172.51
901	811,801	731,432,701	30.0167	9.6585	.001109878	2,830.58	637,587.01
902	813,604	733,870,808	30.0333	9.6620	.001108647	2,833.72	639,003.09
903	815,409	736,314,327	30.0500	9.6656	.001107420	2,836.86	640,420.73
904	817,216	738,763,264	30.0666	9.6692	.001106195	2,840.00	641,839.95
905	819,025	741,217,625	30.0832	9.6727	.001104972	2,843.14	643,260.73
906	820,836	743,677,416	30.0998	9.6763	.001103753	2,846.28	644,683.09
907	822,649	746,142,643	30.1164	9.6799	.001102536	2,849.42	646,107.01
908	824,464	748,613,312	30.1330	9.6834	.001101322	2,852.57	647,532.51
909	826,281	751,089,429	30.1496	9.6870	.001100110	2,855.71	648,959.58
910	828,100	753,571,000	30.1662	9.6905	.001098901	2,858.85	650,388.22
911	829,921	756,058,031	30.1828	9.6941	.001097695	2,861.99	651,818.43
912	831,744	758,550,825	30.1993	9.6976	.001096491	2,865.13	653,250.21
913	833,569	761,048,497	30.2159	9.7012	.001095290	2,868.27	654,683.56
914	835,396	763,551,944	30.2324	9.7047	.001094092	2,871.42	656,118.48
915	837,225	766,060,875	30.2490	9.7082	.001092896	2,874.56	657,554.98
916	839,056	768,575,296	30.2655	9.7118	.001091703	2,877.70	658,993.04
917	840,889	771,095,213	30.2820	9.7153	.001090513	2,880.84	660,432.68
918	842,724	773,620,632	30.2985	9.7188	.001089325	2,883.98	661,873.88
919	844,561	776,151,559	30.3150	9.7224	.001088139	2.887.12	663,316.66
920	846,400	778,688,000	30.3315	9.7259	.001086957	2,890.27	664,761.01
921	848,241	781,229,961	30.3480	9.7294	.001085776	2,893.41	666,206.92
922	850,084	783,777,448	30.3645	9.7329	.001084599	2,896.55	667,654.41
923	851,929	786,330,467	30.3809	9.7364	.001083423	2,899.69	669,103.47
924	853,776	788,889,024	30.3974	9.7400	.001082251	2,902.83	670,554.10
925	855,625	791,453,125	30.4138	9.7435	.001081081	2,905.97	672,006.30
926	857,476	794,022,776	30.4302	9.7470	.001079914	2,909.11	673,460.08
927	859,329	796,597,983	30.4467	9.7505	.001078749	2,912.26	674,915.42
928	861,184	799,178,752	30.4631	9.7540	.001077586	2,915.40	676.372.33
929	863,041	801,765,089	30.4795	9.7575	.001076426	2,918.54	677,830.82
930	864,900	804,357,000	30.4959	9.7610	.001075269	2,921.68	679,290.87
931	866,761	806,954,491	30.5123	9.7645	.001074114	2,924.82	680,752.50
932	868,624	809,557,568	30.5287	9.7680	.001072961	2,927.96	682,215.69
933	870,489	812,166,237	30.5450	9.7715	.001071811	2,931.11	683,680.46
934	872,356	814,780,504	30.5614	9.7750	.001070664	2,934.25	685,146.80
935	874,225	817,400,375	30.5778	9.7785	.001069519	2,937.39	686,614.71
936	876,096	820,025,856	30.5941	9.7829	.001068376	2,940.53	688,084.19
937	877,969	822,656,953	30.6105	9.7854	.001067236	2,943.67	689,555.24

TABLE 45. SQUARES, CUBES, SQUARE AND CUBE ROOTS, CIRCUMFERENCES, AND AREAS (*Continued*)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
938	879,844	825,293,672	30.6268	9.7889	.001066098	2,946.81	691,027.86
939	881,721	827,936,019	30.6431	9.7924	.001064963	2,949.96	692,502.05
940	883,600	830,584,000	30.6594	9.7959	.001063830	2,953.10	693,977.82
941	885,481	833,237,621	30.6757	9.7993	.001062699	2,956.24	695,455.15
942	887,364	835,896,888	30.6920	9.8028	.001061571	2,959.38	696,934.06
943	889,249	838,561,807	30.7083	9.8063	.001060445	2,962.52	698,414.53
944	891,136	841,232,384	30.7246	9.8097	.001059322	2,965.66	699,896.58
945	893,025	843,908,625	30.7409	9.8132	.001058201	2,968.81	701,380.19
946	894,916	846,590,536	30.7571	9.8167	.001057082	2,971.95	702,865.38
947	896,808	849,278,123	30.7734	9.8201	.001055966	2,975.09	704,352.14
948	898,704	851,971,392	30.7896	9.8236	.001054852	2,978.23	705,840.47
949	900,601	854,670,349	30.8058	9.8270	.001053741	2,981.37	707,330.37
950	902,500	857,375,000	30.8221	9.8305	.001052632	2,984.51	708,821.84
951	904,401	860,085,351	30.8383	9.8339	.001051525	2,987.65	710,314.88
952	906,304	862,801,408	30.8545	9.8374	.001050420	2,990.80	711,809.50
953	908,209	865,523,177	30.8707	9.8408	.001049318	2,993.94	713,305.68
954	910,116	868,250,664	30.8869	9.8443	.001048218	2,997.08	714,803.43
955	912,025	870,983,875	30.9031	9.8477	.001047120	3,000.22	716,302.76
956	913,936	873,722,816	30.9192	9.8511	.001046025	3,003.36	717,803.66
957	915,849	876,467,493	30.9354	9.8546	.001044932	3,006.50	719,306.12
958	917,764	879,217,912	30.9516	9.8580	.001043841	3,009.65	720,810.16
959	919,681	881,974,079	30.9677	9.8614	.001042753	3,012.79	722,315.77
960	921,600	884,736,000	30.9839	9.8648	.001041667	3,015.93	723,822.95
961	923,521	887,503,681	31.0000	9.8683	.001040583	3,019.07	725,331.70
962	925,444	890,277,128	31.0161	9.8717	.001039501	3,022.21	726,842.02
963	927,369	893,056,347	31.0322	9.8751	.001038422	3,025.35	728,353.91
964	929,296	895,841,344	31.0483	9.8785	.001037344	3,028.50	729,867.37
965	931,225	898,632,125	31.0644	9.8819	.001036269	3,031.64	731,382.40
966	933,156	901,428,696	31.0805	9.8854	.001035197	3,034.78	732,899.01
967	935,089	904,231,063	31.0966	9.8888	.001034126	3,037.92	734,417.18
968	937,024	907,039,232	31.1127	9.8922	.001033058	3,041.06	735,936.93
969	938,961	909,853,209	31.1288	9.8956	.001031992	3,044.20	737,458.24
970	940,900	912,673,000	31.1448	9.8990	.001030928	3,047.34	738,981.13
971	942,841	915,498,611	31.1609	9.9024	.001029866	3,050.49	740,505.59
972	944,784	918,330,048	31.1769	9.9058	.001028807	3,053.63	742,031.62
973	946,729	921,167,317	31.1929	9.9092	.001027749	3,056.77	743,559.22
974	948,676	924,010,424	31.2090	9.9126	.001026694	3,059.91	745,088.39
975	950,625	926,859,375	31.2250	9.9160	.001025641	3,063.05	746,619.13
976	952,576	929,714,176	31.2410	9.9194	.001024590	3,066.19	748,151.44
977	954,529	932,574,833	31.2570	9.9228	.001023541	3,069.34	749,685.32
978	956,484	935,441,352	31.2730	9.9261	.001022495	3,072.48	751,220.78
979	958,441	938,313,739	31.2890	9.9295	.001021450	3,075.62	752,757.80
980	960,400	941,192,000	31.3050	9.9329	.001020408	3,078.76	754,296.40
981	962,361	944,076,141	31.3209	9.9363	.001019168	3,081.90	755,836.56
982	964,324	946,966,168	31.3369	9.9396	.001018330	3,085.04	757,378.30
983	966,289	949,862,087	31.3528	9.9430	.001017294	3,088.19	758,921.61
984	968,256	952,763,904	31.3688	9.9464	.001016260	3,091.33	760,466.48
985	970,225	955,671,625	31.3847	9.9497	.001015228	3,094.47	762,012.93
986	972,196	958,585,256	31.4006	9.9531	.001014199	3,097.61	763,560.95
987	974,169	961,504,803	31.4166	9.9565	.001013171	3,100.75	765,110.54
988	976,144	964,430,272	31.4325	9.9598	.001012146	3,103.89	766,661.70
989	978,121	967,361,669	31.4484	9.9632	.001011122	3,107.04	768,214.44
990	980,100	970,299,000	31.4643	9.9666	.001010101	3,110.18	769,768.74
991	982,081	973,242,271	31.4802	9.9699	.001009082	3,113.32	771,324.61
992	984,064	976,191,488	31.4960	9.9733	.001008065	3,116.46	772,882.06
993	986,049	979,146,657	31.5119	9.9766	.001007049	3,119.60	774,441.07
994	988,036	982,107,784	31.5278	9.9800	.001006036	3,122.74	776,001.66
995	990,025	985,074,875	31.5436	9.9833	.001005025	3,125.88	777,563.82
996	992,016	988,047,936	31.5595	9.9866	.001004016	3,129.03	779,127.54
997	994,009	991,026,973	31.5753	9.9900	.001003009	3,132.17	780,692.84
998	996,004	994,011,992	31.5911	9.9933	.001002004	3,135.31	782,259.71
999	998,001	997,002,999	31.6070	9.9967	.001001001	3,138.45	783,828.15
1000	1,000,000	1,000,000,000	31.6228	10.0000	.001000000	3,141.59	785,398.16

TABLE 46. DEFLECTIONS FROM THE PI TO THE 10-POINT CIRCULAR CURVE

Vernier reads 0°00′ on the forward tangent

Pts.	Δ = 3°	Diff. per min.	Δ = 4°	Diff. per min.	Δ = 5°	Diff. per min.	Δ = 6°	Diff. per min.
1	0°02.2'	+0.013'	0°03.0'	+0.012'	0°03.7'	+0.012'	0°04.5'	+0.012'
2	0°12.0'	+0.067	0°16.0'	+0.066	0°20.0'	+0.066	0°24.0'	+0.066
3	0°40.5'	+0.224	0°53.9'	+0.224	1°07.4'	+0.223	1°20.8'	+0.223
4	2°23.7'	+0.794	3°11.3'	+0.789	3°58.6'	+0.784	4°45.7'	+0.777
5	88°30.0'	-0.500	88°00.0'	-0.500	87°30.0'	-0.500	87°00.0'	-0.500
6	174°36.3'	-1.794	172°48.7'	-1.789	171°01.4'	-1.784	169°14.3'	-1.777
7	176°19.5'	-1.224	175°06.1'	-1.224	173°52.6'	-1.223	172°39.2'	-1.223
8	176°48.0'	-1.067	175°44.0'	-1.066	174°40.0'	-1.066	173°36.0'	-1.066
9	176°57.8'	-1.013	175°57.0'	-1.012	174°56.3'	-1.012	173°55.5'	-1.012
10	177°00.0'	-1.000	176°00.0'	-1.000	175°00.0'	-1.000	174°00.0'	-1.000
	Δ = 7°		Δ = 8°		Δ = 9°		Δ = 10°	
1	0°05.2'	+0.012'	0°06.8'	+0.012'	0°06.7'	+0.012'	0°07.5'	+0.012'
2	0°27.9'	+0.066	0°31.9'	+0.066	0°35.9'	+0.066	0°39.8'	+0.066
3	1°34.1'	+0.222	1°47.5'	+0.221	2°00.7'	+0.220	2°14.0'	+0.219
4	5°32.3'	+0.770	6°18.5'	+0.762	7°04.2'	+0.752	7°49.3'	+0.742
5	86°30.0'	-0.500	86°00.0'	-0.500	85°30.0'	-0.500	85°00.0'	-0.500
6	167°27.7'	-1.770	165°41.5'	-1.762	163°55.8'	-1.752	162°10.7'	-1.742
7	171°25.9'	-1.222	170°12.5'	-1.221	168°59.3'	-1.220	167°46.0'	-1.219
8	172°32.1'	-1.066	171°28.1'	-1.066	170°24.1'	-1.066	169°20.2'	-1.066
9	172°54.8'	-1.012	171°54.0'	-1.012	170°53.3'	-1.012	169°52.5'	-1.012
10	173°00.0'	-1.000	172°00.0'	-1.000	171°00.0'	-1.000	170°00.0'	-1.000
	Δ = 11°		Δ = 12°		Δ = 13°		Δ = 14°	
1	0°08.2'	+0.012'	0°09.0'	+0.012'	0°09.7'	+0.012'	0°10.4'	+0.012'
2	0°43.8'	+0.065	0°47.7'	+0.065	0°51.6'	+0.065	0°55.5'	+0.065
3	2°27.1'	+0.218	2°40.2'	+0.217	2°53.2'	+0.216	3°06.1'	+0.214
4	8°33.9'	+0.732	9°17.7'	+0.720	10°01.0'	+0.708	10°43.4'	+0.695
5	84°30.0'	-0.500	84°00.0'	-0.500	83°30.0'	-0.500	83°00.0'	-0.500
6	160°26.1'	-1.732	158°42.3'	-1.720	156°59.0'	-1.708	155°16.6'	-1.695
7	166°32.9'	-1.218	165°19.8'	-1.217	164°06.8'	-1.216	162°53.9'	-1.214
8	168°16.2	-1.065	167°12.3'	-1.065	166°08.4'	-1.065	165°04.5'	-1.065
9	168°51.8'	-1.012	167°51.0'	-1.012	166°50.3'	-1.012	165°49.6'	-1.012
10	169°00.0'	-1.000	168°00.0'	-1.000	167°00.0'	-1.000	166°00.0'	-1.000
	Δ = 15°		Δ = 16°		Δ = 17°		Δ = 18°	
1	0°11.2'	+0.012'	0°11.9'	+0.012'	0°12.6'	+0.012'	0°13.4'	+0.012'
2	0°59.4'	+0.064	1°03.3'	+0.064	1°07.1'	+0.064	1°10.0'	+0.064
3	3°19.0'	+0.213	3°31.8'	+0.211	3°44.4'	+0.210	3°57.0'	+0.208
4	11°25.1'	+0.681	12°06.0'	+0.667	12°46.0'	+0.652	13°25.1'	+0.637
5	82°30.0'	-0.500	82°00.0'	-0.500	81°30.0'	-0.500	81°00.0'	-0.500
6	153°34.9'	-1.681	151°54.0'	-1.667	150°14.0'	-1.652	148°34.9'	-1.637
7	161°41.0'	-1.213	160°28.2'	-1.211	159°15.6'	-1.210	158°03.0'	-1.207
8	164°00.6'	-1.064	162°56.7'	-1.064	161°52.9'	-1.064	160°50.0'	-1.064
9	164°48.8'	-1.012	163°48.1'	-1.012	162°47.4'	-1.012	161°46.6'	-1.012
10	165°00.0'	-1.000	164°00.0'	-1.000	163°00.0'	-1.000	162°00.0'	-1.000

TABLE 46. DEFLECTIONS FROM THE PI TO THE 10-POINT CIRCULAR CURVE (*Continued*)

Vernier reads 0°00′ on the forward tangent

Pts.	Δ = 19°	Diff. per min.	Δ = 20°	Diff. per min.	Δ = 21°	Diff. per min.	Δ = 22°	Diff. per min.
1	0°14.1'	+0.012'	0°14.8'	+0.012'	0°15.5'	+0.012'	0°16.2'	+0.012'
2	1°14.8'	+0.063	1°18.6'	+0.063	1°22.3'	+0.063	1°26.1'	+0.062
3	4°09.4'	+0.206	4°21.8'	+0.204	4°34.0'	+0.202	4°46.1'	+0.200
4	14°03.4'	+0.622	14°40.7'	+0.606	15°17.1'	+0.590	15°52.5'	+0.574
5	80°30.0'	-0.500	80°00.0'	-0.500	79°30.0'	-0.500	79°00.0'	-0.500
6	146°56.6'	-1.622	145°29.3'	-1.606	143°42.9'	-1.590	142°07.5'	-1.574
7	156°50.6'	-1.206	155°38.2'	-1.204	154°26.0'	-1.202	153°13.9'	-1.200
8	159°45.2'	-1.063	158°41.4'	-1.063	157°37.7'	-1.063	156°33.9'	-1.062
9	160°45.9'	-1.012	159°45.2'	-1.012	158°44.5'	-1.012	157°43.8'	-1.012
10	161°00.0'	-1.000	160°00.0'	-1.000	159°00.0'	-1.000	158°00.0'	-1.000
	Δ = 23°		Δ = 24°		Δ = 25°		Δ = 26°	
1	0°17.1'	+0.012'	0°17.7'	+0.012'	0°18.4'	+0.012'	0°19.1'	+0.012'
2	1°29.8'	+0.062	1°33.5'	+0.061	1°37.2'	+0.061	1°40.9'	+0.060
3	4°58.1'	+0.197	5°09.9'	+0.195	5°21.7'	+0.193	5°33.2'	+0.190
4	16°26.9'	+0.558	17°00.4'	+0.541	17°32.8'	+0.524	18°04.3'	+0.507
5	78°30.0'	-0.500	78°00.0'	-0.500	77°30.0'	-0.500	77°00.0'	-0.500
6	140°33.1'	-1.558	138°59.6'	-1.541	137°27.2'	-1.524	135°55.7'	-1.507
7	152°01.9'	-1.197	150°50.1'	-1.195	149°38.3'	-1.193	148°26.8'	-1.190
8	155°30.2'	-1.062	154°26.5'	-1.061	153°22.8'	-1.061	152°19.1'	-1.060
9	156°43.0'	-1.012	155°42.3'	-1.012	154°41.6'	-1.012	153°40.9'	-1.012
10	157°00.0'	-1.000	156°00.0'	-1.000	155°00.0'	-1.000	154°00.0'	-1.000
	Δ = 27°		Δ = 28°		Δ = 29°		Δ = 30°	
1	0°19.8'	+0.012'	0°20.5'	+0.011'	0°21.2'	+0.011'	0°21.9'	+0.011'
2	1°44.5'	+0.060	1°48.1'	+0.059	1°51.7'	+0.059	1°55.2'	+0.058
3	5°44.6'	+0.188	5°55.9'	+0.185	6°07.0'	+0.183	6°18.0'	+0.180
4	18°34.7'	+0.491	19°04.2'	+0.474	19°32.6'	+0.457	20°00.0'	+0.441
5	76°30.0'	-0.500	76°00.0'	-0.500	75°30.0'	-0.500	75°00.0'	-0.500
6	134°25.3'	-1.491	132°55.8'	-1.474	131°27.4'	-1.457	130°00.0'	-1.441
7	147°15.4'	-1.188	146°04.1'	-1.185	144°53.0'	-1.183	143°42.0'	-1.180
8	151°15.5'	-1.060	150°11.9'	-1.059	149°08.3'	-1.059	148°04.8'	-1.058
9	152°40.2'	-1.012	151°39.5'	-1.011	150°38.8'	-1.011	149°38.1'	-1.011
10	153°00.0'	-1.000	152°00.0'	-1.000	151°00.0'	-1.000	150°00.0'	-1.000
	Δ = 31°		Δ = 32°		Δ = 33°		Δ = 34°	
1	0°22.5'	+0.011'	0°23.2'	+0.011'	0°23.9'	+0.011'	0°24.6'	+0.011'
2	1°58.7'	+0.058	2°02.2'	+0.057	2°05.6'	+0.057	2°09.0'	+0.056
3	6°28.8'	+0.177	6°39.5'	+0.175	6°49.9'	+0.172	7°00.2'	+0.169
4	20°26.5'	+0.424	20°51.9'	+0.408	21°16.4'	+0.391	21°39.8'	+0.375
5	74°30.0'	-0.500	74°00.0'	-0.500	73°30.0'	-0.500	73°00.0'	-0.500
6	128°33.5'	-1.424	127°08.1'	-1.408	125°43.6'	-1.391	124°20.2'	-1.375
7	142°31.2'	-1.177	141°20.5'	-1.175	140°10.1'	-1.172	138°59.8'	-1.169
8	147°01.3'	-1.058	145°57.8'	-1.057	144°54.4'	-1.057	143°51.0'	-1.056
9	148°37.5'	-1.011	147°36.8'	-1.011	146°36.1'	-1.011	145°35.4'	-1.011
10	149°00.0'	-1.000	148°00.0'	-1.000	147°00.0'	-1.000	146°00.0'	-1.000

TABLE 46. DEFLECTIONS FROM THE PI TO THE 10-POINT CIRCULAR CURVE (*Continued*)

Vernier reads 0°00′ on the forward tangent

Pts.	Δ = 35°	Diff. per min.	Δ = 36°	Diff. per min.	Δ = 37°	Diff. per min.	Δ = 38°	Diff. per min.
1	0°25.2'	+0.011'	0°25.8'	+0.011'	0°26.5'	+0.011'	0°27.2'	+0.011'
2	2°12.4'	+0.056	2°15.7'	+0.055	2°19.1'	+0.054	2°22.3'	+0.054
3	7°10.4'	+0.166	7°20.3'	+0.163	7°30.1'	+0.160	7°39.7'	+0.157
4	22°02.3'	+0.359	22°23.9'	+0.343	22°44.5'	+0.328	23°04.2'	+0.313
5	72°30.0'	-0.500	72°00.0'	-0.500	71°30.0'	-0.500	71°00.0'	-0.500
6	122°57.7'	-1.359	121°36.1'	-1.343	120°15.5'	-1.328	118°55.8'	-1.313
7	137°49.6'	-1.166	136°39.7'	-1.163	135°29.9'	-1.160	134°20.3'	-1.157
8	142°47.6'	-1.056	141°44.3'	-1.055	140°40.9'	-1.054	139°37.7'	-1.054
9	144°34.8'	-1.011	143°34.1'	-1.011	142°33.5'	-1.011	141°32.8'	-1.011
10	145°00.0'	-1.000	144°00.0'	-1.000	143°00.0'	-1.000	142°00.0'	-1.000
	Δ = 39°		Δ = 40°		Δ = 41°		Δ = 42°	
1	0°27.8'	+0.011'	0°28.5'	+0.011'	0°29.1'	+0.010'	0°29.7'	+0.010'
2	2°25.5'	+0.053	2°28.7'	+0.052	2°31.9'	+0.052	2°35.0'	+0.051
3	7°49.1'	+0.153	7°58.3'	+0.150	8°07.3'	+0.147	8°16.1'	+0.144
4	23°22.9'	+0.298	23°40.8'	+0.283	23°57.8'	+0.268	24°13.8'	+0.254
5	70°30.0'	-0.500	70°00.0'	-0.500	69°30.0'	-0.500	69°00.0'	-0.500
6	117°37.1'	-1.298	116°19.2'	-1.283	115°02.2'	-1.268	113°46.2'	-1.254
7	133°10.9'	-1.153	132°01.7'	-1.150	130°52.7'	-1.147	129°43.9'	-1.144
8	138°34.5'	-1.053	137°31.3'	-1.052	136°28.1'	-1.052	135°25.0'	-1.051
9	140°32.2'	-1.011	139°31.5'	-1.011	138°30.9'	-1.010	137°30.3'	-1.010
10	141°00.0'	-1.000	140°00.0'	-1.000	139°00.0'	-1.000	138°00.0'	-1.000
	Δ = 43°		Δ = 44°		Δ = 45°		Δ = 46°	
1	0°30.3'	+0.010'	0°31.0'	+0.010'	0°31.6'	+0.010'	0°32.2'	+0.010'
2	2°38.1'	+0.050	2°41.1'	+0.050	2°44.1'	+0.049	2°47.0'	+0.048
3	8°24.7'	+0.141	8°33.2'	+0.137	8°41.4'	+0.134	8°49.5'	+0.131
4	24°29.1'	+0.240	24°43.5'	+0.226	24°57.0'	+0.213	25°09.8'	+0.199
5	68°30.0'	-0.500	68°00.0'	-0.500	67°30.0'	-0.500	67°00.0'	-0.500
6	112°30.9'	-1.240	111°16.5'	-1.226	110°03.0'	-1.213	108°50.2'	-1.199
7	128°35.3'	-1.141	127°26.8'	-1.137	126°18.6'	-1.134	125°10.5'	-1.131
8	134°21.9'	-1.050	133°18.9'	-1.050	132°15.9'	-1.049	131°13.0'	-1.048
9	136°29.7'	-1.010	135°29.0'	-1.010	134°28.4'	-1.010	133°27.8'	-1.010
10	137°00.0'	-1.000	136°00.0'	-1.000	135°00.0'	-1.000	134°00.0'	-1.000
	Δ = 47°		Δ = 48°		Δ = 49°		Δ = 50°	
1	0°32.8'	+0.010'	0°33.3'	+0.010'	0°33.9'	+0.010'	0°34.5'	+0.009'
2	2°49.9'	+0.047	2°52.7'	+0.047	2°55.5'	+0.046	2°58.3'	+0.045
3	8°57.3'	+0.127	9°04.9'	+0.124	9°12.3'	+0.120	9°19.6'	+0.117
4	25°21.7'	+0.186	25°32.9'	+0.174	25°43.3'	+0.161	25°53.0'	+0.149
5	66°30.0'	-0.500	66°00.0'	-0.500	65°30.0'	-0.500	65°00.0'	-0.500
6	107°38.3'	-1.186	106°27.1'	-1.174	105°16.7'	-1.161	104°07.0'	-1.149
7	124°02.7'	-1.127	122°55.1'	-1.124	121°47.7'	-1.120	120°40.4'	-1.117
8	130°10.1'	-1.147	129°07.3'	-1.047	128°04.5'	-1.046	127°01.7'	-1.045
9	132°27.2'	-1.010	131°26.7'	-1.010	130°26.1'	-1.010	129°25.5'	-1.009
10	133°00.0'	-1.000	132°00.0'	-1.000	131°00.0'	-1.000	130°00.0'	-1.000

TABLE 46. DEFLECTIONS FROM THE PI TO THE 10-POINT CIRCULAR CURVE (*Continued*)

Vernier reads 0°00′ on the forward tangent

Pts.	Δ = 51°	Diff. per min.	Δ = 52°	Diff. per min.	Δ = 53°	Diff. per min.	Δ = 54°	Diff. per min.
1	0°35.1'	+0.009'	0°35.6'	+ 0.009'	0°36.2'	+ 0.009'	0°36.7'	+0.009'
2	3°01.0'	+0.044	3°03.7'	+ 0.043	3°06.3'	+ 0.043	3°08.8'	+0.042
3	9°26.6'	+0.113	9°33.4'	+ 0.110	9°40.0'	+ 0.106	9°46.4'	+0.103
4	26°02.0'	+0.137	26°10.2'	+ 0.126	26°17.8'	+ 0.114	26°24.6'	+0.103
5	64°30.0'	-0.500	64°00.0'	- 0.500	63°30.0'	- 0.500	63°00.0'	-0.500
6	102°58.0'	-1.137	101°49.8'	- 1.126	100°42.2'	- 1.114	99°35.4'	-1.103
7	119°33.4'	-1.113	118°26.6'	- 1.110	117°20.0'	- 1.106	116°13.6'	-1.103
8	125°59.0'	-1.044	124°56.3'	- 1.043	123°53.7'	- 1.043	122°51.2'	-1.042
9	128°24.9'	-1.009	127°24.4'	- 1.009	126°23.8'	- 1.009	125°23.3'	-1.009
10	129°00.0'	-1.000	128°00.0'	- 1.000	127°00.0'	- 1.000	126°00.0'	-1.000
	Δ = 55°		Δ = 56°		Δ = 57°		Δ = 58°	
1	0°37.3'	+0.009'	0°37.8'	+0.009'	0°38.3'	+0.009'	0°38.8'	+0.008'
2	3°11.3'	+0.041	3°13.8'	+0.040	3°16.2'	+0.039	3°18.6'	+0.038
3	9°52.5'	+0.099	9°58.5'	+0.096	10°04.2'	+0.092	10°09.8'	+0.089
4	26°30.8'	+0.092	26°36.4'	+0.082	26°41.3'	+0.072	26°45.6'	+0.061
5	62°30.0'	-0.500	62°00.0'	-0.500	61°30.0'	-0.500	61°00.0'	-0.500
6	98°29.2'	-1.092	97°23.6'	-1.082	96°18.7'	-1.072	95°14.4'	-1.061
7	115°07.5'	-1.099	114°01.5'	-1.096	112°55.8'	-1.092	111°50.2'	-1.089
8	121°48.7'	-1.041	120°46.2'	-1.040	119°43.8'	-1.039	118°41.4'	-1.038
9	124°22.7'	-1.009	123°22.2'	-1.009	122°21.7'	-1.009	121°21.2'	-1.008
10	125°00.0'	-1.000	124°00.0'	-1.000	123°00.0'	-1.000	122°00.0'	-1.000
	Δ = 59°		Δ = 60°		Δ = 61°		Δ = 62°	
1	0°39.3'	+0.008'	0°39.8'	+0.008'	0°40.3'	+0.008'	0°40.8'	+0.008'
2	3°20.9'	+0.037	3°23.1'	+0.037	3°25.3'	+0.036	3°27.4'	+0.035
3	10°15.1'	+0.085	10°20.2'	+0.082	10°25.1'	+0.078	10°29.8'	+0.075
4	26°49.3'	+0.052	26°52.4'	+0.042	26°54.9'	+0.033	26°56.8'	+0.024
5	60°30.0'	-0.500	60°00.0'	-0.500	59°30.0'	-0.500	59°00.0'	-0.500
6	94°10.7'	-1.052	93°07.6'	-1.042	92°05.1'	-1.033	91°03.2'	-1.024
7	110°44.9'	-1.085	109°39.8'	-1.082	108°34.9'	-1.078	107°30.2'	-1.075
8	117°39.1'	-1.037	116°36.9'	-1.037	115°34.7'	-1.036	114°32.6'	-1.035
9	120°20.7'	-1.008	119°20.2'	-1.008	118°19.7'	-1.008	117°19.2'	-1.008
10	121°00.0'	-1.000	120°00.0'	-1.000	119°00.0'	-1.000	118°00.0'	-1.000
	Δ = 63°		Δ = 64°		Δ = 65°		Δ = 66°	
1	0°41.3'	+0.008'	0°41.7'	+0.008'	0°42.2'	+0.007'	0°42.6'	+0.007'
2	3°29.5'	+0.034	3°31.5'	+0.033	3°33.5'	+0.032	3°35.4'	+0.031
3	10°34.3'	+0.071	10°38.6'	+0.068	10°42.6'	+0.064	10°46.5'	+0.061
4	26°58.3'	+0.015	26°59.1'	+0.006	26°59.5'	-0.002	26°59.4'	-0.011
5	58°30.0'	-0.500	58°00.0'	-0.500	57°30.0'	-0.500	57°00.0'	-0.500
6	90°01.7'	-1.015	89°00.9'	-1.006	88°00.5'	-0.998	87°00.6'	-0.989
7	106°25.7'	-1.071	105°21.4'	-1.068	104°17.4'	-1.064	103°13.5'	-1.061
8	113°30.5'	-1.034	112°28.5'	-1.033	111°26.5'	-1.032	110°24.6'	-1.031
9	116°18.7'	-1.008	115°18.3'	-1.008	114°17.8'	-1.007	113°17.4'	-1.007
10	117°00.0'	-1.000	116°00.0'	-1.000	115°00.0'	-1.000	114°00.0'	-1.000

TABLE 46. DEFLECTIONS FROM THE PI TO THE 10-POINT CIRCULAR CURVE (*Continued*)

Vernier reads 0°00′ on the forward tangent

Pts.	Δ = 67°	Diff. per min.	Δ = 68°	Diff. per min.	Δ = 69°	Diff. per min.	Δ = 70°	Diff. per min.
1	0°43.1'	+0.007'	0°43.5'	+0.007'	0°43.9'	+0.007'	0°44.3'	+0.007'
2	3°37.3'	+0.030	3°39.1'	+0.029	3°40.8'	+0.028	3°42.5'	+0.027
3	10°50.1'	+0.057	10°53.5'	+0.054	10°56.7'	+0.050	10°59.7'	+0.047
4	26°58.7'	-0.019	26°57.6'	-0.027	26°56.0'	-0.034	26°53.9'	-0.042
5	56°30.0'	-0.500	56°00.0'	-0.500	55°30.0'	-0.500	55°00.0'	-0.500
6	86°01.3'	-0.981	85°02.4'	-0.973	84°04.0'	-0.966	83°06.1'	-0.958
7	102°09.9'	-1.057	101°06.5'	-1.054	100°03.3'	-1.050	99°00.3'	-1.047
8	109°22.7'	-1.030	108°20.9'	-1.029	107°19.2'	-1.028	106°17.5'	-1.027
9	112°16.9'	-1.007	111°16.5'	-1.007	110°16.1'	-1.007	109°15.7'	-1.007
10	113°00.0'	-1.000	112°00.0'	-1.000	111°00.0'	-1.000	110°00.0'	-1.000
	Δ = 71°		Δ = 72°		Δ = 73°		Δ = 74°	
1	0°44.7'	+0.006'	0°45.1'	+0.006'	0°45.5'	+0.006'	0°45.8'	+0.006'
2	3°44.1'	+0.026	3°45.7'	+0.025	3°47.2'	+0.024	3°48.6'	+0.023
3	11°02.5'	+0.043	11°05.1'	+0.040	11°07.5'	+0.036	11°09.7'	+0.033
4	26°51.4'	-0.049	26°48.5'	-0.056	26°45.1'	-0.063	26°41.3'	-0.070
5	54°30.0'	-0.500	54°00.0'	-0.500	53°30.0'	-0.500	53°00.0'	-0.500
6	82°08.6'	-0.951	81°11.5'	-0.943	80°14.9'	-0.937	79°18.7'	-0.930
7	97°57.5'	-1.043	96°54.9'	-1.040	95°52.5'	-1.036	94°50.3'	-1.033
8	105°15.9'	-1.026	104°14.3'	-1.025	103°12.8'	-1.024	102°11.4'	-1.023
9	108°15.3'	-1.006	107°14.9'	-1.006	106°14.5'	-1.006	105°14.2'	-1.006
10	109°00.0'	-1.000	108°00.0'	-1.000	107°00.0'	-1.000	106°00.0'	-1.000
	Δ = 75°		Δ = 76°		Δ = 77°		Δ = 78°	
1	0°46.2'	+0.006'	0°46.5'	+0.006'	0°46.9'	+0.005'	0°47.2'	+0.005'
2	3°50.0'	+0.022	3°51.3'	+0.021	3°52.6'	+0.020	3°53.8'	+0.019
3	11°11.6'	+0.029	11°13.4'	+0.026	11°15.0'	+0.023	11°16.3'	+0.019
4	26°37.1'	-0.077	26°32.5'	-0.083	26°27.5'	-0.089	26°22.2'	-0.095
5	52°30.0'	-0.500	52°00.0'	-0.500	51°30.0'	-0.500	51°00.0'	-0.500
6	78°22.9'	-0.923	77°27.5'	-0.917	76°32.5'	-0.911	75°37.8'	-0.905
7	93°48.4'	-1.029	92°46.6'	-1.026	91°45.0'	-1.023	90°43.7'	-1.019
8	101°10.0'	-1.022	100°08.7'	-1.021	99°07.4'	-1.020	98°06.2'	-1.019
9	104°13.8'	-1.006	103°13.5'	-1.006	102°13.1'	-1.005	101°12.8'	-1.005
10	105°00.0'	-1.000	104°00.0'	-1.000	103°00.0'	-1.000	102°00.0'	-1.000
	Δ = 79°		Δ = 80°		Δ = 81°		Δ = 82°	
1	0°47.5'	+0.005'	0°47.8'	+0.005'	0°48.1'	+0.005'	0°48.4'	+0.004'
2	3°54.9'	+0.018	3°56.0'	+0.017	3°57.0'	+0.016	3°57.9'	+0.015
3	11°17.5'	+0.016	11°18.5'	+0.013	11°19.2'	+0.009	11°19.8'	+0.006
4	26°16.4'	-0.101	26°10.4'	-0.107	26°03.9'	-0.113	25°57.1'	-0.119
5	50°30.0'	-0.500	50°00.0'	-0.500	49°30.0'	-0.500	49°00.0'	-0.500
6	74°43.6'	-0.899	73°49.6'	-0.893	72°56.1'	-0.887	72°02.1'	-0.881
7	89°42.5'	-1.016	88°41.5'	-1.013	87°40.8'	-1.009	86°40.2'	-1.006
8	97°05.1'	-1.018	96°04.0'	-1.017	95°03.0'	-1.016	94°02.1'	-1.015
9	100°12.5'	-1.005	99°12.2'	-1.005	98°11.9'	-1.005	97°11.6'	-1.004
10	101°00.0'	-1.000	100°00.0'	-1.000	99°00.0'	-1.000	98°00.0'	-1.000

TABLE 46. DEFLECTIONS FROM THE PI TO THE 10-POINT CIRCULAR CURVE (*Continued*)

Vernier reads 0°00′ on the forward tangent

Pts.	Δ = 83°	Diff. per min.	Δ = 84°	Diff. per min.	Δ = 85°	Diff. per min.	Δ = 86°	Diff. per min.
1	0°48.6'	+0.004'	0°48.9'	+0.004'	0°49.1'	+0.004'	0°49.4'	+0.004'
2	3°58.8'	+0.014	3°59.6'	+0.012	4°00.3'	+0.011	4°01.0'	+0.010
3	11°20.2'	+0.003	11°20.3'	-0.000	11°20.3'	-0.003	11°20.1'	-0.007
4	25°50.0'	-0.124	25°42.6'	-0.129	25°34.8'	-0.134	25°26.8'	-0.140
5	48°30.0'	-0.500	48°00.0'	-0.500	47°30.0'	-0.500	47°00.0'	-0.500
6	71°10.0'	-0.876	70°17.4'	-0.871	69°25.2'	-0.866	68°33.2'	-0.860
7	85°39.8'	-1.003	84°39.7'	-1.000	83°39.7'	-0.997	82°39.9'	-0.993
8	93°01.2'	-1.014	92°00.4'	-1.012	90°59.7'	-1.011	89°59.0'	-1.010
9	96°11.4'	-1.004	95°11.1'	-1.004	94°10.9'	-1.004	93°10.6'	-1.004
10	97°00.0'	-1.000	96°00.0'	-1.000	95°00.0'	-1.000	94°00.0'	-1.000
	Δ = 87°		Δ = 88°		Δ = 89°		Δ = 90°	
1	0°49.6'	+0.003'	0°49.8'	+0.003'	0°50.0'	+0.003'	0°50.2'	+0.003'
2	4°01.6'	+0.009	4°02.2'	+0.008	4°02.7'	+0.007	4°03.1'	+0.006
3	11°19.7'	-0.010	11°19.1'	-0.013	11°18.3'	-0.016	11°17.3'	-0.019
4	25°18.4'	-0.145	25°09.7'	-0.149	25°00.8'	-0.154	24°51.5'	-0.159
5	46°30.0'	-0.500	46°00.0'	-0.500	45°30.0'	-0.500	45°00.0'	-0.500
6	67°41.6'	-0.855	66°50.3'	-0.851	65°59.2'	-0.846	65°08.5'	-0.841
7	81°40.3'	-0.990	80°40.9'	-0.987	79°41.7'	-0.984	78°42.7'	-0.981
8	88°58.4'	-1.009	87°57.8'	-1.008	86°57.3'	-1.007	85°56.9'	-1.006
9	92°10.4'	-1.003	91°10.2'	-1.003	90°10.0'	-1.003	89°09.8'	-1.003
10	93°00.0'	-1.000	92°00.0'	-1.000	91°00.0'	-1.000	90°00.0'	-1.000
	Δ = 91°		Δ = 92°		Δ = 93°		Δ = 94°	
1	0°50.3'	+0.003'	0°50.5'	+0.002'	0°50.6'	+0.002'	0°50.8'	+0.002'
2	4°03.4'	+0.005	4°03.7'	+0.004	4°03.9'	+0.002	4°04.1'	+0.001
3	11°16.2'	-0.022	11°14.8'	-0.025	11°13.3'	-0.028	11°11.6'	-0.031
4	24°42.0'	-0.163	24°32.2'	-0.167	24°22.2'	-0.172	24°11.9'	-0.176
5	44°30.0'	-0.500	44°00.0'	-0.500	43°30.0'	-0.500	43°00.0'	-0.500
6	64°18.0'	-0.837	63°27.8'	-0.833	62°37.8'	-0.828	61°48.1'	-0.824
7	77°43.8'	-0.978	76°45.2'	-9.975	75°46.7'	-0.972	74°48.4'	-0.969
8	84°56.6'	-1.005	83°56.3'	-1.004	82°56.1'	-1.002	81°55.9'	-1.001
9	88°09.7'	-1.003	87°09.5'	-1.002	86°09.4'	-1.002	85°09.2'	-1.002
10	89°00.0'	-1.000	88°00.0'	-1.000	87°00.0'	-1.000	86°00.0'	-1.000
	Δ = 95°		Δ = 96°		Δ = 97°		Δ = 98°	
1	0°50.9'	+0.002'	0°51.0'	+0.002'	0°51.1'	0.001'	0°51.2'	+0.001'
2	4°04.2'	+0.000	4°04.2'	-0.001	4°04.1'	-0.002	4°04.0'	-0.003
3	11°09.7'	-0.034	11°07.7'	-0.037	11°05.4'	-0.040	11°03.0'	-0.043
4	24°01.3'	-0.180	23°50.5'	-0.184	23°39.5'	-0.188	23°28.2'	-0.192
5	42°30.0'	-0.500	42°00.0'	-0.500	41°30.0'	-0.500	41°00.0'	-0.500
6	60°58.7'	-0.820	60°09.5'	-0.816	59°20.5'	-0.812	58°31.8'	-0.808
7	73°50.3'	-0.966	72°52.3'	-0.963	71°54.6'	-0.960	70°57.0'	-0.957
8	80°55.8'	-1.000	79°55.8'	-0.999	78°55.9'	-0.998	77°56.0'	-0.997
9	84°09.1'	-1.002	83°09.0'	-1.002	82°08.9'	-1.001	81°08.8'	-1.001
10	85°00.0'	-1.000	84°00.0'	-1.000	83°00.0'	-1.000	82°00.0'	-1.000

TABLE 46. DEFLECTIONS FROM THE PI TO THE 10-POINT CIRCULAR CURVE (*Continued*)

Vernier reads 0°00′ on the forward tangent

Pts.	Δ = 99°	Diff. per min.	Δ = 100°	Diff. per min.	Δ = 101°	Diff. per min.	Δ = 102°	Diff. per min.
1	0°51.2'	+0.001'	0°51.3'	+0.001'	0°51.3'	+0.000'	0°51.4'	+0.000'
2	4°03.8'	-0.004	4°03.6'	-0.005	4°03.2'	-0.007	4°02.9'	-0.008
3	11°00.4'	-0.046	10°57.7'	-0.049	10°54.7'	-0.052	10°51.6'	-0.054
4	23°16.7'	-0.196	23°05.0'	-0.199	22°53.0'	-0.203	22°40.8'	-0.206
5	40°30.0'	-0.500	40°00.0'	-0.500	39°30.0'	-0.500	39°00.0'	-0.500
6	57°43.3'	-0.804	56°55.0'	-0.801	56°07.0'	-0.797	55°19.2'	-0.794
7	69°59.6'	-0.954	69°02.3'	-0.951	68°05.3'	-0.948	67°08.4'	-0.946
8	76°56.2'	-0.996	75°56.4'	-0.995	74°56.8'	-0.993	73°57.1'	-0.992
9	80°08.8'	-1.001	79°08.7'	-1.001	78°08.7'	-1.000	77°08.6'	-1.000
10	81°00.0'	-1.000	80°00.0'	-1.000	79°00.0'	-1.000	78°00.0'	-1.000
	Δ = 103°		Δ = 104°		Δ = 105°		Δ = 106°	
1	0°51.4'	-0.000'	0°51.4'	-0.000'	0°51.3'	-0.001'	0°51.3'	-0.001'
2	4°02.4'	-0.009	4°01.9'	-0.010	4°01.3'	-0.011	4°00.6'	-0.012
3	10°48.4'	-0.057	10°44.9'	-0.060	10°41.3'	-0.063	10°37.6'	-0.065
4	22°28.5'	-0.210	22°15.9'	-0.213	22°03.1'	-0.216	21°50.1'	-0.220
5	38°30.0'	-0.500	38°00.0'	-0.500	37°30.0'	-0.500	37°00.0'	-0.500
6	54°31.5'	-0.790	53°44.1'	-0.787	52°56.9'	-0.784	52°09.9'	-0.780
7	66°11.6'	-0.943	65°15.1'	-0.940	64°18.7'	-0.937	63°22.4'	-0.935
8	72°57.6'	-0.991	71°58.1'	-0.990	70°58.7'	-0.989	69°59.4'	-0.988
9	76°08.6'	-1.000	75°08.6'	-1.000	74°08.7'	-0.999	73°08.7'	-0.999
10	77°00.0'	-1.000	76°00.0'	-1.000	75°00.0'	-1.000	74°00.0'	-1.000
	Δ = 107°		Δ = 108°		Δ = 109°		Δ = 110°	
1	0°51.3'	-0.001'	0°51.2'	-0.001'	0°51.1'	-0.002'	0°51.0'	-0.002'
2	3°59.9'	-0.013	3°59.0'	-0.015	3°58.2'	-0.016	3°57.2'	-0.017
3	10°33.6'	-0.068	10°29.5'	-0.071	10°25.3'	-0.073	10°20.9'	-0.076
4	21°36.9'	-0.223	21°23.6'	-0.225	21°10.1'	-0.230	20°56.3'	-0.232
5	36°30.0'	-0.500	36°00.0'	-0.500	35°30.0'	-0.500	35°00.0'	-0.500
6	51°23.1'	-0.777	50°36.4'	-0.775	49°49.9'	-0.770	49°03.7'	-0.768
7	62°26.4'	-0.932	61°30.5'	-0.929	60°34.7'	-0.927	59°39.1'	-0.924
8	69°00.1'	-0.987	68°01.0'	-0.985	67°01.8'	-0.984	66°02.8'	-0.983
9	72°08.7'	-0.999	71°08.8'	-0.999	70°08.9'	-0.998	69°09.0'	-0.998
10	73°00.0'	-1.000	72°00.0'	-1.000	71°00.0'	-1.000	70°00.0'	-1.000
	Δ = 111°		Δ = 112°		Δ = 113°		Δ = 114°	
1	0°50.9'	-0.002'	0°50.8'	-0.002'	0°50.7'	-0.003'	0°50.5'	-0.003'
2	3°56.2'	-0.018	3°55.1'	-0.019	3°54.0'	-0.020	3°52.7'	-0.022
3	10°16.3'	-0.079	10°11.6'	-0.081	10°06.8'	-0.084	10°01.7'	-0.086
4	20°42.4'	-0.235	20°28.4'	-0.237	20°14.1'	-0.240	19°59.7'	-0.243
5	34°30.0'	-0.500	34°00.0'	-0.500	33°30.0'	-0.500	33°00.0'	-0.500
6	48°17.6'	-0.765	47°31.6'	-0.763	46°45.9'	-0.760	46°00.3'	-0.757
7	58°43.7'	-0.921	57°48.4'	-0.919	56°53.2'	-0.916	55°58.3'	-0.914
8	65°03.8'	-0.982	64°04.9'	-0.981	63°06.0'	-0.980	62°07.3'	-0.978
9	68°09.1'	-0.998	67°09.2'	-0.998	66°09.3'	-0.997	65°09.5'	-0.997
10	69°00.0'	-1.000	68°00.0'	-1.000	67°00.0'	-1.000	66°00.0'	-1.000

TABLE 46. DEFLECTIONS FROM THE PI TO THE 10-POINT CIRCULAR CURVE (*Continued*)

Vernier reads 0°00′ on the forward tangent

Pts.	Δ = 115°	Diff. per min.	Δ = 116°	Diff. per min.	Δ = 117°	Diff. per min.	Δ = 118°	Diff. per min.
1	0°50.4'	-0.003'	0°50.2'	-0.003'	0°50.0'	-0.004'	0°49.8'	-0.004'
2	3°51.5'	-0.023	3°50.1'	-0.024	3°48.7'	-0.025	3°47.2'	-0.026
3	9°56.6'	-0.089	9°51.3'	-0.090	9°45.9'	-0.094	9°40.2'	-0.096
4	19°45.1'	-0.245	19°30.4'	-0.248	19°15.5'	-0.251	19°00.5'	-0.253
5	32°30.0'	-0.500	32°00.0'	-0.500	31°30.0'	-0.500	31°00.0'	-0.500
6	45°14.9'	-0.755	44°29.6'	-0.752	43°44.5'	-0.749	42°59.5'	-0.747
7	55°03.4'	-0.911	54°08.7'	-0.910	53°14.1'	-0.906	52°19.8'	-0.904
8	61°08.5'	-0.977	60°09.9'	-0.976	59°11.3'	-0.975	58°12.8'	-0.974
9	64°09.6'	-0.997	63°09.8'	-0.997	62°10.0'	-0.996	61°10.2'	-0.996
10	65°00.0'	-1.000	64°00.0'	-1.000	63°00.0'	-1.000	62°00.0'	-1.000
	Δ = 119°		Δ = 120°		Δ = 121°		Δ = 122°	
1	0°49.5'	-0.004'	0°49.3'	-0.004'	0°49.0'	-0.005'	0°48.7'	-0.005'
2	3°45.6'	-0.027	3°43.9'	-0.029	3°42.2'	-0.030	3°40.4'	-0.031
3	9°34.4'	-0.098	9°28.5'	-0.101	9°22.5'	-0.103	9°16.3'	-0.105
4	18°45.3'	-0.256	18°30.0'	-0.258	18°14.5'	-0.260	17°58.9'	-0.263
5	30°30.0'	-0.500	30°00.0'	-0.500	29°30.0'	-0.500	29°00.0'	-0.500
6	42°14.7'	-0.744	41°30.0'	-0.742	40°45.5'	-0.740	40°01.1'	-0.737
7	51°25.6'	-0.902	50°31.5'	-0.899	49°37.5'	-0.897	48°43.7'	-0.895
8	57°14.4'	-0.973	56°16.1'	-0.971	55°17.8'	-0.970	54°19.6'	-0.969
9	60°10.5'	-0.996	59°10.7'	-0.996	58°11.0'	-0.995	57°11.3'	-0.995
10	61°00.0'	-1.000	60°00.0'	-1.000	59°00.0'	-1.000	58°00.0'	-1.000
	Δ = 123°		Δ = 124°		Δ = 125°		Δ = 126°	
1	0°48.4'	-0.005'	0°48.1'	-0.005'	0°47.8'	-0.006'	0°47.5'	-0.006'
2	3°38.6'	-0.032	3°36.7'	-0.033	3°34.7'	-0.034	3°32.6'	-0.035
3	9°10.0'	-0.108	9°03.6'	-0.110	8°57.0'	-0.112	8°50.2'	-0.114
4	17°43.1'	-0.265	17°27.2'	-0.267	17°11.2'	-0.269	16°55.0'	-0.271
5	28°30.0'	-0.500	28°00.0'	-0.500	27°30.0'	-0.500	27°00.0'	-0.500
6	39°16.9'	-0.735	38°32.8'	-0.733	37°48.8'	-0.731	37°05.0'	-0.729
7	47°50.0'	-0.892	46°56.4'	-0.890	46°03.0'	-0.888	45°09.8'	-0.886
8	53°21.4'	-0.968	52°23.3'	-0.967	51°25.3'	-0.966	50°27.4'	-0.965
9	56°11.6'	-0.995	55°11.9'	-0.995	54°12.2'	-0.994	53°12.5'	-0.994
10	57°00.0'	-1.000	56°00.0'	-1.000	55°00.0'	-1.000	54°00.0'	-1.000
	Δ = 127°		Δ = 128°		Δ = 129°		Δ = 130°	
1	0°47.1'	-0.006'	0°46.7'	-0.007'	0°46.3'	-0.007'	0°45.9'	-0.007'
2	3°30.5'	-0.037	3°28.3'	-0.038	3°26.0'	-0.039	3°23.7'	-0.040
3	8°43.4'	-0.116	8°36.4'	-0.119	8°29.3'	-0.121	8°22.0'	-0.123
4	16°38.7'	-0.273	16°22.3'	-0.276	16°05.8'	-0.278	15°49.1'	-0.280
5	26°30.0'	-0.500	26°00.0'	-0.500	25°30.0'	-0.500	25°00.0'	-0.500
6	36°21.3'	-0.727	35°37.7'	-0.724	34°54.2'	-0.722	34°10.9'	-0.720
7	44°16.6'	-0.884	43°23.6'	-0.881	42°30.7'	-0.879	41°38.0'	-0.877
8	49°29.5'	-0.963	48°31.7'	-0.962	47°34.0'	-0.961	46°36.3'	-0.960
9	52°12.9'	-0.994	51°13.3'	-0.993	50°13.7'	-0.993	49°14.1'	-0.993
10	53°00.0'	-1.000	52°00.0'	-1.000	51°00.0'	-1.000	50°00.0'	-1.000

Table 47. Direction Angle α and Distance r from the PI to the First (or Third) Quarter-points of a Circular Curve*

Δ	α	r (ft)	Δ	α	r (ft)	Δ	α	r (ft)
10°	1°14.6′	251.4	47°	5°10.8′	1330	84°	6°59.2′	3129
11	1 21.9	276.9	48	5 15.6	1366	85	6 59.9	3197
12	1 29.3	302.4	49	5 20.4	1402	86	7 00.5	3267
13	1 36.6	328.1	50	5 25.1	1438	87	7 01.0	3339
14	1 43.8	353.9	51	5 29.7	1475	88	7 01.4	3412
15	1 51.1	379.8	52	5 34.2	1513	89	7 01.6	3487
16	1 58.3	405.8	53	5 38.6	1551	90	7 01.7	3564
17	2 05.4	432.0	54	5 42.9	1590	91	7 01.8	3643
18	2 12.5	458.3	55	5 47.1	1629	92	7 01.7	3723
19	2 19.6	484.7	56	5 51.2	1669	93	7 01.5	3805
20	2 26.6	511.4	57	5 55.1	1710	94	7 01.2	3889
21	2 33.6	538.2	58	5 59.0	1751	95	7 00.8	3975
22	2 40.5	565.2	59	6 02.8	1793	96	7 00.2	4063
23	2 47.4	592.4	60	6 06.4	1836	97	6 59.5	4154
24	2 54.2	619.8	61	6 09.9	1879	98	6 58.7	4247
25	3 00.9	647.4	62	6 13.3	1923	99	6 57.8	4342
26	3 07.6	675.2	63	6 16.6	1968	100	6 56.7	4439
27	3 14.2	703.2	64	6 19.8	2013	101	6 55.5	4539
28	3 20.8	731.5	65	6 22.9	2060	102	6 54.3	4642
29	3 27.3	760.1	66	6 25.8	2107	103	6 52.9	4748
30	3 33.7	788.9	67	6 28.6	2155	104	6 51.4	4856
31	3 40.1	818.0	68	6 31.4	2204	105	6 49.8	4968
32	3 46.4	847.3	69	6 34.0	2253	106	6 48.1	5083
33	3 52.6	877.0	70	6 36.5	2304	107	6 46.3	5201
34	3 58.7	907.0	71	6 38.9	2356	108	6 44.4	5322
35	4 04.8	937.3	72	6 41.2	2409	109	6 42.3	5447
36	4 10.8	967.9	73	6 43.3	2462	110	6 40.1	5575
37	4 16.7	998.9	74	6 45.3	2517	111	6 37.8	5707
38	4 22.5	1030.2	75	6 47.2	2573	112	6 35.4	5843
39	4 28.2	1061.9	76	6 49.1	2630	113	6 32.9	5984
40	4 33.8	1094.0	77	6 50.8	2688	114	6 30.3	6129
41	4 39.3	1126.4	78	6 52.3	2747	115	6 27.6	6278
42	4 44.8	1159.2	79	6 53.7	2807	116	6 24.8	6432
43	4 50.2	1192.5	80	6 55.1	2869	117	6 21.9	6591
44	4 55.5	1226.2	81	6 56.3	2932	118	6 18.8	6755
45	5 00.7	1260.3	82	6 57.4	2996	119	6 15.6	6925
46	5 05.8	1294.9	83	6 58.3	3062	120	6 12.4	7101

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* See Eqs. (22) and (23).

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1in = 2.54 cm
1ft = 0.3048 m

sight dist. for horizontal cur

$$m = \frac{c^2}{8R}$$

m = dist. from ℄ of road to the obstruction
c = sight dist available in road
R = Radius of curve.

Radius

$R = \frac{18000}{\pi \times D}$

Tangent

$T = R \tan \frac{\Delta}{2}$

External Dist

$E = T \tan \frac{\Delta}{4}$

middle Ordinate

$m = E \cos \frac{\Delta}{2}$

Long chord

$LC = 2R \sin \frac{\Delta}{2}$

$L = 100 \frac{\Delta}{D}$ or $2\pi R \frac{\Delta}{360}$

Arc dist.

Simple Curve.

* Deflection ∠ = ($0.3 \, C_n D$) minutes

$C_t = 2R \sin ∠$

(sub chord)

* Give same answer.

Designing Simple Curve → See pg. 18 Table A

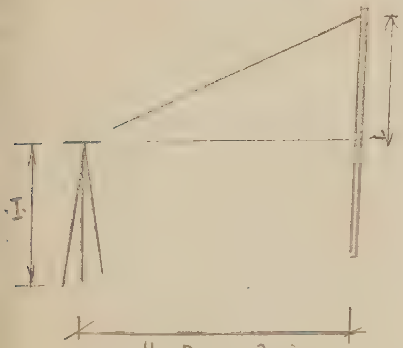
* Deflection ∠ from PC $d_{pc} = \frac{C_n D}{100} \times \frac{1}{2}$

C_n = Chord dist. to pt. on curve from Pc
D = degree of curvature
R = Radius of curve
d_{pc} = deflection from Pc to the point.

Arc distance from Pc
$= 2R \sin d_{pc}$

computations for deflection ∠ and chords on chord-definition curves uses the same formulas as

$R = \frac{50}{\sin D/2}$



$V = \frac{1}{2} Ds \sin 2\theta$

$H = Ds \cos^2 \theta$

$H = Ds + C$

simple curve setting up Precision

$= \frac{\text{Misclosure}}{L + 2T}$

$= 1/x$

D = 100 constant

Format
(i) P.C. below
(ii) Round to tenth the next station.

(For curve layout by offset, see last page)

Simple Curve selected to fit ground conditions:

Given: T_{max} or T_{min} (tangents, site condition)
E_{max} or E_{min} (external dist., site condition)
central ∠
use table 2.
calculate for Deg. of curvature by $D_T = \frac{\text{Tangent}_{table}}{\text{Tangent}_{site}}$

$D_E = \frac{E_{table}}{E_{site}}$

with D_T and D_E and given Central ∠
calculate for Radius, Tangent length, External Dist.
Compare them with site condition.
Then choose the right deg. of curvature for the site condition.

Area by Division into Triangles.

Area = $\sqrt{s(s-a)(s-b)(s-c)}$

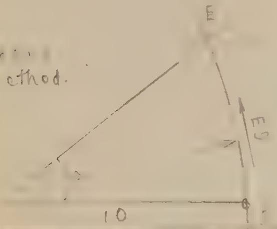
a, b, c = side of triangle

$s = \frac{1}{2}(a+b+c)$

[For the rest [illegible] pg [illegible]]

Area = $\frac{1}{2}ab\sin C$

[illegible] method.



$ED = 2R\sin\frac{1}{2}\alpha$

Vertical Curve (Equal Tangents)

(1) [illegible] HI [illegible] = x_t = $\frac{L\,[illegible]}{G1-G2}$
(from PC) (in ft.)

(2) [illegible] ft. Rate of change of grade [illegible] per station [illegible]

= [illegible] = Elev PC + [illegible] $\left(\frac{x_t}{100}\right)$ = $2a = \frac{100(G2-G1)}{L}$

Slope of the tangent from PC to HI [illegible] curve = $\frac{G1}{2}$ and $\frac{G2}{2}$

(3) whole [illegible] L + $a x^2$ (a = -ve or +ve)
[illegible] tangent) [illegible] with respect to PC elev. x station
e.g. if 400 ft
i.e. 4 sta
∴ x = 4

[illegible] HI [illegible]

Tangent Offsets Elevation

If Table 31 is used — see pg. 154 for format

(4) Design (see pg. 156) L = KA K (table) A = G2 − G1 %

[illegible]

[illegible] ation) = $\sqrt{\frac{8Lh}{[illegible]}}$ h = 3.75 ft

L [illegible]

e.g. If 400 ft = 4
500 ft = 5 } Fo[illegible]

If S > L ; $S = \frac{L}{2} + \frac{4h}{G_1-G_2}$ L = Sta. Dist 1st [illegible]

[illegible]

To convert [illegible]

Parallel curves.

$$\frac{C_{out}}{R_{out}} = \frac{C_{℄}}{R_{℄}} = \frac{C_{in}}{R_{in}} \qquad \frac{L_{out}}{R_{out}} = \frac{L_{℄}}{R_{℄}} = \frac{L_{in}}{R_{in}}$$

chord Length

Radius Radius:

$$\phi_b = (2l_t + l_b)(l_t - l_b)\frac{\theta_s}{3\,l_s^2}$$

minutes

$$\phi_f = (l_f + 2l_t)(l_f - l_t)\frac{\theta_s}{3l_s^2}$$

minutes

Volumes

h_1, h_2, h_3, h_4 = heights (depths to the level)

$$\text{Volume} = \frac{\text{Area of one square lot}}{27}\left[\frac{\sum h_1 + 2\sum h_2 + 3\sum h_3 + 4\sum h_4}{4}\right]$$

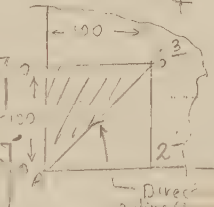
plus

$$\frac{1}{27} \times \frac{100\times100}{2} \times \left[\frac{6\frac{3}{10} + 0 + 0}{3}\right]$$

$$+$$

Vol. of [illegible] lot [illegible] ground volume (one or two excavation depth and the rest zero depth)

$$= \frac{1}{27} \times \frac{100\times100}{2}\left[\frac{6\frac{3}{10} + 2\frac{1}{10}}{3}\right]$$



Direct[illegible] outsid[illegible]

6^3 is $6\frac{3}{10} = 6.3$ A.

2^1 is $2\frac{1}{10} = 2.1$

[illegible] 6.3 and [illegible] are only examples [illegible]

at the special [illegible]

Point 2 arms h ①

Point 3 arms h ②

Point [illegible] h ③ or ④

h(3) ⇒ [illegible]

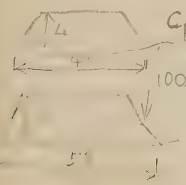
h(4) ⇒ [illegible]

[illegible] formula

$$V_p = \frac{L}{27}\left(\frac{A_1 + 4A_m + A_2}{6}\right)$$ volume

(use this only with 3 transitional sections)

V_e is Ave. End Method



C_p, correction [illegible]

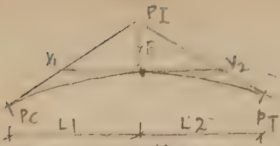
$$= \frac{L}{12\times27}(d_1 - d_2)(W_1 - W_2)$$

$d_1 = 4ft$ $W_1 = 40ft$ $L = 100ft$

$d_2 = 10ft$ $W_2 = 50ft$

Answer in yd^3 (already divided by 27)

Volume by Prismodal formula $= V_e - C_p$



Unequal Tangents

① Intermediate grade $= \frac{G1L1 + G2L2}{L1 + L2}$ =
(slope of $V_1 - V_2$)

+ve measuring fr left to rig end.

② Rate of change of grade ($2a_1$) before PI is $2a_1 = \frac{100(G2-G1)}{L1}$

③ Rate of change of grade ($2a_2$) after PI is $2a_2 = \frac{100(G2-G1)}{L2}$

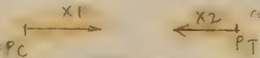
④ E from PI to curve $= \frac{L1L2(G2-G1)}{200L}$

5) Any elevation along the curve before PI
= Elev. on grade line with respect to PC elev. $\pm a_1x^2$
(x sta) If between ch90+00 and ch91+45
∴ x = 91.45 − 90 = 1.45

(6) Any elevation along the curve after PI
= Elev. on grade line with respect to PT elev. $\pm a_2x^2$ (x sta) [illegible] PT

⑦ To determine Highest or Lowest on Vertical Curve
check with $x_1 = \frac{LG1}{G1-G2}\left(\frac{L1}{L2}\right)$. If $x_1 > L1$, then calculate for x_2 (below)
If $x_1 < L_1$, then distance from PC to Highest or Lowest pt. on curve = x1
(Thus, chainage of Hi pt. obtainable consequently to the elevation fo

$$x2 = \left(\frac{L\,G2}{G1-G2}\right)\left(\frac{L2}{L1}\right)$$

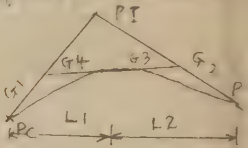


⑧ To get sight distances, S :-
Two formulae
h = 3.75 ft (constant)
L = sta. dist.
1 sta. dist = 100 ft

$$S = \sqrt{\frac{8L_1h}{G1-G4}}$$

Formula applicable only if S < L

S = sta. dist
So S = sta. dist x100 to get distance in ft



After PI $S = \sqrt{\frac{8L2h}{G3-G2}}$

If S > L, use the formula below instead

$S = \frac{L1}{2} + \frac{4h}{G1-G4}$ } Before PI

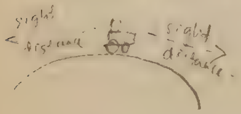
L = Sta. length
e.g. if bet. ch20+00 and 20+50
∴ L = 20.5 − 20 = 0.5

After PI $S = \frac{L2}{2} + \frac{4h}{G3-G2}$

h = 3.75 ft

* The shorter sight distance is more cri

see table in pg. 156 the AASHO spec.



If designed speed = 70 mph
K = 240 for crest curve
K = 140 for sag.

Use formula
Ignore −ve sign if appear in answer.
$L_1 \text{ min.} = K(G4 - G1)$ (Before PI)
$L_2 \text{ min.} = K(G2 - G3)$ (After PI

If L1 > L1 minimum O.K
also L2 > L2 minimum O.K.

format

chainage	x sta.	Elev. on tangent	ax^2	Elev on curve	Difference
		15:			

Dealing with Spiral Curve

You need the detail drawing given in class. Use formulae in the sheet

You need Table A in pg. 18 to solve for Deg. of Curvature, D

Format: Sta Point Deflection Ang's Chord Dist (Program No. 3)

compound Curve

You need detail drawing given in class. You need Table A in pg. 18 to solve for Deg. of Curvature, D

simple curve

1st Program No. 0

Format: Sta Point Deflection Ang's Chord Dist

Traversing

Dist	Azimuth	Departure	Latitude
[illegible]	323°00'	-401.05	+532.21
⋮	⋮	+572.02	+268.97
	⋮	-171.04	-801.25
Tot. Dist		Error -0.07	Error -0.07

$$\text{Error of Closure} = \sqrt{0.07^2 + 0.07^2}$$

$$\text{Relative Accuracy Ratio} = \frac{\text{Error of Closure}}{\text{Total Dist}}$$

when deal with correcting Departure + Latitude:
- Divided by 'Total Dist' — Bowditch/Compass Rule
- Divided by 'Sums of Lat. or Dep' — Transit Rule

Simple Curve set by Tangent offset

i) Find Radius of Curve, R.

ii) chord dist. bet. PC or PT to point on curve, C $= 2R \sin \delta$

(4) δ is the deflection angle from PC or PT to the point on the curve

(5) $\delta = \frac{C_n D}{100} \times \frac{1}{2}$

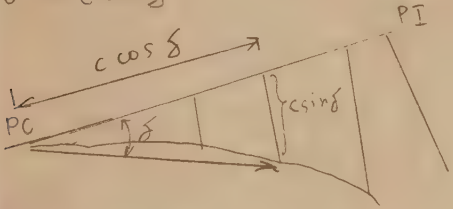
Cn = Chord dist. to point
D = Deg of Curve

(iii) Tangent distance from PC to point along the tangent (towards PI)

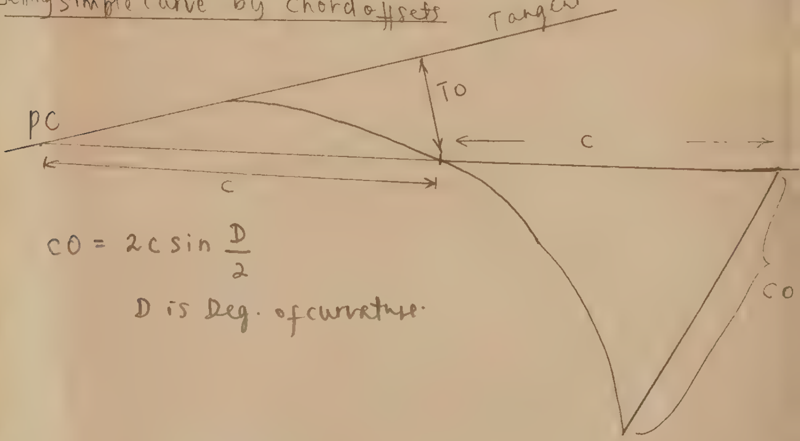
(6) $TD = C \cos \delta$

(7) (iv) Tangent offset from each pt. on the tangent

$TO = C \sin \delta$

(8) 

Setting simple curve by chord offsets



$CO = 2c \sin \frac{D}{2}$

D is Deg. of curvature.

$\frac{TO}{Cho}$

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